



# BAM

## Barolo Astroparticle Meeting

### Aspects of High Scale Leptogenesis with Low-Energy Leptonic CP Violation

A. G., K. Moffat and S. T. Petcov, arXiv:2107.02079

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Supervisor: Prof. Serguey T. Petcov



**SISSA**

## Motivations

- **Neutrino Masses and Mixing:**  $\nu_{\alpha L}(x) = \sum_{a=1}^3 U_{\alpha a} \nu_{aL}(x)$ .
- **Baryon Asymmetry of the Universe (BAU):**  
 $\eta_B \equiv (n_B - n_{\bar{B}})/n_\gamma \cong 6.1 \times 10^{-10}$ .

## Framework

- Type-I seesaw model (with  $n \geq 2$  heavy neutrinos  $N_i$ ):

$$\mathcal{L}_{Y,M}(x) = - (Y_{\alpha i} \overline{\psi_{\alpha L}}(x) i\tau_2 \Phi^*(x) N_{iR}(x) + \text{h.c.}) - \frac{1}{2} M_i \overline{N_i}(x) N_i(x).$$

► Light neutrino mass generation:  $m_\nu \approx -\frac{v^2}{2} YM^{-1} Y^T$ ;

► Casas-Ibarra parametrisation: 
$$Y_{\alpha j} = \pm i \frac{\sqrt{2}}{v} U_{\alpha a} \sqrt{m_a} R_{ja} \sqrt{M_j}$$

$R$ : 3x3 complex orthogonal matrix parametrised by  $\theta_{1,2,3} = x_{1,2,3} + iy_{1,2,3}$ , or just  $\theta = x + iy$  for  $m_1 \simeq 0$  (NH) or  $m_3 \simeq 0$  (IH).

- **Leptogenesis (LG):** lepton asymmetry generation via out-of-equilibrium,  $L$ -,  $C$ - and  $CP$ - violating  $N_i$  decays and inverse decays in the early Universe. The lepton asymmetry is converted into the BAU by the SM sphalerons ( $T \gtrsim 131.7$  GeV).

**Low-energy CP violation ( $CP$ ):** the only sources of CP violation being Dirac phase  $\delta$  and/or the Majorana phases  $\alpha_{21}$  and/or  $\alpha_{31}$  of the PMNS matrix  $U$  (CP-conserving  $R$ -matrix, i.e.  $y = 0$  or  $x = 0, \pi, \dots$ ).

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$N_i \rightarrow \Phi \psi_i$ ,  $\psi_i = \sum_{\alpha=e,\mu,\tau} C_{i\alpha} \psi_\alpha$ : coherent superpositions of lepton flavours.

**Decoherence effects:** the SM  $\tau$ - and  $\mu$ -Yukawa interactions ( $\Gamma_\tau$ ,  $\Gamma_\mu$ ) destroy the coherence making flavours distinguishable.

- **Single-flavour regime:**  $T \gg 10^{12}$  GeV ( $\Gamma_{\tau,\mu} \ll H$ ).
  - ▶ Flavours indistinguishable.
  - ▶ *Single-flavour approx.:* neglect  $\tau$ - and  $\mu$ -Yukawas and *Single-flavoured Boltzmann Equations* (BE1F).
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The DMEs account for quantum decoherence processes due to SM Yukawas.

- Valid also in the intermediate regimes.
- They reproduce the BEs.

