

Hadron spectrum and light pseudoscalar decay constants

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Lattice 2010, Villasimius
June 14 2010



Outline

This review will cover recent results on

- Light quark QCD spectrum
 - Ground states
 - Excited states
- Decay constants f_π and f_K

I will discuss where we stand with respect to

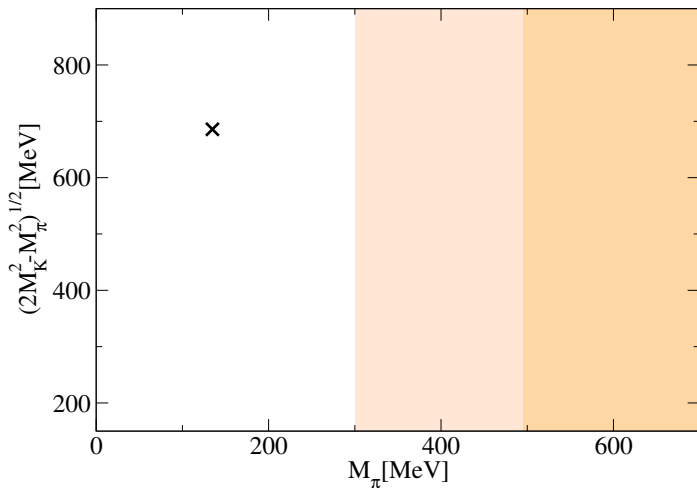
- Reaching the physical point
- Taking the continuum limit
- Taking the infinite volume limit

I will discuss what needs to be done to further increase accuracy

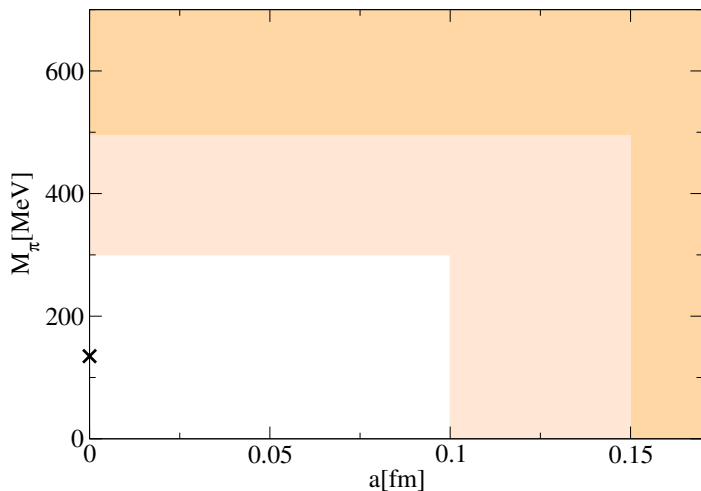
Light flavors: 🗨️ J. Laiho (Sat)

Heavy flavors: 🗨️ J. Heitger (Fri)

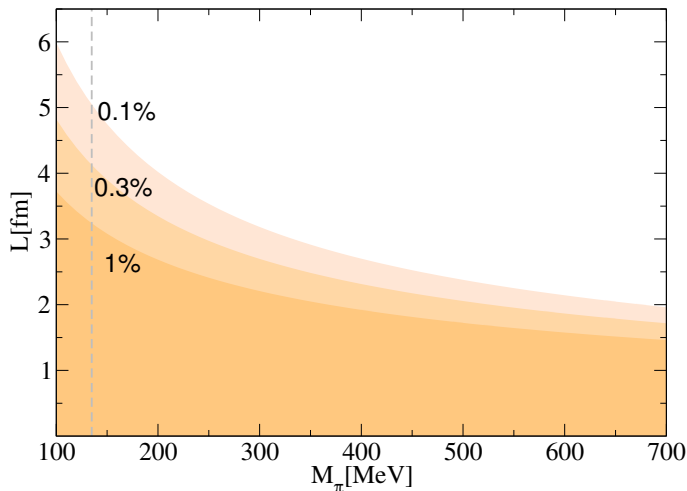
Reaching the physical point



Taking the continuum limit



Taking the infinite volume limit

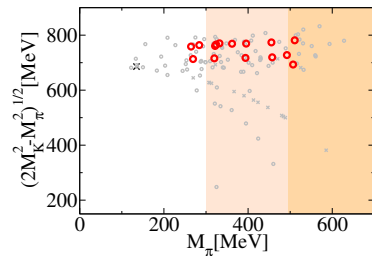
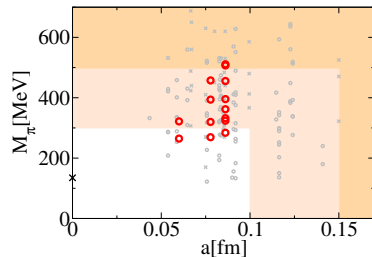
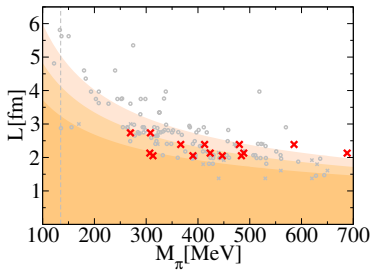


