Theoretical overview on equilibrium and out-of-equilibrium hydrodynamics

Eduardo Grossi
IPhT Saclay, Ecole Polytechnique

E.G., A.Soloviev, D. Teany, F. Yan PRD (2020)
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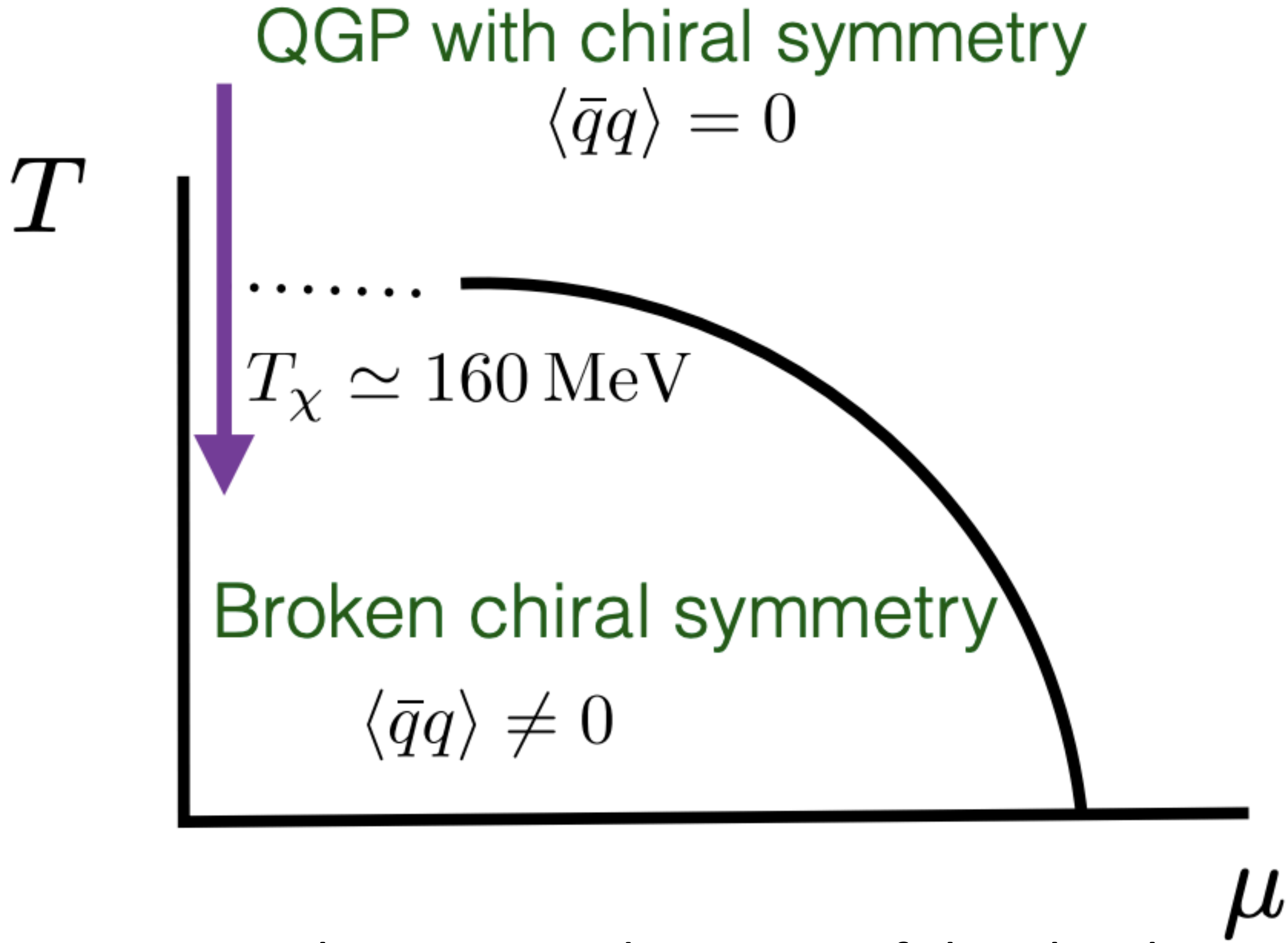
A. Florio, E.G., A. Soloviev, D, Teany 2111.03640

Padova 24/11/2021, Terzo incontro terzo incontro sulla fisica dei ioni pesanti alle alte energie

Motivation 1



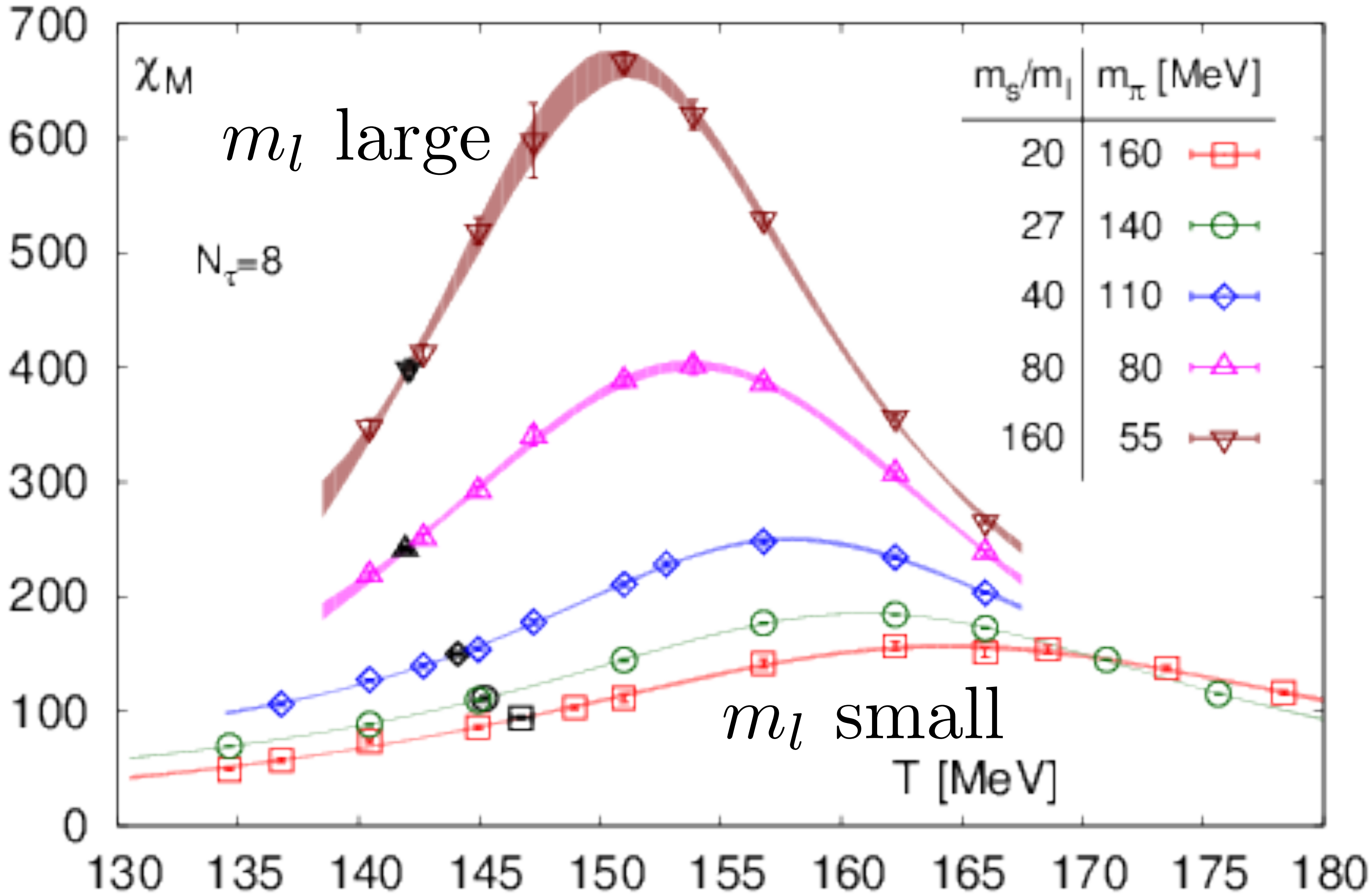
We are neglecting any dynamics of the chiral condensate !

