

Lattice Quantum Gravity - an Update

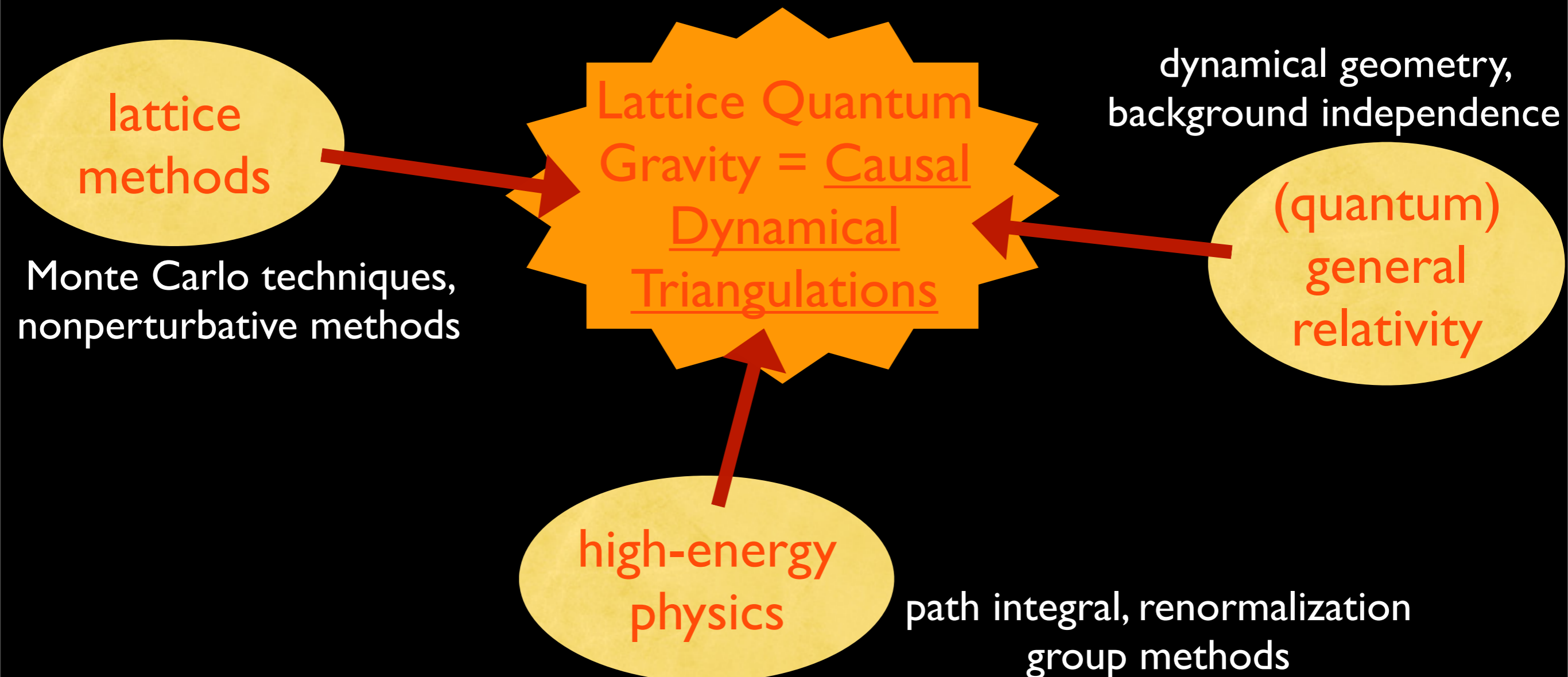
Villasimius, 16 Jun 2010

**Renate Loll, Institute for Theoretical Physics,
Utrecht University**

Update on the many exciting things that have happened since 2001 in

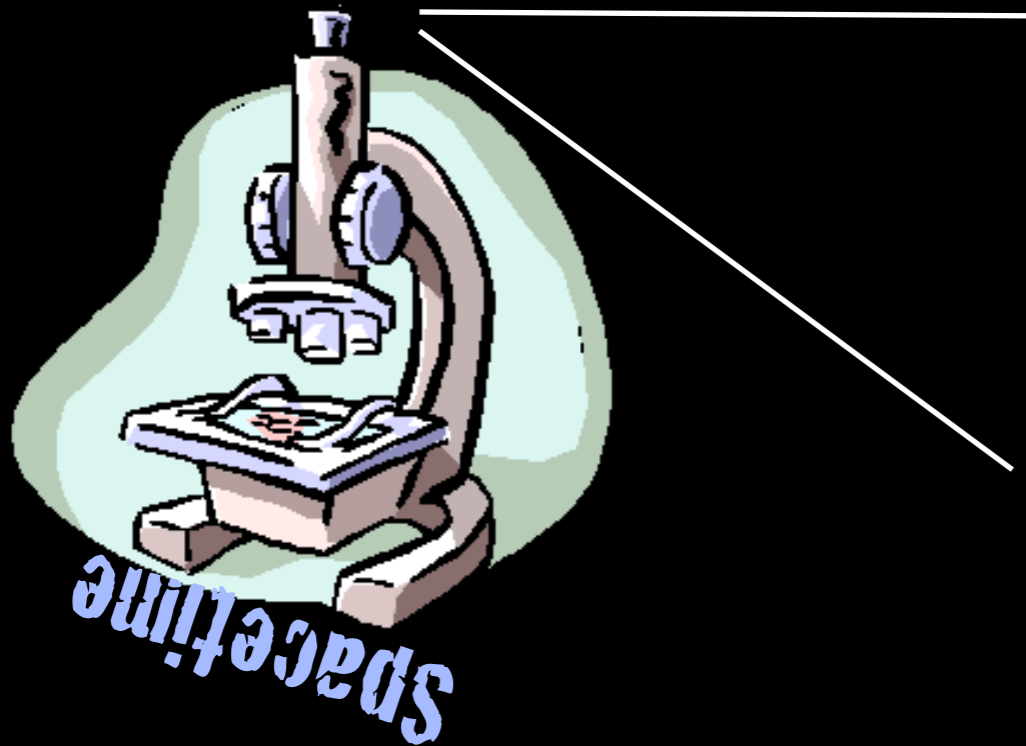
Lattice Quantum Gravity =
using dynamical lattice methods to understand
quantum gravity and its nonperturbative properties

