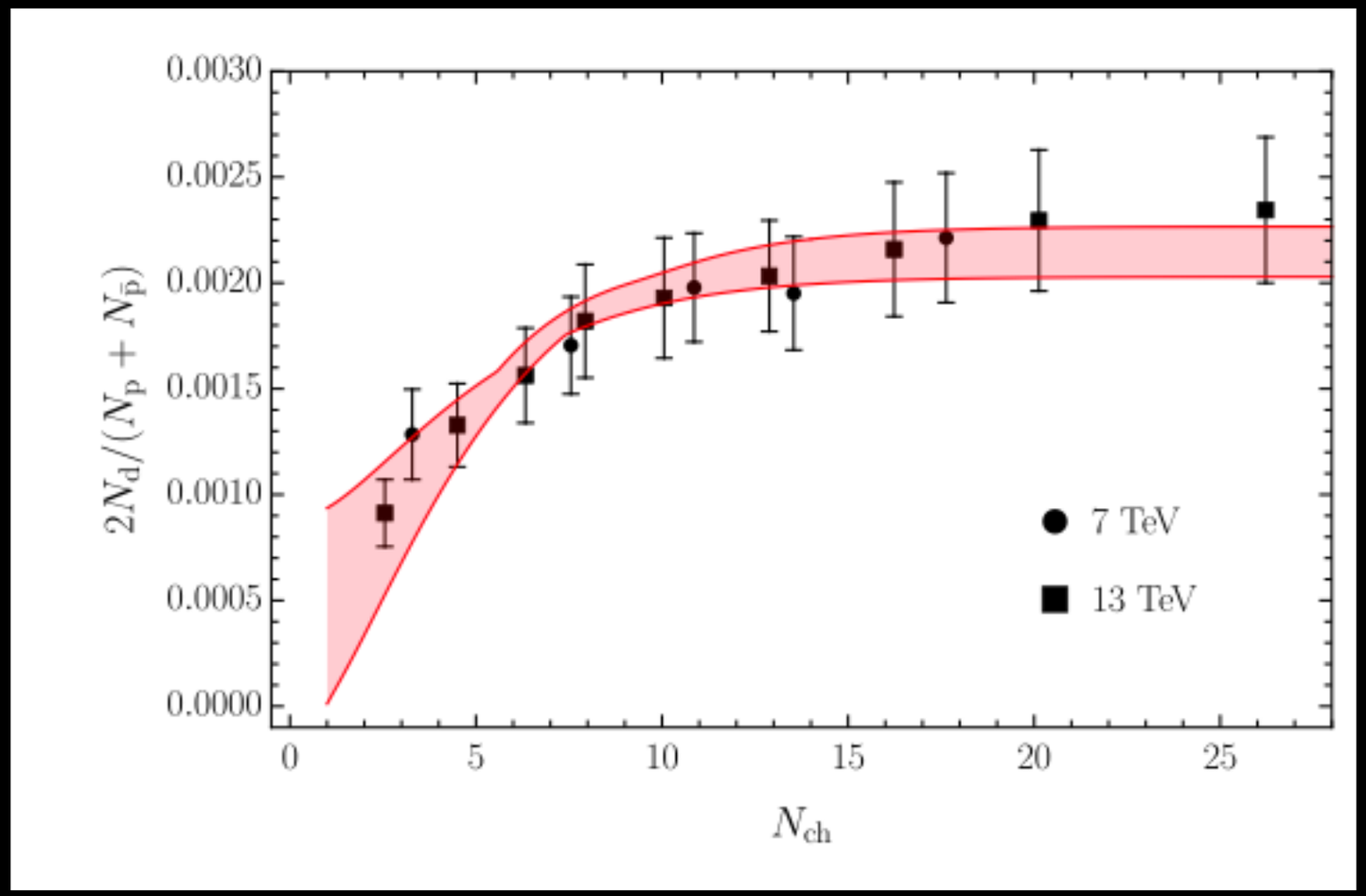
AD POLOSA — SAPIENZA UNIVERSITY OF ROME

SOME REFLECTIONS ON TETRAQUARKS & MOLECULES

Based on Esposito, Maiani, Pilloni, ADP, Riquer, 2108.11413
and ADP, 1505.03083

DEUTERON FROM ALICE

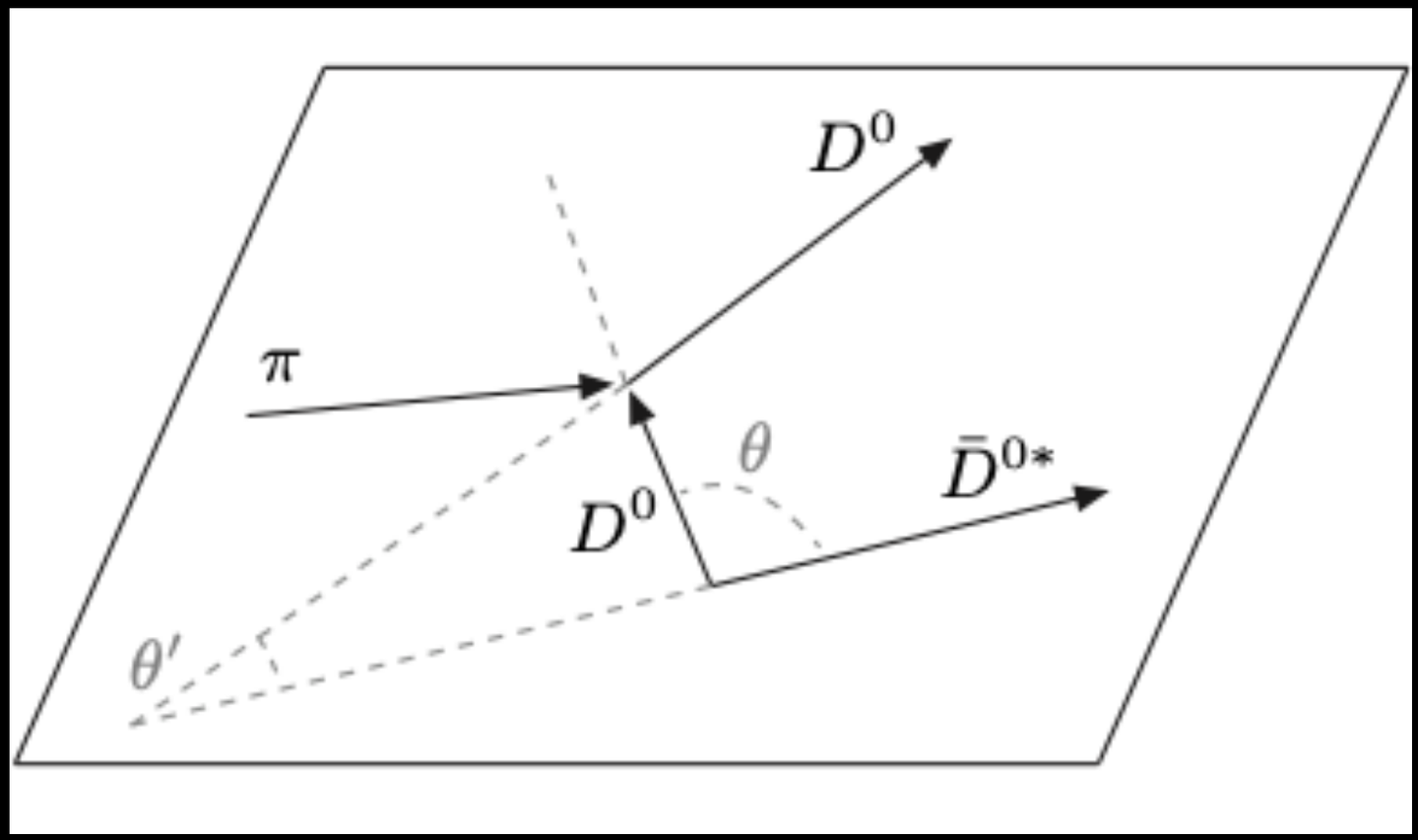
ALICE: 1902.09290; 2003.03184



Esposito, Ferreiro, Pilloni, ADP, Salgado 2006.15044

Numbers of molecules as a function of multiplicity, computed with Boltzman eq. in a coalescence model.

