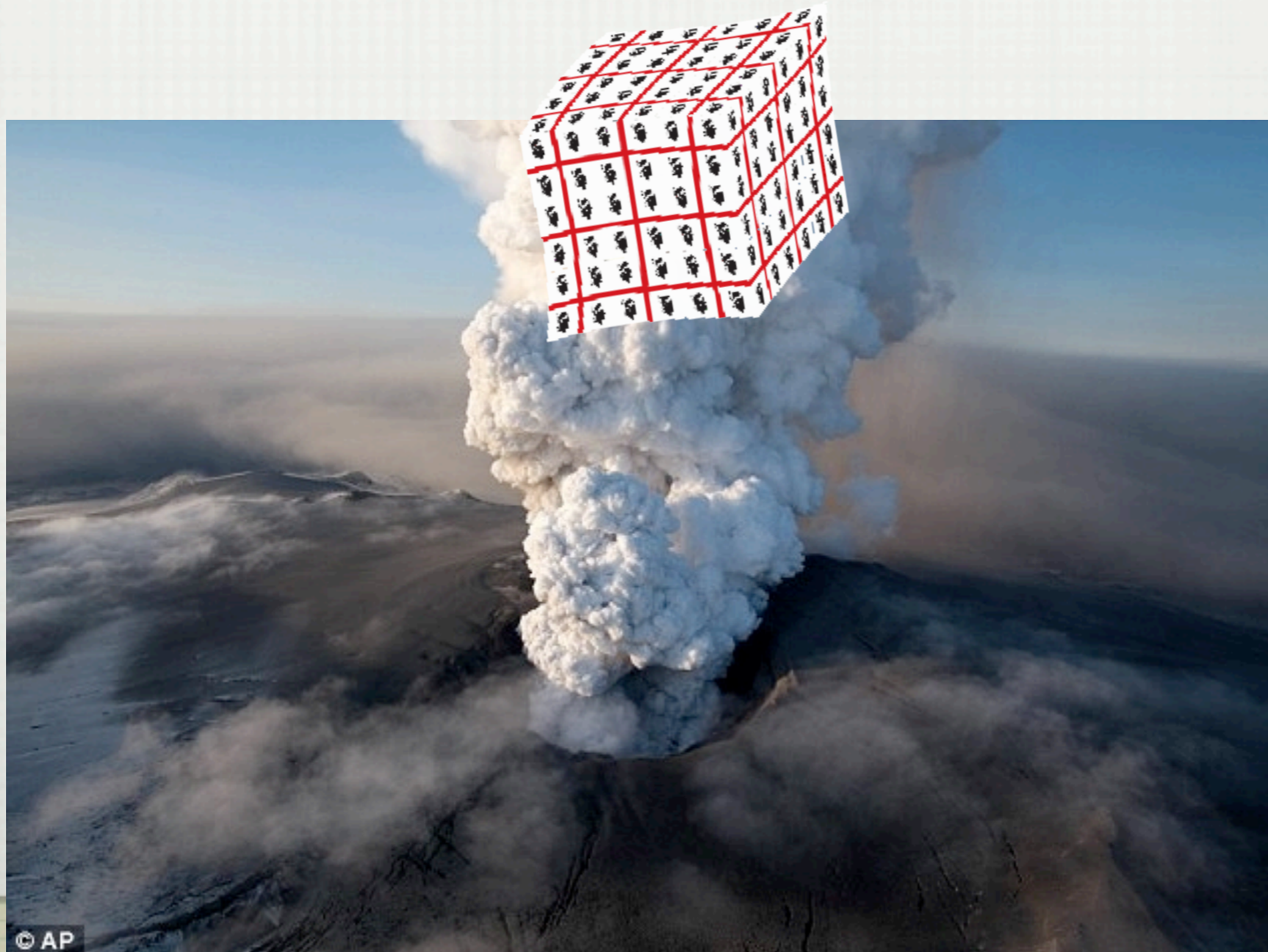


LATTICE 2010 SARDINIA

FINITE TEMPERATURE QCD ON THE LATTICE – STATUS 2010

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- ❖ Lattice quarks
- ❖ Phase structure: T_C & scaling
- ❖ EOS for $N_F = 2+1$
- ❖ Other hot issues

- ❖ Finite density QCD \implies *Soulendu Gupta plenary*

LATTICE QUARKS

Staggered-type quarks

Most successful.

- Merits:**
- ✓ A part of the flavor-chiral symmetry preserved at $a > 0$.
=> location of the chiral limit protected (no additive ren. to m_q)
 - ✓ Light quark simulations less expensive. $\leq \det M$ positive definite

Problems:

- ❖ 4 copies of flavors (“*tastes*”) => **4th root trick** $\det M \Rightarrow [\det M]^{1/4}$
- ❖ **non-local** => **universality arguments fragile**

If a continuum limit exists, and if it belongs to the universality class of QCD ?

Vital discussions: Sharpe@Lat06, Creutz@Lat07, Kronfeld@Lat07, ...

See also Rossi-Testa 1005.3672.

I *assume* that the staggered quarks have the continuum limit in the universality class of QCD with desired N_F .

Message: **Continuum extrapolation must be done first.**

Still a couple of worrisome issues — I come back to them later.

Wilson-type quarks

Most conservative.

Merits: ✓ Describe a single flavor.
 ✓ QCD continuum limit exists.

Problems:

- ❖ Explicit violation of the chiral symmetry at $a > 0$ \Rightarrow Additive m_q renormalization
- ❖ $\det M$ not positive definite \Rightarrow light quarks more expensive.

Phase structure:

S. Aoki PRD30('84)

2nd order transition to explain massless π 's without the chiral symmetry

Sharpe-Singleton PRD58('98)

Wilson term \rightarrow effective int. c_2 the phase structure depends on the sign of c_2 .

$$\mathcal{V}_\chi = -\frac{c_1}{4} \text{Tr}(\Sigma + \Sigma^\dagger) + \frac{c_2}{16} \{\text{Tr}(\Sigma + \Sigma^\dagger)\}^2$$

$c_2 > 0$: 2nd order

$c_2 < 0$: 1st order

Twisting



$$S_q^{\text{TW}} = a^4 \sum_x \bar{\psi}(x) [D_W + m_0 + i\gamma_5 \tau_3 \mu_0] \psi(x)$$

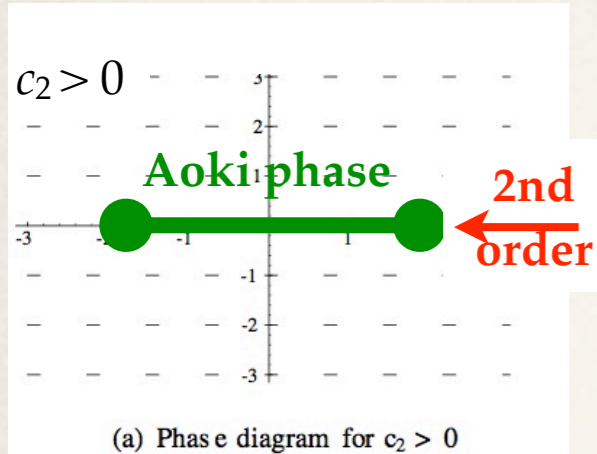
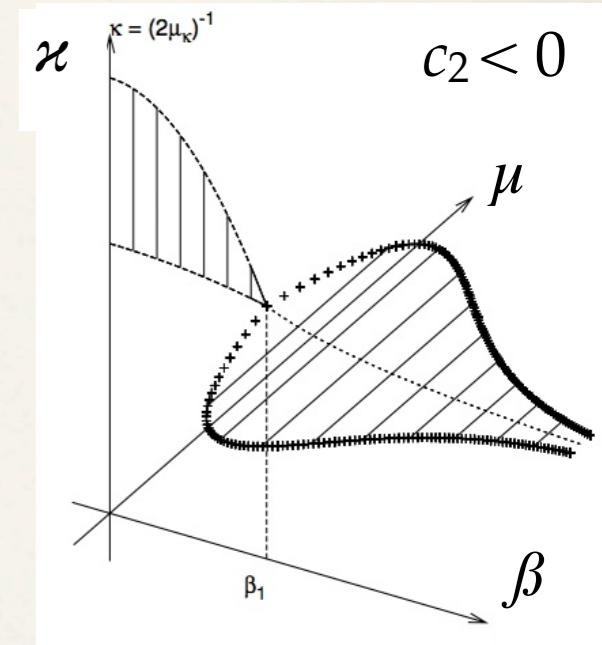
c_2 depends on the lattice action.

various glues + $S_q^{\text{TW}} \Rightarrow$ 1st order

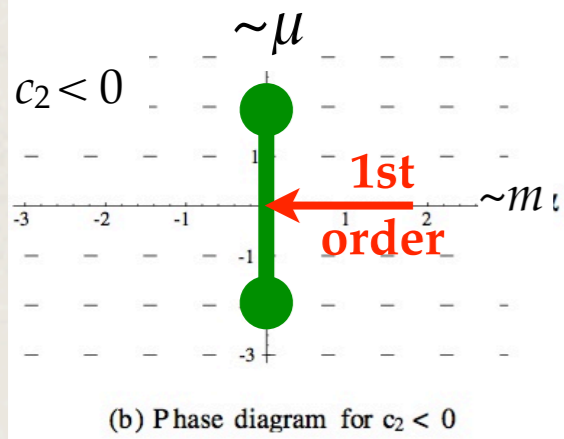
In this case, we have to avoid the 1st order region to approach the continuum limit.

Very light quarks without twisting possible only at small a .

Farchioni et al. EPJ C42('05)



(a) Phase diagram for $c_2 > 0$



(b) Phase diagram for $c_2 < 0$

Sharpe-Wu PRD70('04)

S. Aoki et al. (PACS-CS) PRD79('09); PRD81('01)

- ▶ Iwasaki gauge + Clover ($C_{\text{SW}}^{\text{NP}}$)
- ▶ $N_F = 2+1$, $32^3 \times 64$, $a = 0.09 \text{ fm}$ (π, K, Ω input), MPDDHMC algorithm
- ➔ m_{ud} could be reduced down to the phys. point w/o encountering a 1st order transition

i.e., either $c_2 > 0$, or harmless 1st order at smaller m_q .

Phase structures with tmClover not well clarified yet.

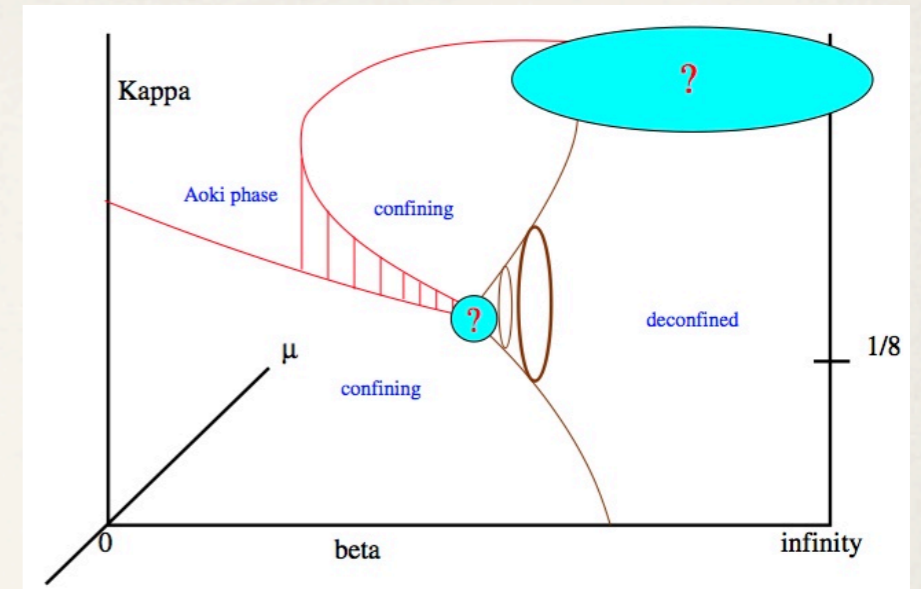
See also Becirevic et al. PRD74('06)
S.Aoki et al. PRD72('05)

$T > 0$

Creutz PRD76('07)

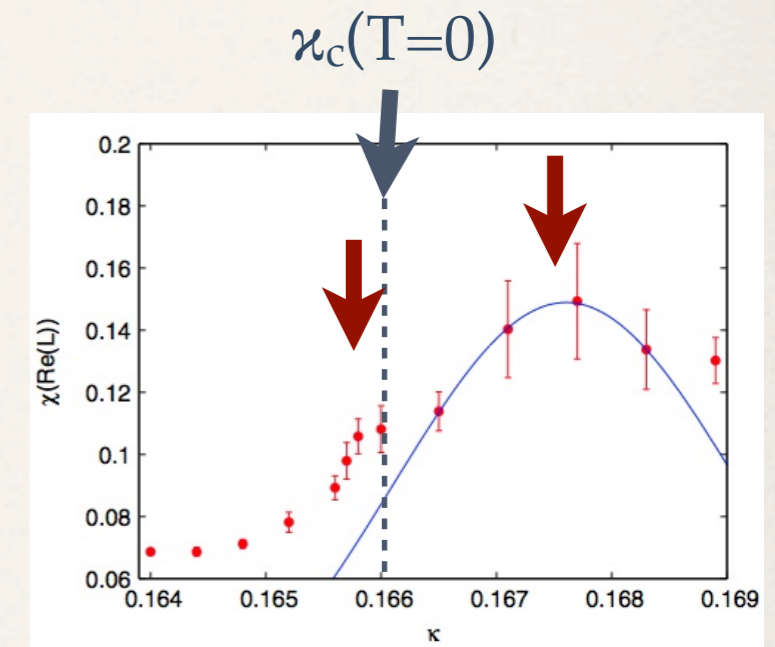
Cone-shaped deconfinement transition plane

Possible 1st order transition ($c_2 < 0$)
hidden in the “?” region.