input from different physics communities:



Questions for Quantum Gravity

- What are the (quantum) origins of space, time and our universe?
- What is the microstructure of spacetime, and can it *explain* the observed large-scale structure?
- Are “space”, “time”, and “causality” fundamental or emergent?



Spacetime Foam?

The most fruitful and concrete ideas on how to make progress come from a “new minimalism” in quantum gravity.

→ forget about strings, loops, branes, extra dimensions, new symmetries, landscapes, multiverses, ...

The future of quantum gravity

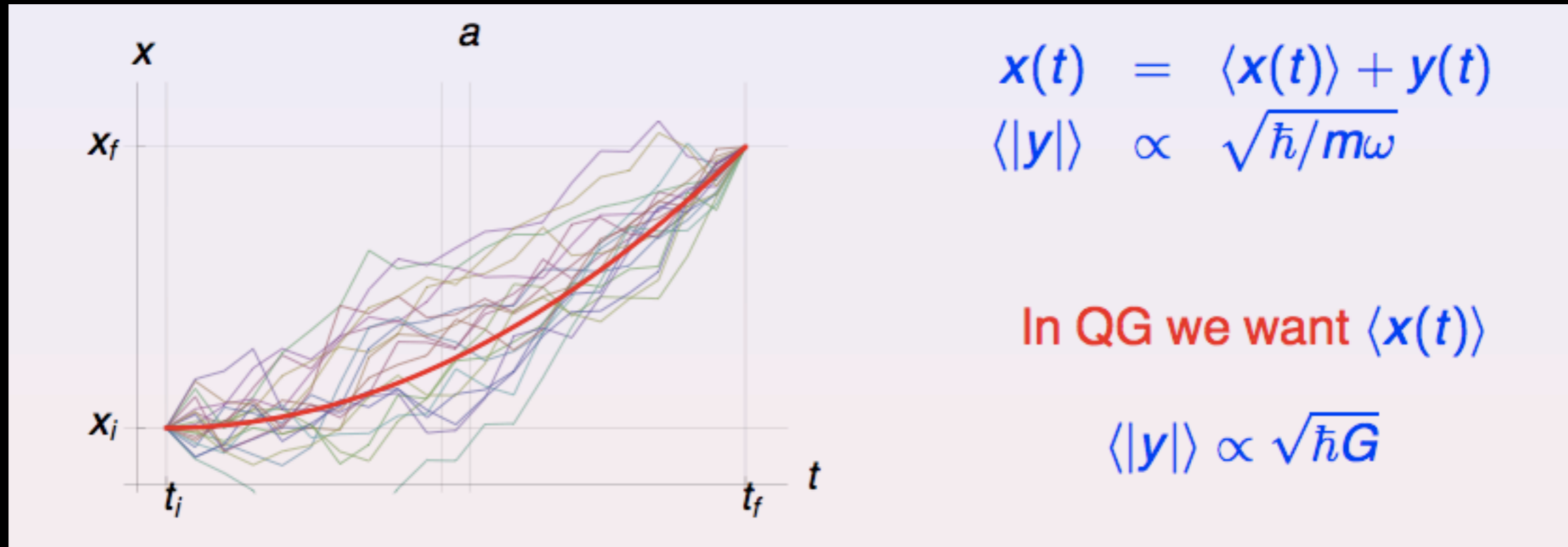
The framework of standard quantum field theory is sufficient to construct and understand quantum gravity as a fundamental theory, if one properly takes into account the dynamical, causal and nonperturbative nature of spacetime.

Significant support for this thesis comes from a new★ candidate theory, “Quantum Gravity from Causal Dynamical Triangulation (CDT)”, a nonperturbative implementation of the gravitational path integral, which has already passed nontrivial tests and produced unprecedented results.

★ birth of idea ~1998; first results in the physical, 4-dim. theory: 2004, main collaborators in 4D: J. Ambjørn, A. Görlich, S. Jordan, J. Jurkiewicz

Basic tool: the path integral

Textbook example: the nonrelativistic particle in one dimension



Quantum superposition principle: the transition amplitude from $x_i(t_i)$ to $x_f(t_f)$ is given as a weighted sum over amplitudes $\exp iS[x(t)]$ of all possible trajectories, where $S[x(t)]$ is the classical action of the path.

