Theoretical overview: Hadronization and coalescence

S. Plumari

Dipartimento di Fisica e Astronomia 'E. Majorana', Università degli Studi di Catania

INFN-LNS





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Ultra-relativistic heavy ion collisions

Hadronization of the QGP:

Transition from a deconfined medium composed of quarks, antiquarks and gluons to hadronic matter (color-neutral)



No first-principle description of hadron formation: Non-perturbative problem

Hadronization schemes



Indipendent fragmentation

Inclusive hadron production from hard-scattering processes (large Q²):

Factorization of: PDFs, partonic cross section (pQCD), fragmentation function

 $\frac{dN_h}{d^2p_h} = \sum_f \int dz \frac{dN_f}{d^2p_f} D_{f \to h}(z) \qquad \begin{array}{l} \mathbf{q} \neq \mathbf{\pi}, \, \mathbf{K}, \, \mathbf{p}, \, \mathbf{\Lambda} \, .. \\ \mathbf{c} \neq \mathbf{D}, \, \mathbf{D}_{\mathrm{s}}, \, \mathbf{\Lambda}_{\mathrm{c}}, \, ... \end{array}$

Fragmentation function

Fragmentation functions $D_{f \rightarrow h}$ are phenomenological functions to parameterize the *non-perturbative parton-to-hadron transition* z = fraction of the parton momentum taken by the hadron h**Fragmentation functions**assumed**universal**among energyand collision systems and constrained from e⁺e⁻and ep

Cluster Fragmentation (HERWIG)

Initially developed for e⁺e⁻ collisions (B.R. Webber, NPB 238 (1984), 492)

- Parton shower of both the initial partons involved in the collision and the particle produced in the collision are evolved perturbative down to a softer scale Q₀
- Multiple scattering
- Non-perturbative gluon splitting in into qq pairs
- Identify colour-singlet clusters of partons
- Final Clusters decay into hadrons



Hadronization: fragmentation and coalescence

Proton to pion ratio Enhancement:

In vacuum from fragmentation functions the ratio is small $\frac{D_{q \to p}(z)}{D_{q \to \pi}(z)} < 0.25$

Elliptic flow splitting:

For $p_T>2$ GeV Both hydro and fragmentation predicts similar v_2 for pions and protons

Another hadronization mechanism is by coalescence:

Formalism originally developed for light-nuclei production from coalescence of nucleons on a freezeout hypersurface.

Extended to describe meson and baryon formation in AA collisions from the quarks of QGP through $2\rightarrow 1$ and $3\rightarrow 1$ processes

V. Greco, C.M. Ko, P. Levai PRL 90, 202302 (2003).
V. Greco, C.M. Ko, P. Levai PRC 68, 034904 (2003).
R.J. Fries, B. Muller, C. Nonaka, S.A. Bass PRL 90, 202303 (2003).
R.J. Fries, B. Muller, C. Nonaka, S.A. Bass PRC 68,044902 (2003).



Hadronization in medium: Coalescence

Phase space at the hadronization is filled with partons

- No partons in the vacuum but a thermal ensemble of partons
- No need to create qq pairs via splitting or string breaking
- Partons that are "close" to each other in phase space (position and momentum) can simply recombine into hadrons

Coalescence

- partons are already there to be close in phase space
- p_h= n p_{T,}, n = 2, 3 baryons from lower p_T (denser)



Fragmentation

energy to create quarks from vacuum

hadrons from higher p_T

Hadronization in medium: Coalescence

0.5

0

0.5

1.0

1.5

 $(m_{\tau} - m_{o})/n (\text{GeV}/c^{2})$

2.0

2.5



Recombination of soft partons (thermal) with mini-jet partons Inclusion of contribution of resonance decays

- Coalescence is dominant at low p_T
- Fragmentation is dominant at high p_T
- Radial flow of partons (from blast-wave) needed to describe the data



Transport approaches

momentum diffusion coeff.

Two main approaches:

1) Fokker-Planck (T<<m_a soft scattering)

[TAMU, Duke, Nantes, Torino, Catania, ...]