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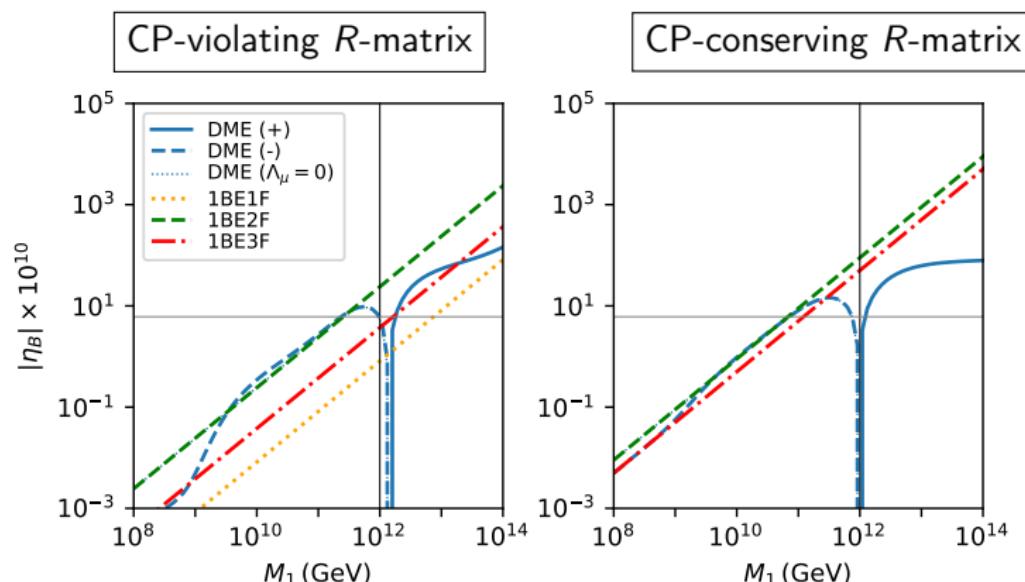
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LG for  $10^8 \lesssim M_1/\text{GeV} \lesssim 10^{14}$  and  $M_1 \ll M_2 \ll M_3$  solving BE1F, BE2F, BE3F and DMEs with ULYSSES Python package (A.G., K. Moffat, Y. Perez-Gonzalez, H. Schulz, J. Turner, *Computer Physics Communication* **262** (2021) 107813, [2007.09150]).

## General features

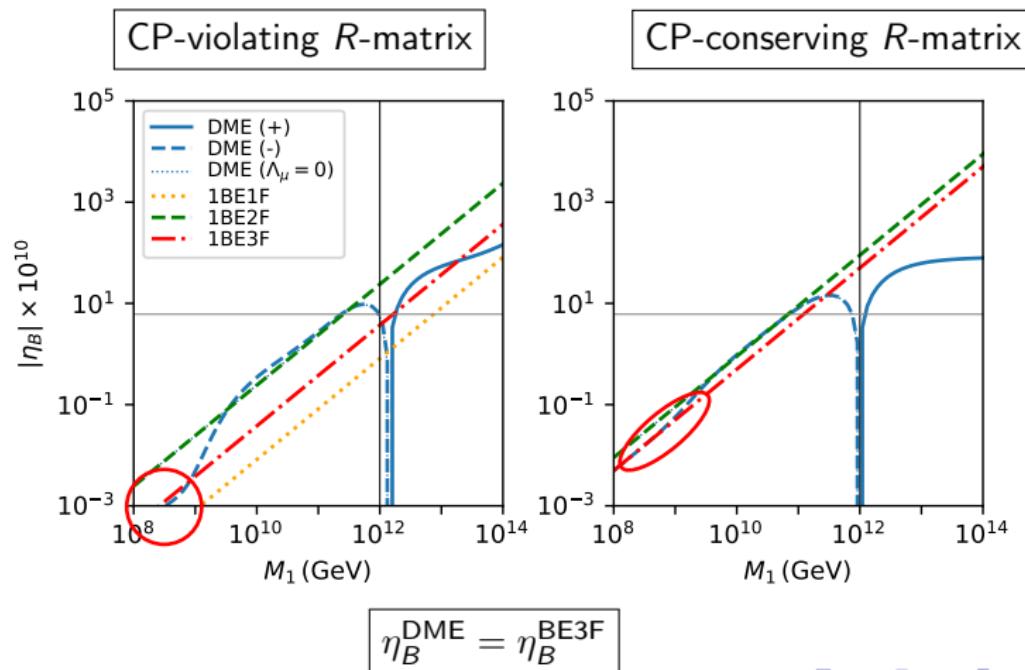


Vanishing Initial Abundance ( $N_{N_1}(z_i) = 0$ ), “standard” behaviour of  $\eta_B$