Error on m_π from χ PT (Collangelo, Durr, Haefeli, '05)

ETMC 2+1+1 Flavor (Baron et. al. '10)

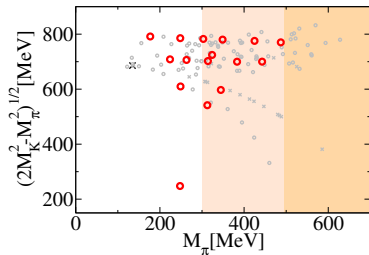
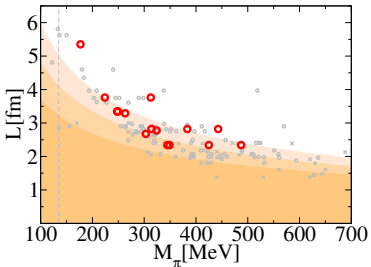
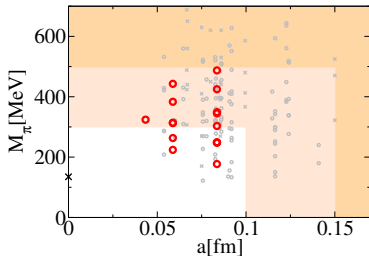
Gauge	Ferm.	N_f
Iwasaki	TM	2+1+1
$a \rightarrow 0$	χ	FV
3	✓	✗

✉ S. Reker (Mon) ✓

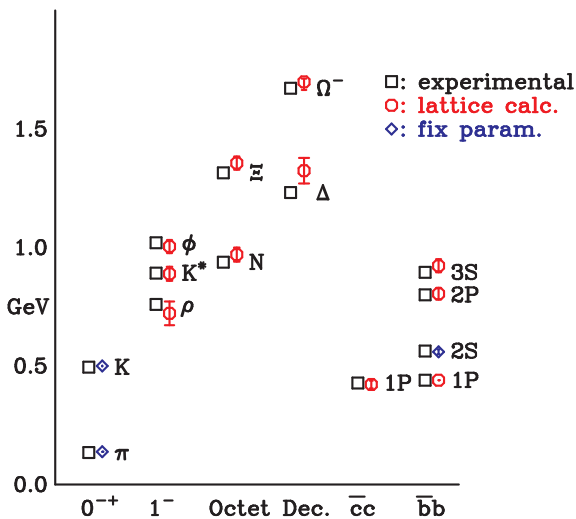


MILC (Bazavov et. al. '09)

Gauge		Ferm.		N_f	
Symanzik		Asqtad		2+1	
$a \rightarrow 0$	χ	FV	FV Res	Taste	
3	✓	✓	✗	✓	



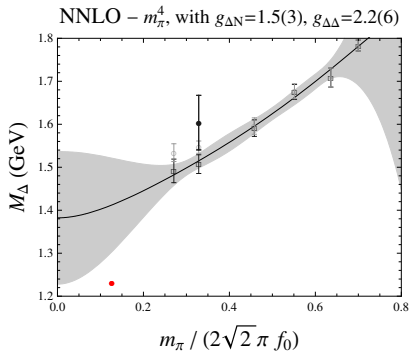
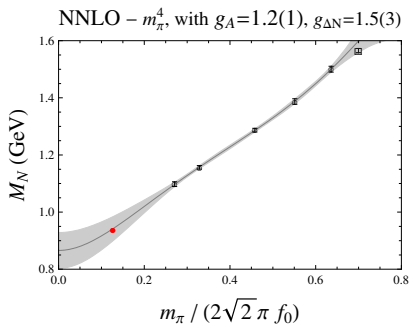
MILC Baryon Spectrum



η' would be interesting for conceptual reasons (check staggered anomaly)

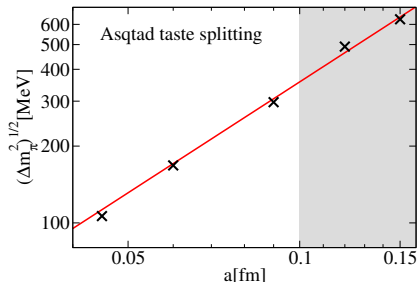
LHP Baryon Spectrum (Walker-Loud et. al. '09)

- DW on Asqtad, $a \approx 0.12\text{fm}$ ensemble
- SU(2)/SU(3) HB χ PT, FV corrections



MILC Decay Constants Setup

- Partially quenched $N_f = 2 + 1$ ensembles
- Cascaded fits:
 - Fix SU(3) LECs with $m < 0.6m_s^{\text{phys}}$ (N²LO)
 - Use LECs and add analytic N³LO, N⁴LO to fit full set
- Crosscheck with SU(2) for light quark observables (up to N²LO)
- Scale set by r_1
- Taste splitting $\sqrt{\Delta m_\pi^2} < m_\pi^{\text{phys}}$.
for finest lattices



