

# Towards four-flavour dynamical simulations

Gregorio Herdoiza



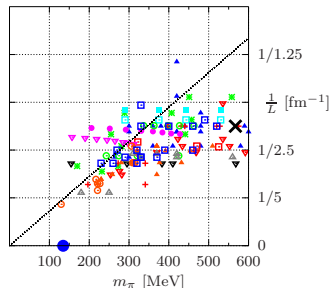
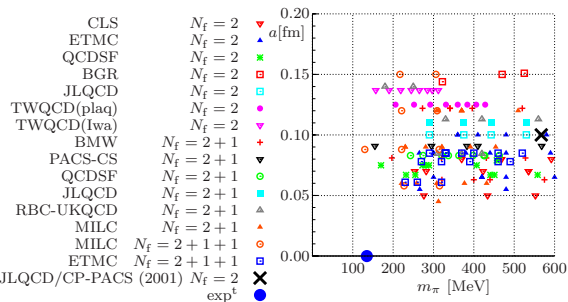
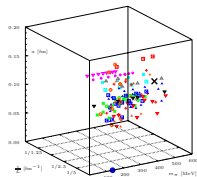
NIC, DESY



Lattice 2010, Villasimius, June 14, 2010

# dynamical simulations : parameters landscape

- number of flavours :  $N_f$
- lattice spacing :  $a$
- lattice size :  $L$
- pion masses :  $m_{\text{PS}}$

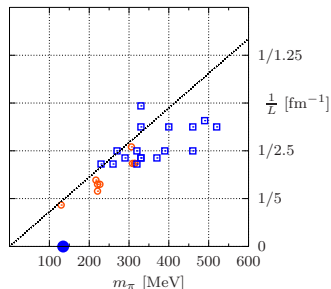
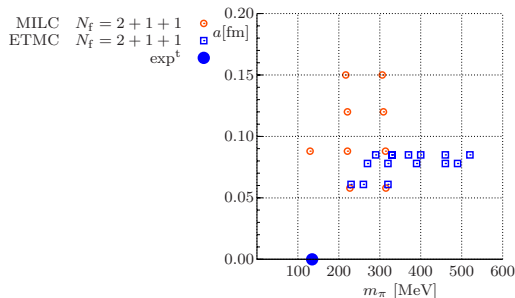
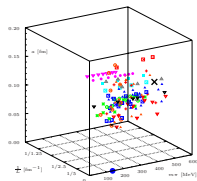


oblique dotted line :  $m_{\pi} L = 3.5$

Caveat in plots : no information on systematic effects (scale setting, cut-off effects, ...),  $m_S, m_C, \dots$

# dynamical simulations : parameters landscape

- number of flavours :  $N_f = 2 + 1 + 1$
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$$N_f = 2 + 1 + 1$$

$u, d, s, c$  dynamical quarks with  $m_u = m_d < m_s < m_c$

### Motivation for including the dynamical charm :

- ▶ expected effect  $O(1/m_c^2)$  and  $O(\alpha_s(m_c))$
- ▶ a dynamical quark is active at energy scales  $E > m_q$
- ▶ running of renormalised quantities with  $N_f = 4$
- ▶ matching to  $\overline{\text{MS}}$  with  $N_f = 4$  at  $\mu > m_c$
- ▶ explore physical effects dynamical  $c$  on hadronic masses and matrix elements
- ▶ remove so far uncontrolled systematic effect of the dynamical charm :  
relevant with increasing precision in lattice computations

### Potential difficulties of including the dynamical charm :

1. are there large cut-off effects in the light sector from  $am_c \lesssim 1$  in the sea?
2. strange and charm : is the tuning effort too heavy?
3. dedicated runs for renormalisation

$$N_f = 2 + 1 + 1$$

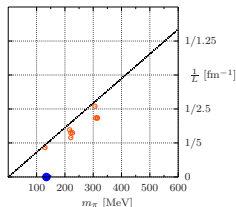
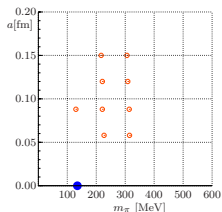
# light sector

large cut-off effects in the light sector ?