(here, time is discretized in steps of length a , and the trajectories are piecewise linear)

The same superposition principle, applied to gravity

"Sum over histories"
a.k.a. gravitational path integral

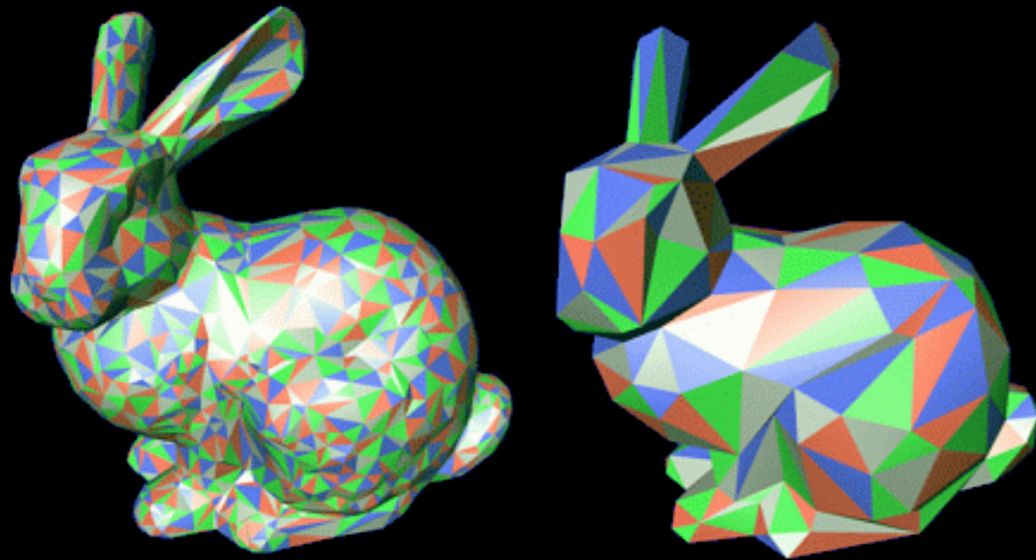
$$Z(G_N, \Lambda) = \int \mathcal{D}g e^{iS_{G_N, \Lambda}^{EH}[g]}$$

cosmol. const. \downarrow
Newton const. \rightarrow
spacetime geom.s $g \in \mathcal{G}$

Each "path" is now a four-dimensional, curved spacetime geometry g , which can be thought of as a three-dimensional, spatial geometry developing in time. The weight associated with each g is given by the corresponding Einstein-Hilbert action $S^{EH}[g]$,

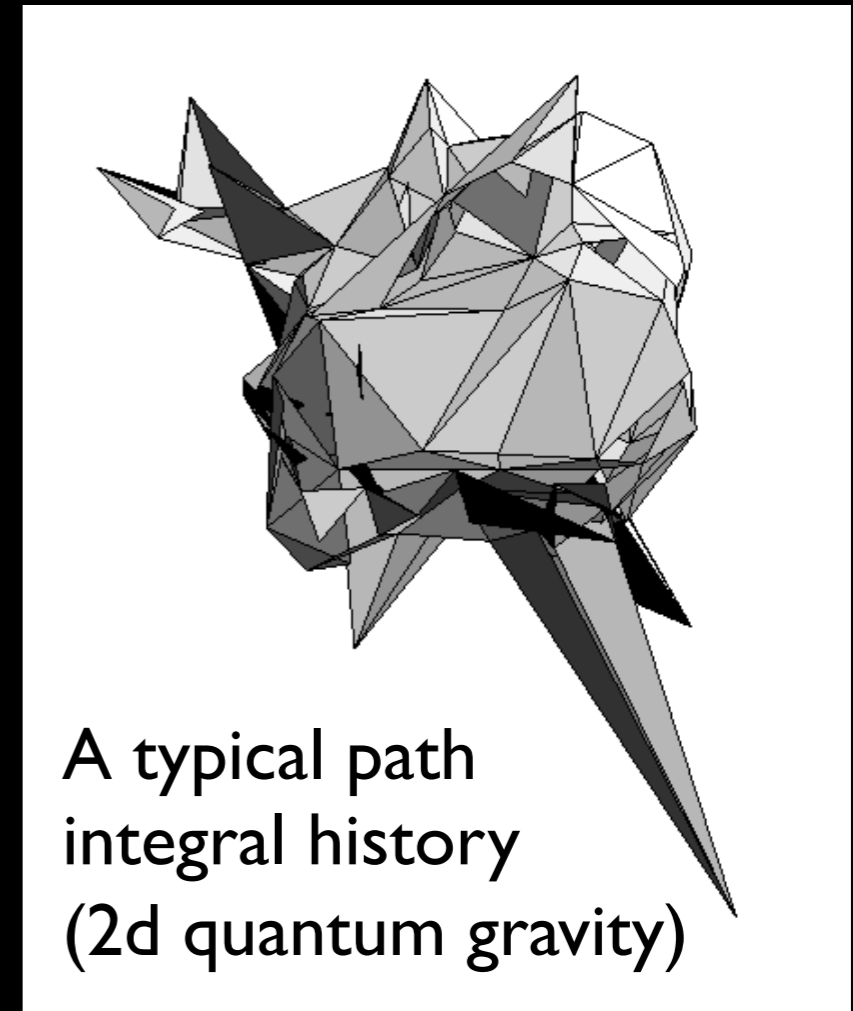
$$S^{EH} = \frac{1}{G_N} \int d^4x \sqrt{-\det g} (R[g, \partial g, \partial^2 g] - 2\Lambda)$$

A key input in dynamical triangulations



approximating *classical* curved surfaces through triangulation

triangulation = regularization



Quantum Theory: approximating the space of all curved geometries by a space of triangulations - one needs to integrate over this space^(*)!