MILC Decay Constants

- $f_\pi = 129.2(4)(1.4)\text{MeV SU}(3)$

$$f_\pi = 130.2(1.4) \begin{pmatrix} 2.0 \\ 1.6 \end{pmatrix} \text{MeV SU}(2)$$

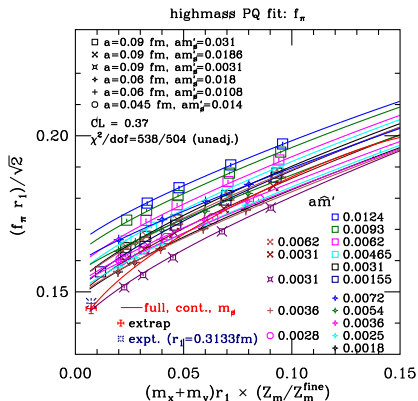
- $f_K = 156.1(4) \begin{pmatrix} 2.0 \\ 1.6 \end{pmatrix} \text{MeV}$

- $f_K/f_\pi = 1.197(2) \begin{pmatrix} 3 \\ 7 \end{pmatrix}$

- $f_3 = 118.0(3.6)(4.6)\text{MeV}$

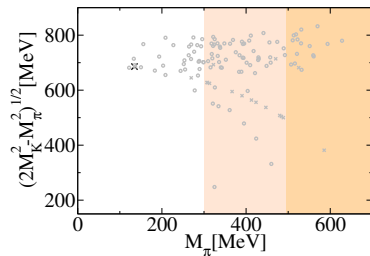
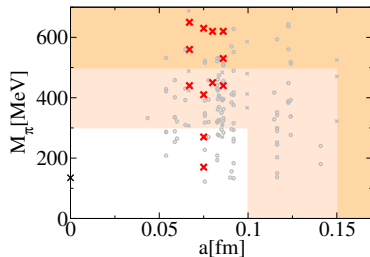
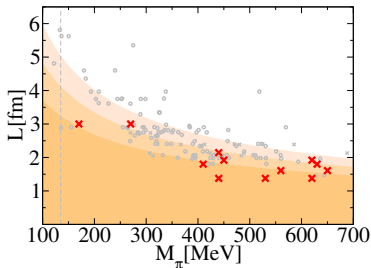
- $f_2 = 123.0(5)(7)\text{MeV}$

$$f_2 = 123.8(1.4) \begin{pmatrix} 1.0 \\ 3.7 \end{pmatrix} \text{MeV}$$



QCDSF 2 Flavor

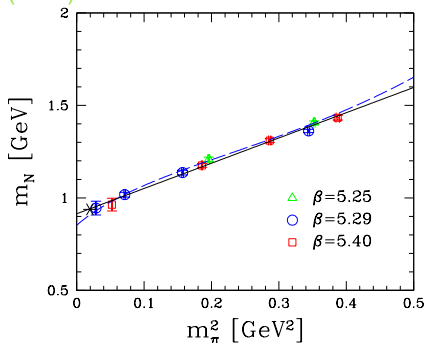
Gauge		Ferm.		N_f
Symanzik		SLiNC		2
$a \rightarrow 0$	χ	FV	FV Res	
3	✓	✓	✓	



QCDSF $N_f = 2$ Baryon Spectrum

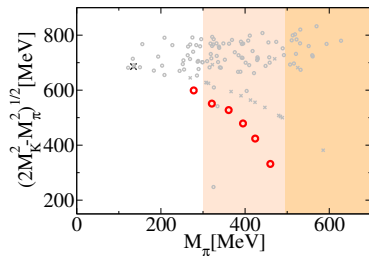
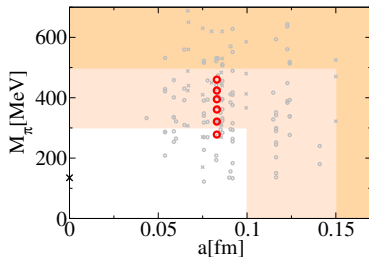
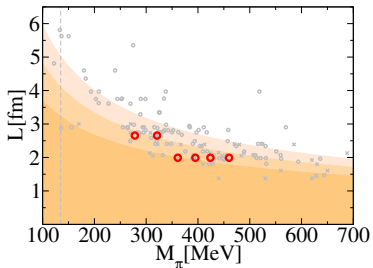
- Minimal discretization effects
- Best fit of nucleon $\propto m_\pi^2$

👉 G. Schierholz (Mon)



QCDSF 2+1 Flavor (Bietenholz et. al. '10)

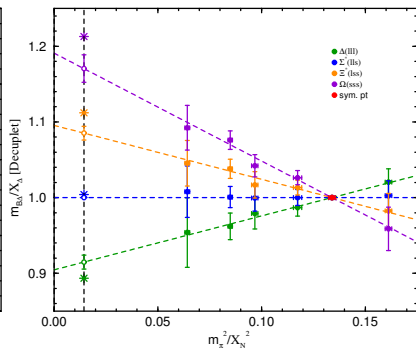
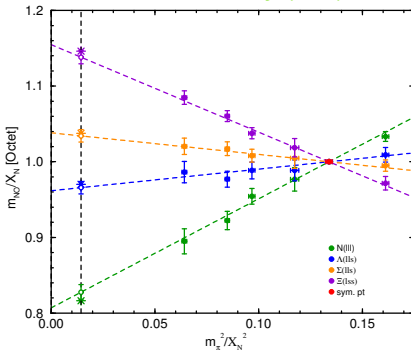
Gauge		Ferm.		N_f
Symanzik		SLiNC		2+1
$a \rightarrow 0$	χ	FV	FV Res	
✗	✓	✗	✗	