examples from MILC and ETMC

# $N_f = 2 + 1 + 1$ MILC ensembles

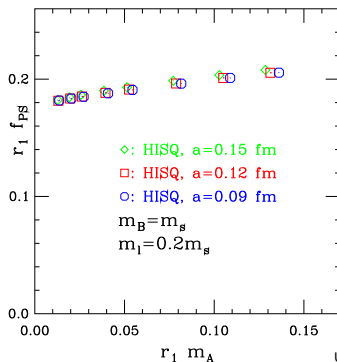
- Symanzik-improved gauge action
- staggered quarks :
  - asqtad ( $N_f = 2 + 1$ )  $\rightsquigarrow$  HISQ ( $N_f = 2 + 1 + 1$ )  
[HPQCD-UKQCD, 2006]
- HISQ : further smearing of gauge links in fermion action  
 $\rightsquigarrow$  reduced taste symmetry violations
- $a = \{0.06, 0.09, 0.12, 0.15\}$  fm
- $L = \{2.4, 5.5\}$  fm ,  $m_{\text{PS}}L > 3.5$
- $m_\pi \in \{130, 315\}$  MeV
- $m_s$  and  $m_c$  from  $2m_K^2 - m_\pi^2$  ,  $1/4(m_{\eta_c} + 3 m_{J/\psi})$   
around physical values
- RHMC
- target : 5000  $\div$  12000 thermalised traj. ,  $\tau = 1$
- while for  $N_f = 4$  rooting trick is avoided,  
still present for  $N_f = 2 + 1 + 1$
- $am_c \lesssim 0.8$
- setup : good conditions for a scaling analysis



# $N_f = 2 + 1 + 1$ MILC : scaling of $f_{PS}$

scaling of partially quenched  $f_{PS}$

$a = 0.09, 0.12, 0.15$  fm



are there large cut-off effects from the dynamical charm ?

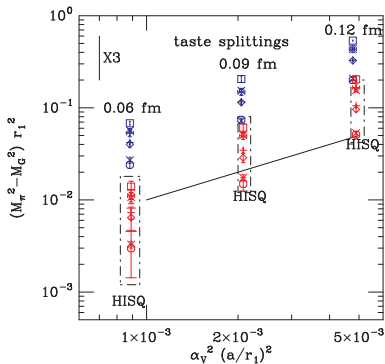
- ▶ not large ...
- ▶ similar behaviour for other observables :  $m_N, m_\rho, \dots$
- ▶ effect of HISQ

talk by Doug Toussaint

# $N_f = 2 + 1 + 1$ MILC : taste splitting

HISQ and comparison to  $asqtad$  ( $N_f = 2 + 1$ )

$a = 0.06, 0.09, 0.12$  fm



- ▶ significant reduction of the size of taste splitting wrt.  $asqtad$
- ▶ improvement also in generic discretisation effects

talk by Doug Toussaint



# $N_f = 2 + 1 + 1$ ETMC ensembles

- Iwasaki gauge action
- Wilson twisted mass  $\rightsquigarrow$  doublets  
[ALPHA, Frezzotti et al., 2001; Frezzotti & Rossi, 2003]
- $a = \{0.06, 0.08, 0.09\}$  fm
- $L = \{1.9, 2.7\}$  fm,  $m_{\text{PS}}L \gtrsim 3.5$
- $m_\pi \in \{230, 520\}$  MeV
- $m_s$  and  $m_c$  from  $2m_K^2 - m_\pi^2$ ,  $m_D$  around physical values
- HMC+PHMC
- target : 5000 thermalised traj.,  $\tau = 1$

