Electromagnetic Splitting of Charged and Neutral Mesons

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BACKGROUND

- Most lattice calculations done at $\alpha_{\text{EM}} = 0$, and $m_u = m_d$
- EM uncertainty in quark mass ratio *from continuum estimates* (MILC, arXiv:0903.3598v1):

$$m_u/m_d = 0.42(0)(1)(4)$$

- Lattice results for spectroscopy $< 1\%$ level
- some meson scattering length calculations at $\sim 1\%$ level
- **bottom line: Need to include EM effects for high precision calculations**

Previous studies:

- Duncan, et.al. (arXiv:hep-lat/9602005)
- Blum et. al. (arXiv:0708.0484v2,arXiv:1006.1311v1)
- MILC initial study Basak, et. al. (arXiv:0812.4486v1)

CHALLENGES

- cannot calculate π^0 mass without disconnected diagrams
- SχPT not developed until very recently (talk by Elizabeth Freeland at Lattice 2010 Tuesday)
- finite-volume corrections and lattice-spacing effects
- quenched photons
- dynamical studies: determinant re-weighting (Duncan, et.al, arXiv:hep-lat/9607032)
  fully dynamical (Blum, et.al, arXiv:0911.1348 [hep-lat])

THIS WORK

- MILC staggered, $NF = 2 + 1$, SU(3) gauge fields
- quenched U(1) fields generated separately
- calculated $m_{\pi^+}, m_{K^+}, m_{K^0}, m_{u\bar{u}}, m_{d\bar{d}}$
- also the ρ^+ (same issue with ρ^0 as π^0)
- electric charges ($e^2 = 4\pi\alpha$):

e	α
± 0.606	$4\alpha_{\text{phys}}$
± 0.303	α_{phys}
0	0

- 7 valence quark masses on the MILC coarse ($a = 0.12$ fm) and fine ensembles ($a = 0.09$ fm),
- propagator inversion using GPU multi-mass inverter

DASHEN'S THEOREM

- $\Delta M_D^2 = (\Delta M_K^2 - \Delta M_\pi^2)_{EM} = 0$ to $\mathcal{O}(\alpha)$:

$$\Delta M_D^2 = \left[m_{K+}^2 - m_{K0}^2\right] - \left[m_{\pi+}^2 - m_{\pi^0}^2\right]$$

$$\Delta E_D = \frac{m_{K+}^2 - m_{K0}^2}{m_{\pi+}^2 - m_{\pi0}^2} - 1$$

- PQχPT (Bijnens, arXiv:0708.0484v2) ΔM^2:

$$\Delta M^2 = \left[M^2(\chi_1,\chi_3,q_1,q_3) - M^2(\chi_1,\chi_3,q_3,q_3)\right] - \left[M^2(\chi_1,\chi_1,q_1,q_3) - M^2(\chi_1,\chi_1,q_3,q_3)\right]$$

$$\Delta E = \frac{M^2(\chi_1,\chi_3,q_1,q_3) - M^2(\chi_1,\chi_3,q_3,q_3)}{M^2(\chi_1,\chi_1,q_1,q_3) - M^2(\chi_1,\chi_1,q_3,q_3)} - 1$$

$\Delta M^2(m_u = m_d) \rightarrow (\Delta M_D^2)_{EM}$ up to very small EM corrections to m_{π^0}

GAUGE FIELDS

Quenched photon configurations generated with non-compact U(1) action (Duncan, et. al. arXiv:hep-lat/9602005v1,Blum, et. al., arXiv:0708.0484v2)

- gaussian distributed in momentum space
- U(1) Coulomb gauge fixed
- FFT → coordinate space
- SU(3) × U(1) gauge fields, (SU(3) Coulomb gauge fixed also)
- quenched U(1): sea quarks have no EM charge, valence quarks (propagators) do

$$\begin{aligned} \mathbf{U}_{j,\mu} &= \hat{U}_{j,\mu} U_{j,\mu} \\ \hat{U}_{j,\mu} \to \mathrm{SU}(3) \quad &, \quad U_{j,\mu} \to \mathrm{U}(1) \end{aligned}$$

STATISTICS

Initial study:

ensemble	m_l/m_s	a (fm)	cfgs	resource
l1648f21b6572m0097m0484	0.2	0.15	400	BigRed