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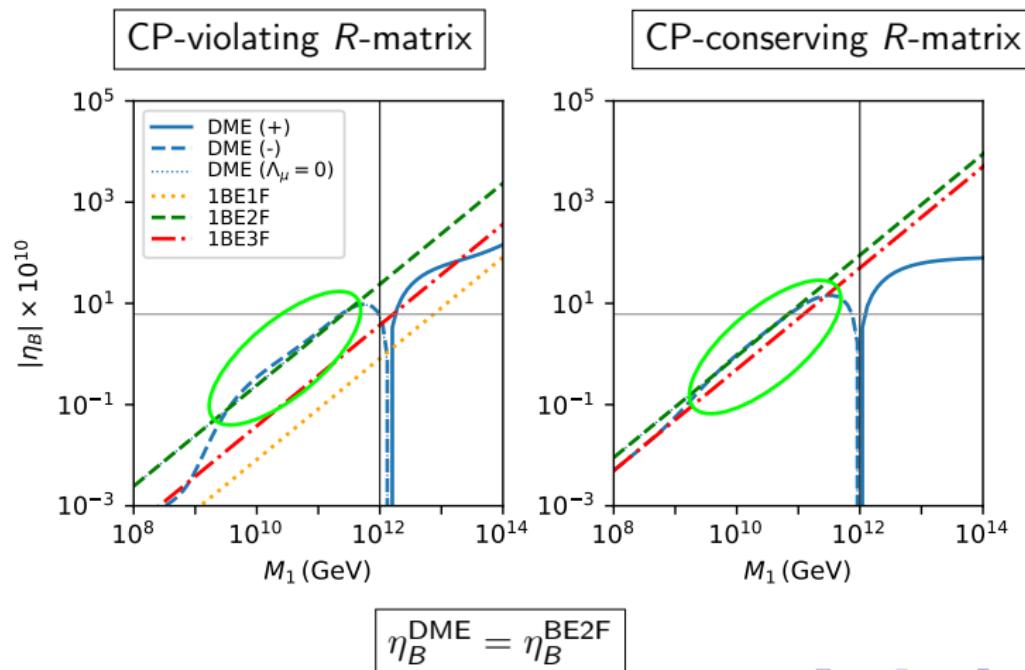
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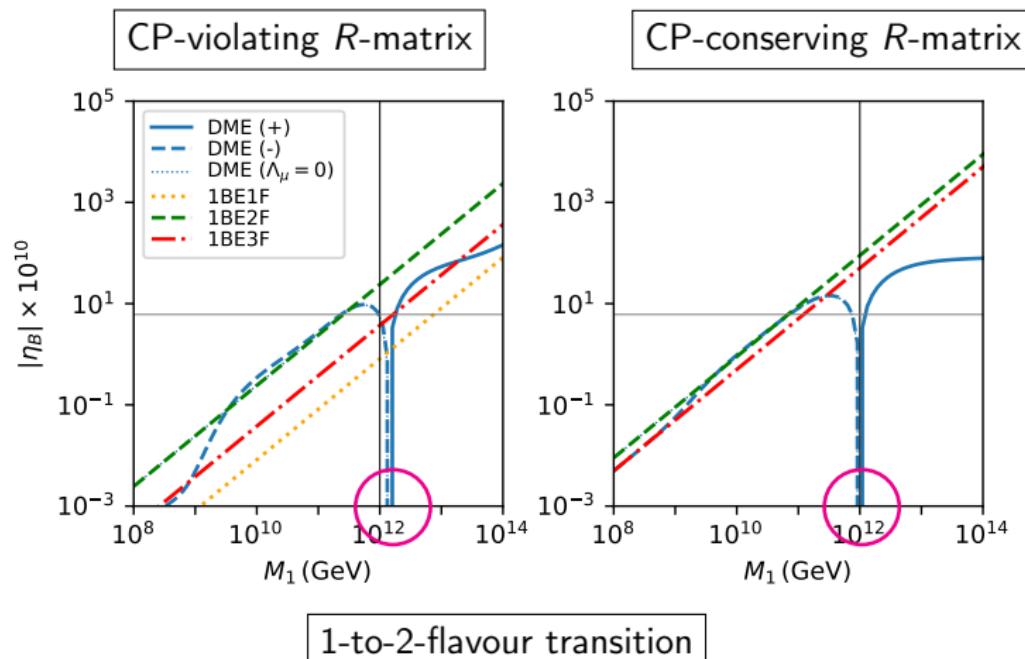
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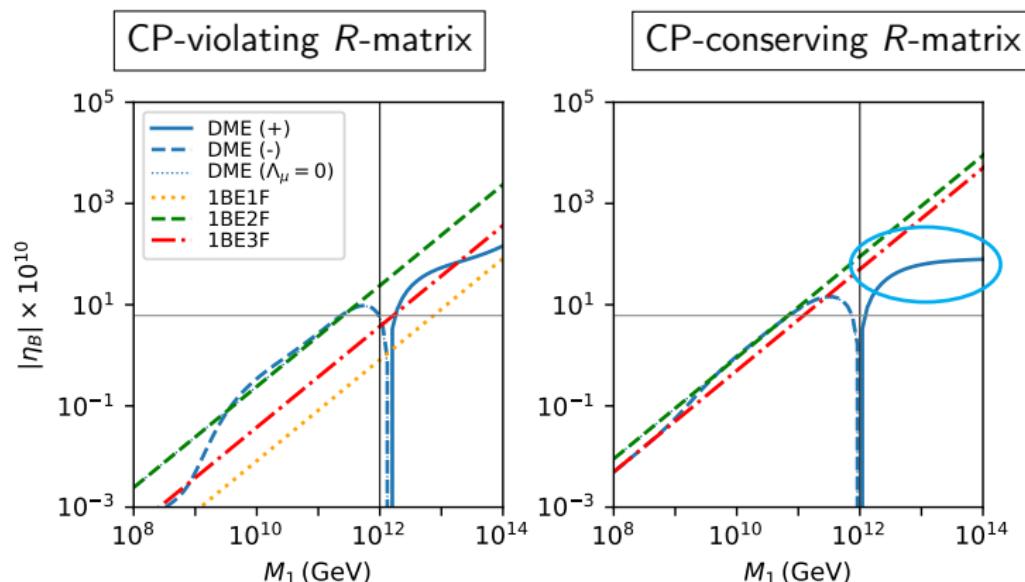
## General features



