Exact lattice supersymmetry and AdS/CFT

Simon Catterall

Syracuse University

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Incomplete list of recent work

- $\mathcal{N} = 1$ SYM - Wilson: Demmouche at al. arXiv:1003.2073
- $\mathcal{N} = 1$ SYM - DWF: Endres arXiv:0902.4267, Giedt et al. arXiv:0810.5746
- Wess-Zumino model using overlap fermions, Giedt et al. arXiv:1005.3276
- 2D $\mathcal{N} = (2, 2)$ SQCD using GW fermions, Sugino, Kikukawa et al. arXiv:0903.5398,0811.0916,0807.2683
- Momentum space approach to lattice $\mathcal{N} = 2$ QM with full SUSY D'Adda et al. arXiv:1006.2046
- $\mathcal{N} = 4$ SYM using SUSY lattices: Catterall et al.arXiv:0903.4881,0909.4947,0811.1203,0803.4273

Note: numerical studies using non lattice methods discussed last year by Nishimura.

Motivation for lattice SUSY

- Rigorous definition of SUSY QFT - like lattice QCD.
- Understand dynamical SUSY breaking. SQCD - predicting soft terms in MSSM ...
- AdS/CFT: Connection between strongly coupled Yang-Mills and (quantum) gravity. Lattice allows:
  - Strong coupling calculations
  - Monte Carlo simulations
  - Geometry from gauge theory ?

Lattice SUSY - problems

- SUSY extends Poincaré – broken by discretization. $\{Q, \overline{Q}\} = \gamma.p$. No p.
- Folklore: Impossible to put SUSY on lattice exactly.
- Leads to (very) difficult fine tuning – lots of relevant SUSY breaking counterterms...

Way out?

Options

- Let SUSY emerge as accidental symmetry in continuum limit
  - Limited possibilities: eg. $\mathcal{N} = 1$ SYM: Only relevant SUSY breaking op. is gaugino mass
  - Tune bare mass (Wilson) or protect with exact chiral symmetry (DWF) - SUSY in chiral continuum limit.
  - SQCD: Using GW fermions with fine tuning of scalar sector (Giedt et al.). Hard - future ...

Or preserve some SUSY exactly in lattice theory

Exact SUSY - new ideas

- Topological twisting (S.C, F. Sugino, N. Kawamoto, A. d'Adda, P. Damgaard, S. Matsuura, J. Giedt, ..)
- Orbifolding/deconstruction (D. B. Kaplan, M. Ünsal, A. Cohen, P.Damgaard, S. Matsuura, J. Giedt, ...)

Two approaches produce identical lattice theories!

Phys Rep. 484:71-130,2009, arXiv:0903.4881

- Focus on former. Emphasizes geometry.
- Warning: Approaches work only if $\mathcal{Q}$ multiple of 2^D

In $D = 4$ unique theory: $\mathcal{N} = 4$ SYM

Twisted $\mathcal{N}=4$ lattice theory

Usual fields	Twisted fields
$A_\mu, \mu = 1\ldots 4 \quad \phi_i, i = 1\ldots 6$	$\mathcal{U}_a, a = 1\ldots 5$
$\Psi^i, i = 1\ldots 4$	$\eta, \psi_a, \chi_{ab}, a, b = 1\ldots 5$

- Fields live on links of A_4^* lattice - 5 basis vectors correspond to vectors from center of hypertetrahedron to vertices.
  - $\eta \quad x \to x$
  - $\psi_a \quad x \to x + \mu_a$
  - $\chi_{ab} \quad x + \mu_a + \mu_b \to x$
- All fields transform like links: $X_p \to G(x)\, X_p\, G^\dagger(x+p)$
- Single exact lattice SUSY $\mathcal{Q}^2 = 0$

Where did this come from ?

- Basic idea: twisted theory formulated in terms of fields transforming under $G = \mathrm{Diag}(SO_{\mathrm{Lorentz}}(4) \times SO_{\mathrm{R}}(4))$
- Point group symmetry A_4^* subgroup twisted rotations G.
- Fermions: spinors under both factors – become integer spin after twisting.
- Scalars transform as vectors under R-symmetry – vectors after twisting.
- Gauge fields remain vectors – combine with scalars to make complex gauge fields. $\mathcal{U} = e^{A+i\phi}$
- Still only $U(N)$ gauge symmetry.

Important: flat space: just a change of variable

Lattice SUSY

Important: twisted action takes $\mathcal{Q}$-exact form $S=\mathcal{Q}\Lambda$ (+$\mathcal{Q}$-closed)

$$
\begin{aligned}
\mathcal{Q}\,\mathcal{U}_a &= \psi_a \\
\mathcal{Q}\,\psi_a &= 0 \\
\mathcal{Q}\,\overline{\mathcal{U}}_a &= 0 \\
\mathcal{Q}\,\chi_{ab} &= -\overline{\mathcal{F}}_{ab} \\
\mathcal{Q}\,\eta &= d \\
\mathcal{Q}\,d &= 0
\end{aligned}
$$