QCDSF Baryon Spectrum

- χ limit with fixed singlet quark mass (LO): $2m_K^2 + m_\pi^2 = \text{const}$
- Gell-Mann Okubo linear fit

👤 P. Rakow, R. Horsley (Tue)



QCDSF Decay Constants

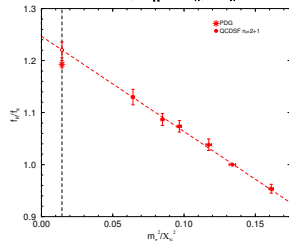
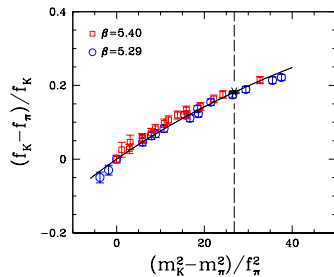
- $N_f = 2$
- Scale set by m_N
- Finite volume, continuum
- Preliminary:

$$f_K/f_\pi = 1.222(6)$$

✉ G. Schierholz (Mon)

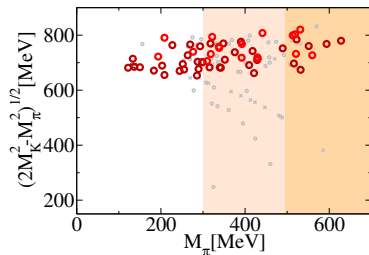
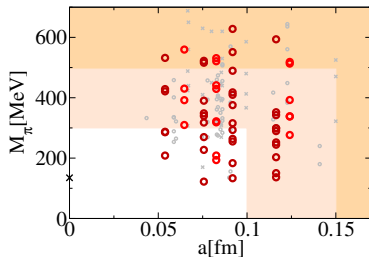
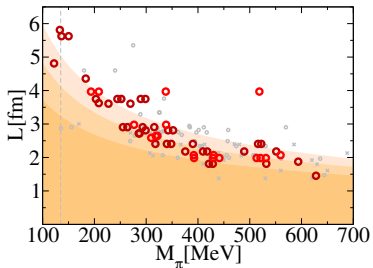
- $N_f = 2 + 1$
- Scale set by $X_N \equiv m_{\text{Octet}}$
- No systematics yet
- Preliminary:

$$f_K/f_\pi = 1.221(15)$$



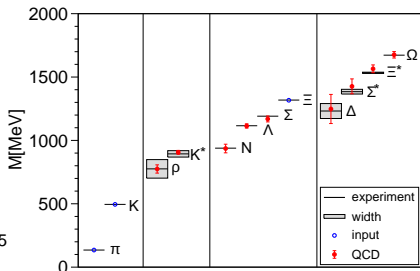
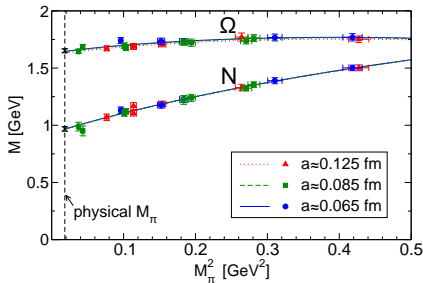
BMW (Durr et.al. '08, Durr et.al. '10)

Gauge		Ferm.		N_f
Symanzik		Clover		2+1
$a \rightarrow 0$	χ	FV	FV Res	
3	✓	✓	✓	



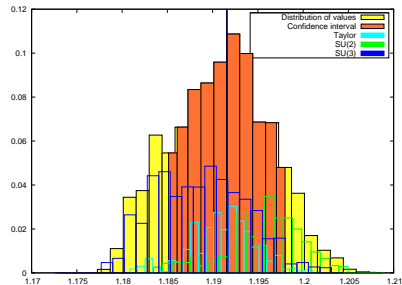
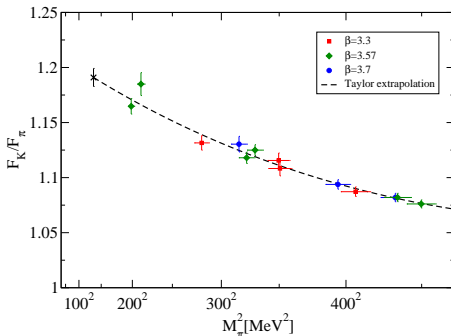
BMW Baryon Spectrum (Durr et.al. '08)

- 3 lattice spacings for continuum extrapolation
- Non-relativistic heavy baryon χ PT and Taylor χ fits
- Resonances: ground state FV energy shift



