Numerical properties of overlap staggered fermions

Philippe de Forcrand
ETH Zürich and CERN

with Aleksi Kurkela & Marco Panero

inspired by David Adams

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Motivation

Most large-scale LQCD projects use [improved] **staggered fermions**: they are cheap to simulate

BUT

- $N_f = 4$ continuum *tastes*, with $\mathcal{O}(a^2)$ taste-symmetry breaking
  $\Longrightarrow$ take $\sqrt{\det}$ non-local ?

- No quartet of low-lying eigenvalues $\leftrightarrow$ no index theorem

Rescued by David Adams (0912.2850 & LAT10)

Construction

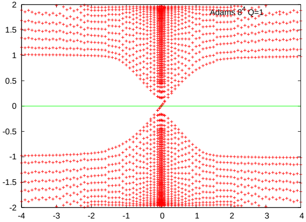
- Index from flow of eigenvalues of $H(m) = \gamma_5(\not{D} + m) = \gamma_5 \not{D} + m\gamma_5$
- Topology comes from gluon field, ie. taste-singlet

$\Longrightarrow$ Need **taste-singlet** γ_5, at least for mass term $\rightarrow \Gamma_5$

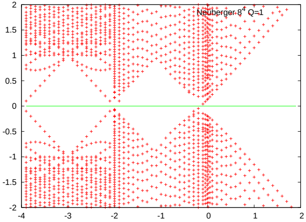
$$H(m) = \gamma_5 D_{st} + m\Gamma_5$$

$D_{st} = \frac{1}{2}\sum_\mu \eta_\mu(x)(U_\mu(x) - U^\dagger_\mu(x-\hat{\mu}))$

$\gamma_5 = (-)^{x+y+z+t}$, $\Gamma_5 = \prod_4 \eta_\mu \times \sum$ 4-link transporters



Adams

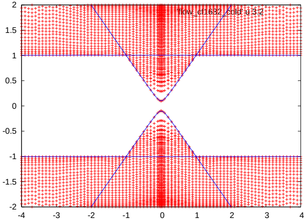


Neuberger

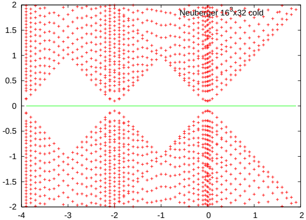
$Q = 1$

More eigenvalue flows

- Cold configuration: agreement with analytic result

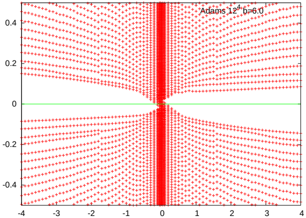


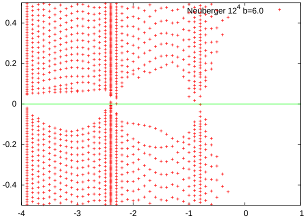
Adams



Neuberger

- $\beta = 6.0$: eigenvalue gap closes, but $|m_0|$ can be *arbit. large* in Adams





Overlap staggered fermions

- Just like Neuberger: $D_{sov} = 1 + \gamma_5 \mathrm{sign}(H(-m_0))$

with $\gamma_5 = (-)^{x+y+z+t}$ (need $\gamma_5^2 = 1$)

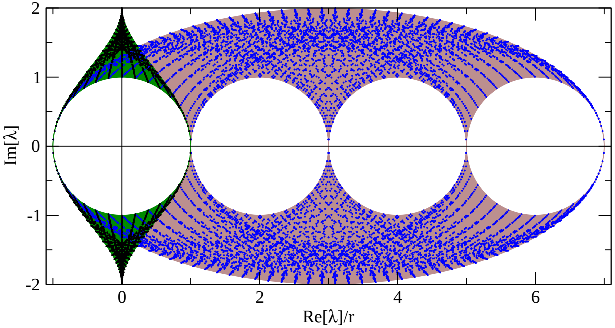
- Potential advantages:

  - cheaper (4 times fewer d.o.f. per site)
  - more robust ($|m_0|$ can be arbitrarily large)

And reduces $N_f = 4$ to $N_f = 2$ tastes.