Note: complexified gauge link: $\mathcal{U}_a(x)$
field strength: $\mathcal{F}_{ab}(x)=\mathcal{U}_a(x)\mathcal{U}_b(x+a)-\mathcal{U}_b(x)\mathcal{U}_a(x+b)$

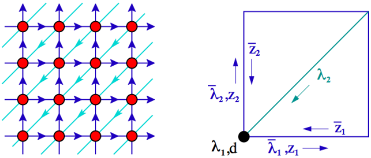
Lattice action

$$S = \beta(S_1 + S_2)$$

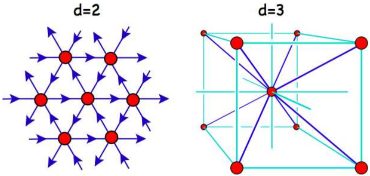
$$\begin{aligned} S_1 &= \sum_{\mathbf{x}} \mathrm{Tr} \left(\alpha_1 \mathcal{F}_{ab}^{\dagger} \mathcal{F}_{ab} + \frac{\alpha_2^2}{2\alpha_1} \left(\overline{\mathcal{D}}_a^{(-)} \mathcal{U}_a \right)^2 \right. \\ &- \left. \alpha_1 \chi_{ab} \mathcal{D}_{[a}^{(+)} \psi_{b]} - \alpha_2 \eta \overline{\mathcal{D}}_a^{(-)} \psi_a \right) \\ S_2 &= -\frac{\alpha_3}{8} \sum_{\mathbf{x}} \mathrm{Tr}\, \epsilon_{abcde} \chi_{de}(\mathbf{x} + \mu_a + \mu_b + \mu_c) \overline{\mathcal{D}}_c^{(-)} \chi(\mathbf{x} + \mu_c) \end{aligned}$$

- Bosonic action is just Wilson plaquette if $\mathcal{U}_a^{\dagger} \mathcal{U}_a = 1$. Fermions: Kähler-Dirac action - no doublers.
- Well defined prescription for all difference ops.
- Classically $\alpha_i = 1$.

More supersymmetric lattices



$\mathcal{Q} = 2 \; D = 2$



$\mathcal{Q} = 8 \; D = 2, 3$

Outstanding questions

Lattice theories are:

local, gauge invariant, doubler free and invariant under one SUSY

Two questions:

- Is rotational symmetry restored as $a \to 0$?
- What about restoration of full SUSY ?

Must understand how lattice theory renormalizes ...

Two approaches

- Examine at 1-loop using p. theory
- Attempt a non-perturbative tuning by measuring broken SUSY Ward identities

Renormalization

Lattice symmetries:

- Gauge invariance
- $\mathcal{Q}$-symmetry.
- Point group symmetry - eg. S^5 for A_4^* - subgroup of $SO(4)'$
- Exact fermionic shift symmetry $\eta \to \eta + \epsilon I$

Conclusion:

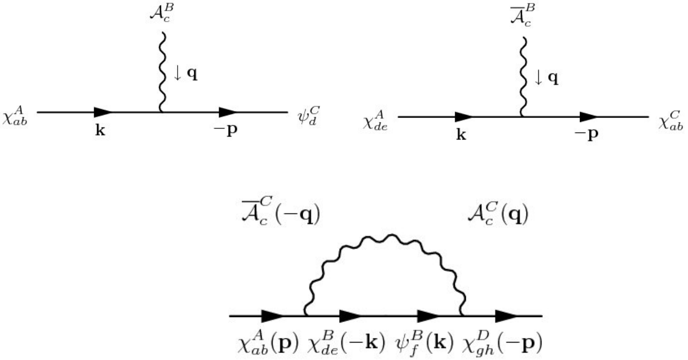
- Twisted $SO(4)'$ restored in continuum limit.
- No mass terms can appear to all orders in perturbation theory! (Topological argument based on exact $\mathcal{Q}$)
- Power counting: only relevant ops that can arise correspond to log corrections to $\alpha_i, i = 1 \dots 3$.

Ingredients for perturbation theory

Lattice rules for A_4^* lattice (Feynman gauge):

- Boson propagator $< \overline{\mathcal{A}}_a^C(k)\mathcal{A}_b^D(-k) >= \frac{1}{\hat{k}^2}\delta_{ab}\delta^{CD}$ with $\hat{k}^2 = 4\sum_a \sin^2(k_a/2)$
- Fermion propagator $M^{-1}_{\mathrm{KD}}(k) = \frac{1}{\hat{k}^2}M_{\mathrm{KD}}(k)$ with $M(k)$ a 16×16 block matrix acting on $(\eta, \psi_a, \chi_{ab})$
- Vertices: $\psi\eta$, $\psi\chi$ and $\chi\chi$.
- Only 4 Feynman graphs needed to find 1-loop contributions to α_i. No tadpoles

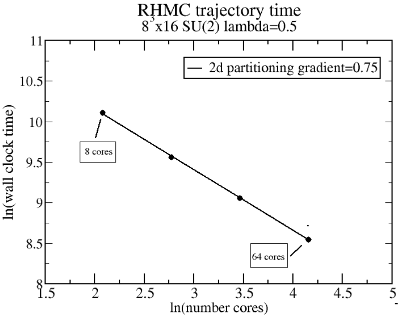
Example: chi-chi propagator



Calc. underway - results soon ... (with Giedt, Joseph, Dzeimkowski)

Simulations

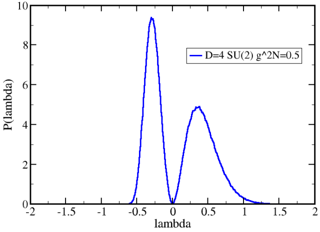
- Parallized, multiple time step RHMC code developed.
- Uses MDP libraries within FermiQCD for communication.



with J. Schneible

Divergent path integral ?