 $\frac{\partial}{\partial t} f_Q = \gamma \frac{\partial}{\partial p_i} [p_i f_Q] + D_p \nabla_p^2 [f_Q] \qquad \text{Background:} \\ \text{Hydro/transport expanding bulk}$

Drag coeff. (thermalization rate)

- Fluctuation dissipation theorem $D_p = ET \gamma$

- Spatial diffusion coefficient
$$D_s = \frac{T}{M_y} = \frac{T^2}{D_p} = \frac{T}{M} \tau_{th} \qquad \langle x^2 \rangle - \langle x \rangle^2 = 6 D_s t$$

a measure of thermalization time

D_c from IQCD

2) Boltzman kinetic transport

(...Kadanoff-Baym-PHSD) [Catania, Nantes, Frankfurt, LBL,...]

 $p^{\mu}\partial_{\mu}f_{O}(x,p)=C[f_{a},f_{a},f_{O}]$

$$\begin{split} &C[f_q, f_g, f_Q] = \frac{1}{2E_1} \int \frac{d^3 p_2}{2E_2(2\pi)^3} \int \frac{d^3 p_1}{2E_1'(2\pi)^3} \\ &\times [f_Q(p_1') f_{q,g}(p_2') - f_Q(p_1) f_{q,g}(p_2)] \\ &\times [M_{(q,g) \rightarrow Q}(p_1 p_2 \rightarrow p_1' p_2')] \\ &\times (2\pi)^4 \delta^4(p_1 + p_2 - p_1' - p_2') \end{split}$$

Transport coefficient



Models not really tested at $p \rightarrow 0$ The new data \rightarrow determine $D_s(T)$ more properly, i.e. $p \rightarrow 0$ where it is defined and computed in IQCD

							2018-2019
	Catania	Duke	$\operatorname{Frankfurt}(\operatorname{PHSD})$	LBL	Nantes	TAMU	
Initial HQ (p)	FONLL	FONLL	pQCD	pQCD	FONLL		Several Collab. In joint activities:
Initial HQ (x)	binary coll.	binaryy coll.	binary coll.	binary coll.		binary coll.	- EMMI-RRTF:
Initial QGP	Glauber	Trento	Lund		EPOS		R. Rapp et al., Nucl. Phys. A 979 (2018)
QGP	Boltzm.	Vishnu	Boltzm.	Vishnu	EPOS	2d ideal hydro	
partons	mass	m=0	m(T)	m=0	m=0	m=0	- HQ-JEIS:
formation time QGP	$0.3~{\rm fm/c}$	$0.6~{\rm fm/c}$	$0.6~{\rm fm/c}$ (early coll.)	0.6 fm/c	0.3 fm/c	0.4 fm/c	S. Cao et al.,Phys. Rev. C 99 (2019)
interactions in between	HQ-glasma	no	HQ-preformed plasma	no		no	- Y. Xu et al., Phys. Rev. C 99 (2019)

Transport coefficient



the extraction of the charm quark diffusion coefficient New joint activity needed



2018-2019 Several Collab. in joint activities:

- EMMI-RRTF:
- R. Rapp et al., Nucl. Phys. A 979 (2018) - HQ-JETS:
 - S. Cao et al., Phys. Rev. C 99 (2019)
- Y. Xu et al., Phys. Rev. C 99 (2019)

Coalescence approach in phase space for HQ



Wigner function <-> Wave function

$$\Phi_M^W(\mathbf{r},\mathbf{q}) = \int d^3 r' e^{-i\mathbf{q}\cdot\mathbf{r}'} \varphi_M\left(\mathbf{r}+\frac{\mathbf{r}'}{2}\right) \varphi_M^*\left(\mathbf{r}-\frac{\mathbf{r}'}{2}\right)$$

 $\varphi_M(\mathbf{r})$ meson wave function Assuming gaussian wave function

$$f_M(x_1, x_2; p_1, p_2) = A_W \exp\left(-\frac{x_{r1}^2}{\sigma_r^2} - p_{r1}^2 \sigma_r^2\right)$$

For baryon $N_q=3$

$$f_H(...) = \prod_{i=1}^{N_q-1} A_W \exp\left(-\frac{x_{ri}^2}{\sigma_{ri}^2} - p_{ri}^2 \sigma_{ri}^2\right)$$