The pions are the goldstone boson of the spontaneous symmetry break

Motivation 2

$$\chi_M = \frac{\partial \langle \bar{q} q \rangle}{\partial m_l}$$

[Ding et al PRL 2019]



The independent left and right rotation of the flavour of the quark is spontaneous broken.

The Universality class is the same of O(4) scalar field

The chiral susceptibility seams to respect the scaling as predicted from O(4) universality class in d=3

Chiral summery

The (approximated) conserved quantities are of 2 flavour QCD

$T^{\mu\nu}$	J_V^μ	J_A^μ
Stress	Iso-vector (isospin)	Iso-axial
(T, u^μ)	μ_V	μ_A
	$\bar{q}\gamma^0 t_I q$	$\bar{q}\gamma^0\gamma_5 t_I q$

The approximate flavour symmetry $SU(2)_L \times SU(2)_R \sim O(4)$

The order parameter is the chiral condensate

$$\langle \bar{q}q \rangle \sim \phi_\alpha = (\sigma, \varphi_\alpha)$$

sigma pions

The pion are the pseudo goldstone boson and they decay at large distance due to the small quark mass

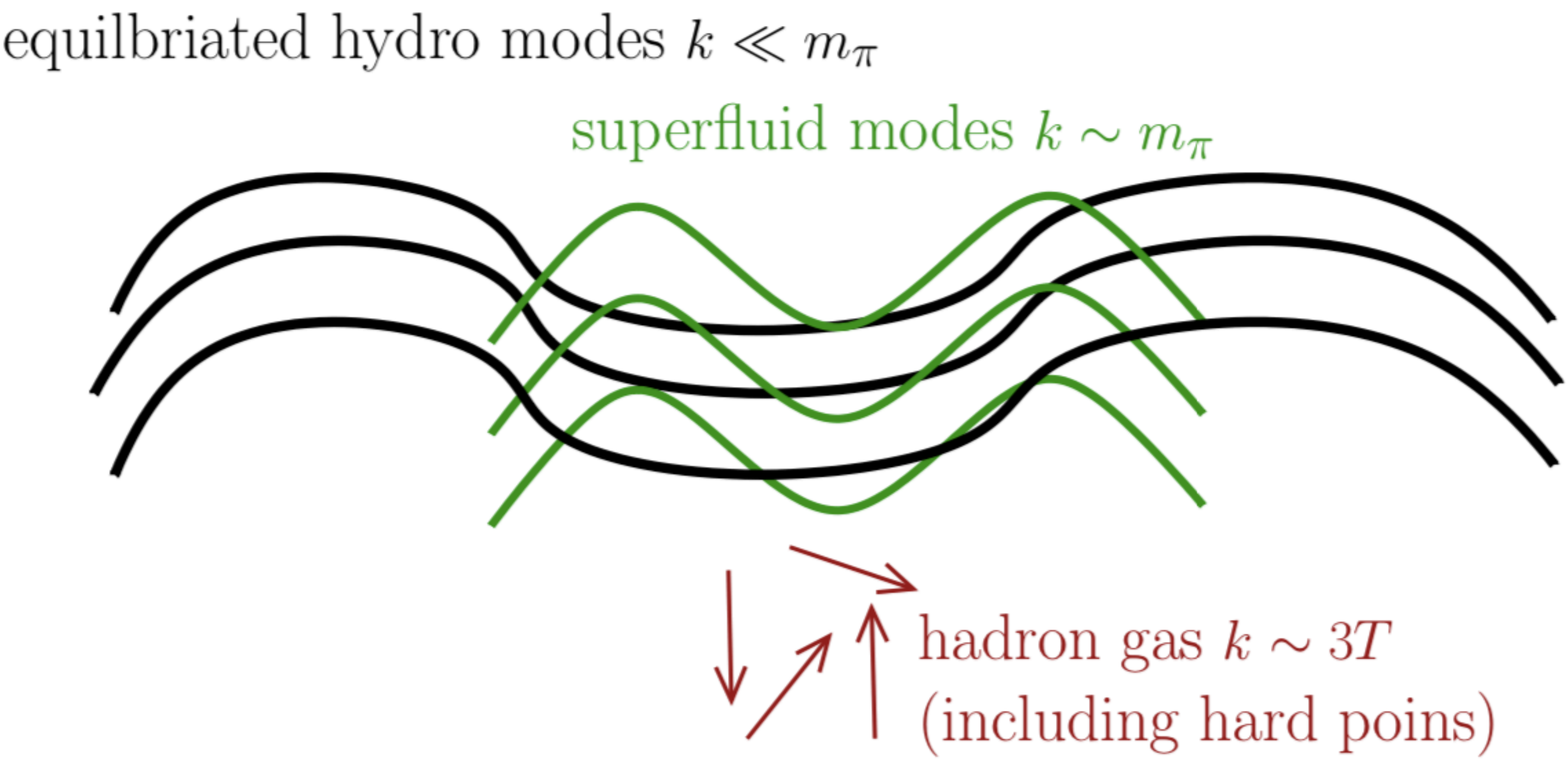
The order parameter can be considered as state variables like temperature