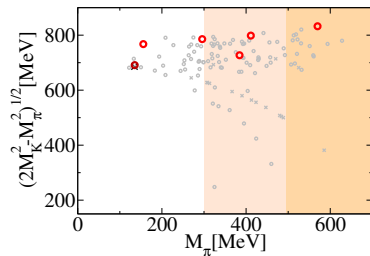
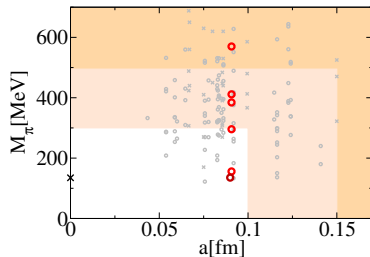
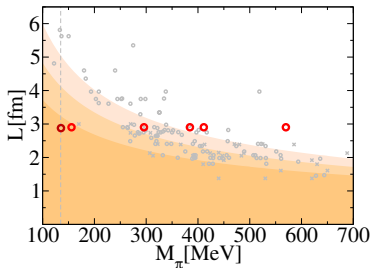
BMW Decay Constants (Durr et.al. '10)

- SU(2), SU(3) and Taylor χ fits
- Full error analysis
- $f_K/f_\pi=1.192(7)(6)$



PACS-CS (Aoki et. al. '09, Aoki et. al. '10)

Gauge	Ferm.		N_f
Iwasaki	Clover		2+1
$a \rightarrow 0$	χ	FV	FV Res
✗	✓	✗	✗



PACS-CS Baryon Spectrum and Decay Constants

- Reweighted to physical point

- 1 lattice spacing

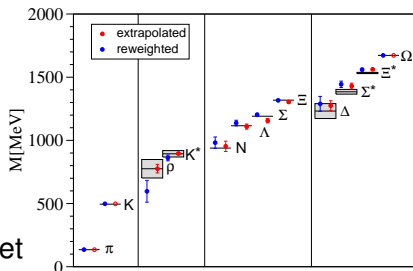
- $m_\pi L \sim 1.97$

- No treatment of resonant states yet

- Still good agreement with resonance spectrum

- Small V: minimum momentum $p_{\min} \equiv \frac{2\pi}{L} > m_\rho^{\text{phys}}/2$

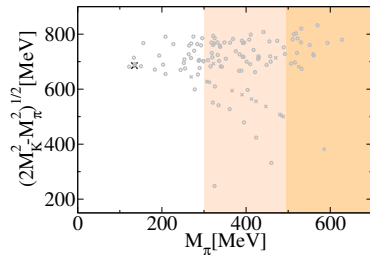
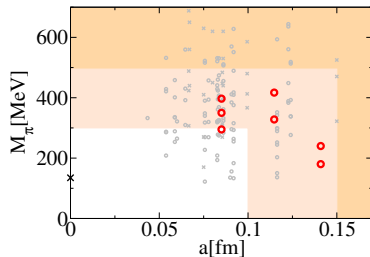
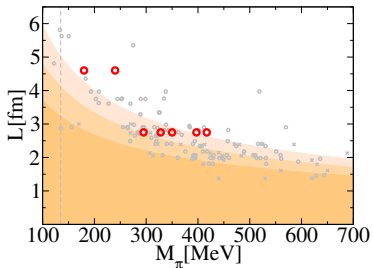
- Decay constants:



	reweighted	extrapolated
f_π [MeV]	124.1(8.5)(0.8)	134.0(4.2)
f_K [MeV]	165.5(3.4)(1.0)	159.4(3.1)
f_K/f_π	1.333(72)	1.189(20)

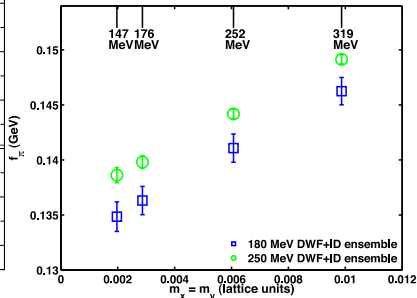
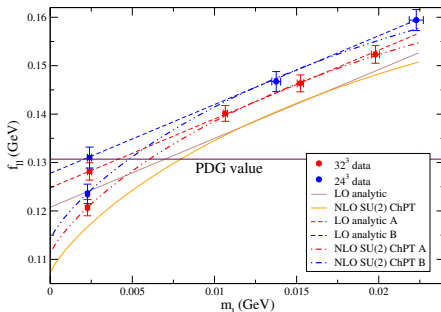
RBC/UKQCD

Gauge	Ferm.		N_f
Iwasaki	DW		2+1
$a \rightarrow 0$	χ	FV	FV Res
3	✓	✓	✗



RBC/UKQCD Decay Constants C. Kelly (Tue)

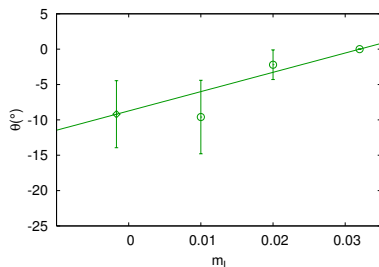
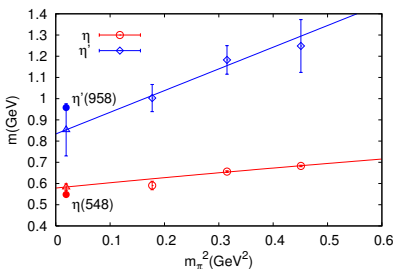
- Continuum, FV, Ω scale
- SU(2) and Taylor fits
- (Prelim. DSDR gauge)
- $f_\pi = 122(2)(5)_\chi(2)_{FV} \text{ MeV}$
- $f_K = 147(2)(4)_\chi(1)_{FV} \text{ MeV}$
- $f_K/f_\pi = 1.208(8)(23)_\chi(14)_{FV}$