COALESCENCE MODEL

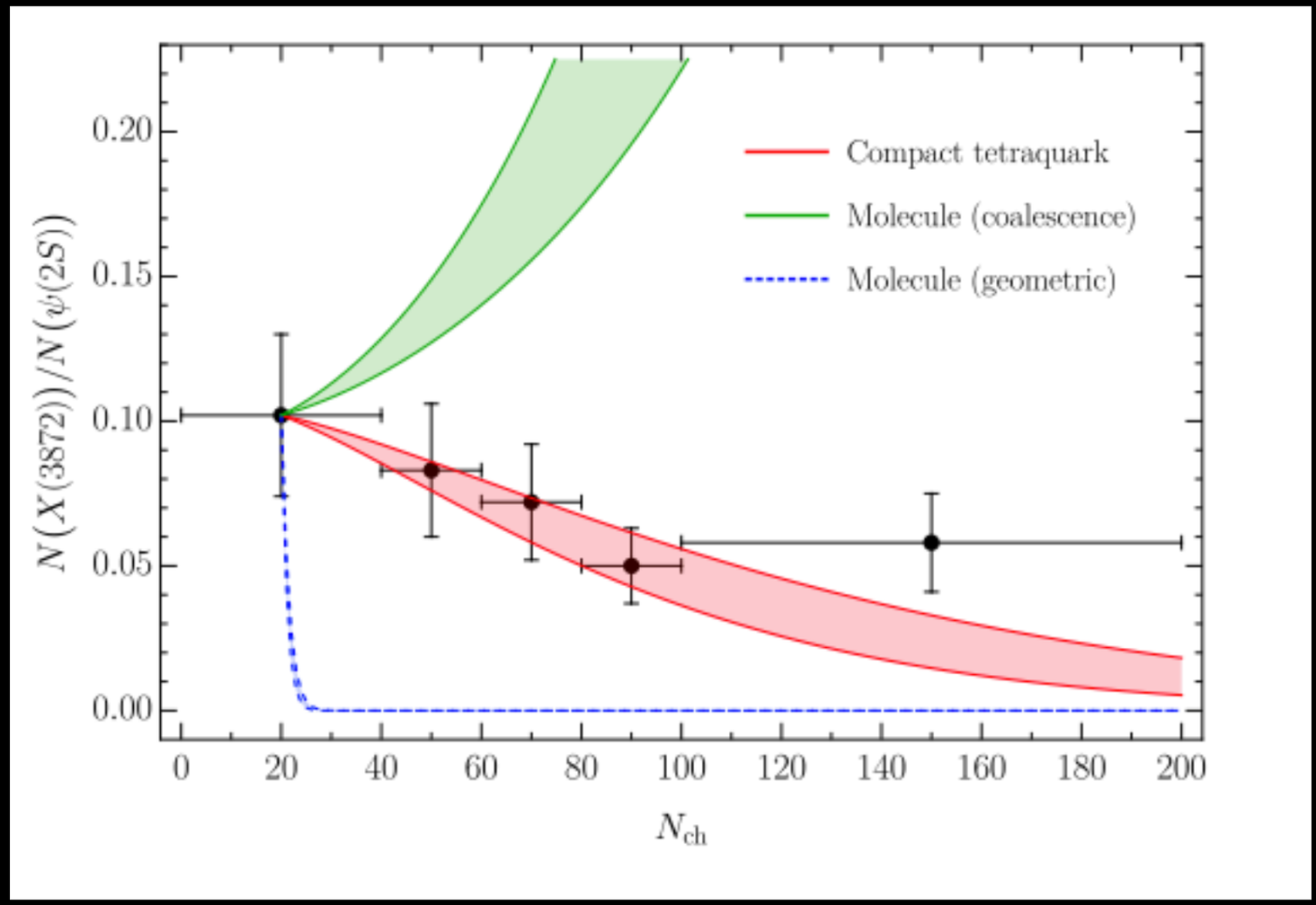


Esposito, Piccinini, Pilloni, ADP, 1305.0527
Guerrieri, Piccinini, Pilloni, ADP, 1405.7929

In final states of hadron collisions, the would-be molecule constituents have large ($k > \Lambda$) relative momenta and, after an interaction with a GeV comovers, the prob. of falling within $k < \Lambda$ is small. On the other hand a bound pair is most likely broken.