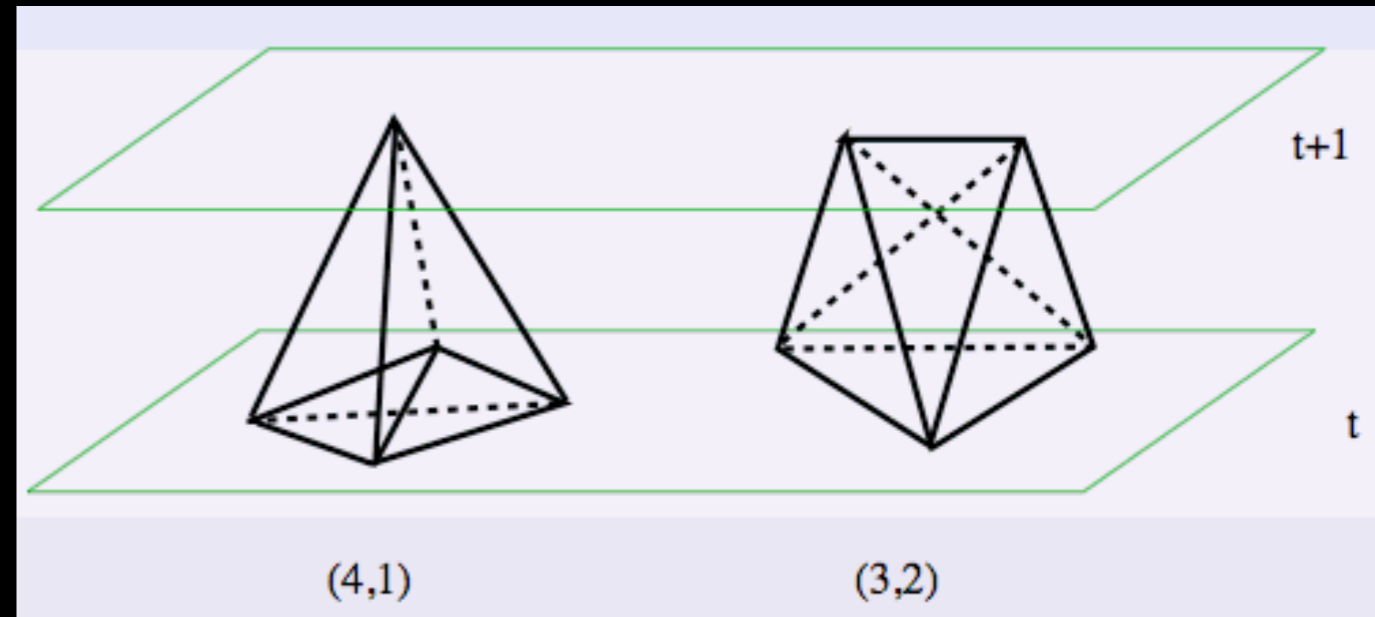
^(*) by Monte Carlo simulations (for CDT models in $d=2, 3$ have also exact stat. mech. solutions methods, see e.g. [D. Benedetti, F. Zamponi, R.L., PRD 76 \(2007\) 104022](#); in $d=2$, the problem becomes purely combinatorial)

How causal dynamical triangulations make sense of the path integral

Elementary four-simplex, building block for a causal dynamical triangulation:

($a \sim$ edge length, UV regulator)

note CDT's proper-time slicing



unphysical! (no assumption about spacetime discreteness)



Sum over histories $Z(G_N, \Lambda)$:

$$\int Dg \, e^{iS^{EH}[g]} \quad \stackrel{\text{CDT}}{=} \quad \lim_{\substack{a \rightarrow 0 \\ N \rightarrow \infty}} \sum_{\substack{\text{inequiv.} \\ \text{triangul.s} \\ T \in \mathcal{G}_{a,N}}} \frac{1}{C(T)} e^{iS^{\text{Regge}}[T]}$$