RBC/UKQCD η and η'

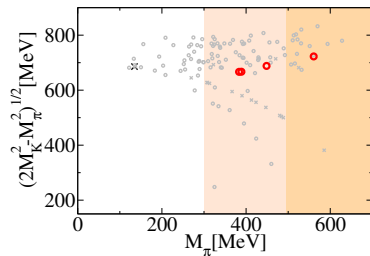
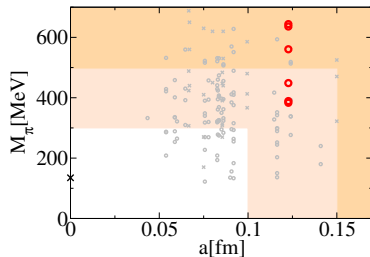
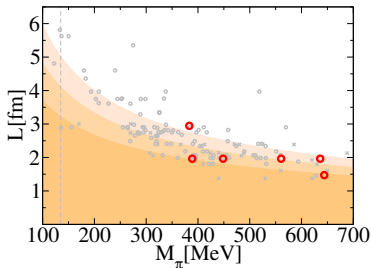
- $a \approx 0.11\text{fm}$, $m_\pi \sim 400 - 700\text{MeV}$
- 2-state operators, compute masses and mixing angle

	m_η [MeV]	$m_{\eta'}$ [MeV]	θ [°]
RBC/UKQCD	583(15)	853(123)	-9.2(4.7)
Expt.	548	958	



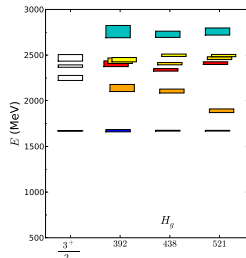
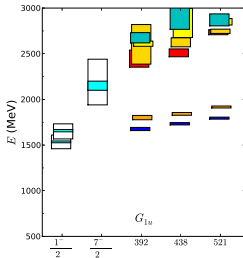
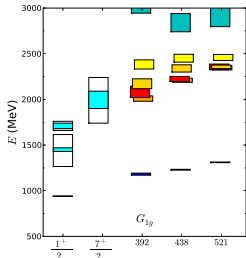
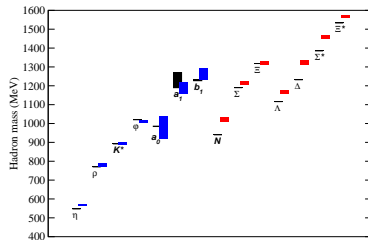
Hadron Spectrum Collaboration (Bulava et. al. '10)

Gauge		Ferm.		N_f
Symanzik		Aniso Clover		2+1
$a \rightarrow 0$	χ	FV	FV Res	
X	X	X	X	



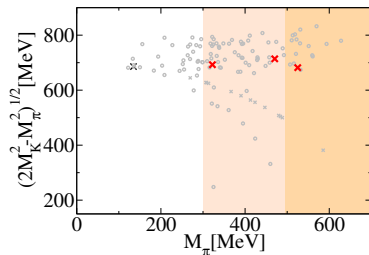
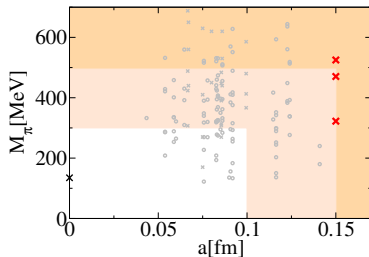
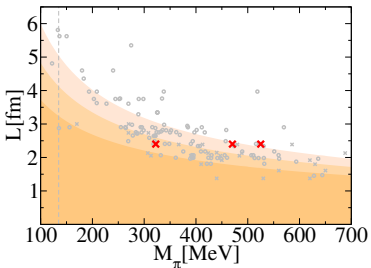
HSC (Excited) Hadron Spectrum

- Anisotropic lattices ($16^3 \times 128$)
- Variational method
- Need for multi-hadron interpolating operators



BGR (Engel et. al. '10)

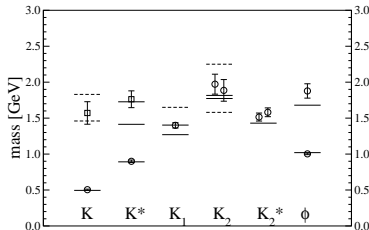
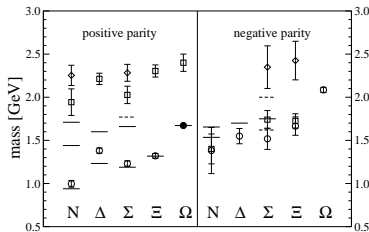
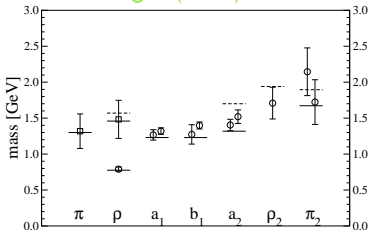
Gauge		Ferm.	N_f
Symanzik		CI	2
$a \rightarrow 0$	χ	FV	FV Res
✗	✓	✗	✗