RECENT IMPLICATIONS

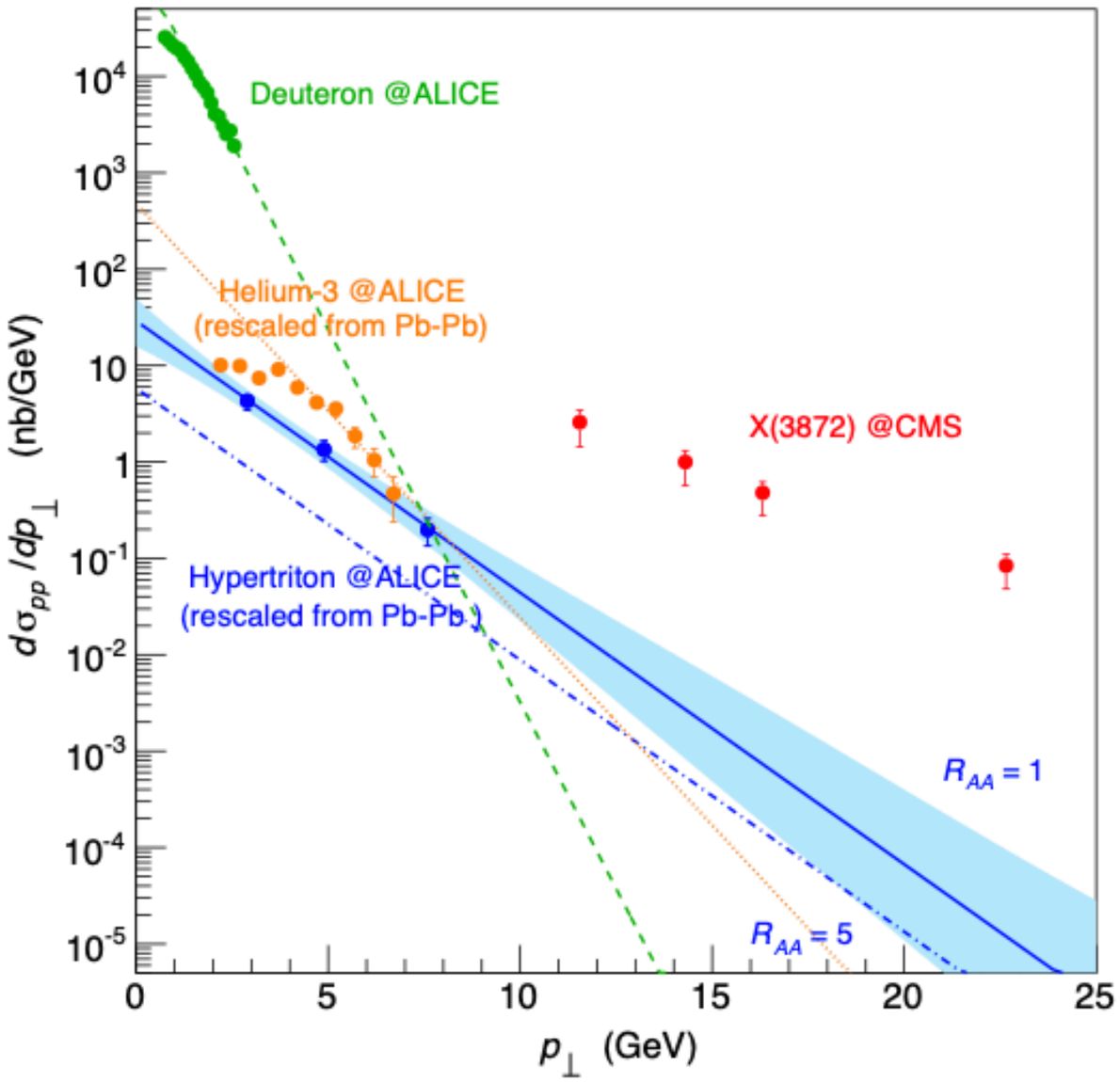
LHCb:2009.06619

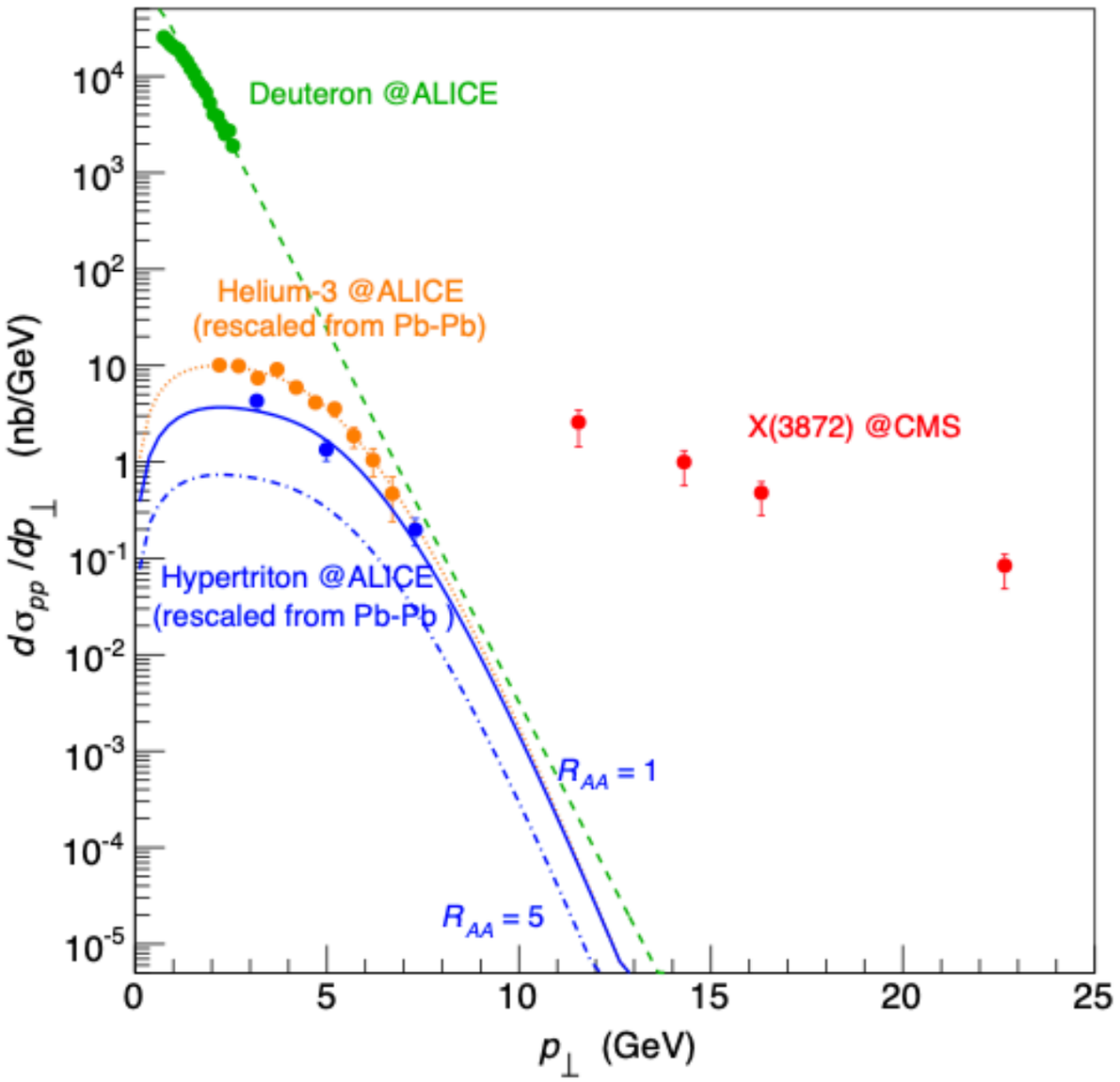


Esposito, Ferreiro, Pilloni, ADP, Salgado 2006.15044

The coalescence picture predicts a behavior (green band) qualitatively different from data.

FROM MULTIPLICITY TO PT





Esposito, Guerrieri, Maiani, Piccinini, Pilloni, ADP, Riquer, 1508.00295

POINTLIKE/COMPOSITE PARTICLES

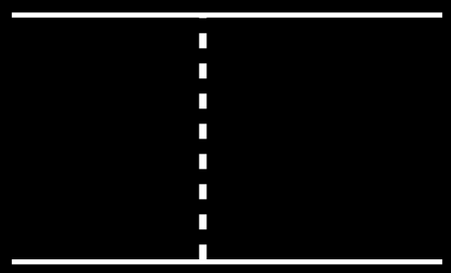
If a pointlike particle is hit by another pointlike particle, the only effect is that its momentum will change and the strength of the interaction is insensitive to the exchanged momentum.

However if the target is composite, a charge distribution in space may result and the coupling is $\mathbf{k}$ dependent differently from the contant e

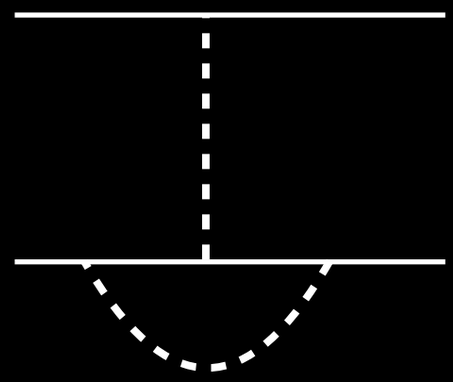
$$\delta\phi(P) = \int d^3k\, e^{i\mathbf{k}\cdot\mathbf{r}} \frac{F(\mathbf{k})}{\mathbf{k}^2}$$

with $F(\mathbf{k})$ in place of e (which is obtained if $\rho(\mathbf{r}) = e\delta^3(\mathbf{r})$)

QUANTUM LOOPS FORM FACTORS



The interaction strength
depends only on g (cubic coupling)



The interaction strength is a function of the exchanged momentum as a result of the loop. A *composite system of virtual* particles.