<u>Note</u>: only σ_r coming from $\varphi_M(\mathbf{r})$ or $\sigma_r^* \sigma_p = 1$ valid for harmonic oscillator with V(r) $\sigma_r^* \sigma_p > 1$ Wigner function **width** fixed by root-mean-square charge radius from **quark model**

Meson	$\langle r^2 \rangle_{ch}$	σ_{p1}	σ_{p2}
$D^+ = [c\bar{d}]$	0.184	0.282	
$D_s^+ = [\bar{s}c]$	0.083	0.404	
Baryon	$\langle r^2 \rangle_{ch}$	σ_{p1}	σ_{p2}
$\Lambda_c^+ = [udc]$	0.15	0.251	0.424
$\Xi_c^+ = [usc]$	0.2	0.242	0.406
$\Omega_c^0 = [ssc]$	-0.12	0.337	0.53

C.-W. Hwang, EPJ C23, 585 (2002); C. Albertus et al., NPA 740, 333 (2004) $\langle r^2 \rangle_{ch} = \frac{3}{2} \frac{m_2^2 Q_1 + m_1^2 Q_2}{(m_1 + m_2)^2} \sigma_{r1}^2$ (8) $+ \frac{3}{2} \frac{m_3^2 (Q_1 + Q_2) + (m_1 + m_2)^2 Q_3}{(m_1 + m_2 + m_3)^2} \sigma_{r2}^2$ $\sigma_{ri} = 1/\sqrt{\mu_i \omega}$ Harmonic oscillator relation $\mu_1 = \frac{m_1 m_2}{m_1 + m_2}, \ \mu_2 = \frac{(m_1 + m_2) m_3}{m_1 + m_2 + m_3}.$

Normalization $f_H(...)$ fixed by requiring $P_{coal}(p>0)=1$ which fixes A_w , additional assumption wrt standard coalescence which does not have confinement

Coalescence approach in phase space for HQ





S. Plumari, V. Minissale et al., Eur. Phys. J. **C78** no. 4, (2018) 348

- ♦ Normalization in $f_W(...)$ fixed by requiring $P_{coal}(p>0)=1$:others modify by hand σ_r to enforce confinement for a charm at rest in the medium
- ♦ The charm not "coalescencing" undergo fragmentation:

$$\frac{dN_{had}}{d^2 p_T \, dy} = \sum \int dz \frac{dN_{fragm}}{d^2 p_T \, dy} \frac{D_{had/c}(z, Q^2)}{z^2}$$

charm number conserved at each p_T, we have employed e⁺e⁻ FF now PYTHIA

RHIC: results



Data from STAR Coll., arXiv:1704.04364 [nucl-ex].

2

2

3

4 p_T (GeV)

10

3

4

p_T (GeV)

5

5

6

6

charm

coalescence

fragmentation coal + fragm

7

8

0

charm

 coalescence
 fragmentation coal + fragm

D_s⁺ STAR (0-10) %

RHIC: Baryon/meson

S. Plumari, et al., Eur. Phys. J. C78 no. 4, (2018) 348

Coalescence

Following: L.W.Chen, C.M. Ko, W. Liu, M. Nielsen, PRC 76, 014906 (2007). K.-J. Sun, L.-W. Chen, PRC 95, 044905 (2017). For hypersurface of proper time τ and non relativistic limit: for $p_T \ll m \frac{\Lambda_c^+}{D^0} \propto \frac{g_\Lambda}{g_D} \left(\frac{m_T^\Lambda}{m_T^D}\right) e^{-(m^\Lambda - m^D)/T_C} \tau \mu_2$ $\mu_2 = \frac{m_3(m_1 + m_2)}{m_1 + m_2 + m_3}$ Is the reduced mass of the baryon

STAR coll. arXiv:1910.14628







RHIC: Baryon/meson

Recent improvements of the coalescence model Wigner function modified including both s and p wave states



Stronger QGP flow boost on heavier hadrons => increasing Λ_c/D^0 ratio with Npart

harder initial charm spectra at LHC reduces the Λ_c/D^0 ratio



LHC: results



wave function widths σ_p of baryon and mesons kept the same at RHIC and LHC!