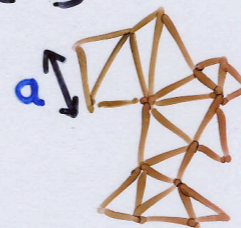
spacetime geom.s $g \in \mathcal{G}_g$

$\frac{1}{C(T)} \leftarrow |\text{Aut}(T)|$

curved spacetime geometry g

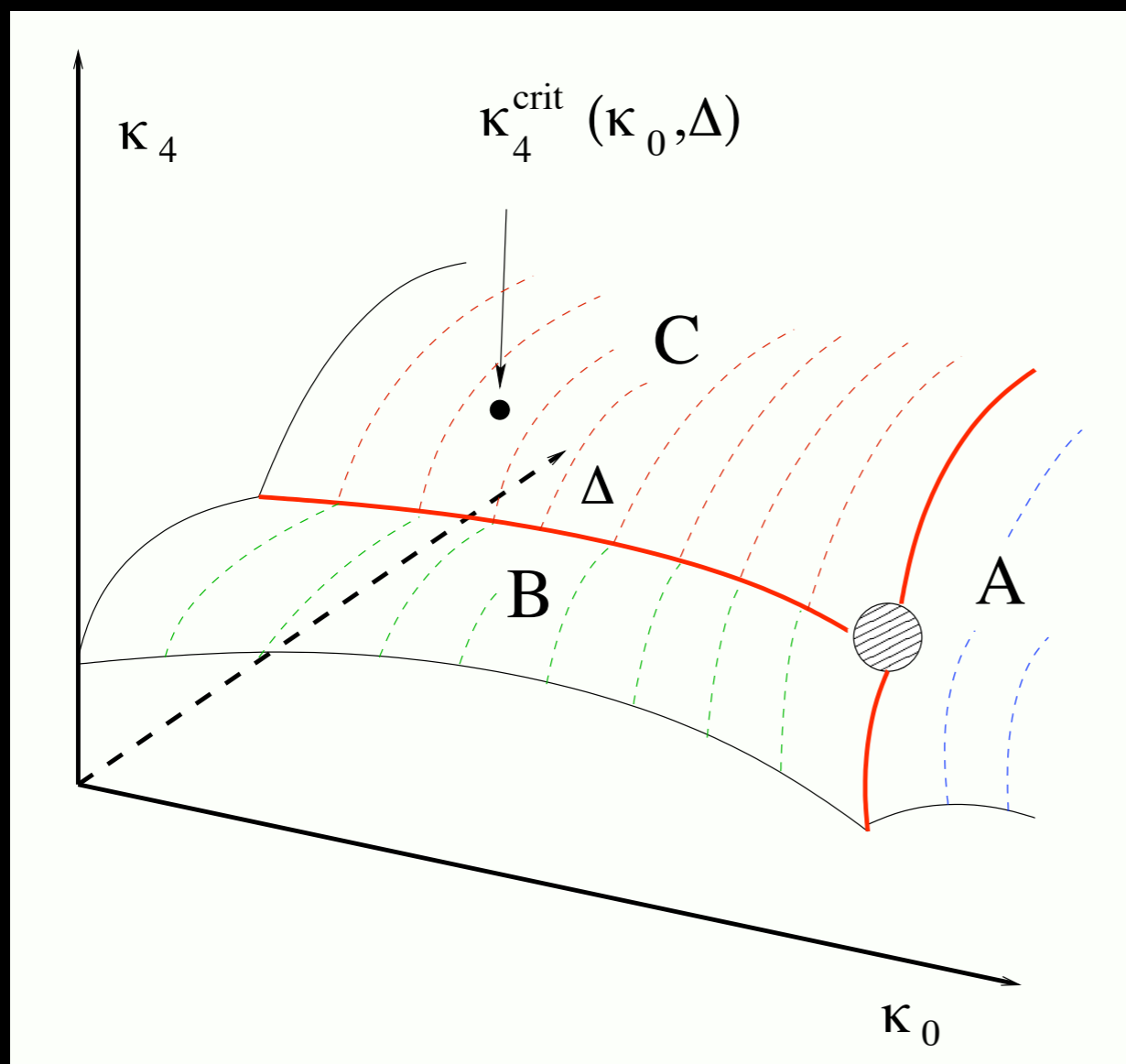
$\xrightarrow{\text{CDT regular.}}$

gluing T of N simplices (piecewise flat manifold)



taking the continuum limit of the statistical system obtained after Wick rotation, one studies its critical behaviour

The phase diagram of Causal Dynamical Triangulations



$\kappa_4 \sim$ cosmological constant
 $\kappa_0 \sim 1/G_N$
 $\Delta \sim$ relative time/space scaling

The partition function is defined for $\kappa_4 > \kappa_4^{\text{crit}}(\kappa_0, \Delta)$;
approaching the critical surface = taking infinite-volume limit.
red lines \sim (first-)order phase transitions

(J. Ambjørn, J. Jurkiewicz, RL, PRD 72 (2005) 064014;