Free field: $U_\mu(x) = \mathbf{1}\ \forall \mathbf{x}, \mu$

Spectrum of kernel: $\gamma_5 H_W(m_0 = -1)$ and $\gamma_5 H_{Adams}(m_0 = -1)$



$\gamma_5 \mathrm{sign}(H) = \frac{D}{\sqrt{D^\dagger D}}$ projects eigenvalues of $D = \gamma_5 H$ on unit circle

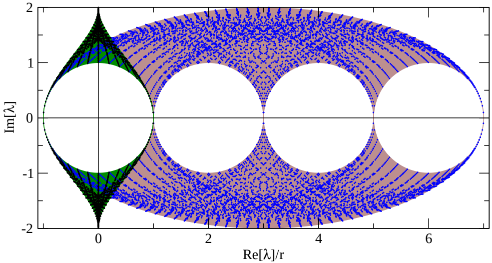
Adams: two low-p eigenmodes projected to -1, two projected to +1 $\Rightarrow$ $N_f = 2$

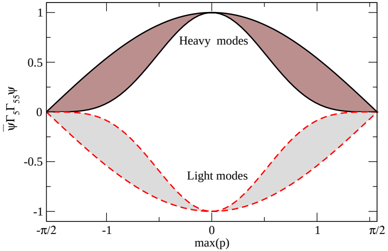
Free field: from $N_f = 4$ to $N_f = 2$

Kernel: $\gamma_5 H_{Adams}(-m_0) = \not{D}_{st} - m_0\gamma_5\Gamma_5$

Low-momentum (up to π/a) $\rightarrow \not{D}_{st}\tilde{\psi} \approx 0$

And $\langle\tilde{\psi}(-m_0\gamma_5\Gamma_5)\tilde{\psi}\rangle \approx -1$ ($N_f = 2$ physical modes)
or $+1$ ($N_f = 2$ doublers)

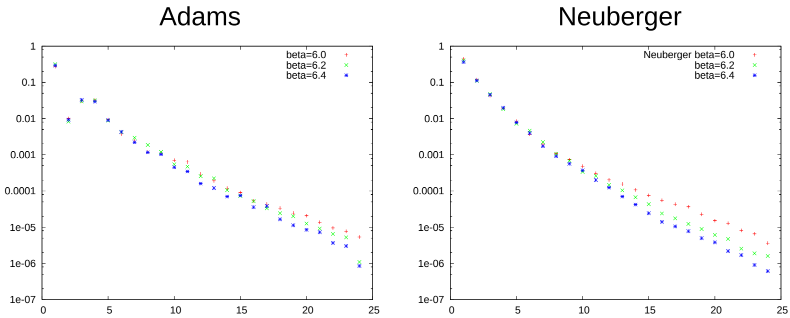




So that $\langle\tilde{\psi}(1 + \gamma_5 \mathrm{sign}(H_{Adams}))\tilde{\psi}\rangle \approx 0$ ($N_f = 2$ physical modes)
or $+2$ ($N_f = 2$ doublers)

Locality of operator?

$\mathrm{Max}_y |M_{xy}|$ versus $|x-y|$ (Manhattan distance) cf. hep-lat/9808010

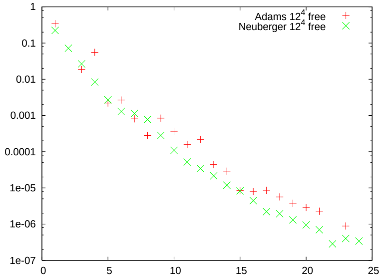


Adams comparable to Neuberger although kernel less local (4-link)

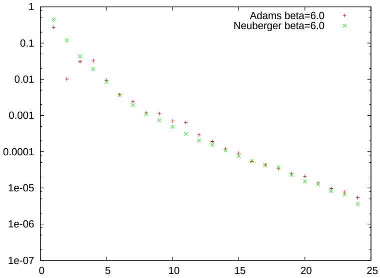
Locality of operator?

$\mathrm{Max}_y |M_{xy}|$ versus $|x - y|$ (Manhattan distance) cf. hep-lat/9808010

Both, cold



Both, $\beta = 6.0$



Adams comparable to Neuberger although kernel less local (4-link)

Cost of applying operator

- Multiplication by D: about **2** times faster for Adams (no Dirac indices)

- Sign(H) [using CG, no deflation]:
  - about **8** times faster for Adams on easy cases
  - about **2-3** times faster on hard cases

A lot of room for optimization of m_0 in Adams' operator: not exploited yet

Also:

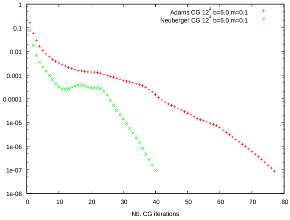
improved kinetic operator, link smearing (kinetic and/or mass), deflation, preconditioning, ...