Particles are composite if their interaction with probes depends on momentum. Then, due to quantum corr., all particles are composite.
What is the meaning of elementary?

DEUTERON

Is there any way to tell if the deuteron is elementary or composite?

Weinberg proposes that the reply could come from the study of the np scattering amplitude.

AN ELEMENTARY DEUTERON?

$$H_0 |\alpha\rangle = E |\alpha\rangle$$

(Continuum spectrum)

Let $|\alpha\rangle$ represent np pairs (at rest) with some relative momentum in the c.o.m., with $J^P = 1^+$ (the deuteron).

$$H_0 |m\rangle = E_m |m\rangle$$

((Bare) Elementary particle states)

$$\sum_m |m\rangle\langle m| + \int d\alpha \, |\alpha\rangle\langle\alpha| = 1$$

(Completeness relation)

See Weinberg Phys. Rev. 137, B672 (1965)

THE ELEMENTARY DEUTERON

The physical normalized $|d\rangle$ deuteron state is

$$\underbrace{|d\rangle}_{\Psi} = \underbrace{\sqrt{Z}\,|\mathfrak{d}\rangle}_{\Psi_Q} + \underbrace{\int d\beta\, C_\beta\, |\beta\rangle}_{\Psi_P}$$

Assume there is one elementary deuteron state $|\mathfrak{d}\rangle$ among the $|m\rangle$ states.

$$\text{Also } \int d\beta\, |C_\beta|^2 = 1 - Z$$

THE ELEMENTARY DEUTERON

If there is no elementary deuteron $\mathfrak{d}$, i.e. $Z = 0$, then we need a V to make a bound state np; $B = 2.2\ \mathrm{MeV}$

$$(H_0 + V)\,|\,d\rangle = -\,B\,|\,d\rangle$$

Otherwise we consider a generic

$$H\Psi = \mathscr{E}\Psi$$

$$(\mathscr{E} - H_{PP})\Psi_P = H_{PQ}\Psi_Q$$

$$(\mathscr{E} - H_{QQ})\Psi_Q = H_{QP}\Psi_P$$

See Weinberg Phys. Rev. 137, B672 (1965)

THE ELEMENTARY DEUTERON

If $Z = 0$ we have a hadron molecule.

If $Z \neq 0$ we have two cases:

1) $Z \neq 0$, 2) Ψ_P is a np bound state.

It is hard to think that $H_{PQ} = H_{QP} = 0$! Thus we should observe **two** states separated in mass (no such pair observed).

2) $Z \neq 0$, 2) Ψ_P it is *not* a np bound state

but the quantum dressing of $\mathfrak{d}$ made by np unbound pairs coupled to $\mathfrak{d}$ by $H_{PQ} = H_{QP} = H_I$. Only one state.

WEINBERG & DEUTERON (1965)

Weinberg finds, for *shallow bound states*, a relation between Z and a quantity known in low energy scattering theory as the *effective range* r_0

$$r_0 = -\frac{Z}{1-Z}R + O(\frac{1}{m_\pi})$$

$$R = \frac{1}{\sqrt{2mB}}$$

The "molecule" has $Z = 0$ thus $r_0 = O(1/m_\pi)$.

What is the sign of these corrections?

See Weinberg Phys. Rev. 137, B672 (1965)

BETHE ('49), LANDAU-SMORODINSKY ('48)

Results on two body shallow bound states:

$$\mathbf{r_0 > 0} \text{ (indeed } r_0 = +\,1.74 \text{ fm for deuteron)}$$

There is a general theorem for this, together with case by case analyses (see e.g. Blatt & Weisskopf).

This agrees with original Weinberg's $Z = 0$ molecule:

"…an elementary deuteron would have $0 < Z < 1$"

"The true token that the deuteron is composite is an r_0 small and positive rather than large and negative "

"…an elementary deuteron would entail a large and negative r_0"

Esposito et al. 2108.11413

LHCB (2020)

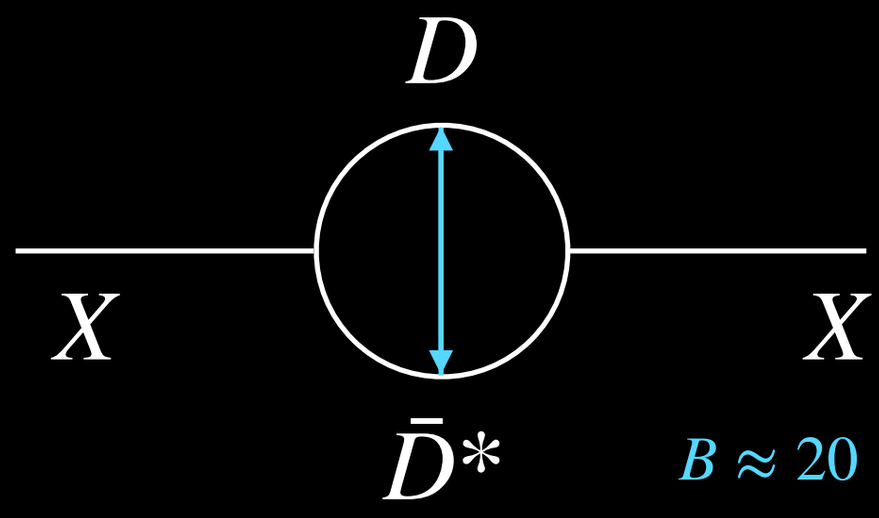
arXiv:2005.13419

Allows to compute the effective range r_0 for the $X(3872)$. This was dubbed as the "deuson", a $D\bar{D}^*$ mesonic molecule analogue of deuteron: a viable option

iff $Z = 0$ or $r_0 > 0$ and $O(1/m_\pi)$.

However we find $r_0 = -5.43$ fm and $|r_0| > 1/m_\pi$!

"…an elementary deuteron would entail a large and negative r_0"



Esposito et al. 2108.11413

FINITE Z

Measuring a finite, although ``small`` Z

$1 - Z$ is the `weight` of the molecular component, which then prevails and determines the nature of the state

Esposito et al. 2108.11413

FINITE Z

Measuring a finite, although ``small`` Z (say 0.01!)

$\Rightarrow$

the state is elementary — the only conclusion that can be formulated safely.
The molecular component is not necessarily needed (two states?).

Esposito et al. 2108.11413

THE MOST RECENT TETRAQUARKS

DIQUARK-ANTIDIQUARK

$$X(1^{++}) = [cq][\bar{c}\bar{q}] = \frac{1}{\sqrt{2}}(|1,0\rangle_1 + |0,1\rangle_1) \qquad X(3872)$$

$$X(1^{+-}) = [cq][\bar{c}\bar{q}] = \frac{1}{\sqrt{2}}(|1,0\rangle_1 - |0,1\rangle_1) \qquad Z(3900)$$

$$X'(1^{+-}) = [cq][\bar{c}\bar{q}] = \frac{1}{2\sqrt{2}}(|1,1\rangle_1) \qquad Z(4020)$$

Here $q = u, d$. What about strange quarks?