J. Ambjørn, A. Görlich, S. Jordan, J. Jurkiewicz, RL, arXiv 1002.3298)

Key Achievements of CDT Quantum Gravity I

The dynamical emergence of spacetime as we know it

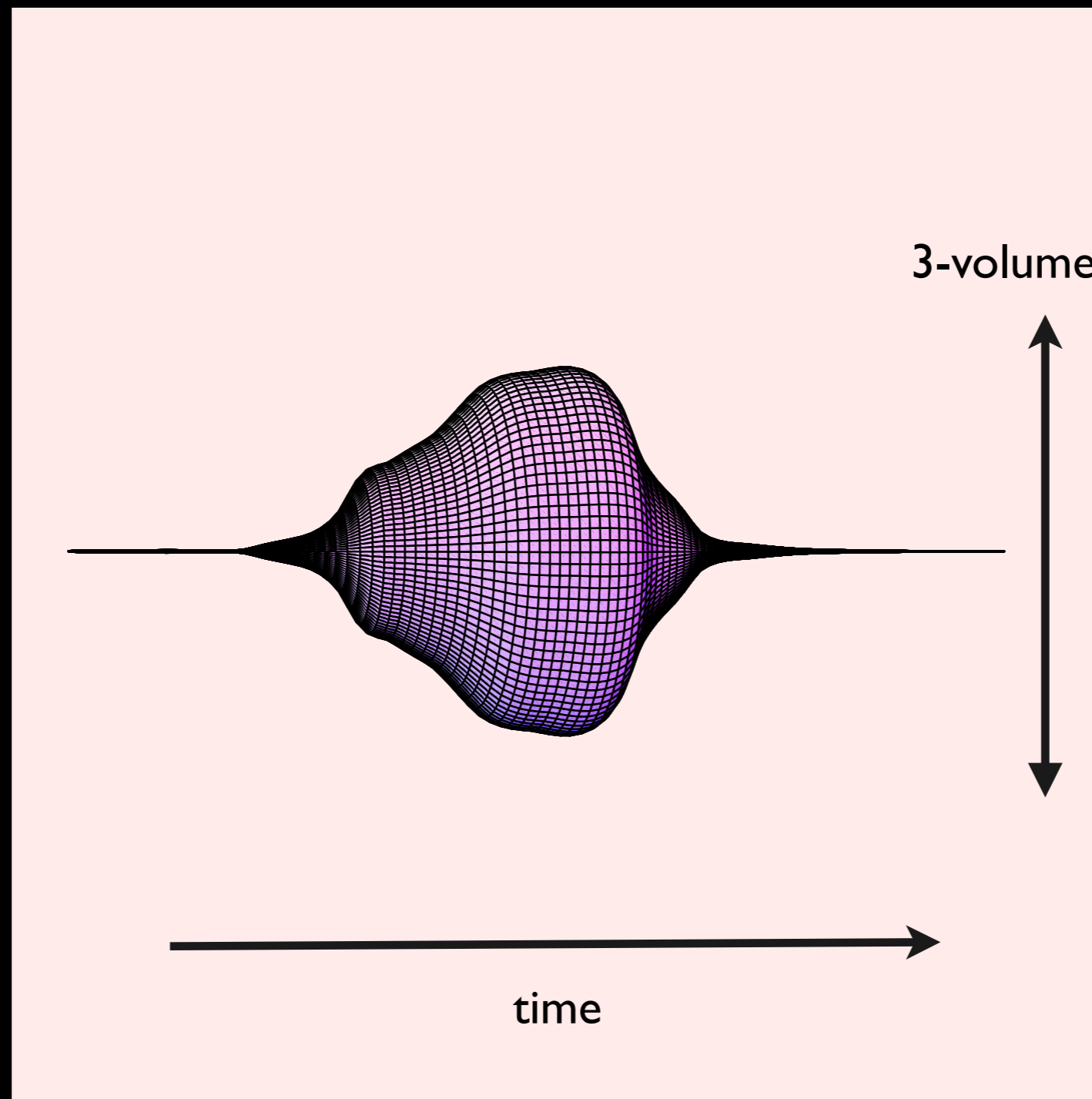
In phase C of the CDT phase diagram (which has no analogue in previously studied *Euclidean* dynamical triangulations) we find an spacetime whose microscopic shape is that of a well-known cosmology.

CDT is the so far only candidate theory of quantum gravity where a classical extended geometry is generated from nothing but Planck-scale quantum excitations.

This happens by a nonperturbative, entropic mechanism.

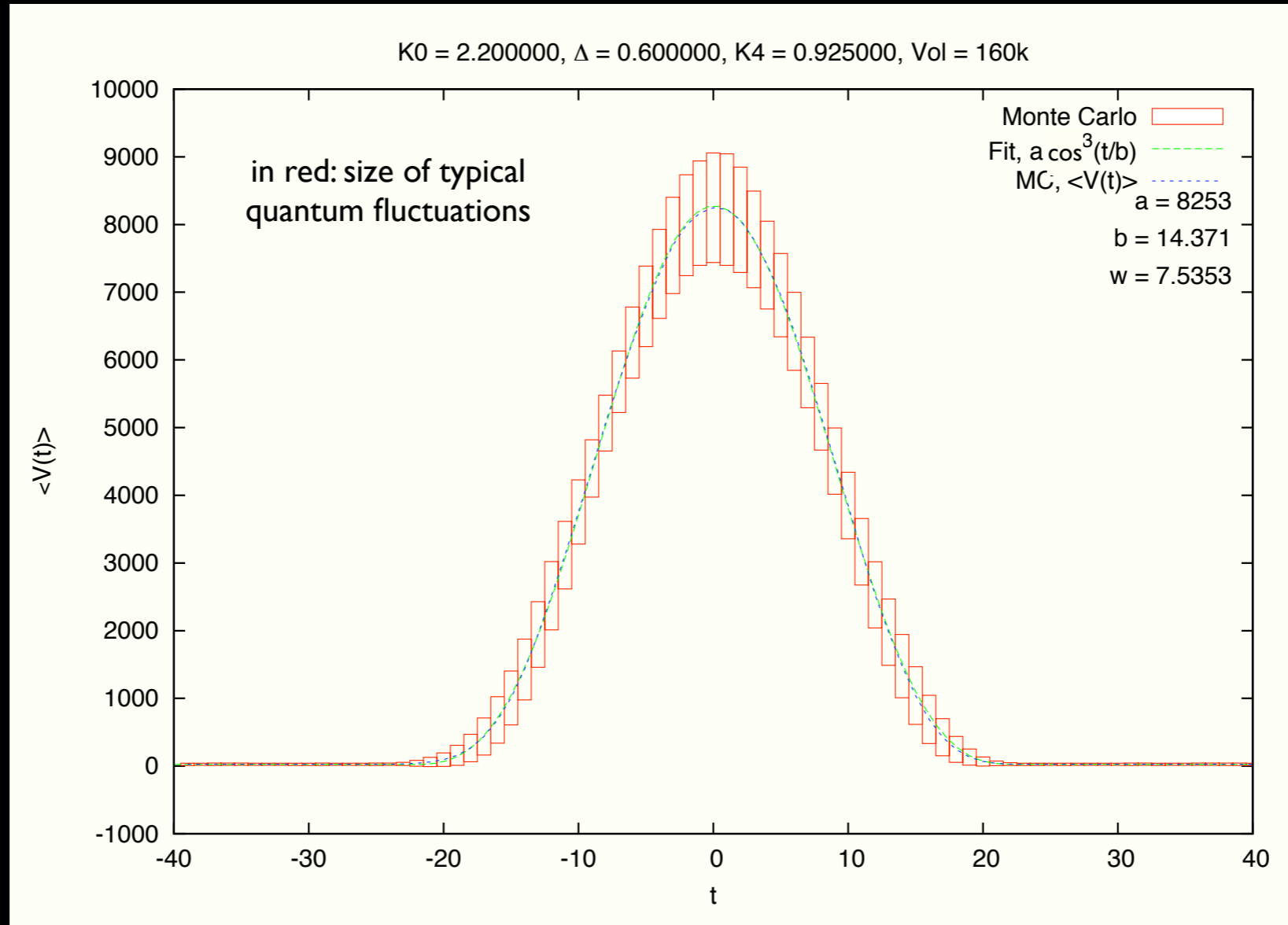
When, from all the gravitational degrees of freedom present, we monitor only the spatial three-volume $\langle V_3(t) \rangle$ of the universe as a function of proper time t , we find a distinct “volume profile”.