BGR (Excited) Hadron Spectrum

- Variational method
- Reasonable ground states
- Weak signals for excited and scattering states
- Finite volume effects

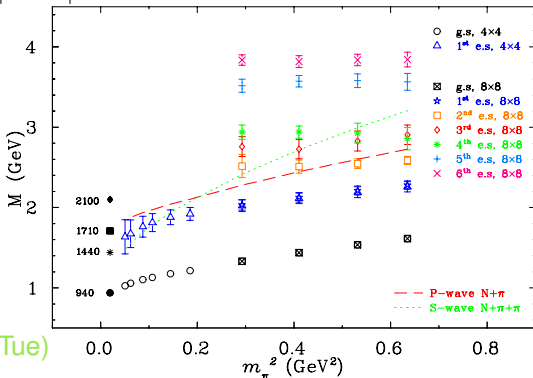
👉 G. Engel (Mon)



CSSM Nucleon Excitations (Mahbub et. al. '10)

Gauge	Ferm.	N_f
DBW2	FLIC	0
$a \rightarrow 0$	χ	FV
X	✓	X
		FV Res
		X

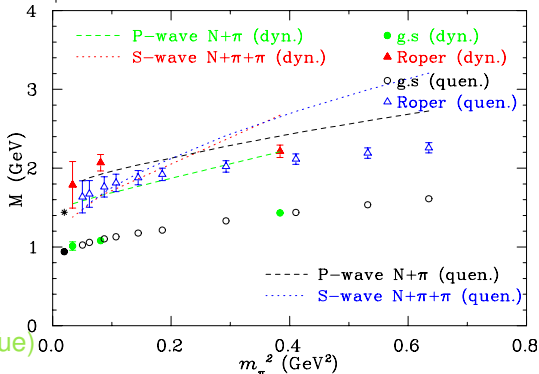
- Variational method (up to 8×8)
- $16^3 \times 32$, $a \sim 0.13\text{fm}$
- Signal for 3 excited states



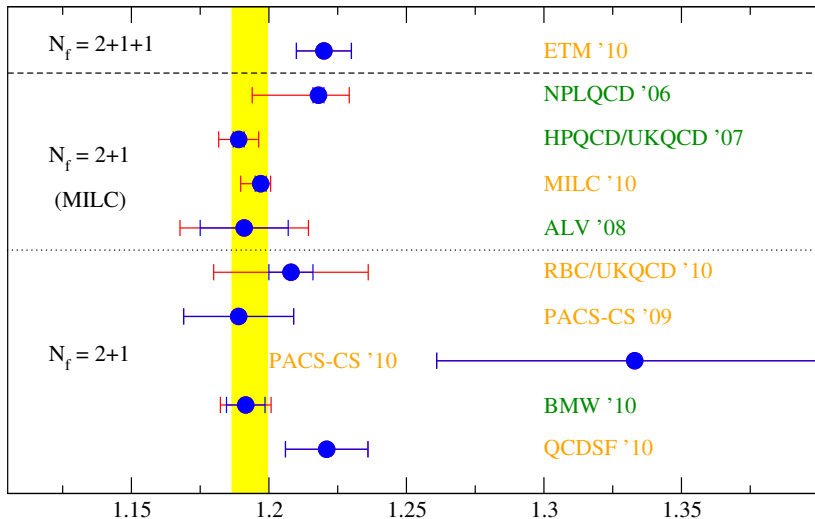
👤 D. Leinweber (Tue)

CSSM Nucleon Excitations $N_f = 2 + 1$

Gauge		Ferm.		N_f
Iwasaki		FLIC on Clover		2+1
$a \rightarrow 0$	χ	FV	FV Res	● PACS-CS ensembles
✗	✓	✗	✗	



✉ D. Leinweber (Tue)

F_K/F_π Summary

Sources of Errors

Currently, the leading systematics for ground state masses and decay constants come from

- Reaching the physical point
- Taking the continuum limit
- Taking the infinite volume limit
- Resonances: FV interaction with scattering states

Subleading:

- QED effects
- Isospin breaking effects

Chiral Extrapolation - Interpolation

We have calculations

- close to the physical point ($m_\pi < 200\text{MeV}$)
 - Clover (QCDSF, PACS-CS, BMW)
 - Staggered (MILC)
 - Domain Wall (RBC)
- reweighted to the physical point (PACS-CS)

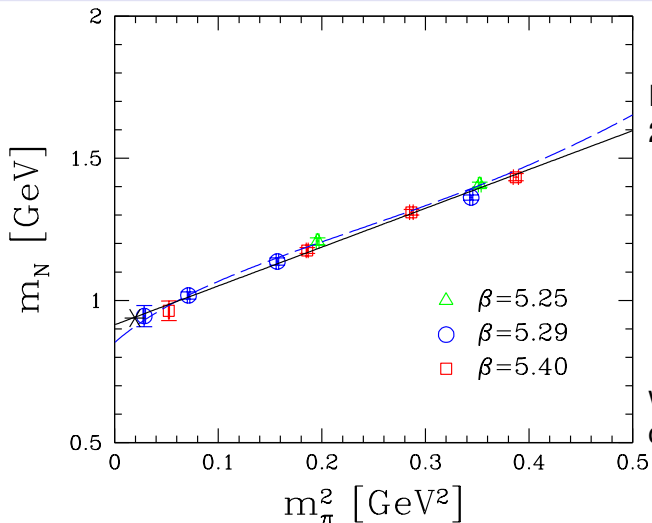
Different extrapolation methods agree

- Extrapolation is tiny
- Taylor expansion and χ PT
 - Order ($N^n\text{LO}$) depends on quality of data / external input

This is an observable dependent statement!