We need the theory of superfluid hydro (Son '99)

Physical picture T<<Tc

Working regime

$$k \ll m_\pi \ll \pi T \sim \pi \Lambda_{QCD}$$



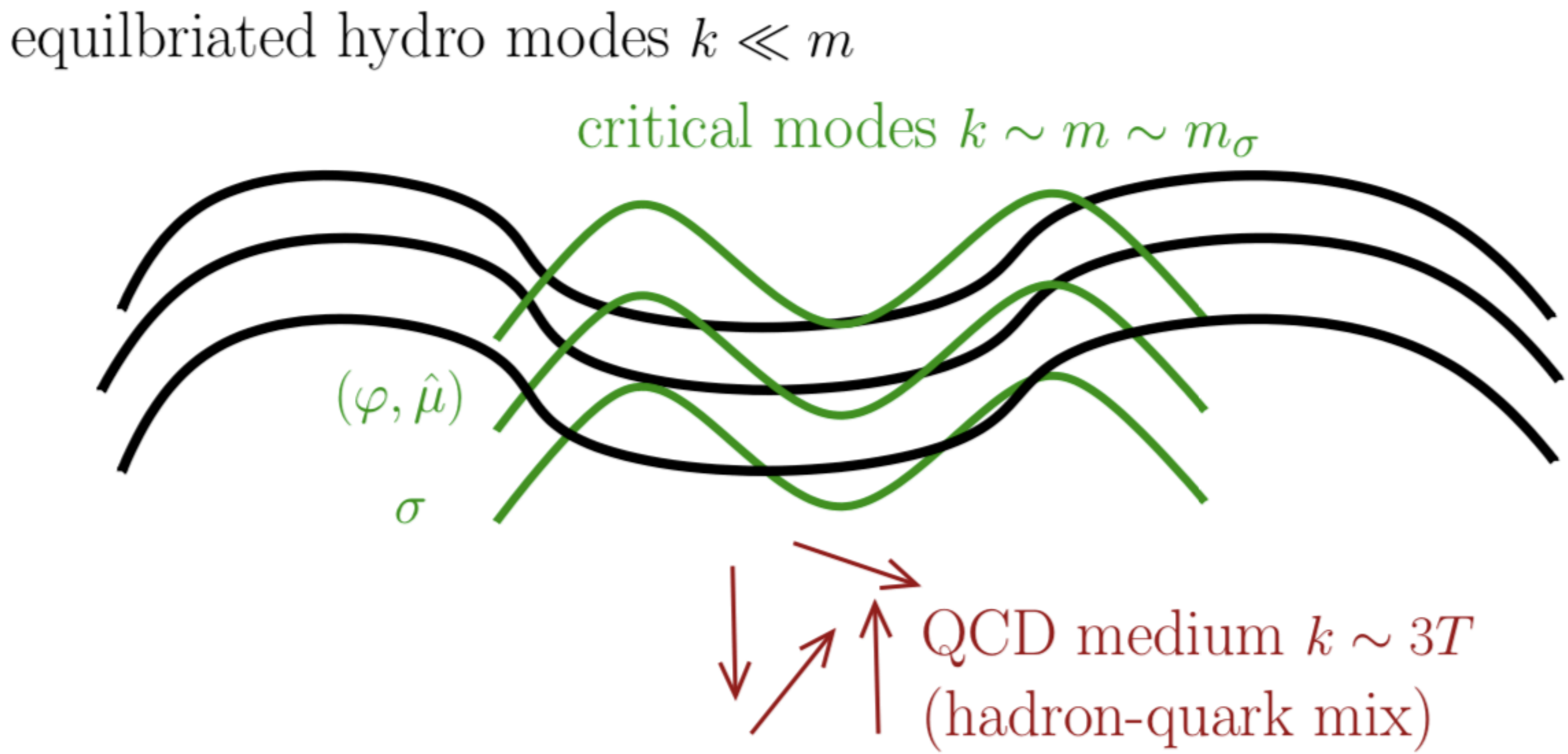
Soft pions mode on Hydro

Son hep-ph/9912267;
Son and Stephanov hep-ph/020422
Wilczek-Rajagopal hep-ph/9210253 + little bit us

Physical picture T~Tc

Working regime

$$k \ll m \sim m_\sigma \ll \pi T_C \sim \pi \Lambda_{QCD}$$



Soft pions+sigma mode on Hydro

Son hep-ph/9912267;

Son and Stephanov hep-ph/020422

Wilczek-Rajagopal hep-ph/9210253 + little bit us

Linearized equation

We linearize the equation around equilibrium (mean field)

$$\phi_\alpha = (\bar{\sigma} + \delta\sigma, \bar{\sigma}\varphi_a)$$

Axial charge and pion equation (assuming zero vector chemical potential)

$$\partial_t \varphi = -\mu_A + \Gamma\left(\nabla^2 - m^2\right)\varphi,$$

$$\partial_t \mu_A = v^2(-\nabla^2 + m^2)\varphi + D_0 \nabla^2 \mu_A,$$

Sigma equation

$$\partial_t \delta\sigma = \Gamma\left[\nabla^2 - m_\sigma^2\right]\delta\sigma,$$

The parameter depends on the temperature and on the value of the fields at the minimum

$$v^2 = \frac{\bar{\sigma}^2}{\chi} \qquad m^2 = \frac{H}{\bar{\sigma}} \qquad \Gamma = \text{const} \qquad D_0 = \text{const}$$