Dynamically generated four-dimensional quantum universe,
obtained from a path integral over causal spacetimes



This is a Monte Carlo “snapshot” - still need to average to obtain the
expectation value of the volume profile.

The dynamical emergence of de Sitter space!



The shape $\langle V_3(t) \rangle$ of the universe, as
function of Euclidean proper time $t = i\tau$,
fitted to Euclidean de Sitter space,

$$ds^2 = dt^2 + c^2 \cos^2\left(\frac{t}{\ell}\right) d\Omega_{(3)}^2$$

↖ squared "scale factor" $a(t)$

note: a maximally symmetric
spacetime!

Key Achievements of CDT Quantum Gravity II

Getting a handle on Planckian physics

(or, another nonperturbative surprise!)

A diffusion process is sensitive to the dimension of the medium where the “spreading” takes place. We have implemented such a process on the quantum superposition of spacetimes. By measuring a suitable observable[★], we have extracted the spectral dimension D_s of the quantum spacetime.



Quite remarkably, we find that it depends on the length scale probed: D_s changes smoothly from 4 on large scales to ~ 2 on short scales.

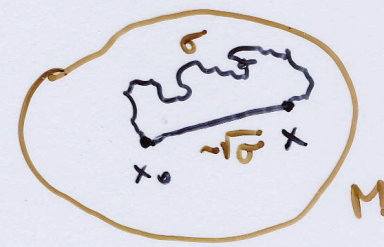
J. Ambjørn, J. Jurkiewicz, R.L.,
PRL 95 (2005) 171301