Chiral Extrapolation



Example: QCDSF
2-flavor

which begs the
question...



Finite Volume

The real challenge

- Physical Point
- Infinite volume
- Continuum

In principle, infinite volume is easy for many observables:

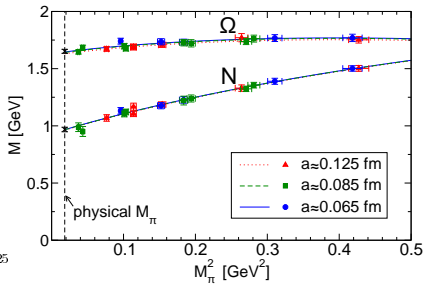
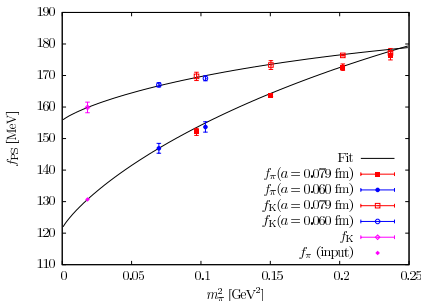
- Leading corrections vanish exponentially in L
- Just need large enough volumes
- Carefull: This is not true for all observables!
 - Resonances: mixing with scattering states
 - FV can be usefull for determining widths etc.
(Lüscher '85-'91)



Continuum Extrapolation

Continuum extrapolation:

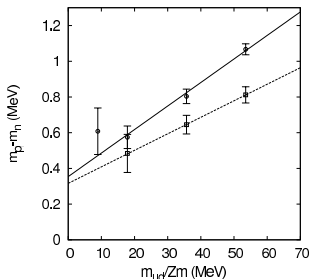
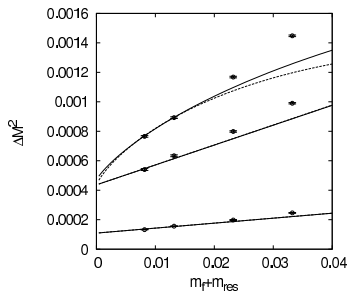
- Mild for ratios of hadron masses and decay constants
- Observable dependent
- Action dependent (interplay with flavor/taste splitting)



still extrapolation needed \rightarrow will be leading systematics

QED Effects (Blum et. al. '10)

- PQ DW, $a \approx 0.11\text{fm}$
- $16^3 \times 32$ and $24^3 \times 32$
- $m_\pi \sim 250 - 400\text{MeV}$
- Quenched, non-compact QED
- NLO χ PT
- $(m_{\pi^+} - m_{\pi^0})_{\text{QED}} = 4.50(23)\text{MeV}$
- $(m_{K^+} - m_{K^0})_{\text{QED}} = 1.33(4)\text{MeV}$
- $(m_n - m_p)_{\text{QED}} = -0.38(7)\text{MeV}$

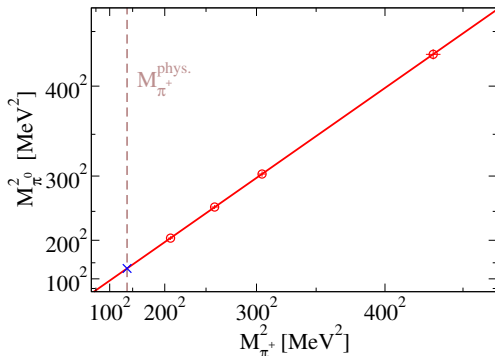




QED Effects (BMW)

- BMW Clover, $a \approx 0.115\text{fm}$
- $m_\pi \sim 200 - 400\text{MeV}$
- Quenched, non-compact QED
- Second order Taylor

- $(m_{\pi^+} - m_{\pi^0})_{\text{QED}} = 4.2(4)\text{MeV}$
- $(m_{K^+} - m_{K^0})_{\text{QED}} = 2.2(2)\text{MeV}$



- 4σ discrepancy with Blum. et. al.

✉ A. Portelli(Thu)

Conclusion

Ground state light Hadron spectrum

- Reproduced to few % accuracy
- On that level: systematics under control

Light decay constants

- Lattice results in good agreement
- F_K/F_π Error competitive with experiment
- Lattice compatible with first-row unitarity

Higher precision:

- Physical point at large volume, more statistics
- Multi-state treatment of resonances

Excited state light Hadron spectrum

- Qualitative agreement
- Improve excited state treatment (scattering states, FV)

Thanks for sharing preliminary results

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BACKUP

Tunneling at $a \approx 0.054\text{fm}$, $m_\pi = 220\text{MeV}$