- $O(a)$  improvement at maximal twist

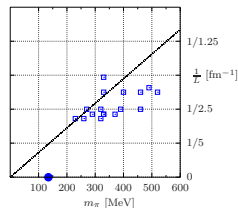
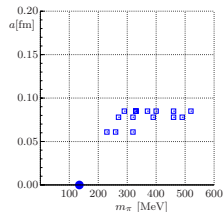
[Frezzotti &amp; Rossi, 2003]

$$m_{\text{PCAC},\ell} = 0 \quad \text{at each } \{\mu_\ell, \mu_\sigma, \mu_\delta\}$$

$$\hat{m}_s = 1/Z_P (\mu_\sigma - Z_P/Z_S \mu_\delta)$$

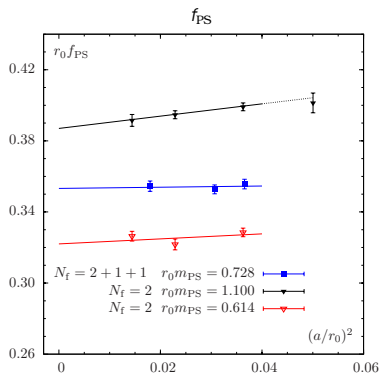
$$\hat{m}_c = 1/Z_P (\mu_\sigma + Z_P/Z_S \mu_\delta)$$

- $am_c \lesssim 0.3$



ETMC : continuum limit scaling of  $f_{PS}$  and  $m_N$ 

$$N_f = 2 + 1 + 1 \quad a = 0.06, 0.08, 0.09 \text{ fm}$$



talk by Siebren Reker

talks by Vincent Drach and Simon Dinter

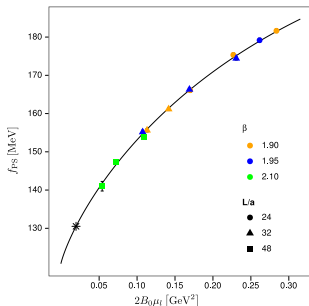
ETMC :  $\chi$ PT results $m_{PS}, f_{PS}$ 

- ▶ SU(2)  $\chi$ PT at NLO
- ▶ central values + stat. error :  $\beta = 1.95$  ( $a = 0.08$  fm)
- ▶ estimate systematic effects : lattice artifacts, FSE

$$a = \{0.06, 0.08, 0.09\} \text{ fm}$$

$$f_\pi = 130.4(2) \text{ MeV} \rightsquigarrow \text{scale}$$

	$N_f = 2 + 1 + 1$	$N_f = 2$
$\bar{\ell}_3$	3.70(27)	3.50(31)
$\bar{\ell}_4$	4.67(10)	4.66(33)
$f_\pi/f_0$	1.076(3)	1.076(9)

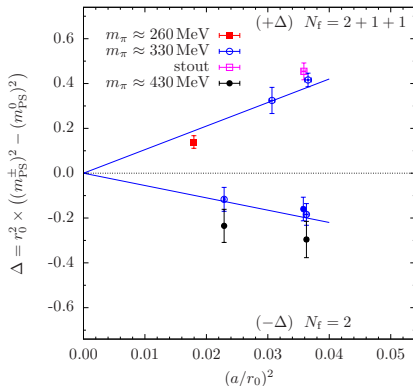


comparison to other groups :

talk by Siebren Reker  
review talk by Christian Hoelbling

# neutral pion mass

- ▶ flavour symmetry breaking at  $\mathcal{O}(a^2)$   
 $\rightsquigarrow m_{\pi^0} \neq m_{\pi^\pm}$
- ▶ while  $m_{\pi^\pm}$  has a mild  $a^2$  dependence  
**large cut-off effects** are observed in  $m_{\pi^0}$
- ▶ evidence that  $\mathcal{O}(a^2)$  effects in  $m_{\pi^0}$  grow with  $N_f$   
 slope :  $c_{N_f=2} \approx 6$   $c_{N_f=2+1+1} \approx 10$  [PRELIMINARY]