Extension of the initial study:

ensemble	m_l/m_s	a (fm)	cfgs	resource
l2064f21b676m007m050	0.14	0.12	1261	AC
l2464f21b676m005m050	0.1	0.12	1274	Lincoln
l2896f21b709m0062m031	0.2	0.09	331	FNAL
l2896f21b711m0124m031	0.4	0.09	331	BigRed

- $m_{val} = 0.005, 0.007, 0.010, 0.020, 0.030, 0.040, 0.050$
- $m_{val} = 0.0031, 0.0062, 0.0093, 0.0124, 0.0155, 0.0186, 0.031$
- NCSA

GPU INVERTER

MILC multi-mass inverter code ported by Guochun Shi to QUDA for NVIDIA GPUs (talk by Steve Gottlieb, Friday, Room3: 15:50 - 16:10)

Machine	t (h)	size	cpu cores	gpu	nodes	core-hr	node-hr
Big Red	13.3	$28^3 \times 96$	32	none	8	426	106
jpsi at FNAL	7.8	$28^3 \times 96$	1	S1070	1	7.8	7.8
jpsi at FNAL	0.8	$20^3 \times 64$	64	none	8	51.2	6.4
ac at NCSA	1.5	$20^3 \times 64$	1	S1070	1	1.5	1.5

- NVIDIA Tesla S1070 4 gpu per node, 4GB per gpu
- multi-gpu code would make this much better!
- factor of $\sim$34 speed up for 1 core, $\sim$4 for a node
- current code limited by cpu memory per node
- multi-gpu code → huge speed-up per node

OBTAINING MASS DIFFERENCES

- different U(1) gauge cfg for every SU(3) gauge cfg
- Average over $\pm e$ for each correlator per cfg
- single-elimination jackknife
- correlated fit to $A(e^{-mt} + e^{-m(T-t)})$
- form $\Delta M^2_{\text{meson}}$ from jackknifed masses

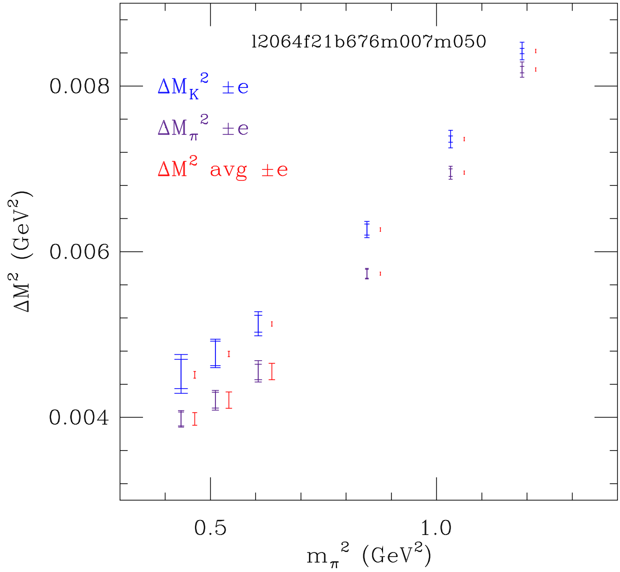
AVERAGE OVER $\pm e$

Averaging over $\pm e$ in propagator calculation can cancel $\mathcal{O}(e)$ U(1) gauge field noise (Blum, et. al, arXiv:0708.0484v2, arXiv:1006.1311v1)

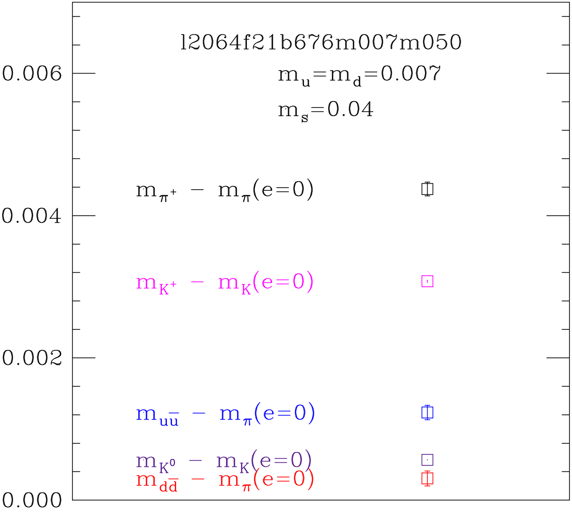
$\mathbf{e}$	α
± 0.606	$4\alpha_{\text{phys}}$
± 0.303	α_{phys}
0	0

$$\begin{aligned} U_{j,\mu} &= e^{i\mathbf{q}\mathbf{e}A_{j,\mu}} \\ \mathbf{q} &= \frac{2}{3},\ -\frac{1}{3} \end{aligned}$$