Illgenfritz et al. (tmfT) PRD80('09) [Zeidlewicz \(Mon\)](#)

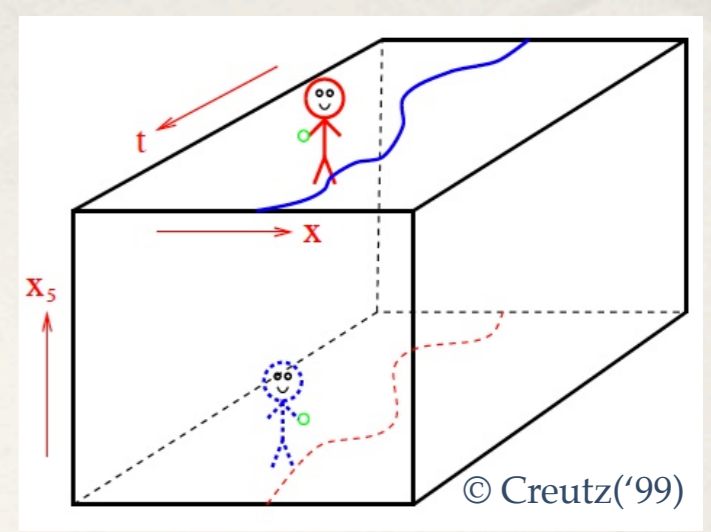
- ▶ tree-level Symanzik gauge + tmWilson i.e. a $c_2 < 0$ case
- ▶ $N_F = 2, N_t = 8$
- ➔ consistent with cone-shaped deconfinement transition plane



$$\mu_0 = 0.005$$

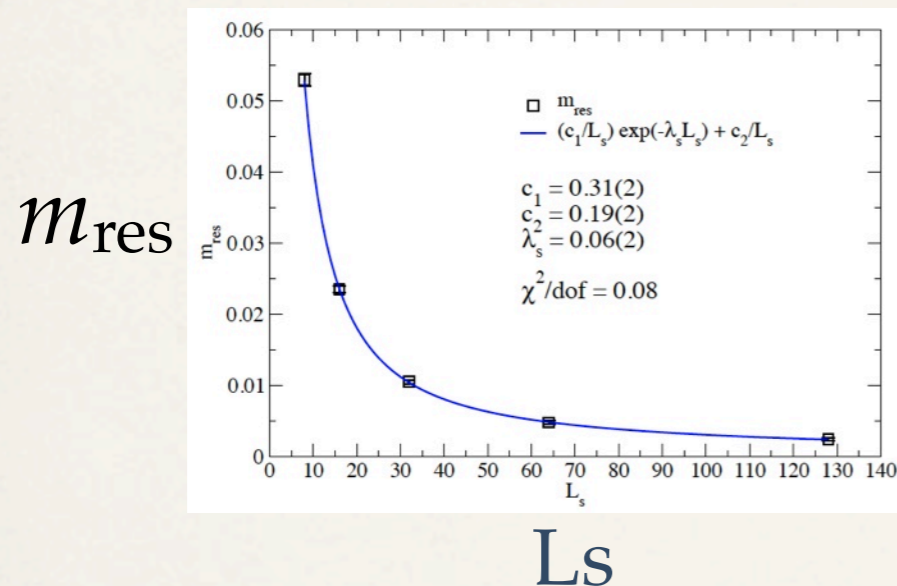
Chiral quarks

Most canonical.



❖ Domain-wall

Chirality realized in the limit $L_s = \infty$. L_s : lattice size in the 5th direction.



Finite $L_s \Rightarrow$ chiral violations $\Rightarrow m_{res}$

$$m_q^{ren} = m_q^{bare} + m_{res} \quad (\text{a la Wilson quarks})$$

$$m_{res} \sim 1/L_s \quad \Leftarrow \text{mobility edge}$$

$T > 0$ simulations usually require coarse lattices
 \Rightarrow Control of chiral violations is a big issue.

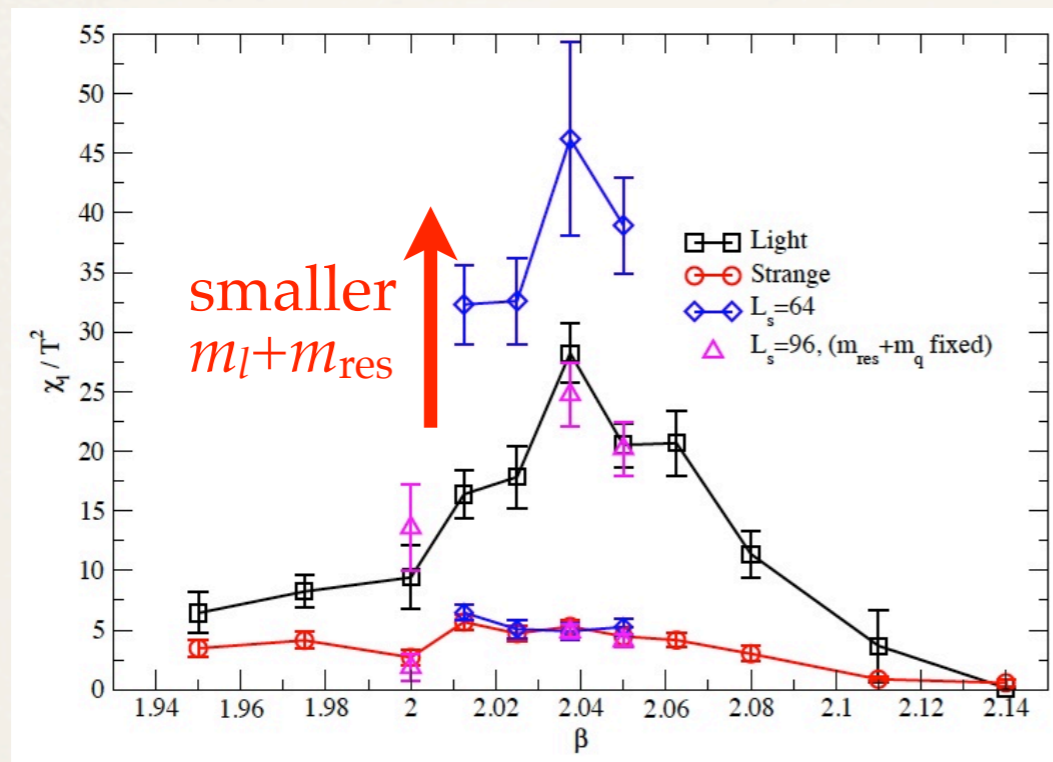
First study: Chen et al. PRD64('01)

- ▶ plaquette gauge + DW
- ▶ $N_F = 2$, $N_t = 4$ ($8^3 \times 4$), $L_s =$ mainly 8
- ➔ Chiral modes confirmed, but large m_{res} effects.

New: Cheng et al. PRD81('10)

- ▶ Iwasaki gauge + DW
- ▶ $N_F = 2+1$, $N_t = 8$ ($16^3 \times 8$), $L_s =$ mainly 32
- ▶ $a \sim 0.15\text{fm}$, $m_\pi \sim 308\text{ MeV}$, $(m_l + m_{\text{res}})/(m_s + m_{\text{res}}) \sim 0.25$

Improved gauge & finer lattice & larger L_s
=> better control of chiral violations.



Qualitatively consistent with expectations.

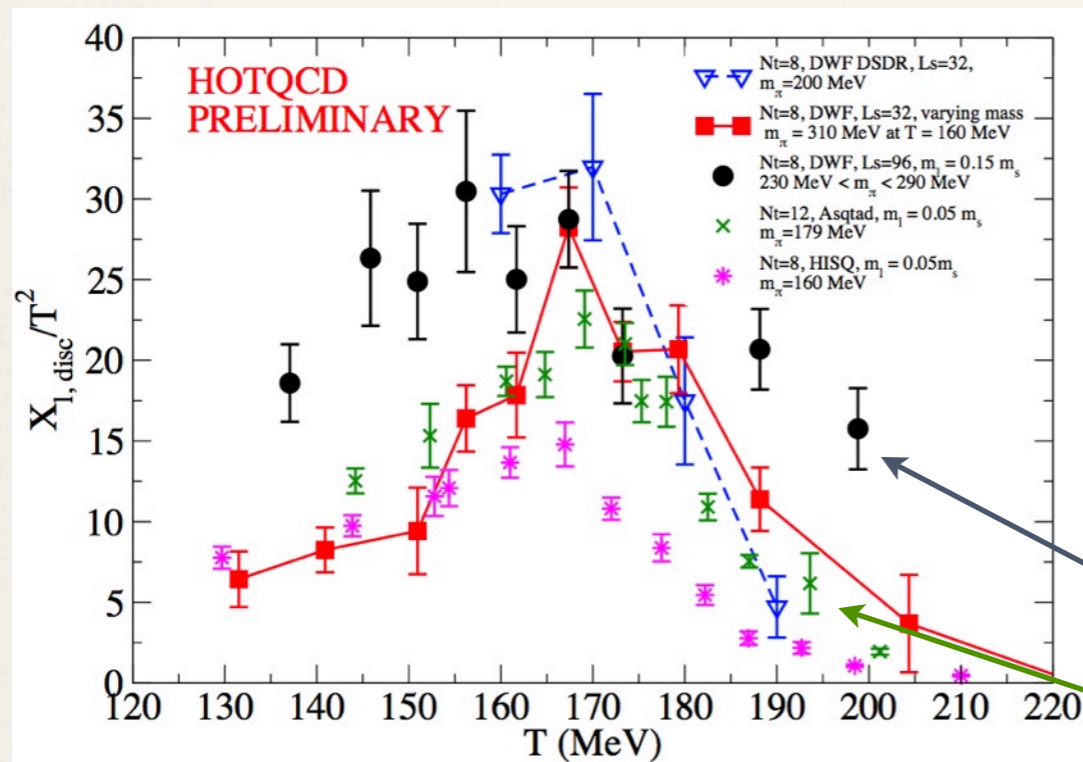
Chiral violations not small enough.

$$m_{\text{res}} \sim 0.008 \gg m_l = 0.003$$

New: Cheng et al. PRD81('10)

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Qualitatively consistent with expectations.

Chiral violations not small enough.

$$m_{\text{res}} \sim 0.008 \gg m_l = 0.003$$

Next steps (HotQCD):

- $L_s = 96$
- ▶ improved action dedicated for DW (“dislocation suppressing determinant ratio”)

❖ Overlap (fixed Q)

Cossu @ Lat10

- ▶ Iwasaki gauge + Overlap + Fukaya-term to suppress topology flips
- ▶ $N_F = 2$, $N_t = 8$, $Q=0$ sector

PHASE STRUCTURE

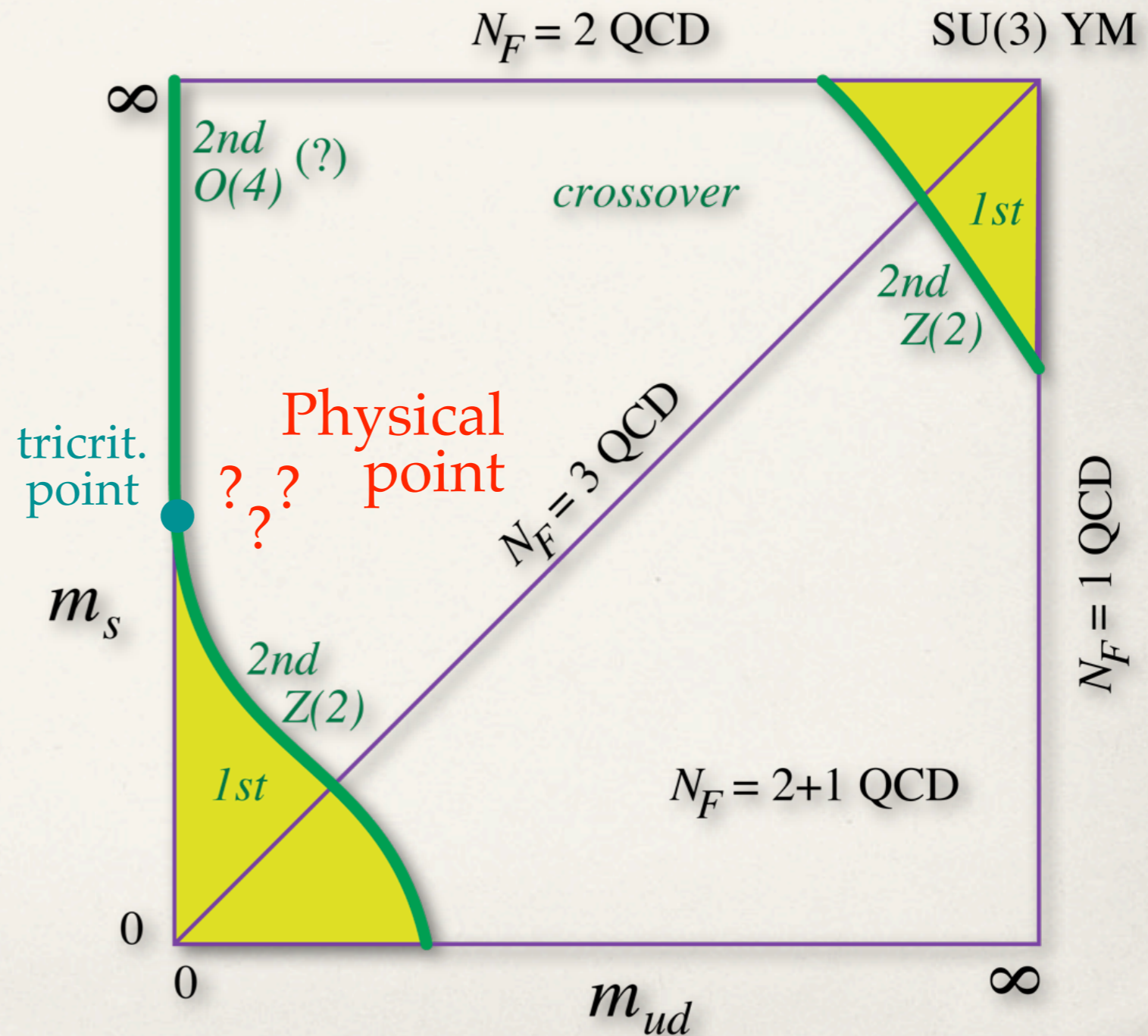
– T_c & SCALING –

Phase structure at $\mu = 0$

Staggered simulations

=> The physical point locates in the crossover region.

We want to know the properties of this transition.