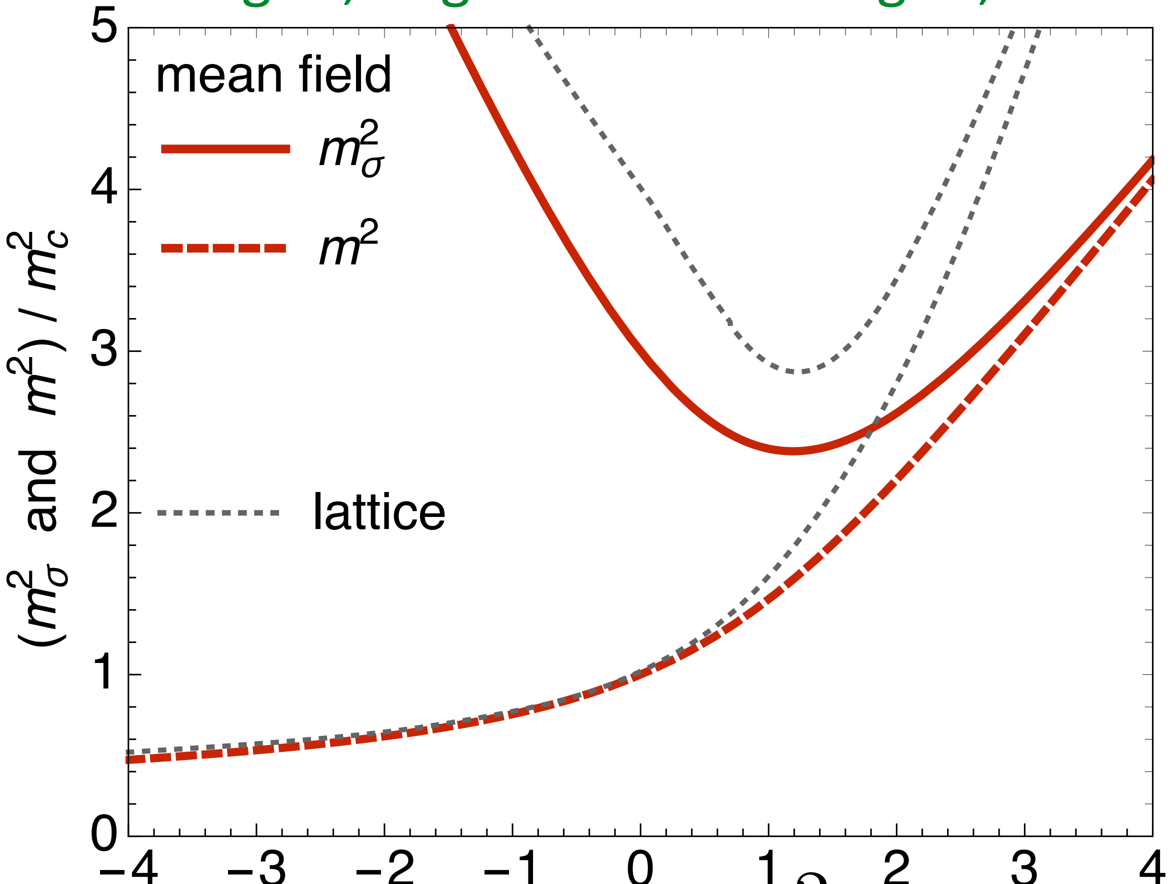
Solving for the pion we get a dumped Klein Gordon equation

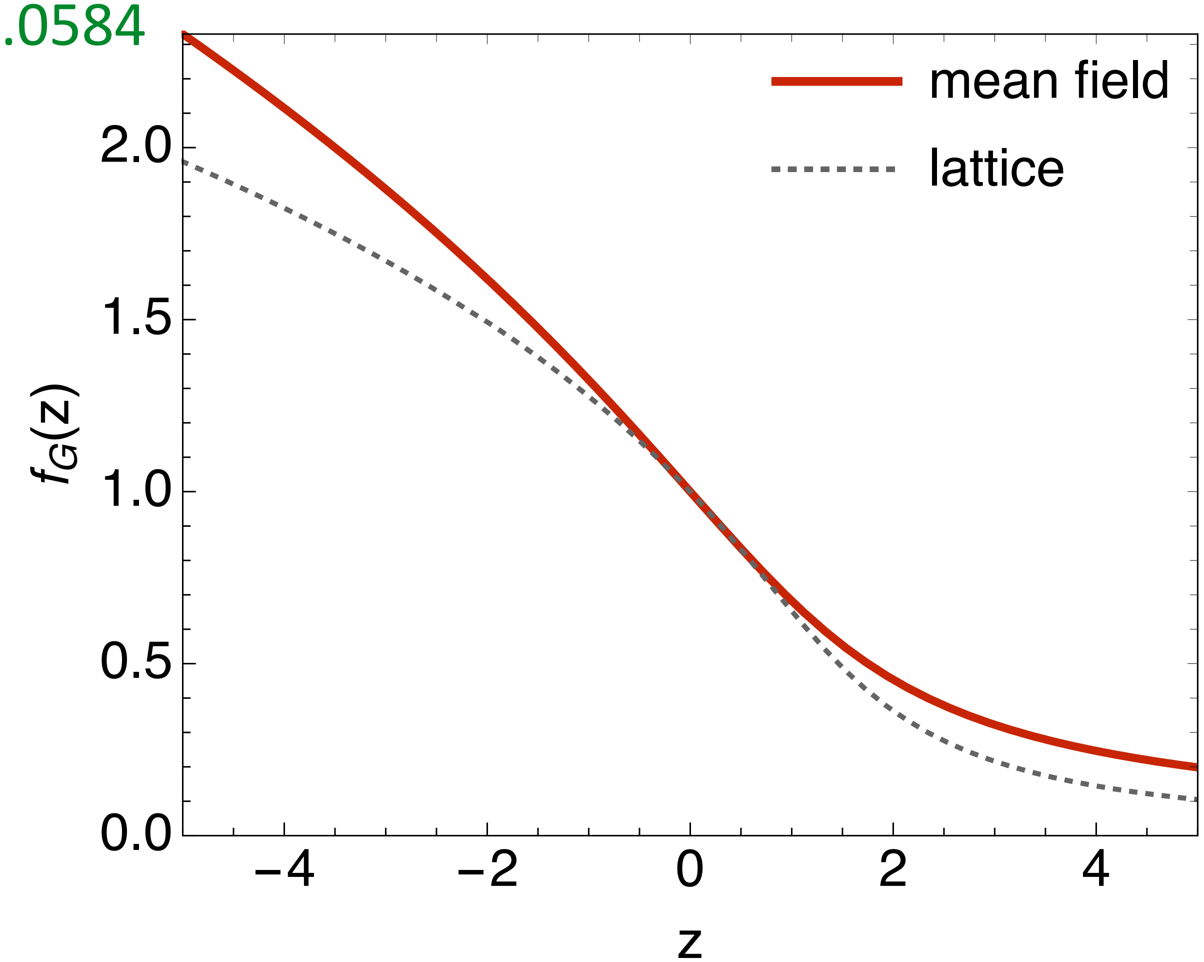
Mean field equation

In mean field one can easily find how the minimum of the potential change and the mass of pions and sigma