Maiani, Piccinini, ADP, Riquer, Phys. Rev. D71, 014028 (2005); D89 114010 (2014); PLB778, 247 (2018)

THE EQUAL SPACING RULE

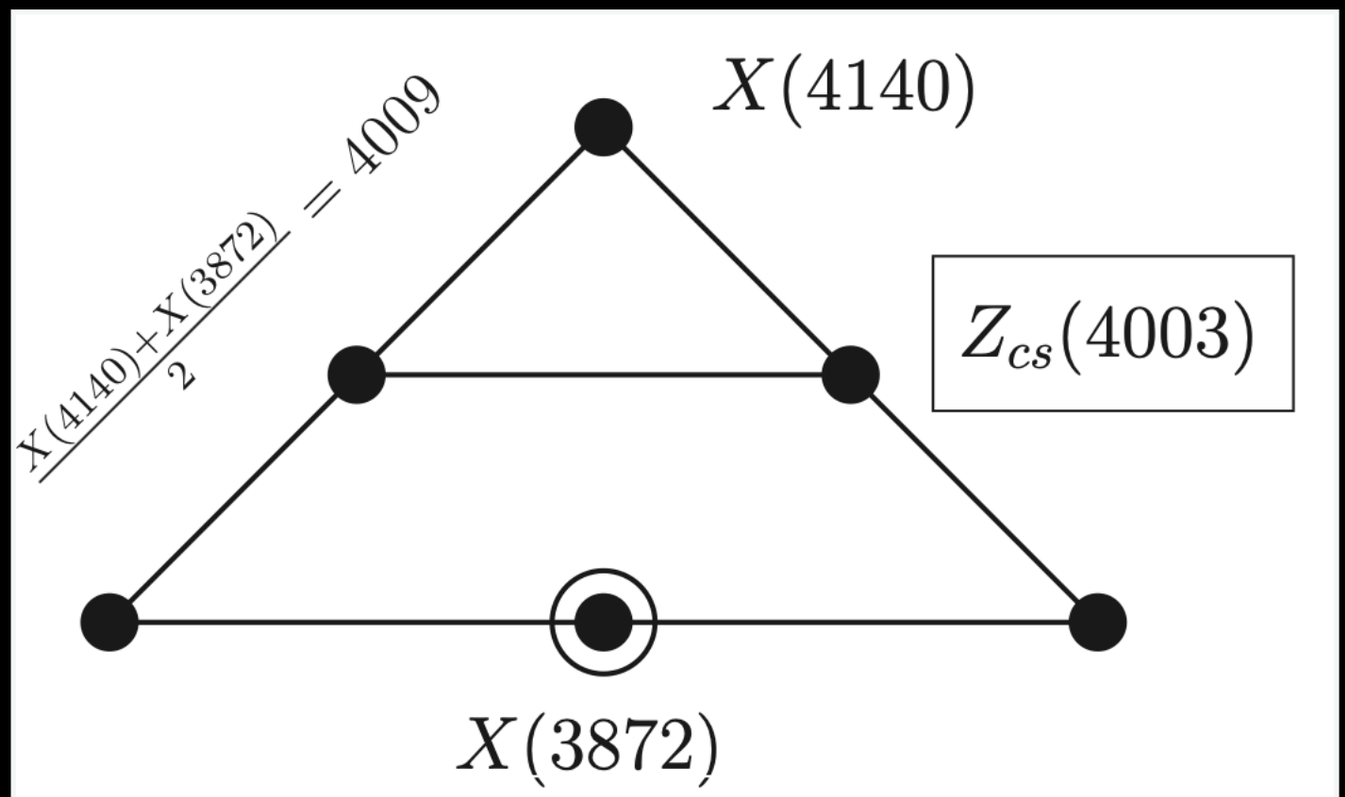
$$K^* \approx (\phi + \rho)/2$$

$$X(1^{++}) = [cs][\bar{c}\bar{s}] \qquad X(4140)$$

To first order of SU(3) flavor symmetry breaking we predict

$$Z_{cs} \stackrel{!}{=} (X(4140) + X(3872))/2 = 4009 \text{ MeV}$$

Spacing = 275 MeV
wrt 244 MeV for $\phi - \rho$



$[cs][\bar{c}\bar{q}] \vee [cq][\bar{c}\bar{s}]$

Maiani, ADP, Riquer
arXiv:2103.08331
Sci. Bulletin 66, 1616 (2021)

LHCB (2021)

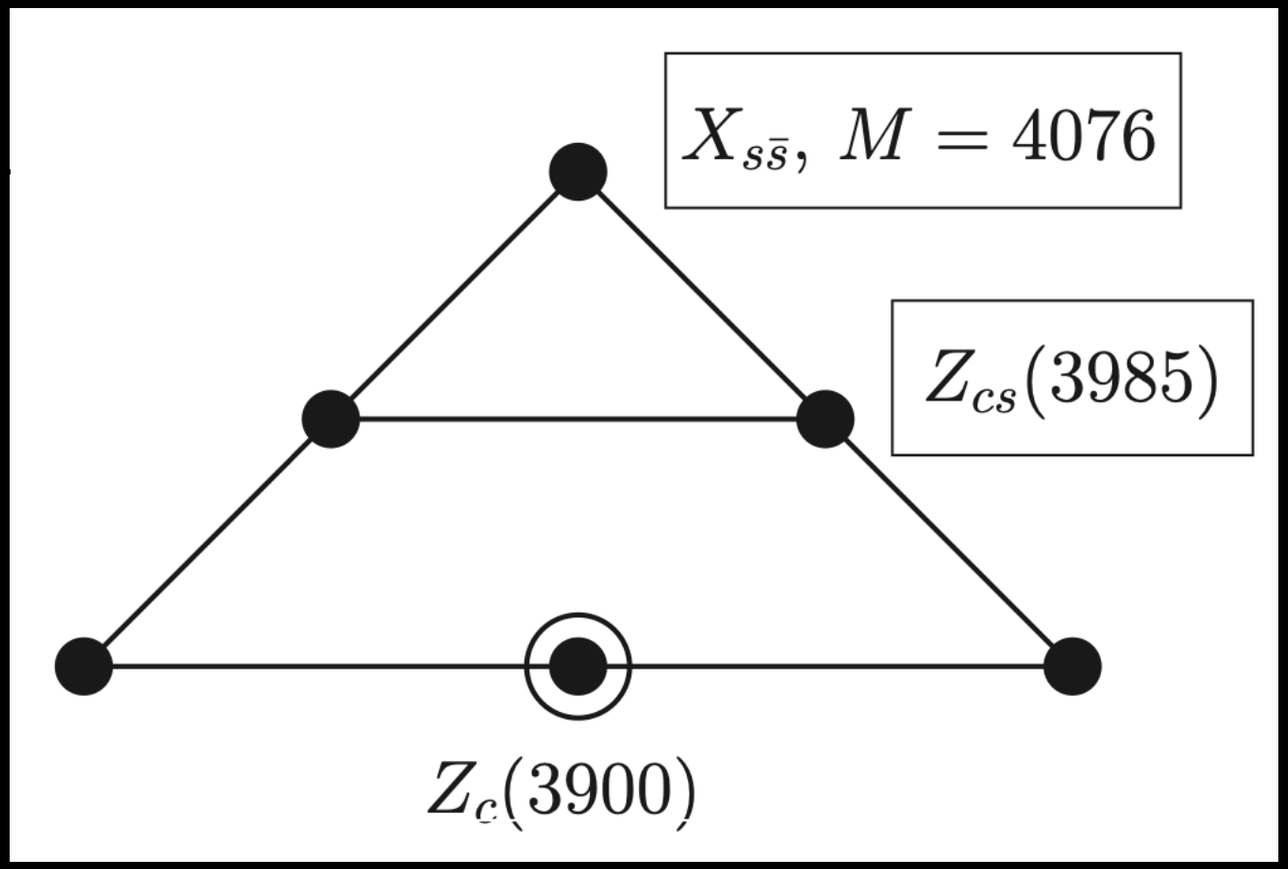
The $Z_{cs}(4003)$ was observed by LHCb in the decay

$$B^+ \to \phi + Z_{cs}^+(4003) \to \phi + K^+ + J/\psi$$

At a mass very close to that given by the equal spacing rule

NEGATIVE CHARGE CONJUGATION

The diquark-antidiquark model requires the quasi-degeneracy $M(X(1^{++})) = M(Z(1^{+-}))$ and we expect a similar multiplet for 1^{+-}