T_c

Disagreement among staggered groups:

stout

$N_F=2+1$, $N_t=6-10$, $m_l/m_s=0.11-0.37$: Y.Aoki et al. (Wu-Bu) PLB643('06)

$T_c = 151(3)(3)$ MeV chiral susceptibility

$175(2)(4)$ strange quark number

at the physical point in the continuum limit.

p4/asqtad

$N_F=2+1$, $N_t=4,6$, $m_l/m_s=0.05-0.5$, p4: Cheng et al. (RBC-Bi) PRD74('06)

$T_c = 192(4)(7)$ MeV chiral susceptibility + Polyakov loop

asqtad supports p4 (HotQCD) PRD77('08)

T_c

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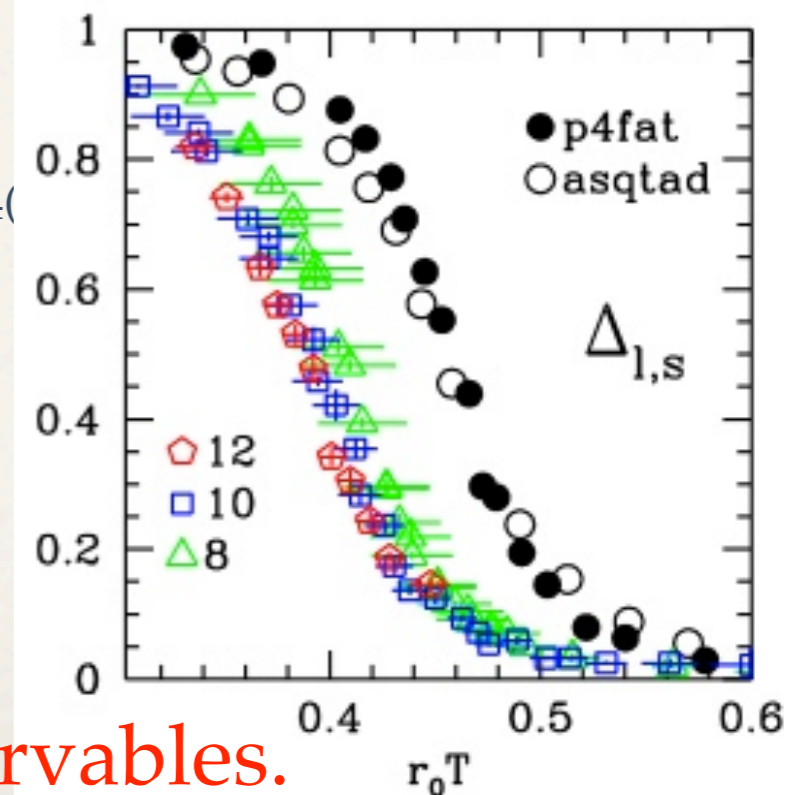
$T_c = 192(4)(7)$ MeV chiral susceptibility + Polyakov loop

asqtad supports p4 (HotQCD) PRD77('08)

Crossover: value of T_c depends on the observables.

Ambiguities in scale setting, LCP definition etc.

Discrepancies remain even with the same observables.



T_c

Disagreement among staggered groups:

stout

$N_F=2+1$, $N_t=6-10$, $m_l/m_s=0.11-0.37$: Y.Aoki et al. (Wu-Bu) PLB643('06)

$T_c = 151(3)(3)$ MeV chiral susceptibility

$175(2)(4)$ strange quark number

at the physical point in the continuum limit.



$147(2)(3)$ $N_t=6-16$ [Fodor \(Mon\)](#)

$165(5)(3)$ Borsányi et al. arXiv:1005.3508

p4/asqtad

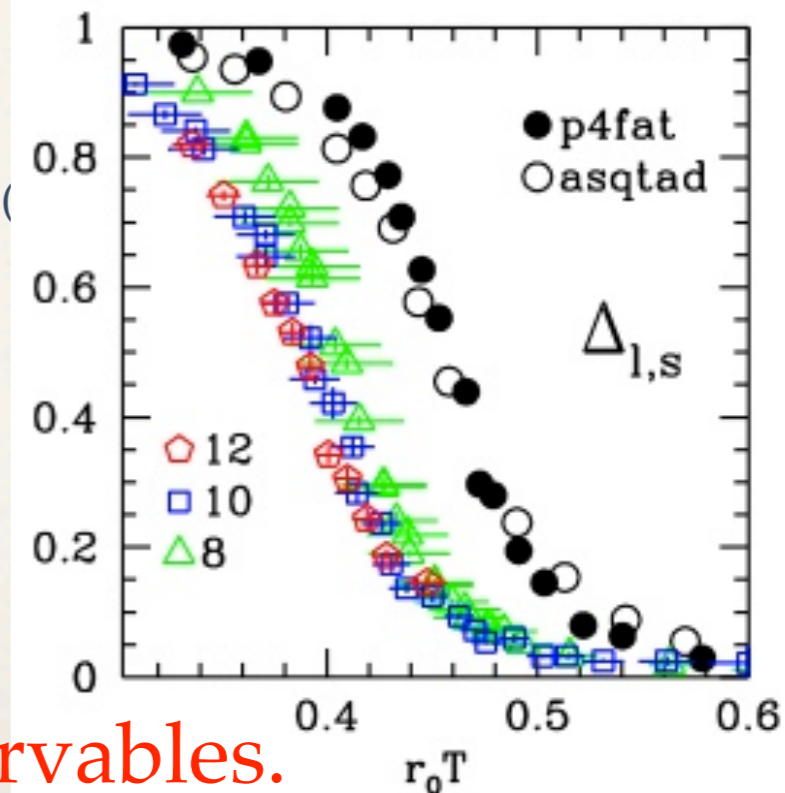
$N_F=2+1$, $N_t=4,6$, $m_l/m_s=0.05-0.5$, p4: Cheng et al. (RBC-Bi) PRD74('08)

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p4/asqtad

$N_F=2+1$, $N_t=6$, $m_l/m_s=0.1$: Cheng et al. (HotQCD)

$[T_c = 196(3) \text{ MeV} @ m_\pi^{\text{pNG}} \approx 220 \text{ MeV}]$ PRD77('08)

$-(5-7) \text{ MeV}$ from $N_t=6$ to $N_t=8$ @ $m_\pi^{\text{pNG}} \approx 220 \text{ MeV}$ PRD80('09)

-5 MeV from $m_\pi^{\text{pNG}} \approx 220 \text{ MeV}$ to 160 MeV p4, PRD81('10)

→ $N_t=12$, $m_l/m_s=0.05$ asqtad:
Bazavov, Söldner (Mon)
 $T_c = 164(6) \text{ MeV}$

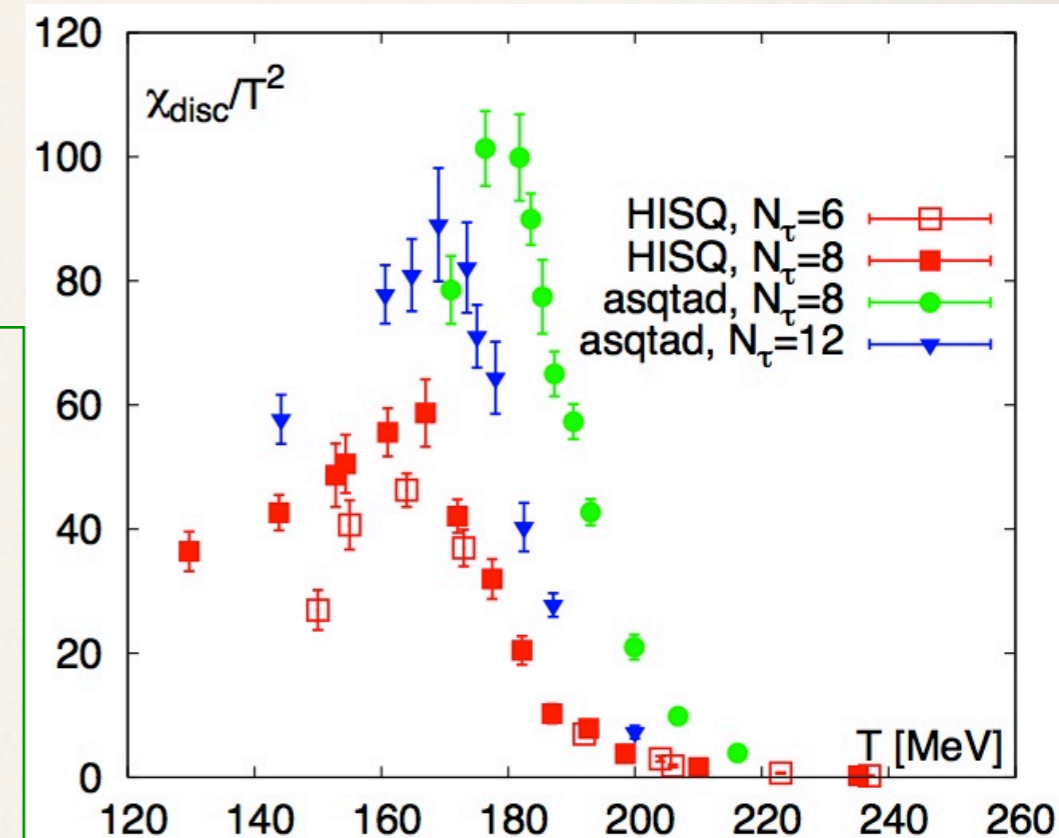
HISQ

$N_F=2+1$, $N_t=6,8$, $m_l/m_s=0.05$

Bazavov, Petreczky (HotQCD) [ArXiv:1005.1131]

Bazavov, Söldner (Mon)

$[T_c \sim 170 \text{ MeV} @ m_\pi^{\text{pNG}} \approx 160 \text{ MeV}]$ by chiral suscept.



p4/asqtad

$N_F=2+1, N_t=6, m_l/m_s=0.1$: Cheng et al. (HotQCD)

$[T_c = 196(3) \text{ MeV} @ m_\pi^{\text{pNG}} \approx 220 \text{ MeV}]$ PRD77('08)

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$N_t=12, m_l/m_s=0.05$ asqtad:

Bazavov, Söldner (Mon)

$T_c = 164(6) \text{ MeV}$

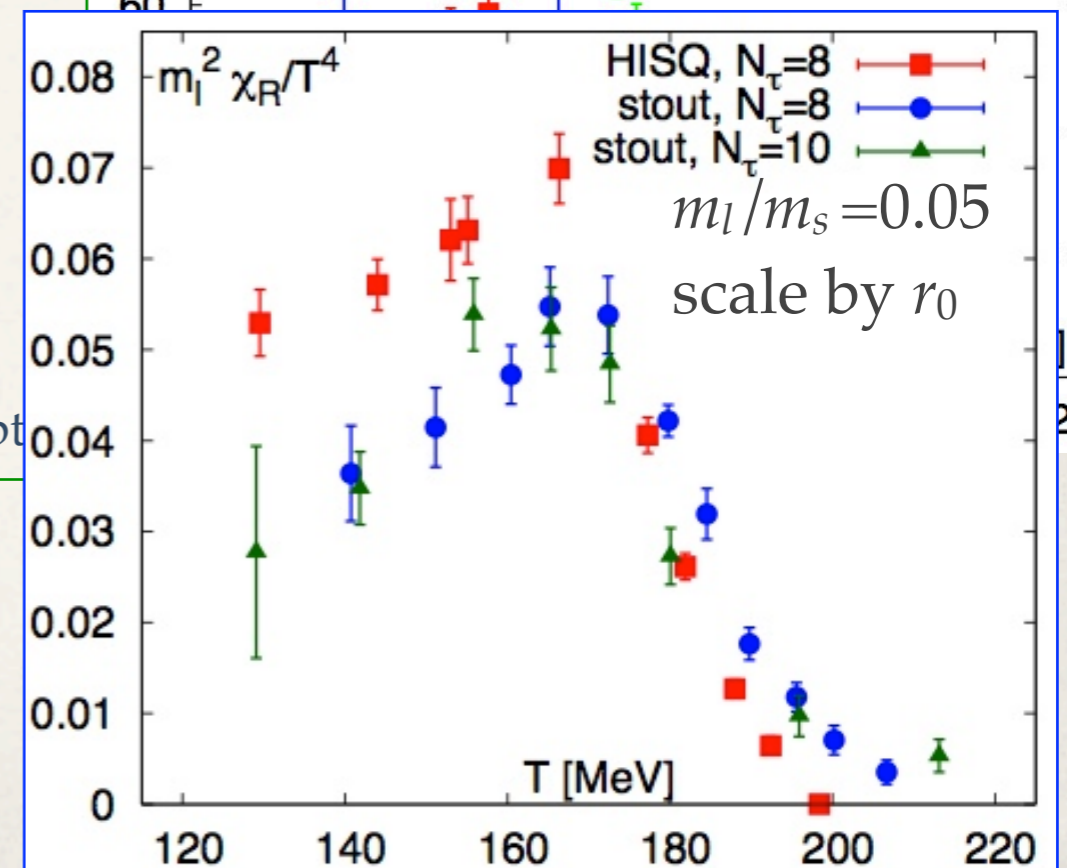
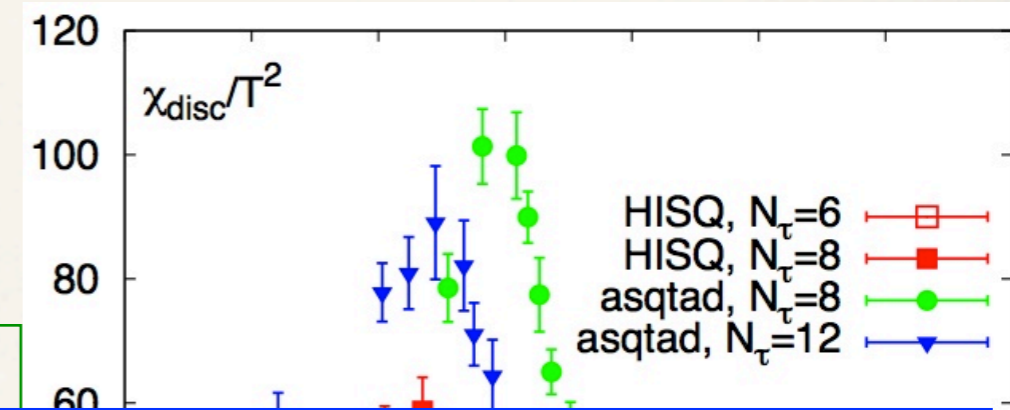
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$N_F=2+1, N_t=6,8, m_l/m_s=0.05$

Bazavov, Petreczky (HotQCD) [ArXiv:1005.1131]

Bazavov, Söldner (Mon)

$[T_c \sim 170 \text{ MeV} @ m_\pi^{\text{pNG}} \approx 160 \text{ MeV}]$ by chiral suscept