Effects of flavour breaking :

- ▶  $N_f = 2$  data : **large  $\mathcal{O}(a^2)$**  effects observed **only** in  $m_{\pi^0}$   
 [Frezzotti & Rossi, 2007; Dimopoulos et al., 2009]
- ▶ explicit checks needed + continuum limit
- ▶ finite size effects  $\rightsquigarrow$  tm $\chi$ PT  
 [Colangelo, Wenger, Wu, 2010]

$$N_f = 2 + 1 + 1$$

$$m_s, m_c$$

# strange & charm sectors

what is the tuning effort?

# tuning of $m_s$ & $m_c$

quality of tuning is only known a posteriori ...

the tuning effort depends on the target accuracy and on the observable

- ▶ ETMC : tune to maximal twist at each  $\{\mu_\ell, \mu_\sigma, \mu_\delta\} \rightsquigarrow$  more effort than in  $N_f = 2$
- ▶ MILC : fix  $\beta$  when changing  $m_\ell \rightsquigarrow$  less effort than in  $N_f = 2 + 1$
- ▶ tuning of  $m_s, m_c$ 
  - ▶ interpolation to  $m_s$  and  $m_c$  :  $\sim 4$  times more effort than in  $N_f = 2$
  - ▶ cost of including the charm is not an issue
  - ▶ reweighting can help

[Chulwoo Jung, lat09]

- ▶ dedicated simulations for renormalisation

overall, yes, some extra effort is needed for  $N_f = 2 + 1 + 1$  simulations ...

... but not out of reach

- ▶ alternative strategy to fix  $m_s$  via the singlet quark mass [  $N_f = 2 + 1$ , QCDSF-UKQCD, 2010 ]

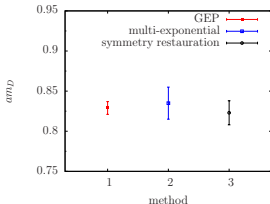
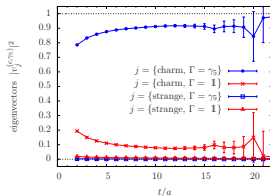
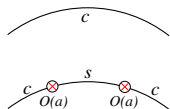
talks by Roger Horsley and Paul Rakow

# $N_f = 2 + 1 + 1$ ETMC : unitary K and D-mesons

Wilson twisted mass Dirac operator for  $(c, s)$  pair :

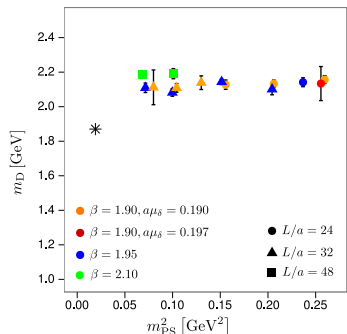
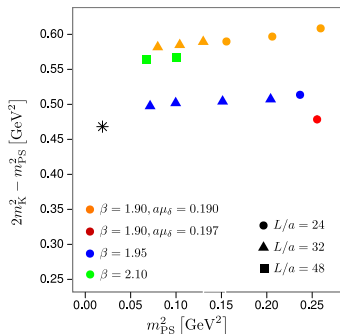
$$D_h = \begin{pmatrix} \gamma_\mu \tilde{\nabla}_\mu + \mu_\sigma + \mu_\delta & -i\gamma_5 \left( -r \frac{a}{2} \nabla_\mu^* \nabla_\mu + M_{cr} \right) \\ -i\gamma_5 \left( -r \frac{a}{2} \nabla_\mu^* \nabla_\mu + M_{cr} \right) & \gamma_\mu \tilde{\nabla}_\mu + \mu_\sigma - \mu_\delta \end{pmatrix}$$