The Λ_c/D^0 ratio is smaller at LHC energies: fragmentation play a role at intermediate p_T

S. Plumari, et al., Eur. Phys. J. C78 no. 4, (2018) 348

Resonance Recombination Model (RRM)

Alternative dynamical realization of the coalescence approach

Hadronization proceeds via formation of resonant states when approaching the critical temperature

Starting point is the Boltzmann equation for the meson

$$\left(\frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla}\right) F_M(t, \vec{x}, \vec{p}) = -\frac{\Gamma}{\gamma_p} F_M(t, \vec{x}, \vec{p}) + \beta(\vec{x}, \vec{p}) \quad \begin{array}{l} \Gamma \text{ width attributed to 2-body decays} \\ \mathsf{M} \to \mathsf{q} + \bar{\mathsf{q}}, \end{array}$$

The gain term

$$g(\vec{p}) = \int \mathrm{d}^3 x \beta(\vec{x}, \vec{p}) = \int \frac{\mathrm{d}^3 p_1 \mathrm{d}^3 p_2}{(2\pi)^6} \int \mathrm{d}^3 x \ f_q(\vec{x}, \vec{p}_1) \ f_{\bar{q}}(\vec{x}, \vec{p}_2) \ \sigma(s) \ v_{\mathrm{rel}}(\vec{p}_1, \vec{p}_2) \ \delta^{(3)}(\vec{p} - \vec{p}_1 - \vec{p}_2)$$

The cross section (**q+q** \rightarrow **M**) is approximated $\sigma(s) = g_{\sigma} \frac{4\pi}{k^2} \frac{(\Gamma m)^2}{(s-m^2)^2 + (\Gamma m)^2}$ by a relativistic Breit-Wigner

By imposing the stationarity condition at the equilibrium

$$\begin{split} f_M(\vec{x},\vec{p}) &= \frac{\gamma_M(p)}{\Gamma_M} \int \frac{d^3 \vec{p_1} d^3 \vec{p_2}}{(2\pi)^3} f_q(\vec{x},\vec{p_1}) f_{\bar{q}}(\vec{x},\vec{p_2}) \ \sigma_M(s) v_{\rm rel}(\vec{p_1},\vec{p_2}) \delta^3(\vec{p}-\vec{p_1}-\vec{p_2}) \\ \text{L. Ravagli and R. Rapp, Phys. Lett. B 655, 126 (2007).} \end{split}$$

L. Ravagli, H. van Hees and R. Rapp, Phys. Rev. C 79, 064902 (2009).

Resonance Recombination Model (RRM)

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The gain term

$$g(\vec{p}) = \int \mathrm{d}^3 x \beta(\vec{x}, \vec{p}) = \int \frac{\mathrm{d}^3 p_1 \mathrm{d}^3 p_2}{(2\pi)^6} \int \mathrm{d}^3 x \ f_q(\vec{x}, \vec{p}_1) \ f_{\bar{q}}(\vec{x}, \vec{p}_2) \ \sigma(s) \ v_{\mathrm{rel}}(\vec{p}_1, \vec{p}_2)$$

The cross section $(q+q \rightarrow M)$ is approximated by a relativistic Breit-Wigner



By imposing the stationarity condition at the equilibrium

$$f_M(\vec{x}, \vec{p}) = \frac{\gamma_M(p)}{\Gamma_M} \int \frac{d^3 \vec{p_1} d^3 \vec{p_2}}{(2\pi)^3} f_q(\vec{x}, \vec{p_1}) f_{\bar{q}}(\vec{x}, \vec{p_2}) \ \sigma_M(s) v_{\rm rel}(\vec{p_1}, \vec{p_2}) \delta^3(\vec{p} - \vec{p_1} - \vec{p_2})$$

L. Ravagli and R. Rapp, Phys. Lett. B 655, 126 (2007).L. Ravagli, H. van Hees and R. Rapp, Phys. Rev. C 79, 064902 (2009).