Infinite number classical vacua corresponding to constant diagonal matrices



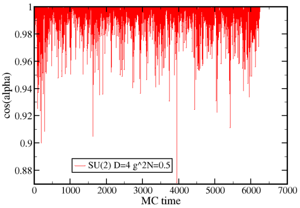
$SU(2)$ $P(\lambda)$ vs λ

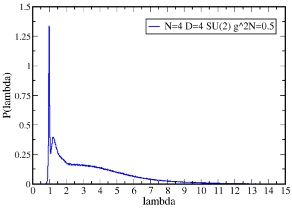
no divergence scalars localized close to origin with power law tails

caveat : still need to introduce mass for scalar trace mode

Sign problem ?

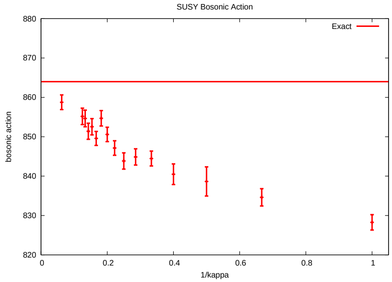
- Observed no significant sign problem with small volume simulations over most parameter range.
- In the case of periodic bcs we understand this: $< e^{i\alpha(\mathrm{Pf}(M))} >_{\text{phase quenched}} = Z_{\text{unquenched}}$ But Z is a topological invariant - can be computed exactly at 1-loop where one finds $< e^{i\alpha} >= 1$.





Bosonic Action

SUSY predicts: $\kappa S_B = \frac{9}{2}(N^2 - 1)V$



$2^3 \times 8$ lattice

κ	κS_B	exact
1.0	13.67(4)	13.5
10.0	13.52(2)	13.5
100.0	13.48(2)	13.5

$D = 0 \; SU(2)$

Gauge-gravity dualities

Original AdS/CFT correspondence:

Quantum $\mathcal{N} = 4$ YM dual Semiclassical strings +D3-branes

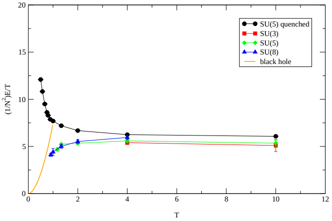
Many other examples eg. Low temperature thermodyamics of dimensionally reduced theory

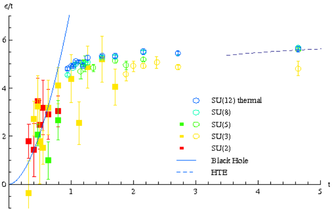
$\mathcal{N} = 4$ SYM in $D = (p + 1)$ and Semiclassical black Dp-branes

Explore using lattice actions ...

Black holes from YM

$p = 0$ case: black holes in type II SUGRA – dual to large N low T $\mathcal{N} = 4$ on circle (with T .Wiseman, Imperial)





Energy vs temperature for SYMQM system+BH prediction using semiclassical Bekenstein-Hawking.

Single deconfined phase.

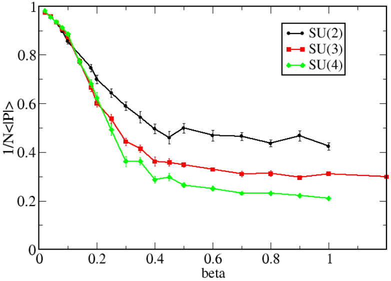
Black Strings

$p = 1$ case: black string in type II SUGRA – dual to large N $\mathcal{N} = 4$ on 2D torus

- New possibility: If event horizon of black hole wraps spatial circle – black hole less stable than black string – Gregory-LaFlamme gravitational transition
- In dual gauge theory see thermal phase transition associated with spatial Polyakov line.

work with A. Joseph and T. Wiseman

Black hole-black string phase transition



$\frac{1}{N}|TrP|$ versus inverse temperature β on 2×8 lattice.
Small β phase - 5D black hole
Large β phase - 5D black string.

Prospects

- Exciting time for lattice SUSY - much activity, many developments. Plenty of opportunities; BSM physics for LHC, connections to strings.
- Lattice actions retaining some exact SUSY possible. Describe topologically twisted SYM continuum limit.
- Possibility for nonperturbative exploration $\mathcal{N} = 4$ YM. Tests of AdS/CFT.
- Dimensional reductions – duality between strings with Dp-branes and $(p+1)$-SYM eg. thermal D0/D1/D2 branes
- For $D = 4$ theory Most pressing question – what residual (fine) tuning needed to get full SUSY as $a \to 0$?