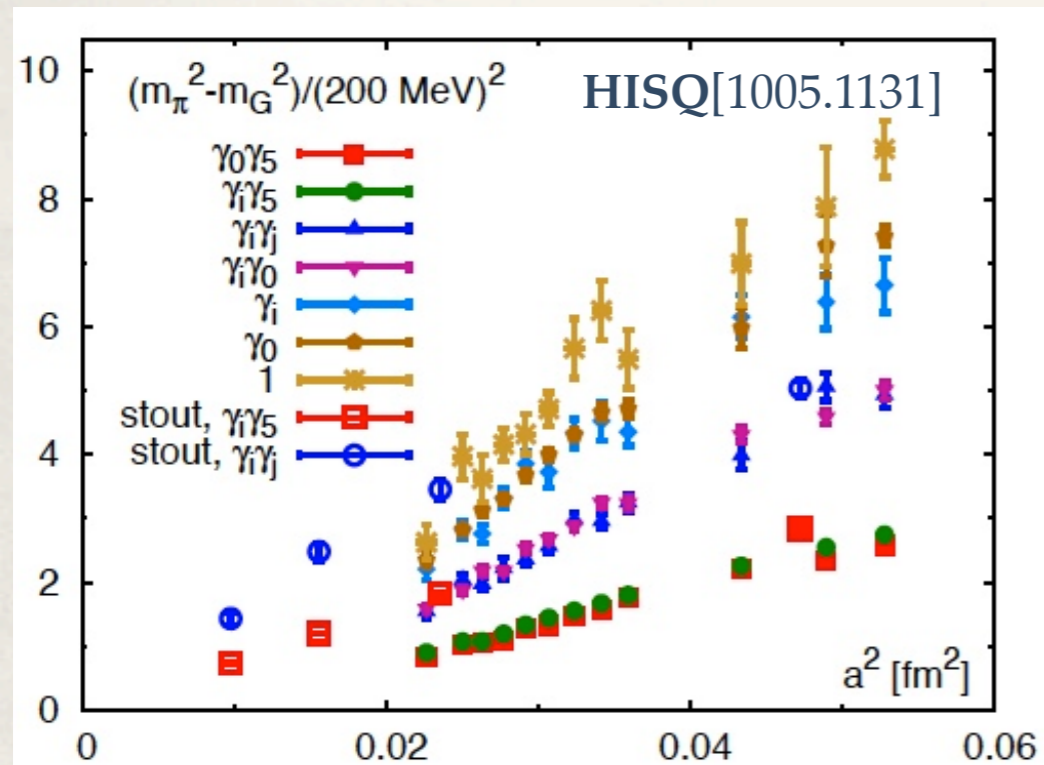
Most discrepancies removed!

Main difference among staggered studies: the magnitude of the taste violation

masses of heavy π 's (at around $T \sim 170 \text{ MeV}$ with fixed $m_{\pi}^{\text{pNG}} \sim 135 \text{ MeV}$)

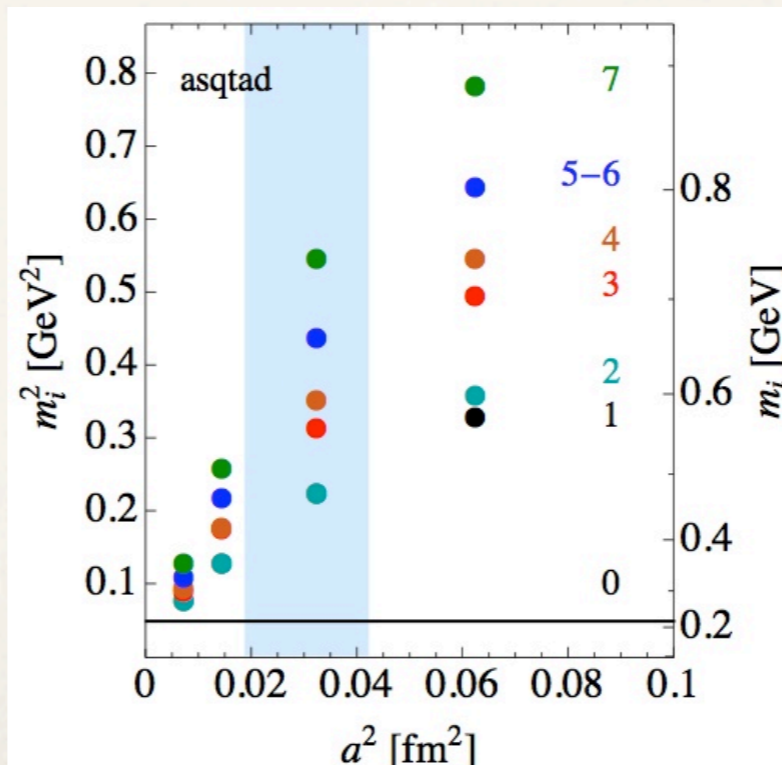
Nt~8	~ 400-600	asqtad < p4
	~ 300-500	stout
	~ 200-400	HISQ
Nt~12	~ 200-350	stout

arXiv:1005.1131

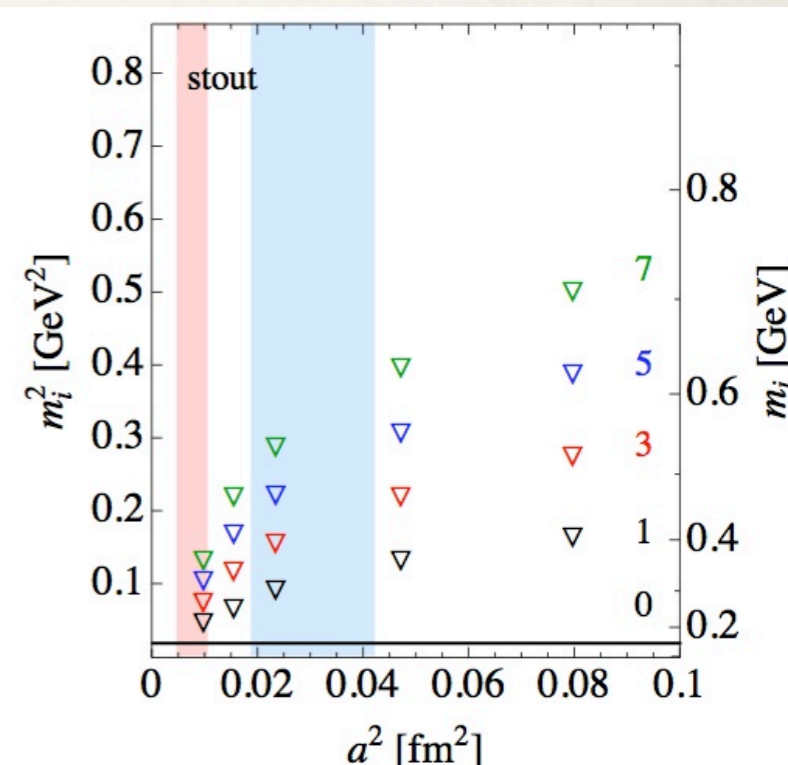


Nt~12 8

arXiv:1005.3508



Nt~12 8

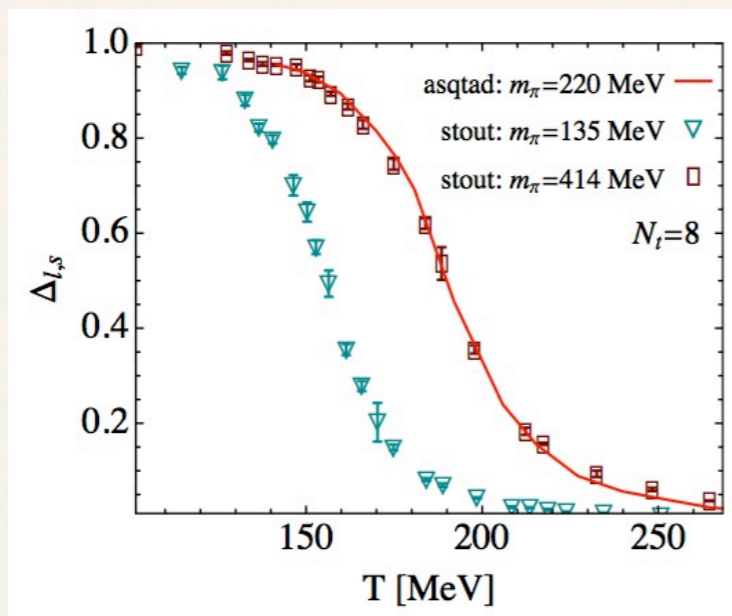


Nt~12 8

Contamination of heavier π 's.

Conventionally, the lightest pNG π is treated as the physical π . However, unlike spectroscopy studies in which we can choose operators, in $T>0$ physics, all heavier π 's directly contribute to the observables.

RMS π mass more appropriate to consult (DeTar @ Lat 08).



Shift can be recovered by adjusting “average pion mass”.

Borsányi et al. arXiv:1005.3508 [Fodor \(Mon\)](#)

Identification of “physical point” (and its LCP) with m_π^{pNG} problematic for $T>0$ physics.

The physical point for $T>0$ should be identified by averaged masses.

=> This will make T_c for HISQ/asqtad actions at finite a even smaller, thus may explain the remaining small discrepancies.

T_c from other quarks

DW: Cheng et al. (HotQCD) arXiv:0911.3450

- ▶ Iwasaki gauge + DW
- ▶ $N_F = 2+1$, $N_t = 8$, $L_s = 32$ etc.
- ➔ $T_c = 171(10)(17)$ MeV chiral suscept., r_0 scale

Wilson ($N_F = 2$):

Bornyakov (Mon) (QCDSF-DIK) arXiv:0910.2392

- ▶ plaquette + Clover (C_{SW}^{NP})
- ▶ $N_F = 2$, $N_t = 8, 10, 12$, + 14 (prelim.)
- ➔ $T_c = 174(3)(6)$ MeV chiral condensate + Polyakov, r_0 scale, $O(4)$

Ejiri et al. (WHOT) arXiv:0909.2121

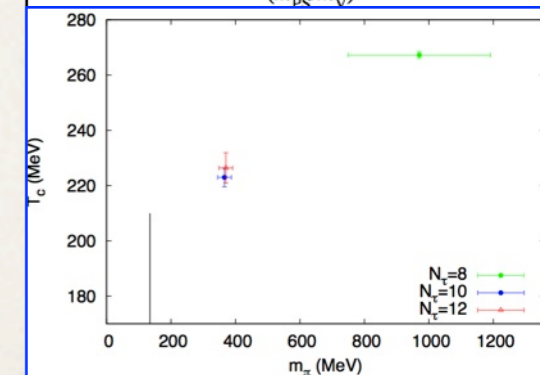
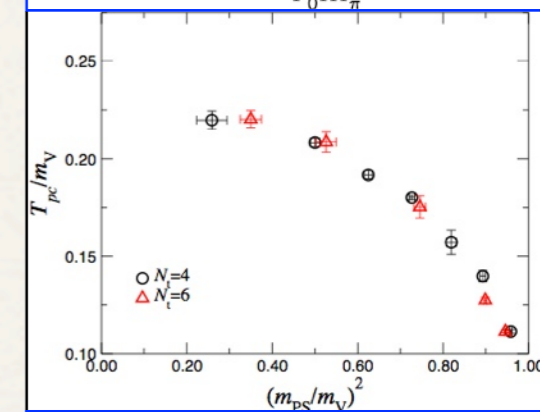
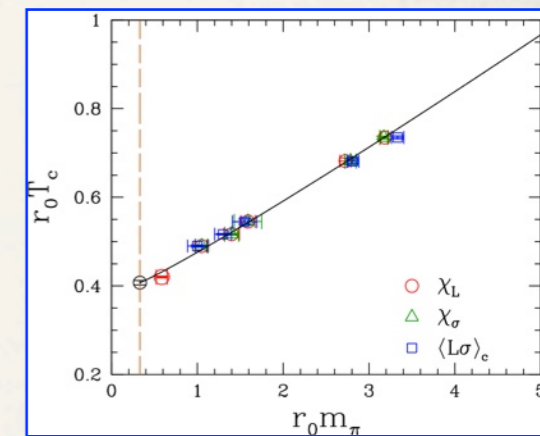
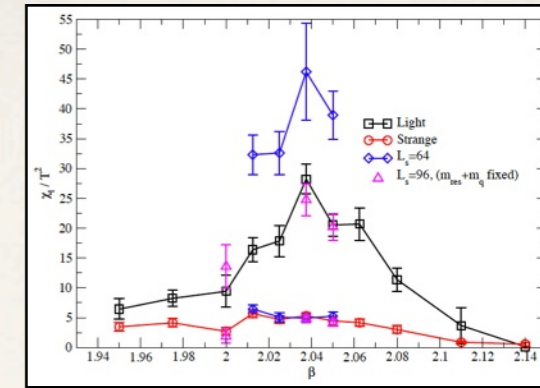
- ▶ Iwasaki gauge + Clover (C_{SW}^{MF})
- ▶ $N_F = 2$, $N_t = 4, 6$
- ➔ $T_c \approx 160-184$ MeV $N_t = 6$ chiral condensate + Polyakov, m_Q scale, $O(4)$

Zeidlewicz (Mon) (tmfT)

- ▶ tree level Symanzik gauge + mtmWilson
- ▶ $N_F = 2$, $N_t = 8, 10, 12$
- ➔ [$T_c \approx 241(10), 244(10)$ MeV @ $m_\pi \approx 380$ MeV, $N_t = 10, 12$] Polyakov

Brandt (Mon)