- ▶ mixing of  $c$  and  $s$  flavour and of parity
- ▶ Kaon is the ground state : good precision
- ▶ D meson appears as an excited state
- ▶ three independent methods to identify the  $D$  meson :
  - ▶ generalised eigenvalue problem
  - ▶ multi-exponential fits
  - ▶ imposing parity and flavour restoration at finite  $a$
- ▶ they provide consistent results for  $m_D$   
 poster by Elisabetta Pallante
- ▶ to overcome the mixing of flavour  $\rightsquigarrow$  mixed action



# $N_f = 2 + 1 + 1$ ETMC : tuning $m_s$ and $m_c$

$a = 0.06, 0.08, 0.09$  fm



further tuning runs are ongoing



# mixed action : OS valence quarks

- ▶ Osterwalder Seiler (OS) valence quarks are the building blocks of twisted mass valence quarks at maximal twist (Mtm)
- ▶ individual valence flavour  $q$

$$S_{\text{OS}} = \bar{q}(x) \left[ \gamma_\mu \tilde{\nabla}_\mu - i r \gamma_5 \left( -\frac{\alpha}{2} \nabla_\mu^* \nabla_\mu + M_{\text{cr}}(r=1) \right) + \mu \right] q(x)$$

[Osterwalder & Seiler, 1978]

- ▶ Mtm corresponds to a pair of OS fermions with  $+r$  and  $-r$  (OS,Mtm)
- ▶ benefits :
  - OS action is flavour diagonal
  - $O(\alpha)$  improvement [Frezzotti & Rossi, 2004]
  - Mtm and OS fermions share the same renormalisation factors : matching is simplified
- ▶ applications to  $N_f = 2$  and  $N_f = 2 + 1 + 1$

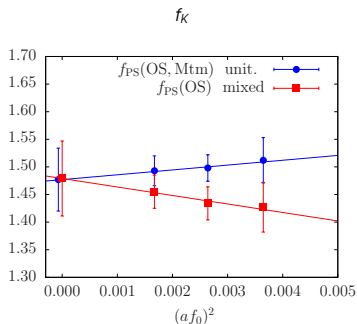
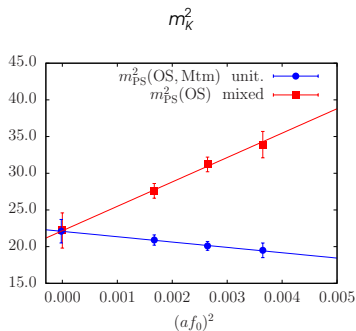
talks about overlap valence on dynamical Wilson type fermions

talk by Fabio Bernardoni

talk by Krzysztof Cichy

ETMC  $N_f = 2$ : mixed action OS valence quarks

continuum limit scaling

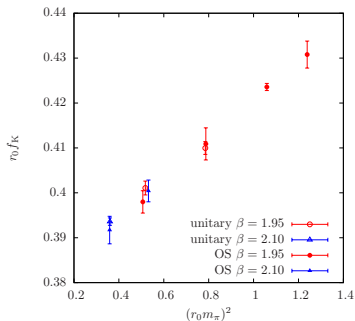
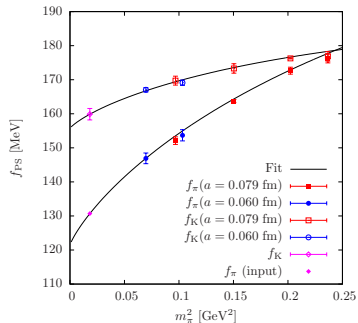
Benefit:  $B_K$   $O(\alpha)$  improved and absence of mixing due to breaking of chiral symmetry

poster by Petros Dimopoulos

 $N_f = 2 + 1 + 1$  mixed action: use of "OS, Mtm" valence action in strange and charm sector ...