Cost of inversion: compare with Neuberger

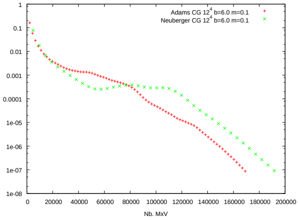
Apples with apples:

- same gauge field ($12^4, \beta = 6.0$)
- same basic algorithm (CG inner, CG outer)

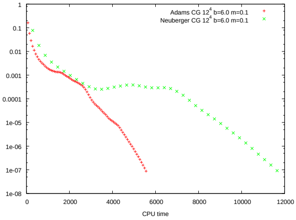
Adams versus Neuberger



Outer CG iter.



MxV



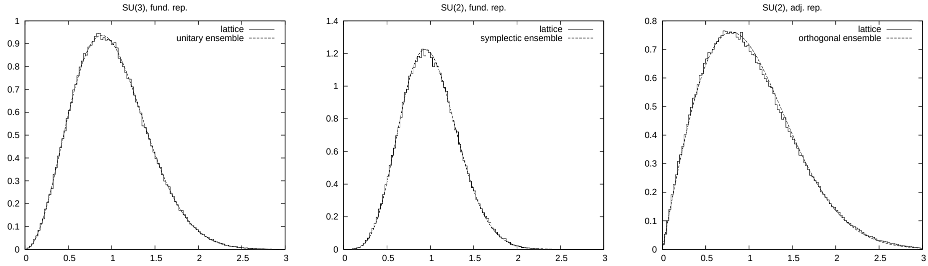
CPU

First application: comparison with RMT

Level spacing densities (dist. of unfolded eigenvalue spacing)

$\beta = 0$ Y-M, 4^4 lattice

parameter-free curve for ensemble with given symmetry



$SU(3)$ — chUE

$SU(2)$ fund. — chSE

$SU(2)$ adj. — chOE

Same as standard staggered (cf 0804.3929), but should change as $\beta \to \infty$

Conclusions

- Works as advertised: $N_f = 2 \quad \rightarrow \quad$ no more evil rooting!
- Sound approach to chiral & continuum limits

  Compare with Wilson & Wilson-based (Neuberger, Domain-wall)

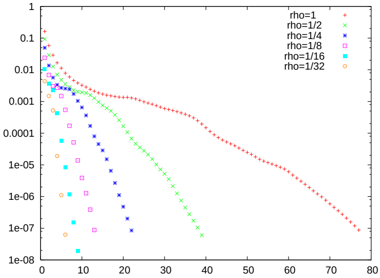
- How efficient? - cheaper than Neuberger
  - but not dramatically so yet
  - optimization (esp. m_0)
- Only a dream?

  zero-modes are chiral, and localized on even (or odd) sites

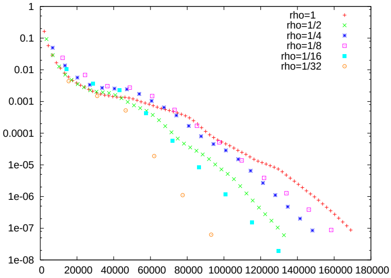
  $\rightarrow$ couple gauge fields to left-handed modes only ?

Backup slide: optimization of m_0 (preliminary)

Solve $[m_q + m_0(1+\gamma_5 \mathrm{sign}(H(-m_0)))]X = \delta_{x_0}$ with $m_q = 0.1$ fixed, vary m_0



CG iter.



MxV

Backup slide: where are the physical d.o.f.?

The overlap operator splits the $N_f = 4$ tastes into
$N_f = 2$ with mass ≈ 0 and $N_f = 2$ with mass $\approx 2/a$

Where are the light and heavy d.o.f. ?

Take $|m_0|$ very large: kinetic operator is $\frac{1}{m_0}$ perturbation of mass operator

- Mass operator Γ_5 is block-diagonal (**8** blocks):
  4-link transporter $\Rightarrow$ parities of (x,y,z,t) *all changed*
- Leading-order perturbation: at most one application of kinetic operator $\not{D}$
- (Any nb. of Γ_5 4-hops + at most one $\not{D}$ single-hop) $\rightarrow$ **bipartite lattice**