Masses $$\left.\frac{\partial V}{\partial \sigma}\right|_{\bar\sigma} = H$$ Minimum

Lattice: Engels, Vogt 0911.1939. Engels, Karsch 1105.0584





$$m^2 \equiv \frac{H}{\bar\sigma} = \frac{m_c^2}{f_G(z)} \qquad z = h^{2/3}\left(\frac{T-T_c}{T_c}\right) \qquad \bar\sigma = h^{1/3} f_G(z)$$

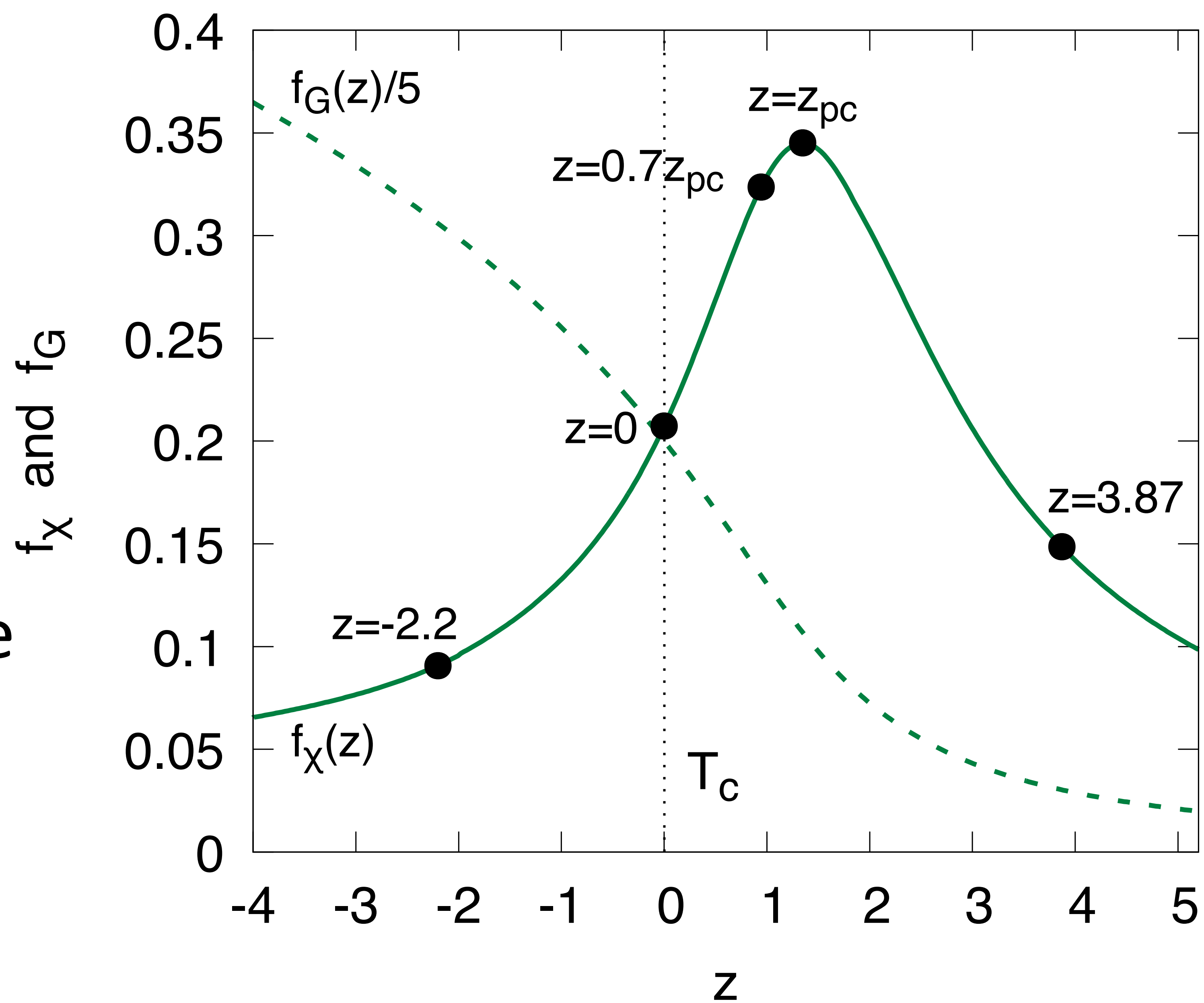
$$m_\sigma^2 \equiv m_c^2\left(z + 3 f_G^2(z)\right)$$

Real time lattice simulation

Florio et al 2111.03640

We perform real time lattice simulation for the model in equilibrium (called Model G)

The static property are the same the introduction of the conserved charged change the dynamic universality class (the scaling exponent)



$$\partial_t \phi_a + g_0\, \mu_{ab}\phi_b = \Gamma_0 \nabla^2 \phi_a - \Gamma_0 (m_0^2 + \lambda\phi^2)\phi_a + \Gamma_0 H_a + \theta_a\,,$$