Prediction

Spacing = 188 MeV
wrt 200 MeV for $f_2' - a_2$

$[cs][\bar{c}\bar{q}] \vee [cq][\bar{c}\bar{s}]$

Maiani, ADP, Riquer
arXiv:2103.08331
Sci. Bulletin 66, 1616 (2021)

BESIII recently observed $e^+e^- \to K^+Z_{cs}^-(3985) \to K^+(D_s^{*-}D^0 + D_s^-D^{*0})$

FURTHER IMPLICATIONS

— BESIII observed $e^+e^- \to K^+Z_{cs}^-(3985) \to K^+(D_s^{*-}D^0 + D_s^-D^{*0})$

$Z_{cs}(4003)$ should appear here too

— Viceversa $Z_{cs}(3985)$ should be observed by LHCb

— A multiplet associated to $Z(4020)$ should also be observed

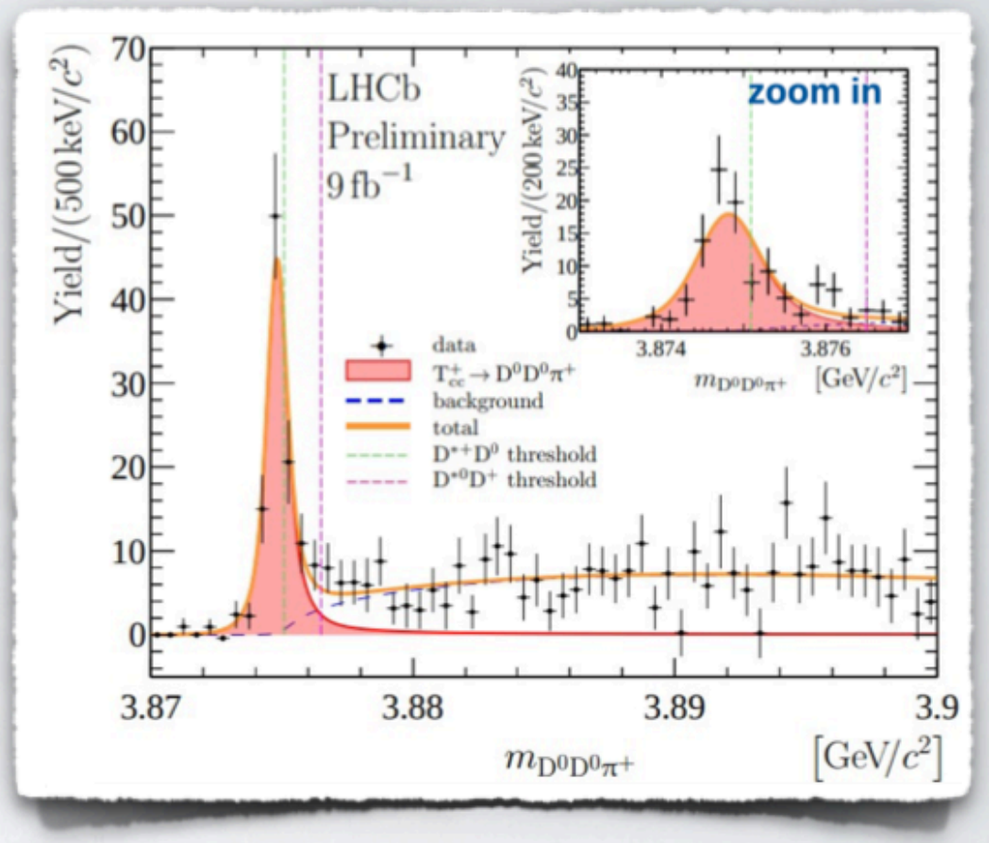
AN EMERGING PATTERN

Doubly charmed

- LHCb observed the first doubly charmed tetraquark $T_{cc}^{+}(cc\bar{u}\bar{d}) \to D^0 D^0 \pi^+$

- These states have been predicted/studied in several works

[see e.g. del Fabbro et al. – PRD (2005), hep-ph/0408258; Carames et al. – PLB (2011); Hyodo et al. – PLB (2013), 1209.6207; AE et al. – PRD (2013), 1307.2873]



[LHCb preliminary – talk presented at EPS-HEP 2021]

- Important for two reasons:

  1. Should be paired with a T_{bb} state far away from threshold

  2. $SU(3)$ multiplet contains doubly-charged states ⟶ cannot be meson molecules because of Coulomb repulsion

CONCLUSIONS

- It is always mentioned that SU(3) predicts ‘too many’ states; recently a new kind of tetraquark $[cc][qq']$ has been reported at EPS together with the evidence for a new pentaquark also.

- In quark models the vicinity to threshold is natural, from below and from above…

$X(3872)$	$Z_c^{0\pm}(3900)$	$Z_c^{0\pm}(4020)$	$Z_b^{0\pm}(10610)$	$Z_b^{0\pm}(10650)$
$D^0\bar{D}^{*0}$	$D^0\bar{D}^{*0\pm}$	$D^{*0}\bar{D}^{*0\pm}$	$B^0\bar{B}^{*0\pm}$	$B^{*0}\bar{B}^{*0\pm}$
$\delta \approx 0$	$+7.8$	$+6.7$ (MeV)	$+2.7$	$+1.8$

ADDITIONAL SLIDES

LINESHAPE OF THE $X(3872)$

The LHCb data

- To apply the composite criterion we must set $\Gamma_\rho^0 = \Gamma_\omega^0 = \Gamma_0^0 = 0$:

$$f\left(X \to J/\psi\pi^+\pi^-\right) \simeq -\frac{N\frac{2}{g_{LHCb}}}{\frac{2}{g_{LHCb}}\left(E - m_X^0\right) - \sqrt{2\mu_+\delta} + E\sqrt{\frac{\mu_+}{2\delta}} + ik}$$

- From here we extract

$$r_0 = -\frac{2}{\mu g_{LHCb}} - \sqrt{\frac{\mu_+}{2\mu^2\delta}} \simeq -5.34 \text{ fm}$$

[Esposito, Maiani, Pilloni, Polosa, Riquer – 2108.11413]

- The effective range is negative and well beyond m_π^{-1} $\longrightarrow$ the dynamics can only be explained if the X is an elementary, interacting tetraquark