- ▶ plaquette + Clover (C_{SW}^{NP})
- ▶ $N_F = 2$, $N_t = 12, 16$



Scaling

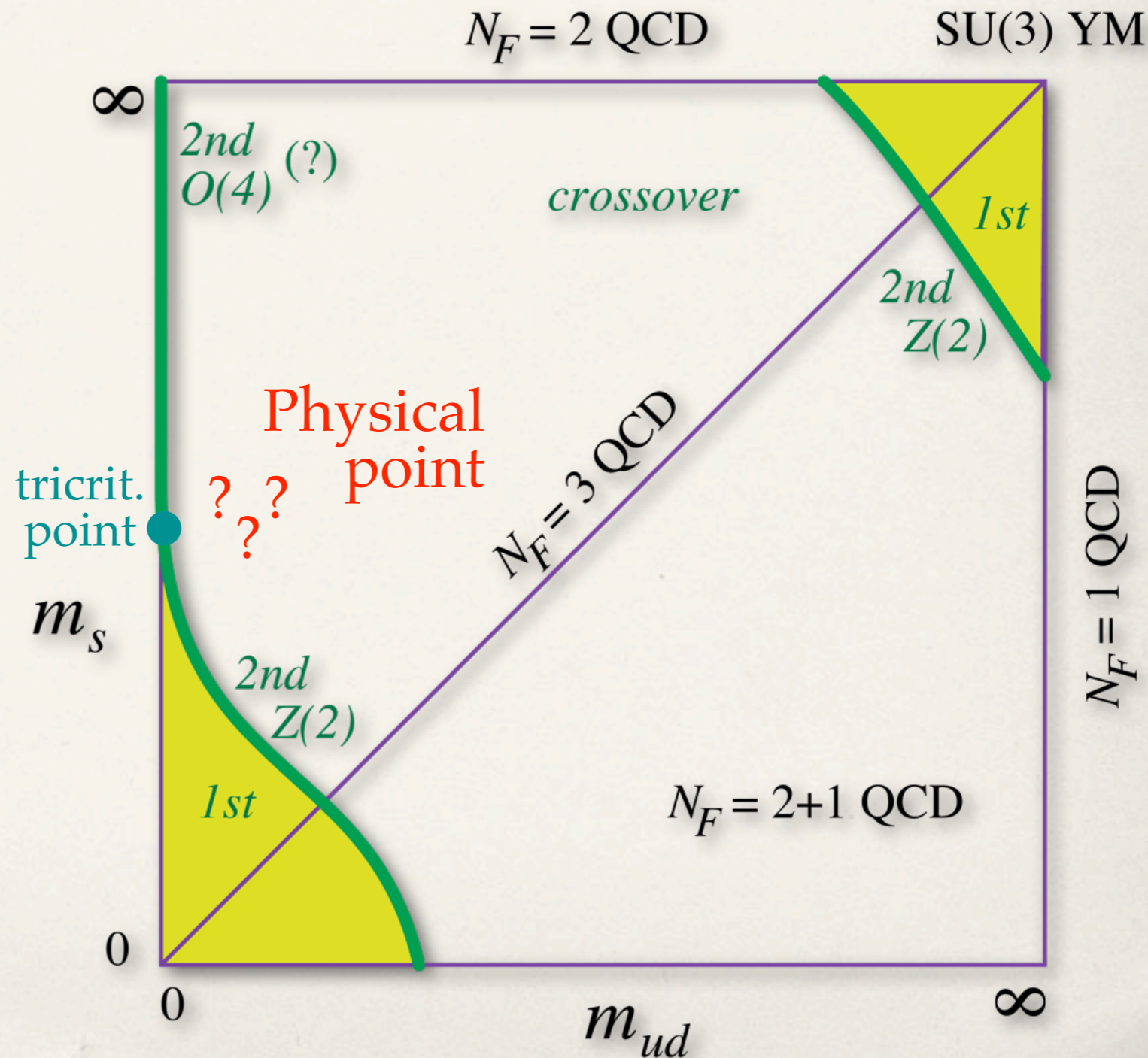
effective G-L model
for massless N_F -flavor QCD

$N_F \geq 3$: 1st order

$N_F = 2$: 2nd order $\approx O(4)$

or 1st order

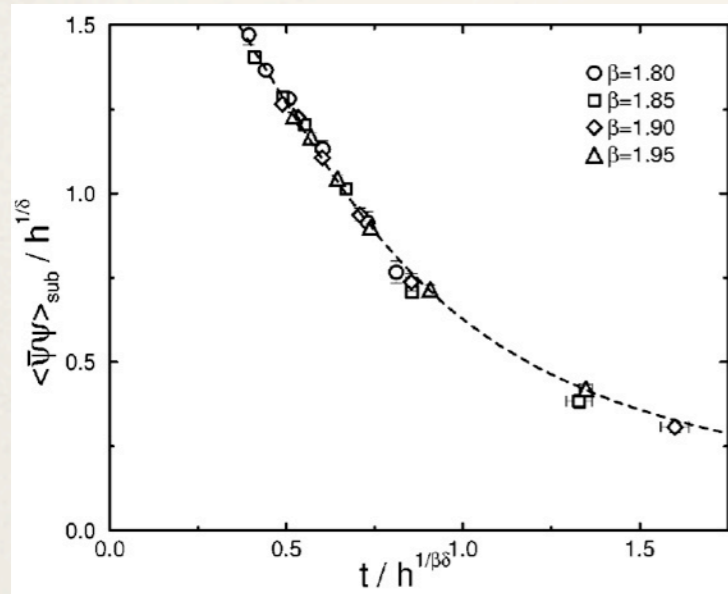
when the effective $U(1)_A$
breaking interaction is
weak at T_c .



❖ Previous $N_F=2$ studies with Wilson-type quarks

$$\langle \bar{\Psi}\Psi \rangle_{\text{sub}} = 2m_q aZ \sum_x \langle \pi(x)\pi(0) \rangle$$

: subtracted chiral condensate via axialvector W.I.
Bochicchio et al.('85)



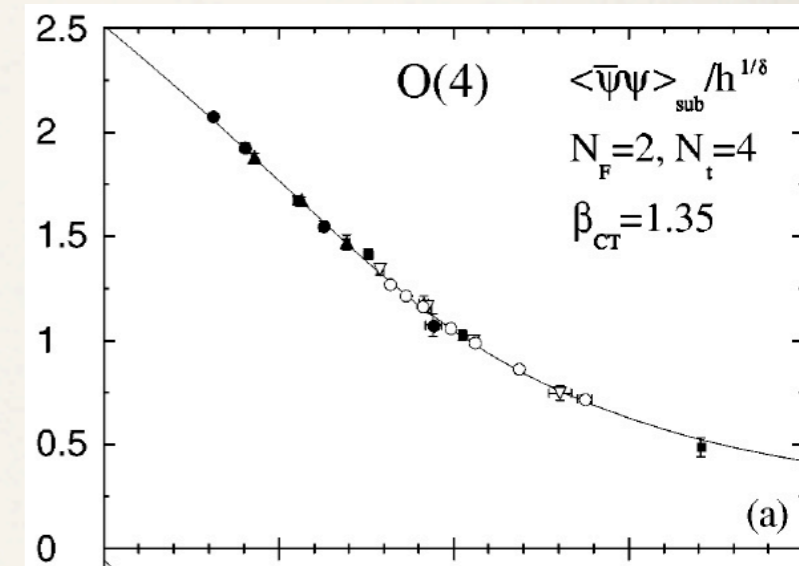
Iwasaki et al. PRL78('97)

- ▶ Iwasaki gauge + Wilson
- ▶ $Nt=4, m_\pi \sim 600-900$ MeV

AliKhan et al.(CP-PACS) PRD63('01)

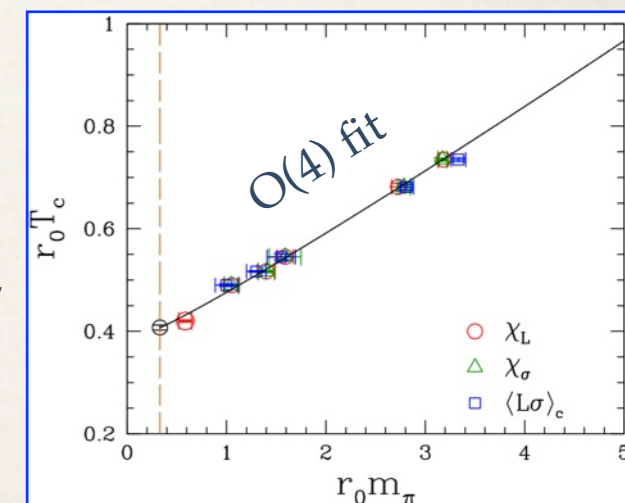
- ▶ Iwasaki gauge + Clover
- ▶ $Nt=4, m_\pi \sim 600-1000$ MeV

See also [Ejiri \(Poster\)](#)



[Bornyakov \(Mon\)](#) (QCDSF-DIK) [arXiv:0910.2392]

- ▶ plaquette gauge + Clover ($C_{\text{SW}}^{\text{NP}}$)
- ▶ $N_F=2, Nt=8,10,12, m_\pi \approx 420-1300$ MeV
- ➔ “in accord with the predictions with the $O(4)$ Heisenberg model”
- ➔ “a first order transition is very unlikely”



➔ Consistent with $O(4)$ scaling, though quarks are heavy.

❖ Previous $N_F=2$ with staggered quarks

★ $O(4)$ vs. $O(2)$

Realize desired N_F through the 4th root trick: $\det M \Rightarrow [\det M]^{1/4}$

Sym. of the system = sym. of $M = O(2)$ for any $a > 0$ and any N_F

\Rightarrow When the chiral transition is 2nd order, we expect

$O(2)$ scaling for any N_F

if the non-locality does not affect the universality at $a > 0$ too.

★ Results of previous efforts: *puzzling*
(all: plaq. gauge + unimproved staggered)

\Rightarrow Transition looks continuous, but
neither $O(2)$ nor $O(4)$

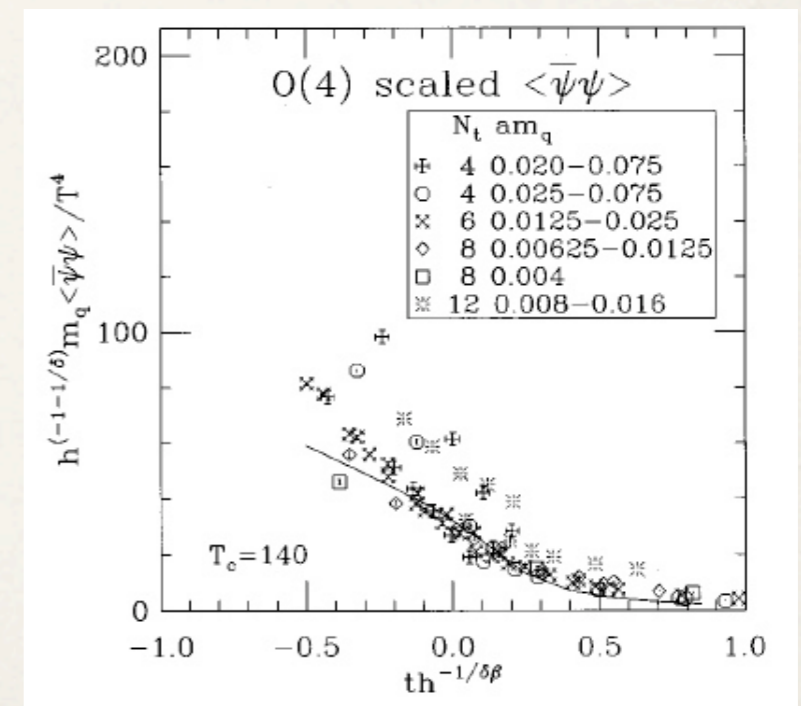
Bielefeld ('94): $m_q a = 0.02-0.075$, $N_t = 4-8$

MILC ('94-96): $m_q a = 0.008-0.075$, $N_t = 4-12$

JLQCD ('98): $m_q a = 0.01-0.075$, $N_t = 4$

\Rightarrow 1st order?

Cossau et al. ('08): $m_q a = 0.01335--$, $N_t = 4$

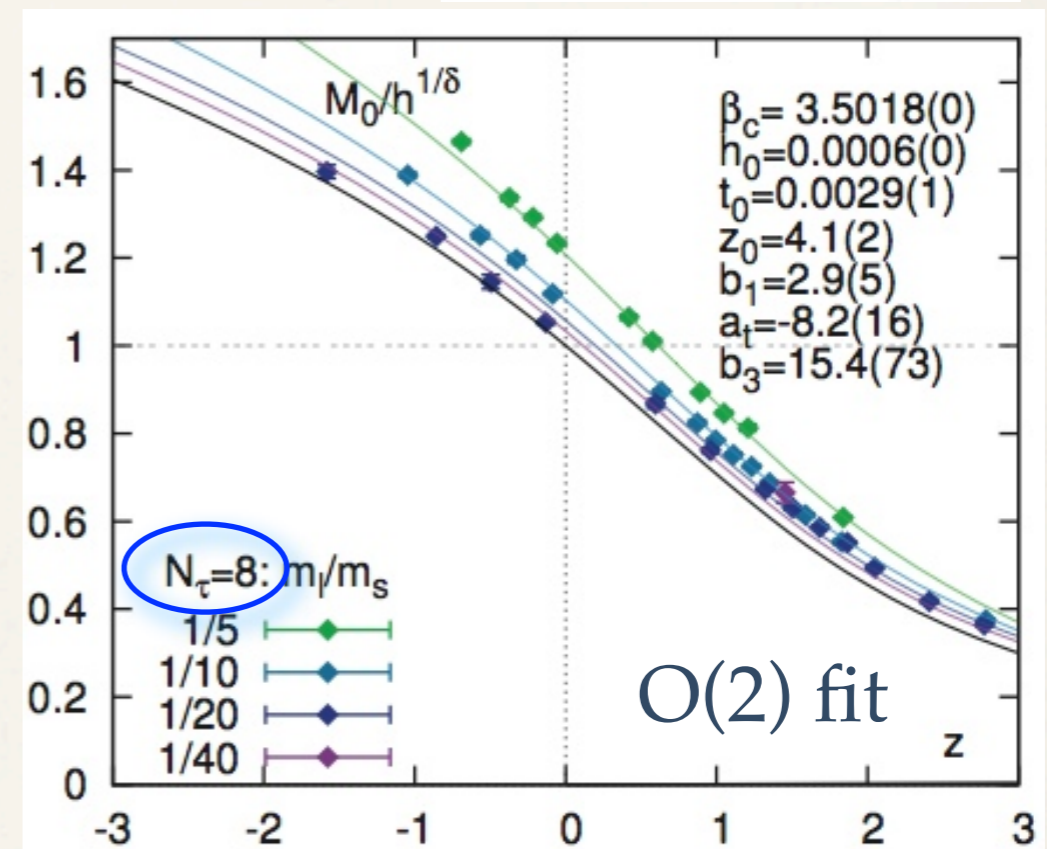
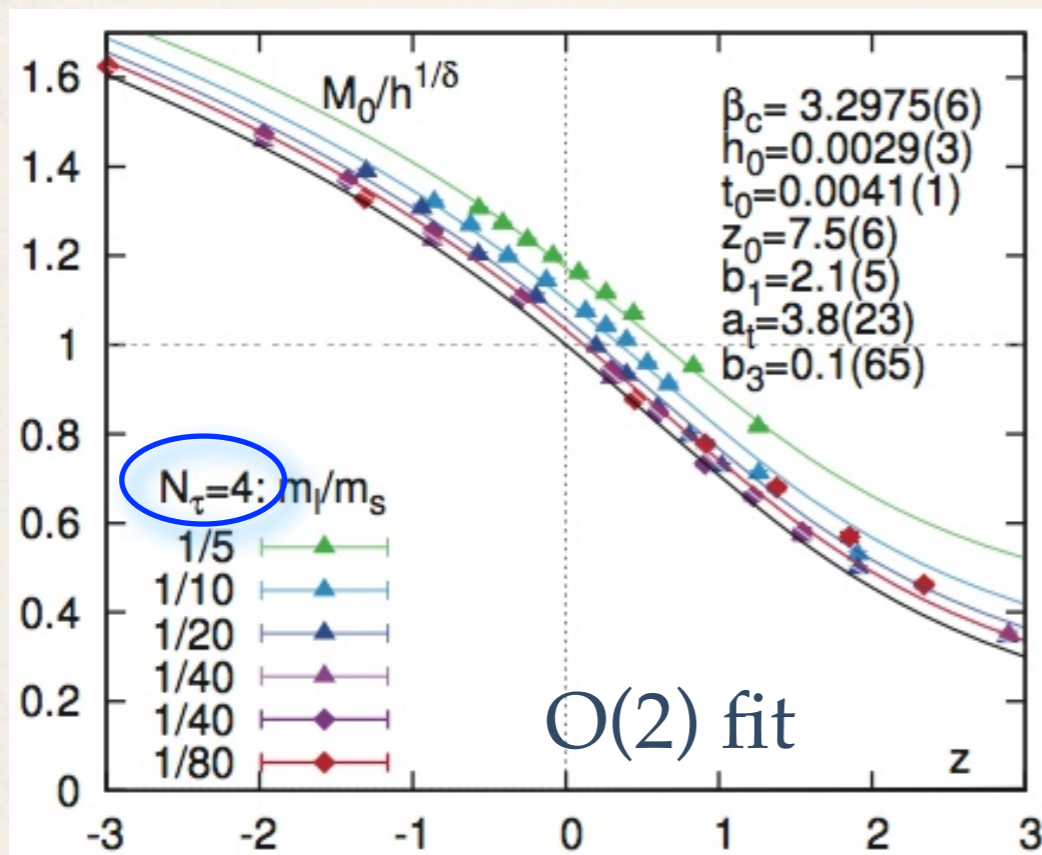