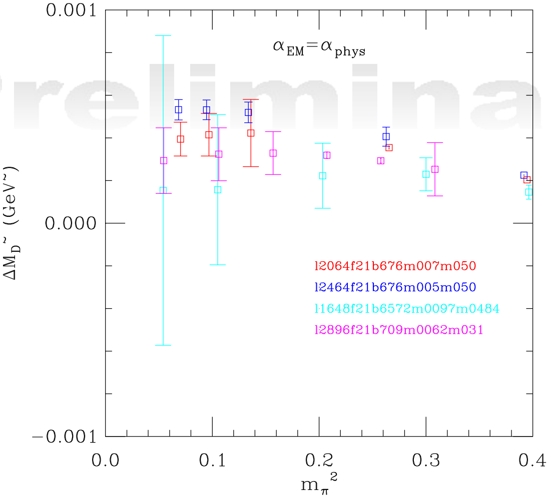
Average over $\pm e$



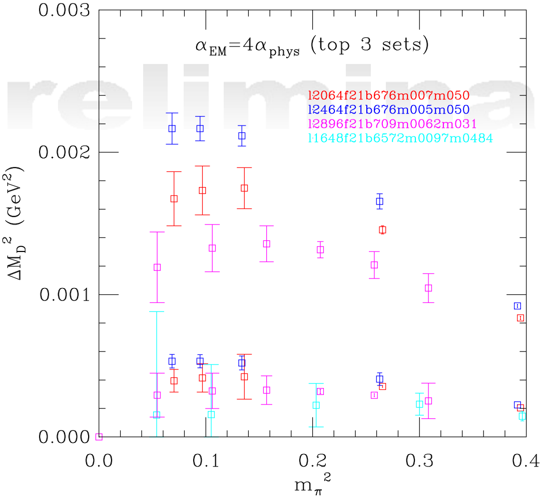
MASS DIFFERENCE FROM $e = 0$



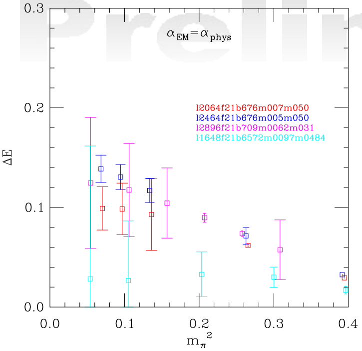
$$\Delta M^2 = \left[M^2(\chi_1,\chi_3,q_1,q_3) - M^2(\chi_1,\chi_3,q_3,q_3)\right] - \left[M^2(\chi_1,\chi_1,q_1,q_3) - M^2(\chi_1,\chi_1,q_3,q_3)\right]$$

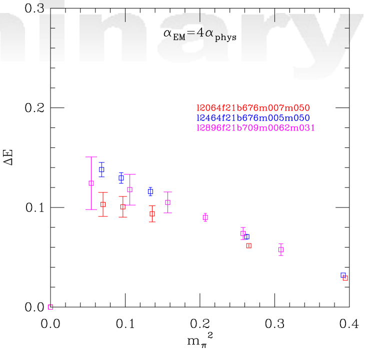


$$\Delta M^2 = \left[M^2(\chi_1,\chi_3,q_1,q_3) - M^2(\chi_1,\chi_3,q_3,q_3)\right] - \left[M^2(\chi_1,\chi_1,q_1,q_3) - M^2(\chi_1,\chi_1,q_3,q_3)\right]$$



$$\Delta E_{\mathrm{EM}} \equiv \frac{M^2(\chi_1,\chi_3,q_1,q_3) - M^2(\chi_1,\chi_3,q_3,q_3)}{M^2(\chi_1,\chi_1,q_1,q_3) - M^2(\chi_1,\chi_1,q_3,q_3)} - 1$$





CONCLUSION AND FUTURE WORK

- it appears from the plots that the result for Δ_E and/or ΔM^2 for physical masses & in the continuum limit will be well-controlled
- fit LEC's from PQχPT with SχPT corrections
- we have the ρ data, analysis in progress
- staggered baryons
- effect on f_π and f_K
- generate dynamical SU(3)×U(1) gauge fields → changes fermion force calculation

THANKS TO:

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MILC collaboration

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