BETHE-LANDAU-SMORODINSKY

$$r_0 = -\frac{Z}{1-Z}R + O(\frac{1}{m_\pi})$$

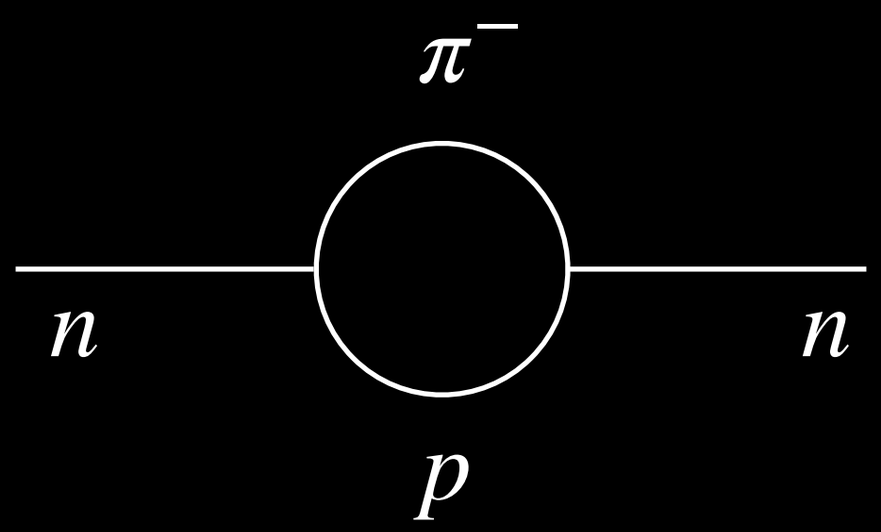
Fix the sign of corrections by the exact relation

$$r_0 \equiv 2\int_0^\infty (\psi_0^2 - u_0^2)\, dr > 0$$

Where u_0 is the solution to the S. equation with V and $k = 0$, whereas ψ_0 is the solution to the free equation, with $k = 0$.

For a proof see Esposito et al. 2108.11413

THE $\pi^- p$ MOLECULE EXAMPLE



$$|n, \text{in}\rangle = \sqrt{Z}\,|n, \text{bare}\rangle + \int_{\mathbf{k}} \Psi_\pi(\mathbf{k})\,|p\,\pi^-(\mathbf{k}), \text{bare}\rangle$$

$$\int_{\mathbf{k}} |\Psi_\pi(\mathbf{k})|^2 = 1 - Z$$

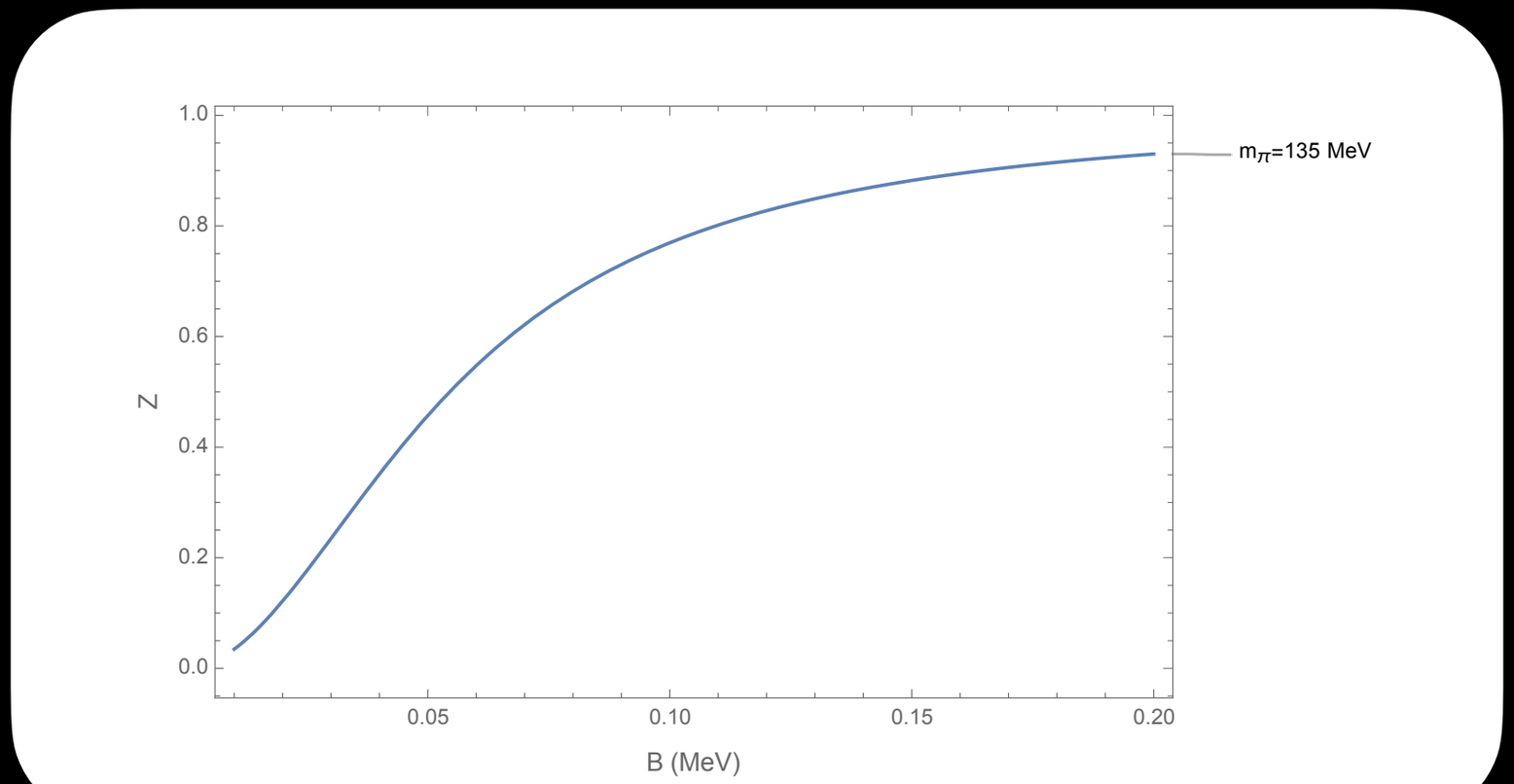
Same equations as Weinberg's

See the "Lee-model" ('54) in Henley & Thirring, Elementary Quantum Field Theory, McGraw-Hill
T.D. Lee, Phys. Rev. 95, 1329 (1954)

THE $\pi^- p$ MOLECULE EXAMPLE

$$Z^{-1} = 1 + \left(\frac{M_p}{F_\pi} g_A\right)^2 \frac{1}{2\pi^2} \int_0^\infty \frac{k^2}{(\sqrt{2m_\pi})^2 (B + \frac{k^2}{2(M_p + m_\pi)})^2 (1 + \frac{k^2}{M_A^2})^4} \, dk$$

Tune the mass of the neutron, i.e. tune B



See the "Lee-model" ('54) in Henley & Thirring, Elementary Quantum Field Theory, McGraw-Hill
T.D. Lee, Phys. Rev. 95, 1329 (1954)