ETMC  $N_f = 2 + 1 + 1$ : mixed action  $f_K, f_D, f_{D_s}$ 

mixed action in the strange and charm sector

 $a = 0.060, 0.079$  fm $f_K$  unitary and mixed $f_K$  and  $f_\pi$ PRELIMINARY  $N_f = 2 + 1 + 1$  results :

stat. errors only

$$f_K/f_\pi = 1.22(1)$$

$$f_D = 204(3) \text{ MeV}$$

$$f_{D_s} = 251(3) \text{ MeV}$$

talk by Carsten Urbach

review talks by Jochen Heitger and Christian Hoelbling

$$N_f = 3 \quad \& \quad N_f = 4$$

dedicated runs for  
renormalisation

# non-perturbative renormalisation

mass-independent scheme :

- ▶ anomalous dimensions independent of quark masses
- ▶ use of  $N_f = 2 + 1$  or  $N_f = 2 + 1 + 1$  ensembles to renormalise scale-dependent operators :
  - ▶  $m_s \rightarrow 0$  and in particular,  $m_c \rightarrow 0$ , are long extrapolations
  - ▶ when extrapolation is not performed : estimate systematic effect
- ▶ dedicated simulations with  $N_f = 3$  or  $N_f = 4$  **degenerate** flavours :
  - ▶ Schrödinger functional (SF) :  $m_q = 0$
  - ▶ RI-MOM :  $m_q \rightarrow 0$

recent work from several groups :

- ▶ PACS-CS :  $N_f = 3$  clover fermions with SF talk by Yusuke Taniguchi
- ▶ ALPHA :  $N_f = 4$  clover fermions with SF talk by Rainer Sommer
- ▶ Pérez Rubio & Sint :  $N_f = 4$  staggered quarks with SF talk by Paula Pérez Rubio
- ▶ ETMC :  $N_f = 4$  twisted mass fermions with RI-MOM talk by David Palao

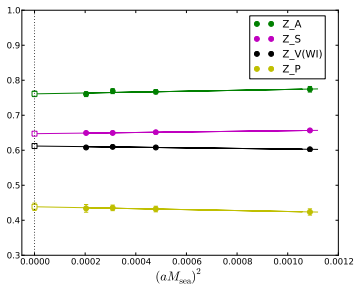
# $N_f = 4$ ETMC

renormalisation factors for  $N_f = 4$  flavours of Wilson fermions and Iwasaki gauge action

- RI-MOM at non zero values of both the standard and twisted mass parameters

$$M_R = \frac{1}{Z_P} \sqrt{(Z_A m_{\text{PCAC}})^2 + \mu_Q^2} \rightarrow 0$$

- $O(a)$  improvement via average of simulations with  $+m_{\text{PCAC}}$  and  $-m_{\text{PCAC}}$
- study at  $\beta = 1.95$ :  $a = 0.08$  fm,  $L = 1.9$  fm



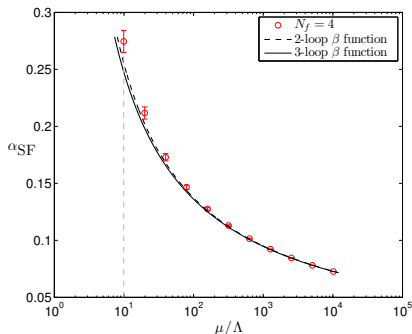
talk by David Palao

# $N_f = 4$ QCD running of the coupling

two recent studies in the Schrödinger functional scheme

- four flavors of  $O(a)$  improved Wilson quarks

[ALPHA, Tekin, Sommer, Wolff, 2010]



talk by Rainer Sommer

- single naive staggered fermion field (no rooting trick)

[Pérez Rubio & Sint]

talk by Paula Pérez Rubio

# conclusions

- ▶ first steps with **four-fermion** simulations
- ▶ **first results** in the light, strange and charm sector
- ▶ no clear signs of large cut-off effects from dynamical charm in light-quark observables
- ▶ some extra effort is needed for tuning . . .  
. . . but not a strong limitation
- ▶ dedicated runs for renormalisation  $\rightsquigarrow$  results from several groups!