❖ New $N_F=2+1$ with improved staggered quarks

[Schmidt \(Mon\)](#) Ejiri et al. (BNL-Bi) PRD80('09)

- ▶ tree-level Symanzik gauge + p4
- ▶ $m_s \approx$ physical, m_l/m_s down to $1/80 - 1/20$ ($m_{\pi}^{\text{pNG}} \approx 75 - 150$ MeV)
- ▶ $N_t = 4, 8$ (preliminary)
- ▶ crossover region

$$M_0 = m_s \langle \bar{\psi}\psi \rangle_t / T^4$$



- ➔ Consistent with both O(2) and O(4) scalings
- ➔ Deviation for $m_l/m_s > 1/20$

★ Why successful this time?

$$\Leftarrow m_l/m_s < 1/20 + \text{improved action}$$

★ Is this O(4) ?

though it is numerically difficult to discriminate between O(2) and O(4)

=> It will be O(2), because O(2) is the exact symmetry for all $a > 0$.

★ Will this O(2) gradually transforms into O(4) near the cont. limit?

=> Probably No, because O(2) is the exact symmetry for all $a > 0$.

➔ Suggests a continuum transition in the chiral limit.

➔ Tricritical point may be lower than m_s^{phys} .

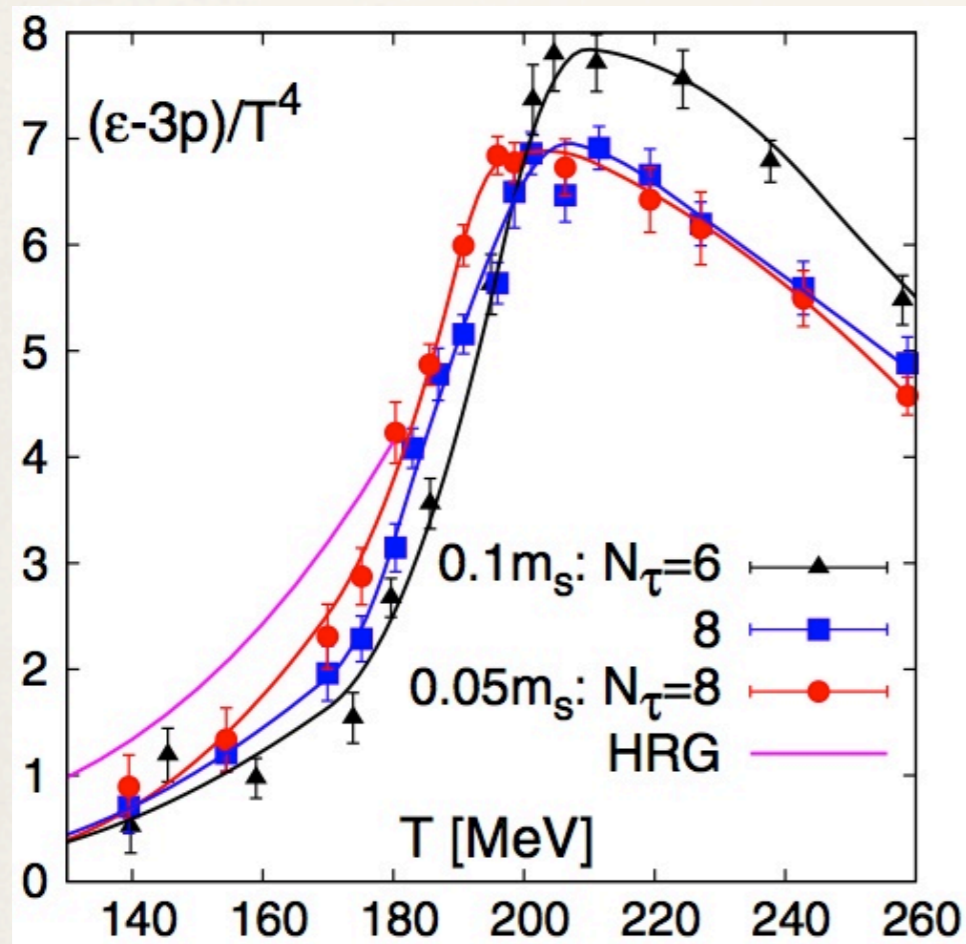
EQUATION OF STATE

$$\text{--- } N_F = 2 + 1 \text{ ---}$$

❖ $N_F = 2+1$ p4 at the “physical point”

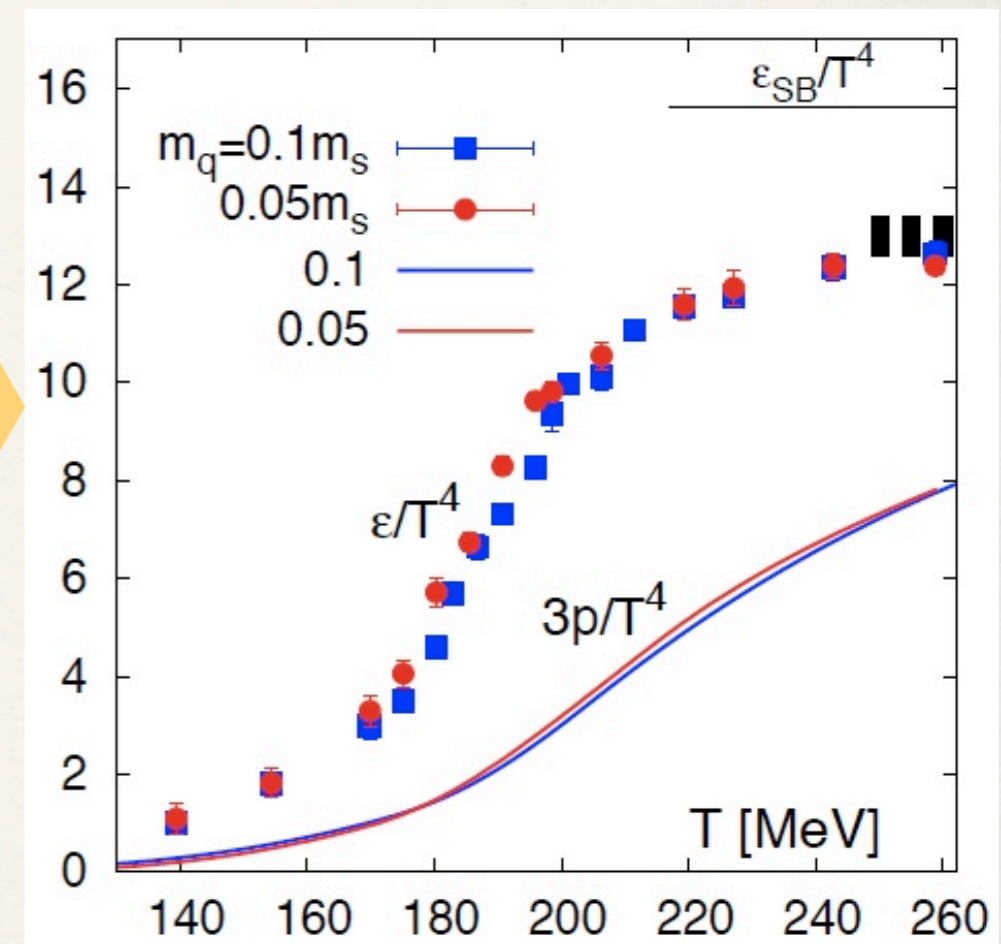
Chen et al. (HotQCD) PRD81('10), [Schmidt \(Mon\)](#)

- ▶ tree-level Symanzik gauge + p4
- ▶ $m_s \approx$ “physical”, $m_l/m_s = 0.05$ ($m_{\pi}^{\text{pNG}} \approx 154$ MeV)
- ▶ $N_t = 8$



$$p = \frac{T}{V} \int_{b_0}^b db \frac{1}{Z} \frac{\partial Z}{\partial b}$$

integral
method



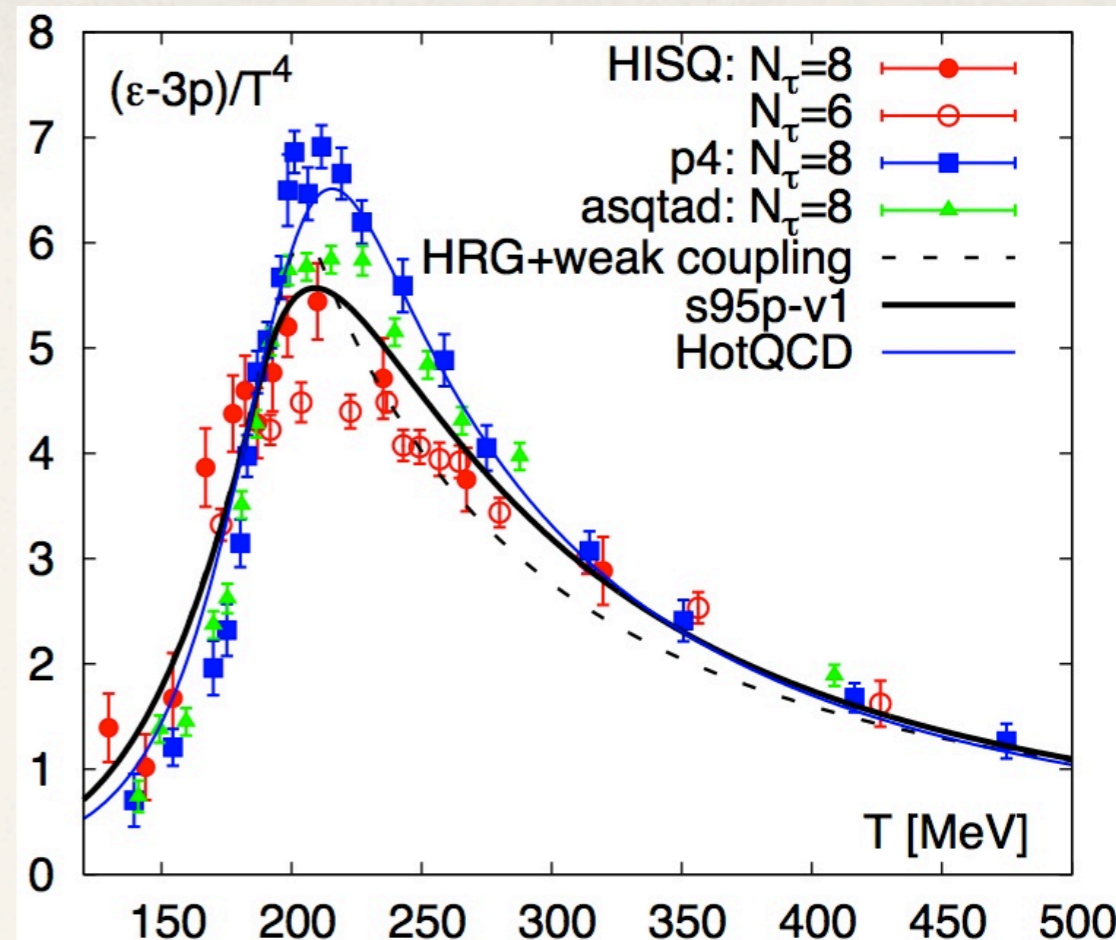
-5 MeV from $m_l/m_s = 0.1$

Caveat: physical point identified by m_{π}^{pNG} .