FORM FACTORS & COMPOSITENESS

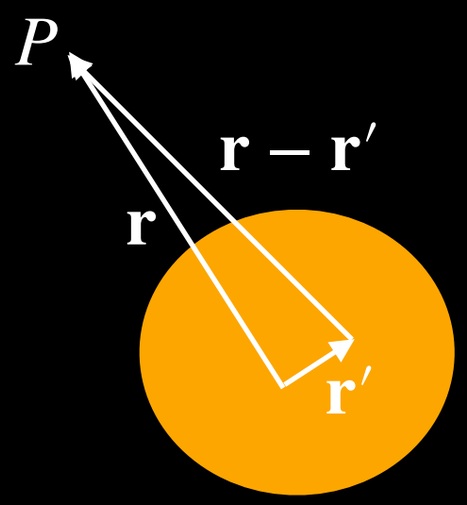
$$\delta\phi(P) = \int d^3r' \frac{\rho(\mathbf{r}')}{4\pi|\mathbf{r}-\mathbf{r}'|} = \int d^3r' \frac{1}{4\pi|\mathbf{r}-\mathbf{r}'|} \int d^3k\, e^{i\mathbf{k}\cdot\mathbf{r}'} F(\mathbf{k})$$

$$= \int d^3k\, F(\mathbf{k}) \int d^3\mathbf{r}' \frac{e^{i\mathbf{k}\cdot\mathbf{r}'}}{4\pi|\mathbf{r}-\mathbf{r}'|} = \int d^3k\, e^{i\mathbf{k}\cdot\mathbf{r}} \frac{F(\mathbf{k})}{\mathbf{k}^2}$$

$$= \int d^4x'\, i\Delta(x-x')\, J(x') \text{ with } J(x') \equiv \rho(\mathbf{r}')$$

$F(\mathbf{k})$ in place of e ⇒

the strength of the inter. depends on $\mathbf{k}$



THE ELEMENTARY DEUTERON

If $\Psi_Q = 0$ then $\mathscr{E} = -B$ ($B = 0$ if continuum)

If $\Psi_P = 0$ then $\mathscr{E} = \epsilon$ (positive or negative)

$$\mathscr{M} = \begin{pmatrix} -B & H' \\ H' & \epsilon \end{pmatrix}$$

$$\mathscr{M}_1 = \frac{1}{2}\left(-B + \epsilon - \sqrt{B^2 + 4H'^2 + 2B\epsilon + \epsilon^2}\right)$$

$$\mathscr{M}_2 = \frac{1}{2}\left(-B + \epsilon + \sqrt{B^2 + 4H'^2 + 2B\epsilon + \epsilon^2}\right)$$

NB: THE DERIVATION OF $r_0(Z)$ REQUIRES AT SOME STEP THAT $\mathscr{E} = -B$ (STRICTLY VALID IF $Z = 0$ OR $H_{PQ} = 0$).

$$|d\rangle = \sqrt{Z}\,|\mathfrak{d}\rangle + \int d\beta\, C_\beta\,|\beta\rangle \text{ with } \int d\beta\,|C_\beta|^2 = 1 - Z$$

$$\Rightarrow \int d\alpha\,|\langle\alpha|d\rangle|^2 = 1 - Z$$

then use

$$(H_0 + V)\,|d\rangle = \mathscr{E}\,|d\rangle$$

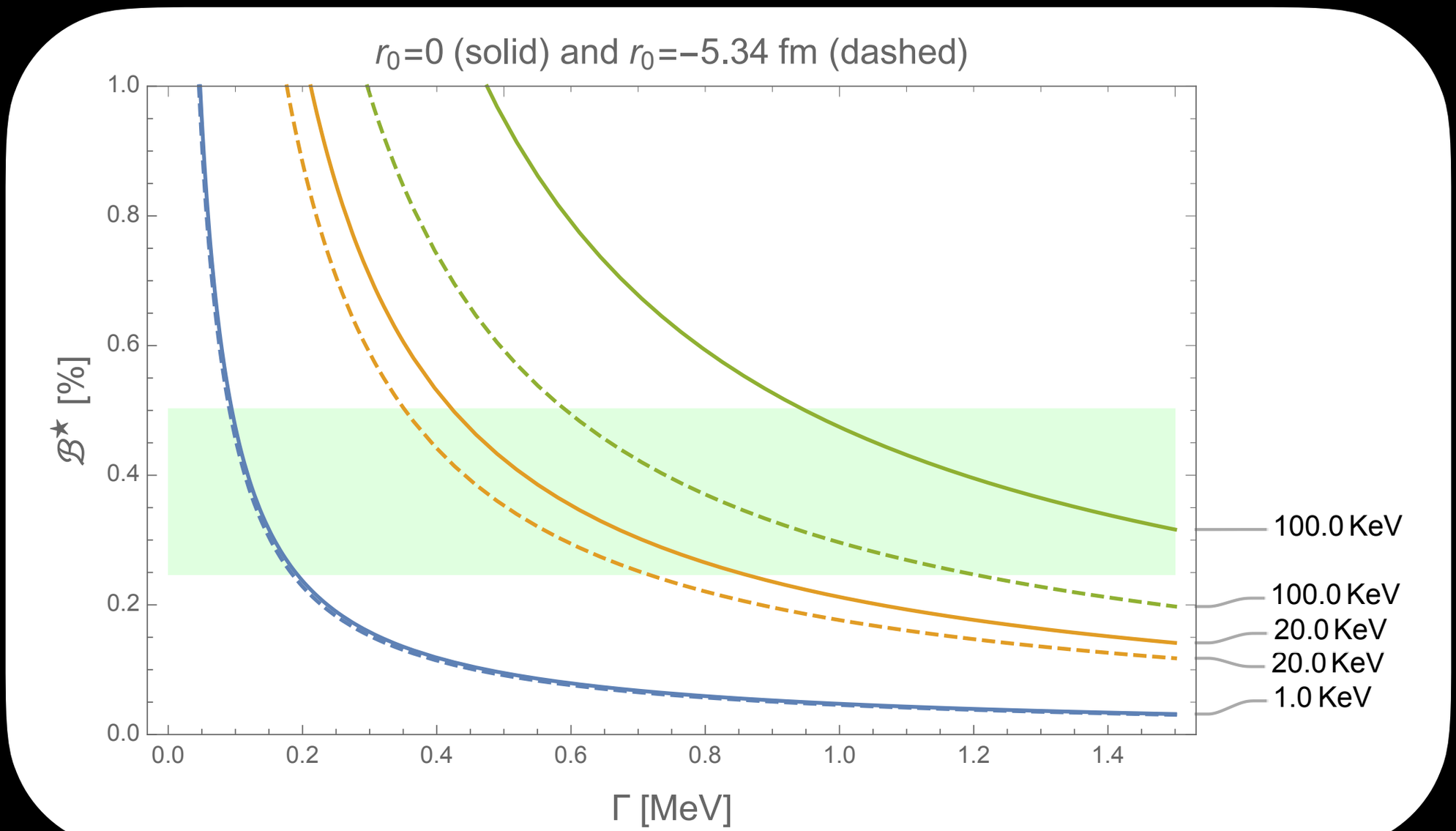
$$\Rightarrow \int d\alpha\,\frac{|\langle\alpha|V|d\rangle|^2}{E(\alpha) + B} = 1 - Z \text{ if } \mathscr{E} = -B$$

From here obtain a formula for $g \equiv |\langle\alpha|V|d\rangle|$;

$g = 0$ if $Z = 1$ (no cubic interaction!)

THE LANDAU COUPLING & $X \to DD\pi$

$$g^2 = 16\pi\sqrt{\frac{2B}{m}}\,\frac{(m_a+m_b)^2}{1-r_0\sqrt{2mB}}$$

To compute $\Gamma(X \to DD\pi)$

ADP, 1505.03083