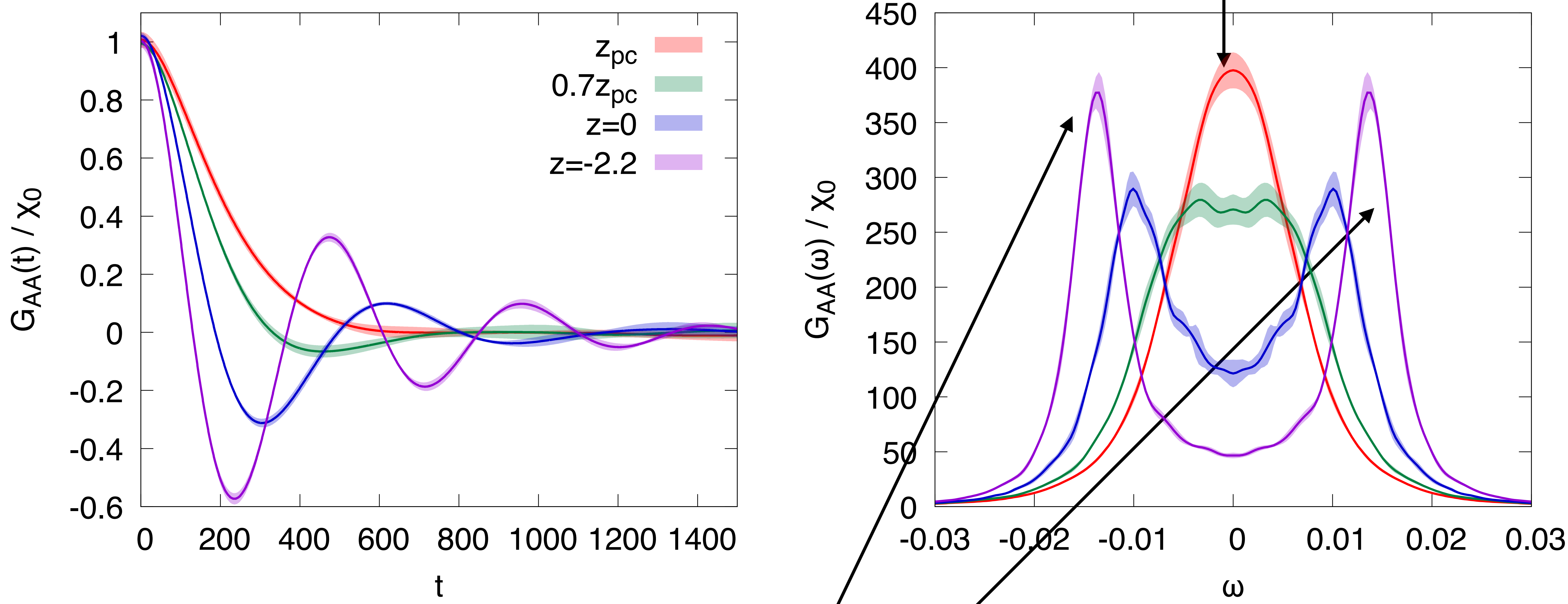
$$\partial_t n_{ab} + g_0\, \nabla\cdot(\nabla\phi_{[a}\phi_{b]}) + H_{[a}\phi_{b]} = D_0 \nabla^2 n_{ab} + \partial_i \Xi^i_{ab}\,.$$

Gaussian Noise

Iso axial spectral function

$$\frac{\rho_{AA}(\omega,\mathbf{q})}{\omega} = \frac{1}{T}\int d^4x e^{-i\omega t+i\mathbf{q}\cdot\mathbf{x}}\langle J_A(x)J_A(0)\rangle$$

Quark diffusion T>>Tc



Quasiparticle Pions T<<Tc

We can se the transition between QGP to propagating pions form EOM

Effective Boltzmann equation for soft pions

From the linear propagator we can define (using the Wigner transform) and effective kinetic description of the soft pions distribution function

$$\partial_t f_\pi + \frac{\partial E_{\mathbf{p}}}{\partial \mathbf{q}} \frac{\partial f_\pi}{\partial \mathbf{x}} - \frac{\partial E_{\mathbf{p}}}{\partial \mathbf{x}} \frac{\partial f_\pi}{\partial \mathbf{q}} = \text{dumping term}$$

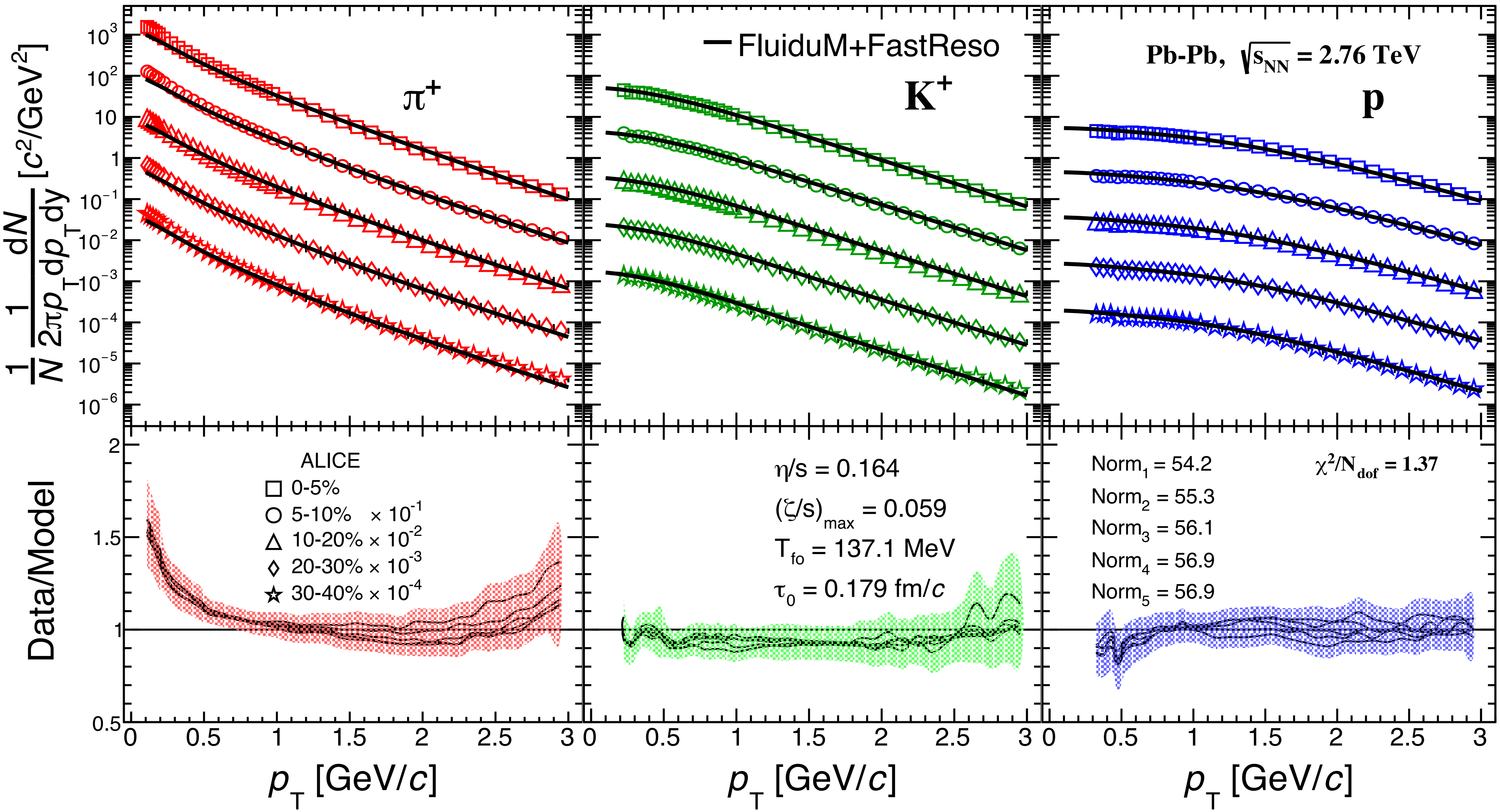
Well below the phase transition the pions propagate like quasipartilce with a modified energy dispersion from the medium