❖ $N_F = 2+1$ HISQ / asqtad

Bazavov, Söldner (Mon)

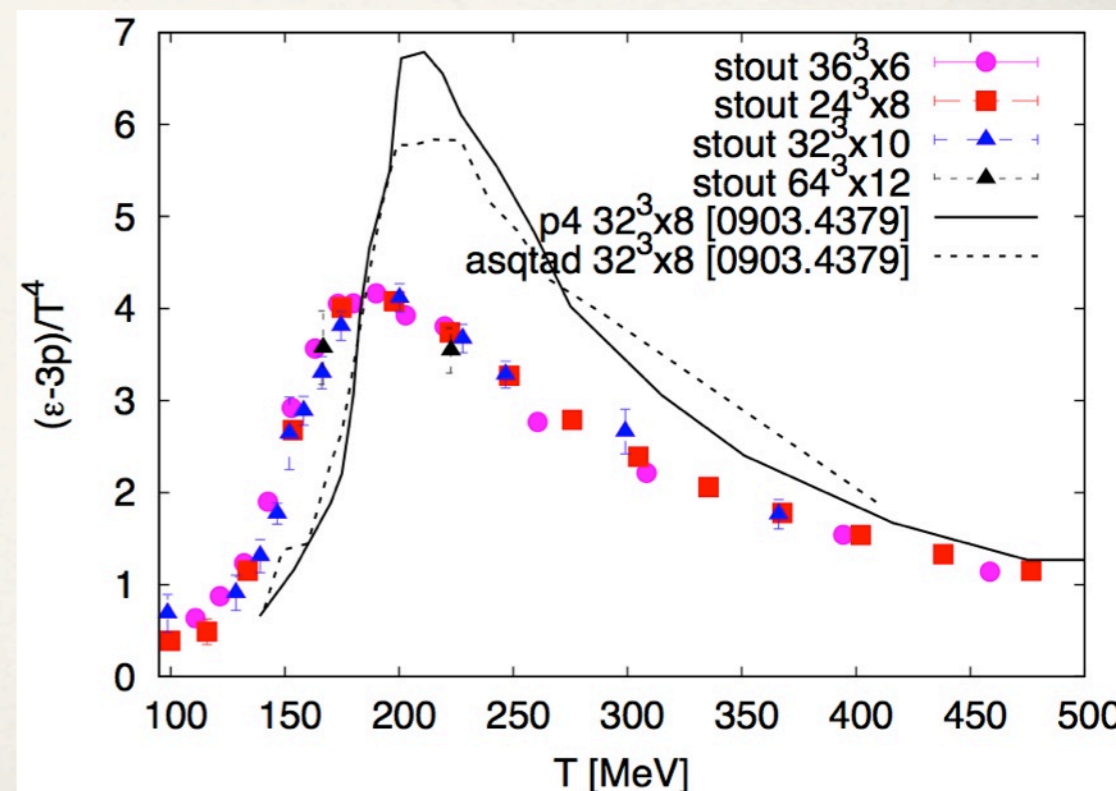
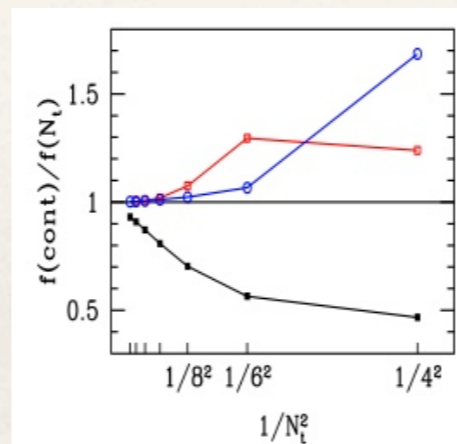
- ▶ tree-level Symanzik gauge + HISQ / asqtad
- ▶ $m_s \approx$ "physical", $m_l/m_s = 0.05$
($m_{\pi}^{\text{pNG}} \approx 160$ MeV)
- ▶ $N_t = 8$



❖ $N_F = 2+1$ stout

Szabo (Mon)

- ▶ tree-level Symanzik gauge + stout
- ▶ $m_s \approx$ "physical", $m_l \approx$ "physical"
- ▶ $N_t = 6-12$
- ▶ tree-level improvement factor multiplied



❖ $N_F = 2+1$ clover

★ Fixed-scale approach + T-integral method [Umeda et al. (WHOT) PRD79('09)]

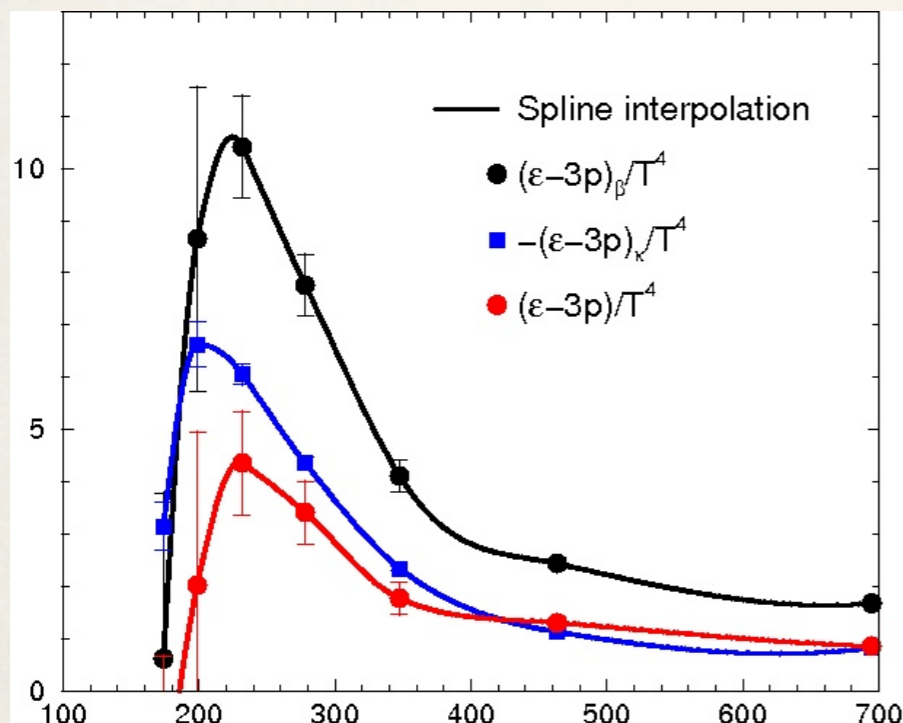
Vary $T = \frac{1}{N_t a}$ by varying N_t with all coupling params. fixed.

- ✓ LCP automatically guaranteed / purely vary T only
- ✓ dedicated $T=0$ simulations needed only at one point
- ✓ keep a small around $T \sim T_c$ at the cost due to large N_t

See also [Gavai \(Tue\)](#)

★ $N_F = 2+1$ study at a CP-PACS+JLQCD $T=0$ point [Umeda \(Mon\)](#)

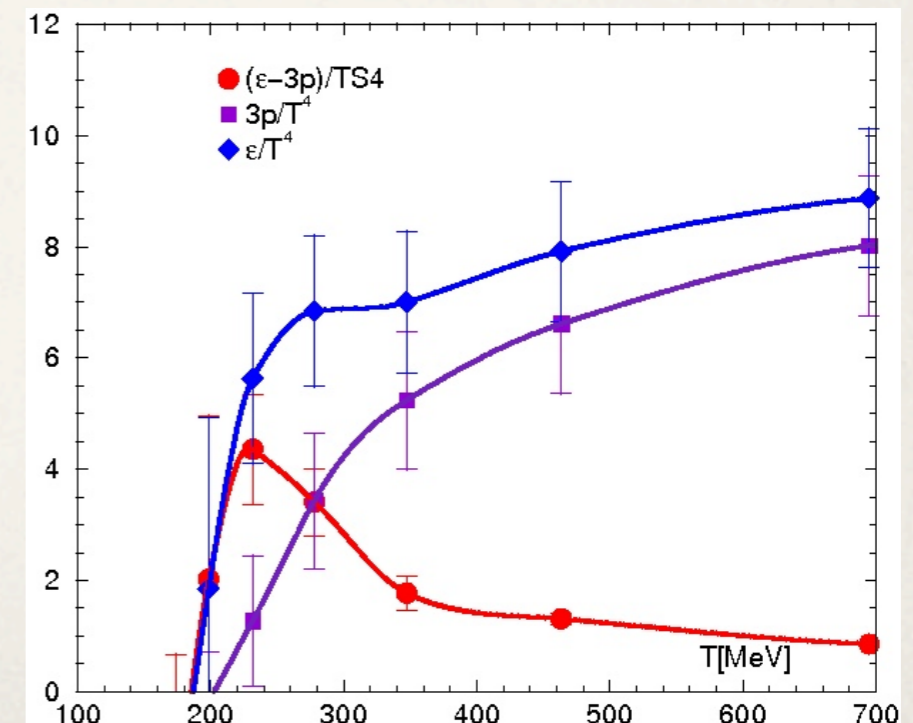
- ▶ Iwasaki gauge + Clover(C_{SW}^{NP})
- ▶ $a \approx 0.07$ fm, $m_s \approx$ physical, $m_\pi/m_\rho = 0.63$, $T=0$ configurations on ILDG.
- ▶ $N_t = 4 - 16$



$$\frac{p}{T^4} = \int_{T_0}^T dT \frac{\epsilon - 3p}{T^5}$$

T-integral
method

$r_0 = 0.5$ fm

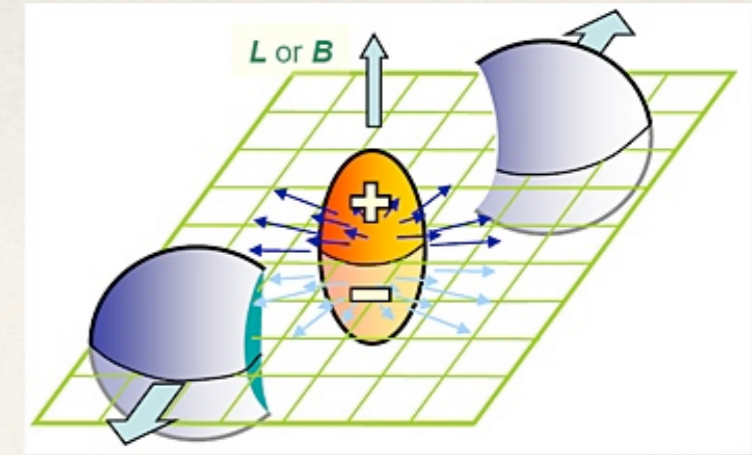


OTHER HOT ISSUES



Chiral magnetic effect

Local strong parity violation

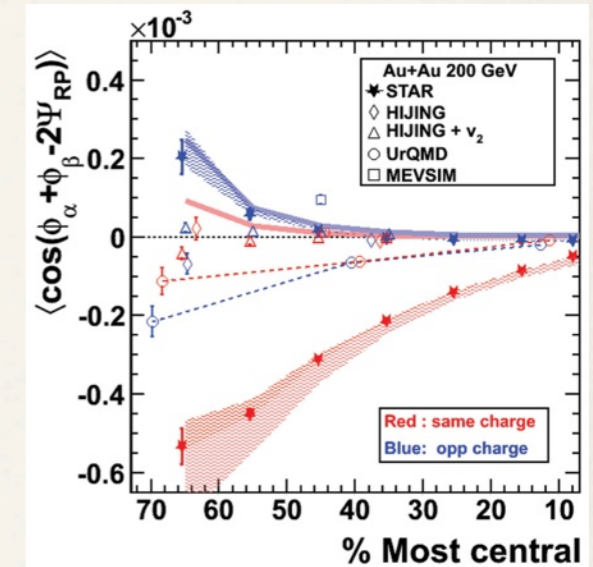


Collision of high energy nuclei with small offset

=> strong magnetic field $B \sim 10^{15}$ T

- local fluctuation of gluonic topological charge

=> electric charge asymmetry w.r.t. the reaction plane



Buividovic (Tue) PRD80('09), arXiv:1003.2180

quenched SU(2) QCD + valence overlap quark + background B

=> enhancement of electric current in the direction of B

=> electric conductivity along the direction of B at $T < T_C$

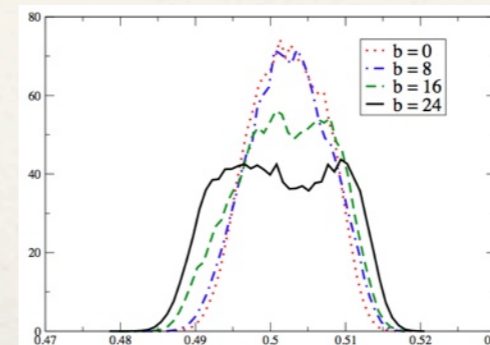
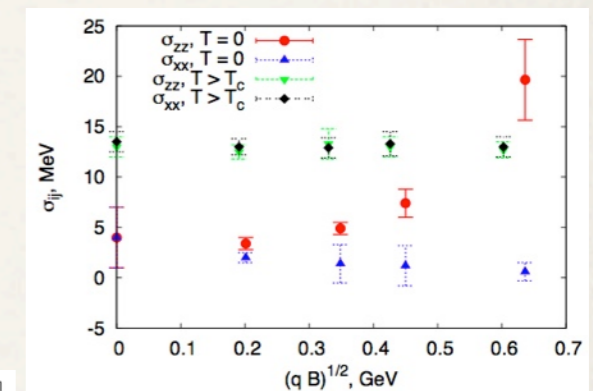
D'Ellia (Poster) arXiv:1005.5365

$N_F = 2$ staggered QCD + uniform background B (quantized)

=> T_c increases; the trans. becomes sharper

Kalaydzhan (Mon)

Lüscher-Weisz + Overlap => chiral condensation



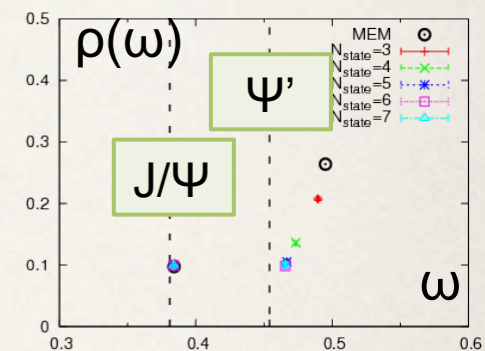
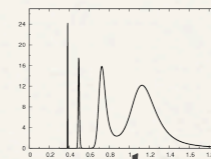
Nature of the QGP phase

★ Charmonia spectral fn's [Ding, Nonaka \(Tue\)](#) see also Oktay-Skullerud 1005.1209

★ Spectral fn's from variational method [Ohno \(Tue\)](#)

=> ground state: mass and area consistent with MEM

=> 1st excited state: improvement by adding smeared operators



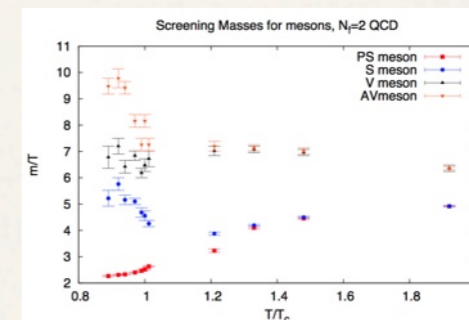
★ Dilepton rates, electric conductivity [Karsch \(Tue\), Francis \(Poster\)](#)

★ Scalar meson above T_c [Banerjee \(Fri\)](#)

$N_F=2$ staggered

Scalar show chiral restor. only above $1.33 T_c$

=> scalar mesons does not decay up to $\approx 1.3 T_c$



★ Hadronic correlation functions [Allton \(Fri\), Loan \(Poster\)](#)

★ Transport coeff's [Khono \(Fri\), Maezawa \(Poster\)](#)

Nature of the QGP phase

★ Chiral transition as Anderson localization?