Baryons in Resonance Recombination Model (RRM)

The 3-body hadronization process in RRM are conducted in 2 steps

□ STEP 1

quark-1 and quark-2 recombine into a diquark, $q1(p1) + q2(p2) \rightarrow dq(p12)$ The diquark spectrum in analogy to meson formation

STEP 2

the diquark recombines with quark-3 into a baryon $dq1(p12) + q3(p3) \rightarrow B$

The baryon spectrum in analogy to meson formation

Space-momentum correlations included



- low-p_T(0-1GeV) c quarks preferentially populate the inner regions of the fireball
- higher-p_T (3-4GeV) c quarks populate the outer regions of the fireball



Baryons in Resonance Recombination Model (RRM)

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The baryon spectrum in analogy to meson formation

$$f_B(\vec{x}, \vec{p}) = \frac{\gamma_B}{\Gamma_B} \int \frac{d^3 \vec{p}_1 d^3 \vec{p}_2 d^3 \vec{p}_3}{(2\pi)^6} \frac{\gamma_{dq}}{\Gamma_{dq}} f_1(\vec{x}, \vec{p}_1) f_2(\vec{x}, \vec{p}_2) \\ \times f_3(\vec{x}, \vec{p}_3) \sigma_{dq}(s_{12}) v_{\rm rel}^{12} \sigma_B(s) v_{\rm rel}^{dq3} \delta^3(\vec{p} - \vec{p}_1 - \vec{p}_2 - \vec{p}_3)$$

HF hadro-chemistry improved by employing a large set of "missing" HF baryon states not listed by PDG, but predicted by the relativistic-quark model

M. He, R. Rapp, Phys. Rev. Lett. **124** (2020) no.4, 042301



Hadronization: SHMc (Baryon/meson)

charmed hadrons included into the SHM with thermal distributions, while the total charm content of the fireball is fixed by the measured open charm cross section.

SHMc yields + blast wave $\rightarrow p_{T}$ spectra

-low pT near fully thermalized

A. Andronic et al, JHEP 07 (2021) 035





P-Glasma + MUSIC (3+1)D

10 r (fm)

Centrality 0-10% |y| < 0.9

0.5

Small systems



ALICE coll. Nature Phys. 13 (2017) 535

Small systems

Traditional view:

- QGP in Pb+Pb
- no QGP in p+p ("baseline")



- Too few particles, cannot be collective
- System not in equilibrium

ALICE coll, arXiv:2105.06335



- Indication that fragmentation depends on the collision system
- Assumption of their universality not supported by the measured cross sections

Grand canonical SHM + fragmentation

M. He & R. Rapp, PLB795(2019)117-121





assuming independent fragmentation of charm quarks but with the hadronic ratios fixed by the SHM, and then excited states decayed into ground state charm-hadrons

$$n_i = \frac{d_i}{2\pi^2} m_i^2 T_H K_2(\frac{m_i}{T_H})$$

the enhanced feeddown from excited charm baryons can account for the Λc /D ratio measured

$n_i \; (\cdot 10^{-4} \; {\rm fm}^{-3})$	D^0	D^+	D^{*+}	D_s^+	Λ_c^+	$\Xi_c^{+,0}$	Ω_c^0
PDG(170)	1.161	0.5098	0.5010	0.3165	0.3310	0.0874	0.0064
PDG(160)	0.4996	0.2223	0.2113	0.1311	0.1201	0.0304	0.0021
RQM(170)	1.161	0.5098	0.5010	0.3165	0.6613	0.1173	0.0144
RQM(160)	0.4996	0.2223	0.2113	0.1311	0.2203	0.0391	0.0044

Small systems: Coalescence in pp?







Small systems: Coalescence in pp?



Conclusion

• Light flavour production at intermediate p_T

Intermediate p_T sensitive to hadronization via recombination baryon/meson enhancement described by coalescence+fragmentation

• Charm hadronization in AA different than in e⁺e⁻ and ep collisions

-Coalescence+fragmentation/Resonance Recombination Model enhancement of Λ_c production at intermediate $p_T \rightarrow \Lambda_c/D^0 \sim 1$ for $p_T \sim 3 \text{ GeV}$ -SHM with charm provide information on charm quark thermalization at low p_T

• In p+p assuming a medium:

- Coal.+fragm. good description of heavy baryon/meson ratio (closer to the data for $\Lambda_{\rm c}/{\rm D^0}$, $\Xi_{\rm c}/{\rm D^0}$, $\Omega_{\rm c}/{\rm D^0}$)
- SHM+fragmentation able to capture the $\Lambda_{\rm c}$ production