$$E_{\mathbf{p}} = v^2(p^2 + m^2)$$

Depends on the position of the minimum $\bar{\sigma}$

Why all of this should matter

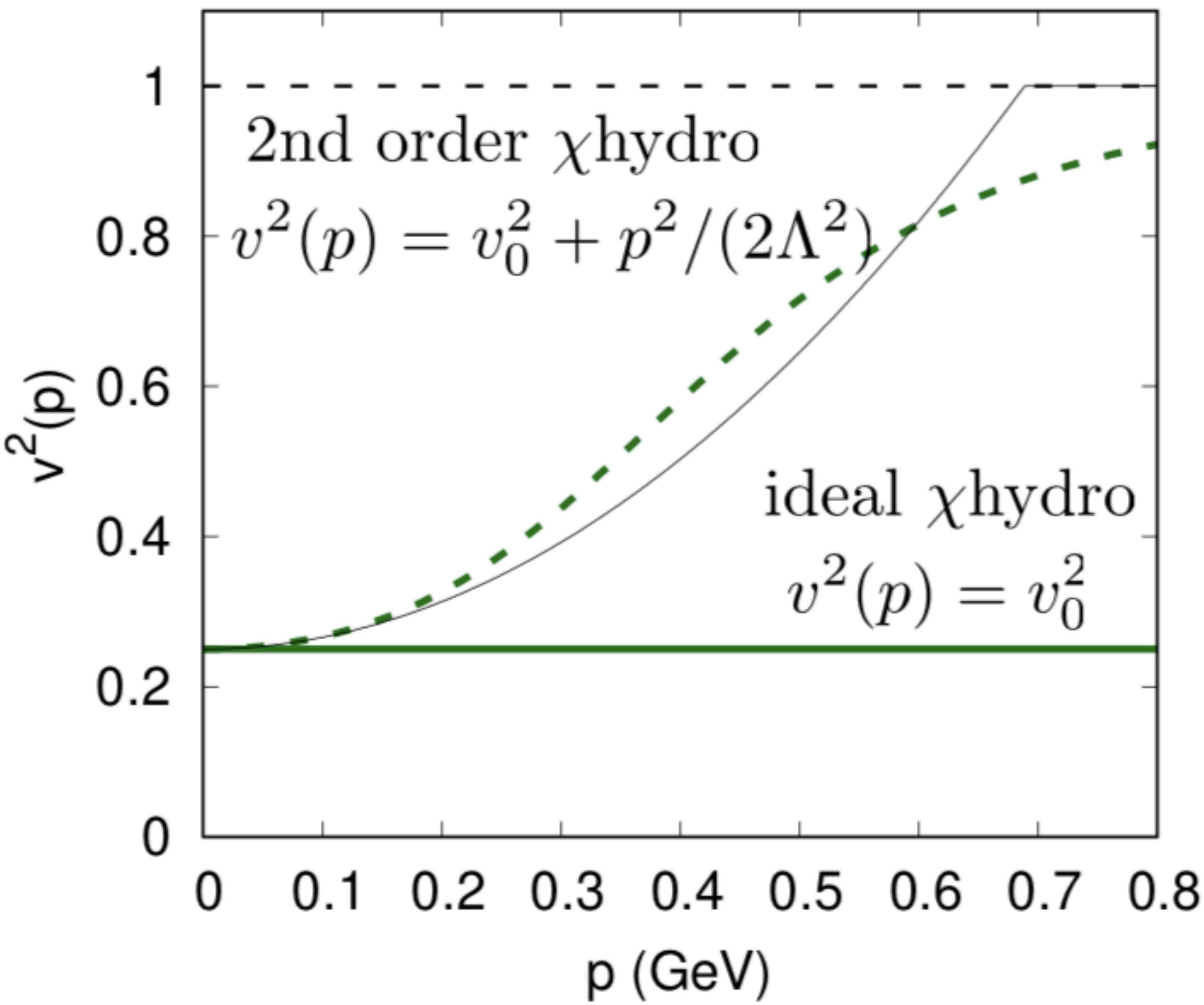
Devetak et al 1909.10485



We solved normal hydro equation + a lot of particles in the feed down we were not able to produce as that many pions

Estimation of diversion curve

The pion velocity and the mass scale with the condensate and are reduce next to the phase transition



$$E_{\mathbf{p}} = v^2(p)(p^2 + m^2)$$

Cut off the χhydro for:

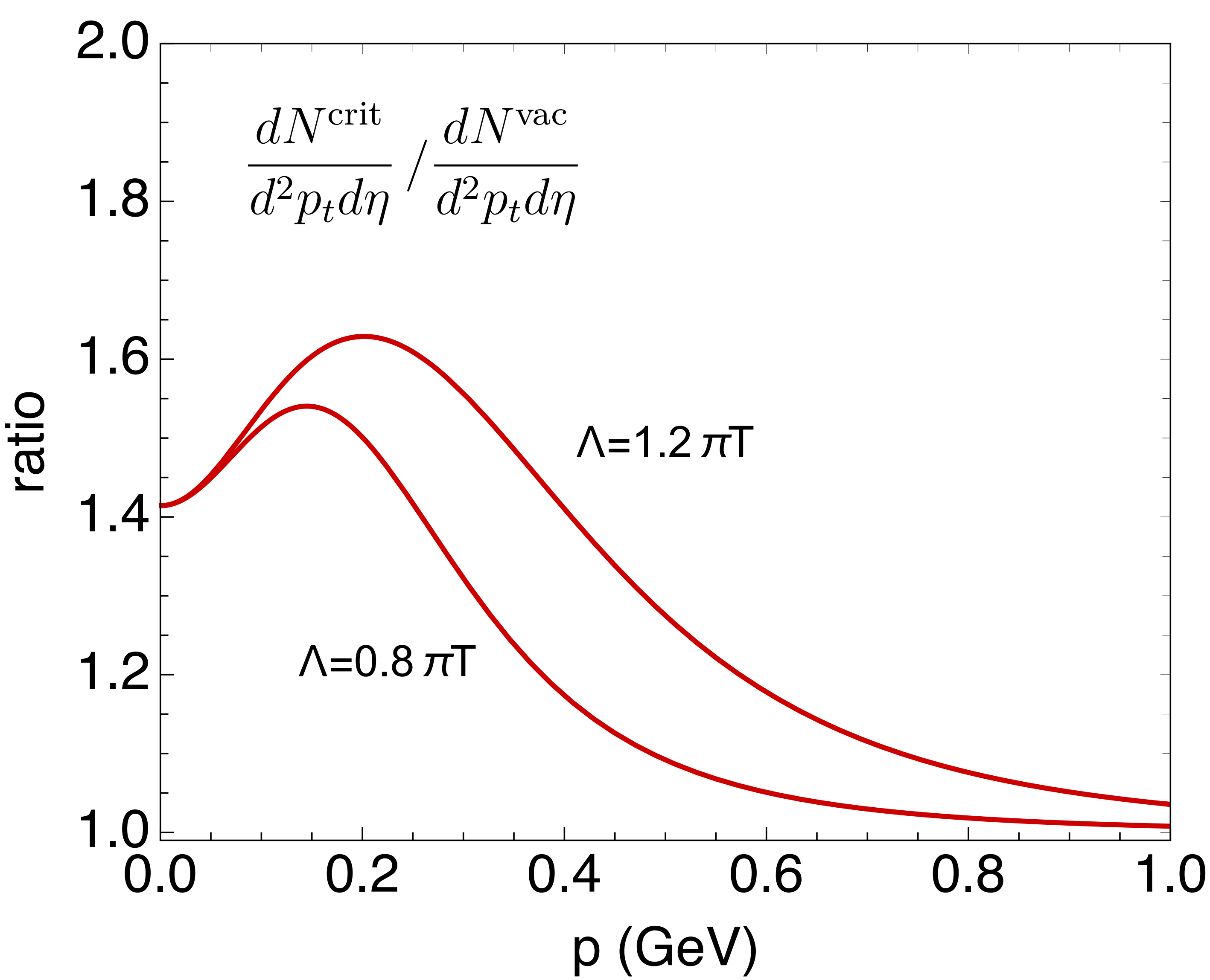
$$p \sim \Lambda = \pi T_{\rm pc}$$

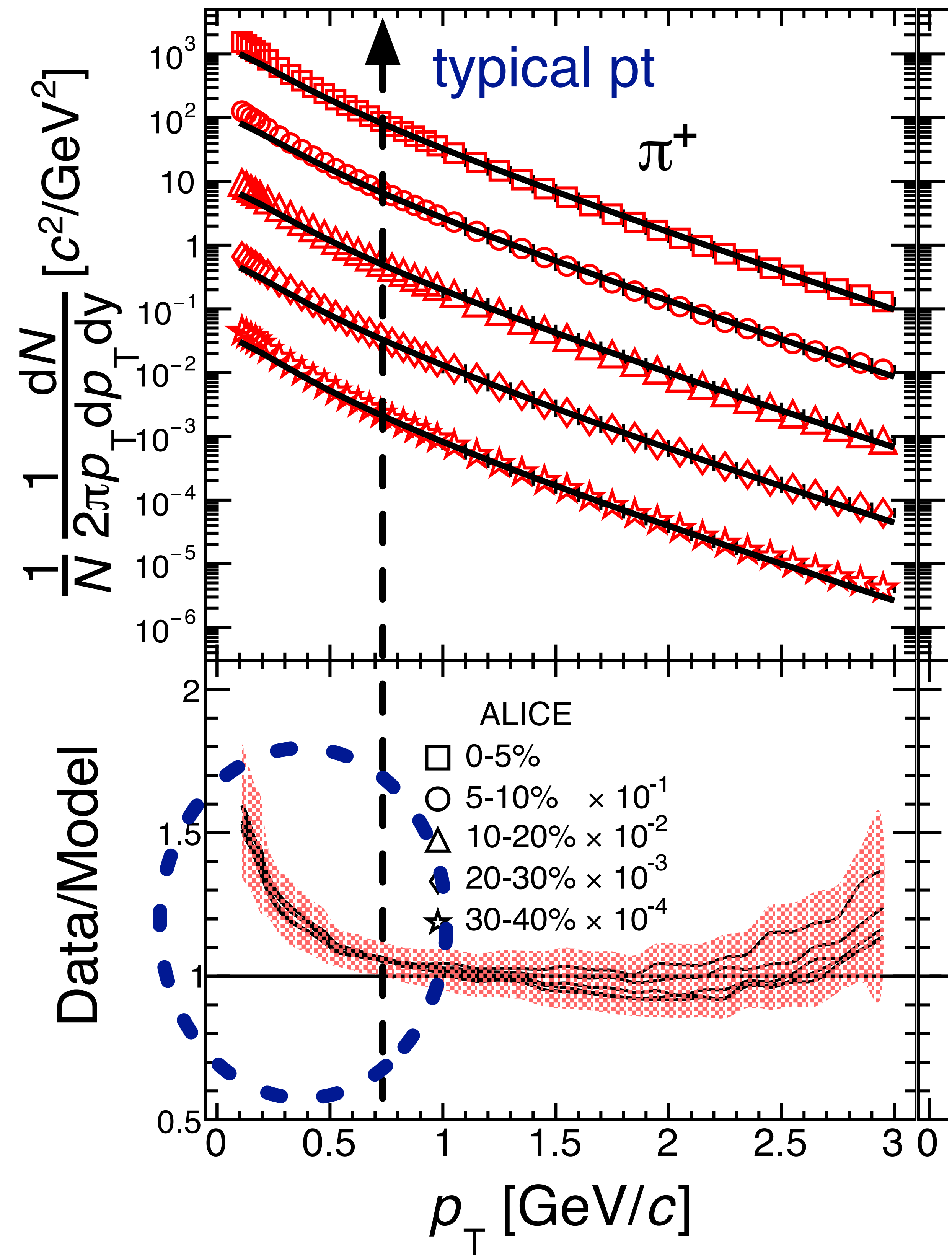
Possible enhancement at low pT

With the modified dispersion curve

$$n(E_p) = \frac{1}{e^{E_p/T} - 1}$$

Devetak et al 1909.10485





Outlook

- The dynamics of the chiral condensate has to been included in the hydrodynamic description of Heavy ion Collision

- The Static universality class is the O(4) model in 3 dimension

- The dynamic universality class is so called Model G

- We perform the first lattice simulation and determine the spectral density

- Naturally the pion propagation in the medium are modified such for low pt

- More distintive signal could be detected in the two point function

Thank you!