★ average return probability

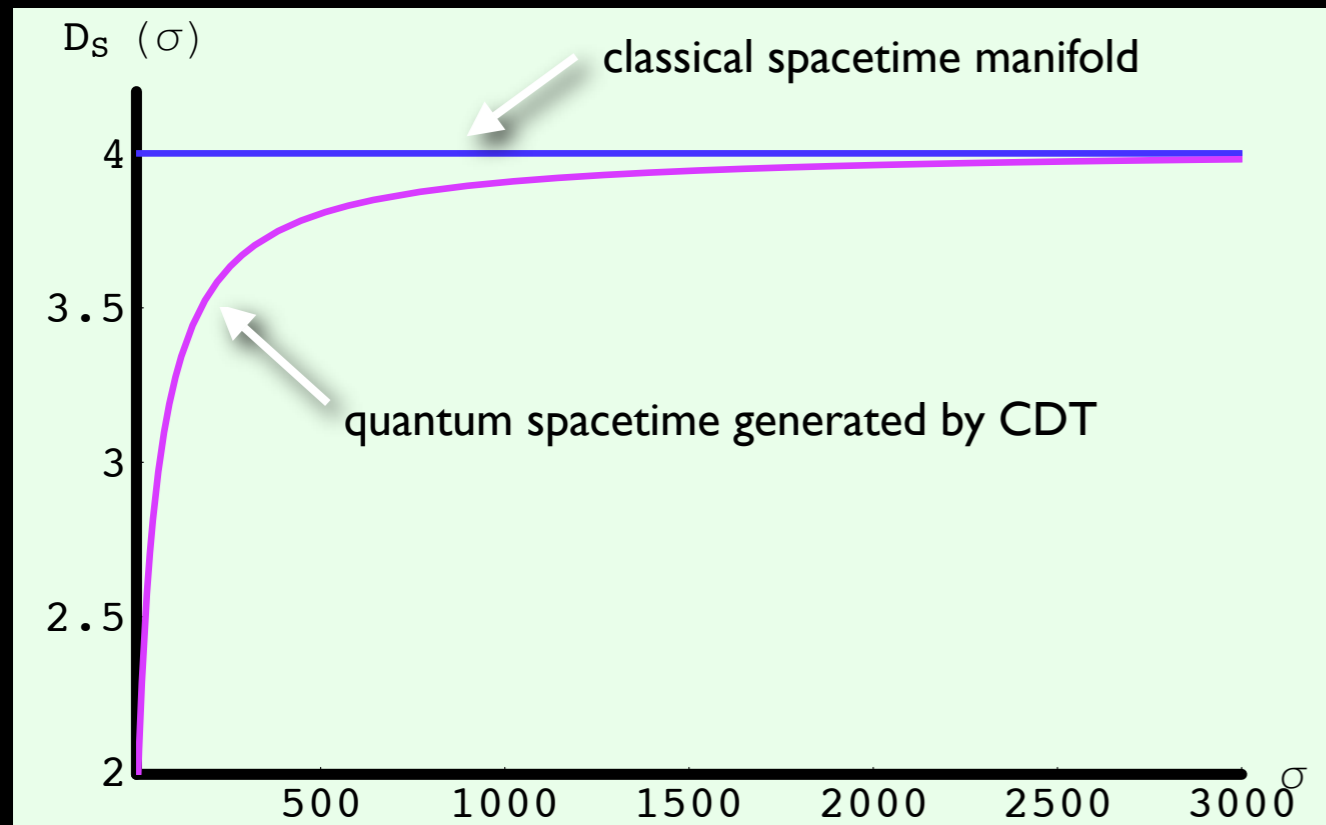
$$\mathcal{R}_v(\sigma) := \frac{1}{V(M)} \int_M d^d x P(x, x; \sigma) \sim \frac{1}{\sigma^{D_s/2}}$$

↑ diffusion time

↑ sol.n to heat eqn.



$D_S(\sigma)$ probes properties of the geometry on linear length scales $\sim \sigma^{1/2}$:



→ on short scales, our “ground state of geometry” is definitely *not* a classical manifold.

Instead, we find evidence for the presence of a random fractal structure.

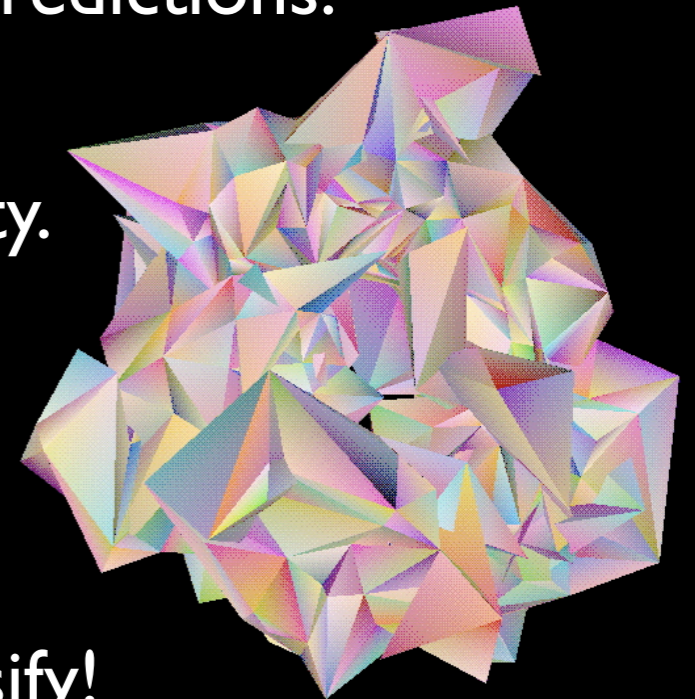
More recently, the same short-scale “*dynamical dimensional reduction*” has been found across a variety of disparate approaches:

- nonperturbative renormalization group flow analysis (M. Reuter, O. Lauscher, [hep-th/0508202](#))
- nonrelativistic “Lifshitz quantum gravity” (P. Hořava, [arXiv 0902.3657](#))
- non-commutative geometry/ κ -Minkowski space (D. Benedetti, [arXiv 0811.1396](#))

Quantum Gravity - quo vadis?

For long, there has been plenty of abstract reasoning on “the nature of quantum gravity”. Now, we have an “experimental lab” - a nonperturbative calculational handle on (near-)Planckian physics (c.f. lattice QCD), namely, *Causal Dynamical Triangulation* plus its associated toolbox.

- We have begun to make quantitative statements/predictions.
- We can also test nonperturbative predictions from other fundamental theories containing gravity.



Presently under investigation:

- (i) relate with asymptotic safety scenario; verify or falsify!
- (ii) matter coupling - extracting Newton's law and studying the early universe
- (iii) investigate short-scale quantum structure of spacetime

Where to learn more

- CDT - light: “The self-organizing quantum universe”, by J. Ambjorn, J. Jurkiewicz, RL (Scientific American, July 2008)
- A reasonably nontechnical review in Contemp. Phys. 47 (2006) [arxiv: hep-th/0509010]
- both review and popular science material on my homepage <http://www.phys.uu.nl/~loll>

Lattice Quantum Gravity - an Update

Villasimius, 16 Jun 2010

The End