García-García, Osborn, PRD75('07)

Quenched (1-loop Symanzik gauge) $(16,20)^3 \times 4$; $N_F=2+1$ (1-loop Symanzik gauge + asqtad) $(12,16)^3 \times 4$

\Rightarrow Chiral trans. \approx metal-insulator trans. driven by Anderson localization

\Leftarrow distribution of spectra and spatial sizes of low-lying eigenstates of Dirac operator at $\sim T_c$

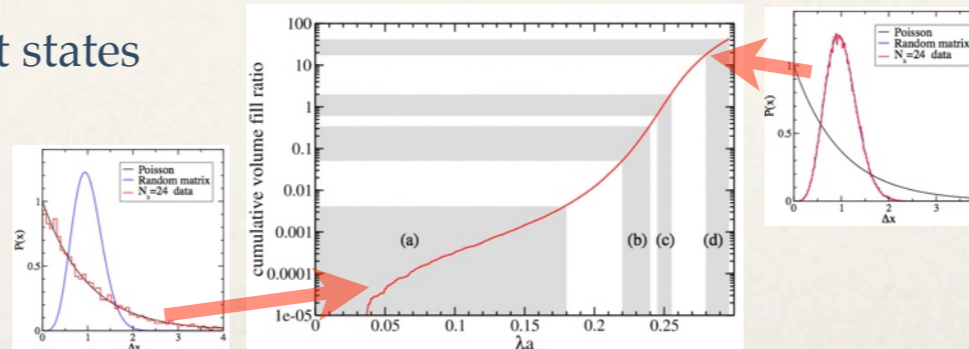
Gavai et al. PRD77('08) $N_F=2$ (plaquette gauge + staggered) $(8-24)^3 \times 4$

\Rightarrow Localization of low-lying eigenstates is a finite volume artifact.

Kovács (Thu)

Quenched $SU(2)_c$ $(24-32)^3 \times 4 \Rightarrow$ In favor of Anderson localization picture

localized independent states
Poisson distribution



spatially overlapping states
RMT statistics

Around the heavy quark limit

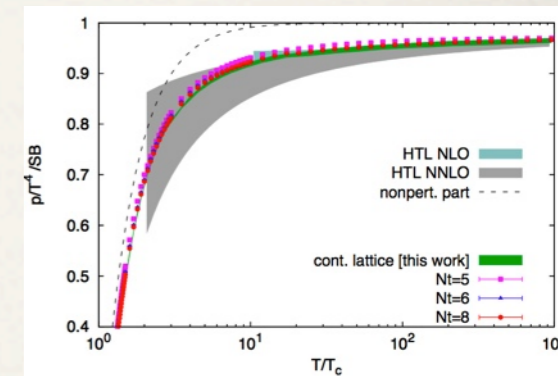
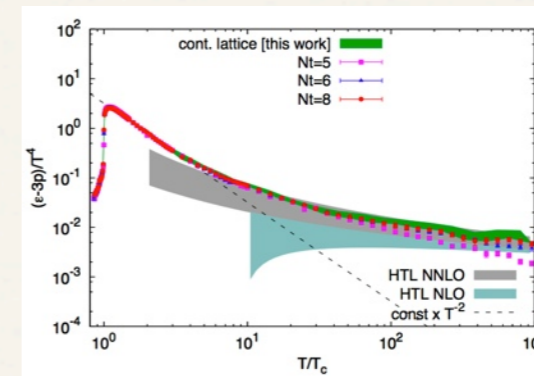
★ SU(3) YM 1st order deconf. trans. \approx Z(3) Potts

Danzer (Thu) Gattringer PLB690('10)

\Rightarrow percolating Z(3) clusters at $T > T_c$

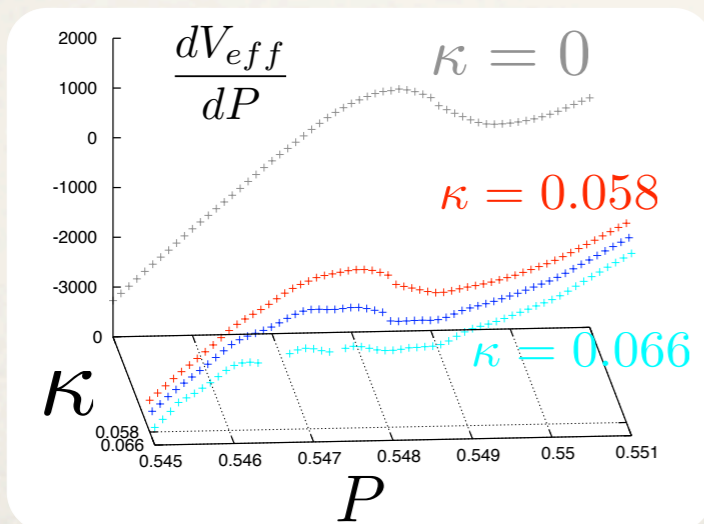
Borsányi (Thu)

\Rightarrow continuum extrapolation of EOS at high T

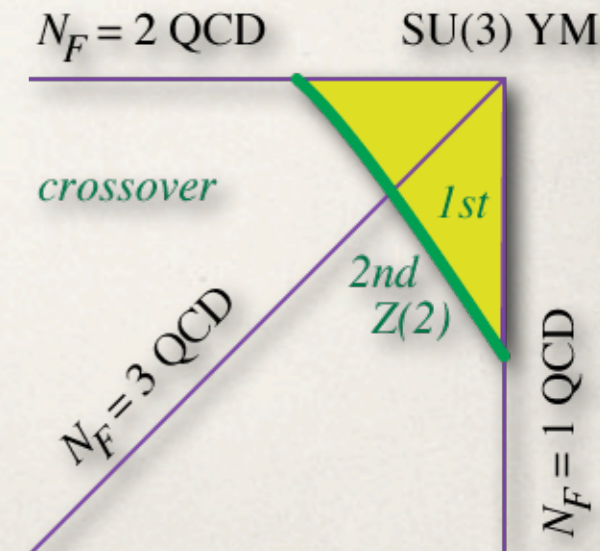


★ With heavy quarks

H. Saito (Tue)



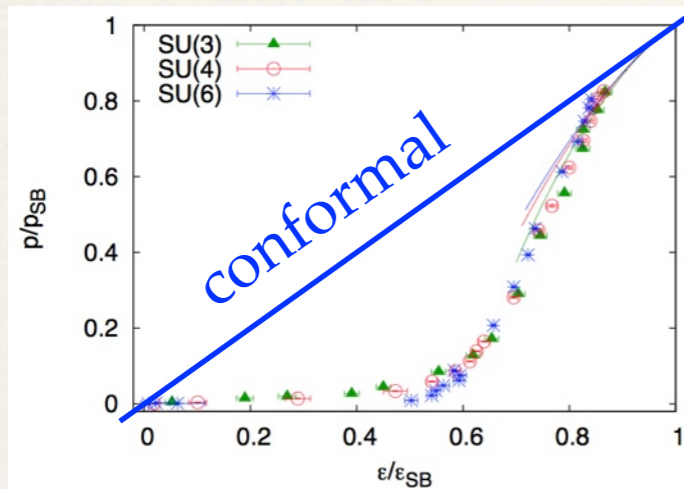
- ▶ plaquette gauge + Wilson, $N_t=4$
- ▶ effective potential + reweighting + hopping param. expansion
- ➔ $\kappa_{EP} = 0.081(8)$ for $N_F = 1$
- $0.068(7)$ $N_F = 2$
- $0.061(6)$ $N_F = 3$



QCD-like theories

★ Varying N_c

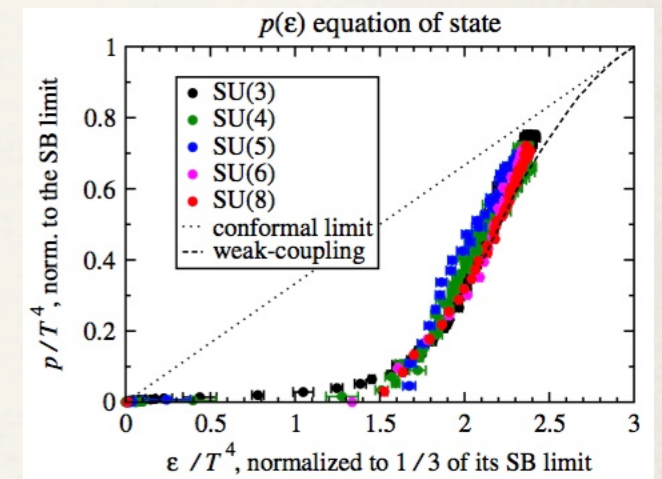
Datta (Thu) $N_c=3,4,6$ YM EOS



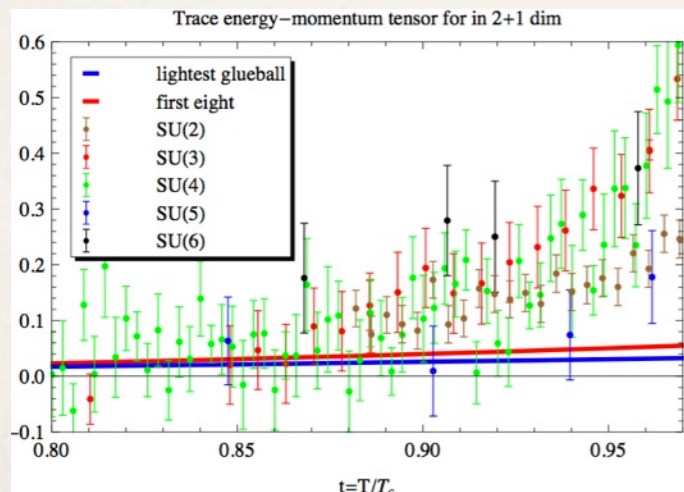
No signs of speculated
“strongly coupled conformality”

Good agreement with a
model of holographic QCD

Panero PRL103('09) $N_c=3-8$



Feo (Thu) $N_c=2-6$, $D=2+1$



Low-lying glueballs do not explain the trace anomaly at $T < T_c$.

See also

Kiskis, Narayanan 0906.3015 $D=3$ string tens.,
Langfeld et al. 0906.5554 even N_c ,
Jenkins et al. 0907.0529 Baryon spectrum,
Bringholtz, Sharpe 0906.3538 Adjoint quarks,
etc.

Further issues

★ Polyakov loop effective theories / dual observables

[Langelage, Lottini, Christoforetti \(Tue\), Sakai \(Poster\)](#)

See also Bilgici et al. FewBodySyst.47('10) dressed Polyakov loop,

Nishimura et al. 0911.2696 Adjoint quarks,

Smith 0911.4037 D=3 effective theory,

Fischer et al. 1003.1960 Landau gauge MC + Dyson-Schwinger, quenched SU(2), SU(3)

★ Strong coupling expansion [Miura, Nakano \(Tue\), Ohnishi \(Fri\)](#)

★ Unitary fermi gas [Gurco, Endres, Nicholson, Lee \(Fri\)](#)

★ 3d U(1) at $T > 0$ [Gravina \(Poster\)](#)

★ SU(2) at $T > 0$ on GPU [Bicudo \(Poster\)](#)

★ hadronic strings Bialas et al. 0912.0206, Bakry et al. 1004.0782, Caselle 1004.3875

CONCLUSIONS

❖ Big steps forward with staggered-type quarks

- * The fruitful conflict about T_c (almost) settled
converging towards $\sim 150\text{--}160$ MeV depending on the operators.
- * $O(2)/O(4)$ scaling finally observed
- * $N_F=2+1$ EOS at close to the “physical point” obtained

We have learned an important effect of the taste violation.

By correctly handling it, e.g. by adopting averaged masses, staggered simulations will achieve a quite high precision.

Continuum extrapolation should be done first.

❖ Steady advances with Wilson-type and DW / Overlap quarks

- * DW entering a quantitative level, will open applications beyond QCD
- * $N_F=2+1$ EOS by the fixed-scale approach

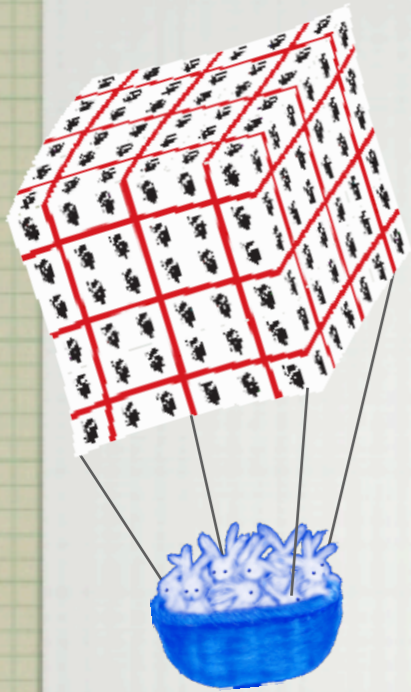
Need to reduce the quark masses towards the physical point.

Important to crosscheck among different quarks.

❖ Many more advances

- * New insights on the QGP matter / high T phase
chiral magnetic effect / charmonium spectral functions / scalar meson / dilepton rate / transport coeff's. / hadronic strings / dual observables / Anderson localization? / phase structure at heavy quarks / large N_c and QCD-like theories / ...
- * New methods
spectral functions from variational method / SC-QCD / ...

Even hotter years will be coming.



I thank D. Banerjee, A. Bazavov, P. Bicudo, S. Borsányi, F. Bruckmann, M. Cheng, M. Chernodub, S. Datta, M. D’Ella, C. DeTar, A. Feo, Z. Fodor, C. Gattlinger, F. Karsch, T. Kovacs, A. Maas, M. Panero, O. Philipsen, K. Szabo, G. Schierholz, W. Soeldner for sending me news/materials.

THANK YOU

I apologize anyone I missed / I could not mention sufficiently / correctly.