# Aspects of High-Scale LG with Low-Energy Leptonic $\mathcal{CP}$

LG for  $10^8 \lesssim M_1/\text{GeV} \lesssim 10^{14}$  and  $M_1 \ll M_2 \ll M_3$  solving BE1F, BE2F, BE3F and DMEs with ULYSSES Python package (A.G., K. Moffat, Y. Perez-Gonzalez, H. Schulz, J. Turner, *Computer Physics Communication* **262** (2021) 107813, [2007.09150]).

## General features



Pleateau region due to  $\tau$ -decoherence effects

# High-Scale LG with Low-Energy Leptonic $\mathcal{CP}$

Sign Change

$$\eta_{B-L}(z_f) \propto \Lambda_\tau \mathcal{I}_2(\kappa_1; z_f) \epsilon_{\tau\tau}^{(1)}(p_{1\tau^\perp} - p_{1\tau}) + \mathcal{O}(\Lambda_\tau^2), \text{ with } \Lambda_\tau \equiv \Gamma_\tau / \text{Hz}.$$

**Strong wash-out regime ( $\Gamma_N \gg H$  at  $z = 1$ )**

**Vanishing Initial Abundance**

$$\eta_{B-L}^{1BE2F}(z_f) \propto \epsilon_{\tau\tau}^{(1)}(p_{1\tau^\perp} - p_{1\tau})$$

$$\mathcal{I}_2(\kappa_1; zf) < 0$$

Sign change:

**Thermal Initial Abundance**

$$\eta_{B-L}^{1BE2F}(z_f) \propto \epsilon_{\tau\tau}^{(1)}(p_{1\tau^\perp} - p_{1\tau})$$

$$\mathcal{I}_2(\kappa_1; zf) > 0$$

Sign change:

**Weak wash-out regime ( $\Gamma_N \ll H$  at  $z = 1$ )**

**Vanishing Initial Abundance**

$$\eta_{B-L}^{1BE2F} \propto -\epsilon_{\tau\tau}^{(1)}(p_{1\tau^\perp} - p_{1\tau})$$

$$\mathcal{I}_2(\kappa_1; zf) < 0$$

Sign change:

**Thermal Initial Abundance**

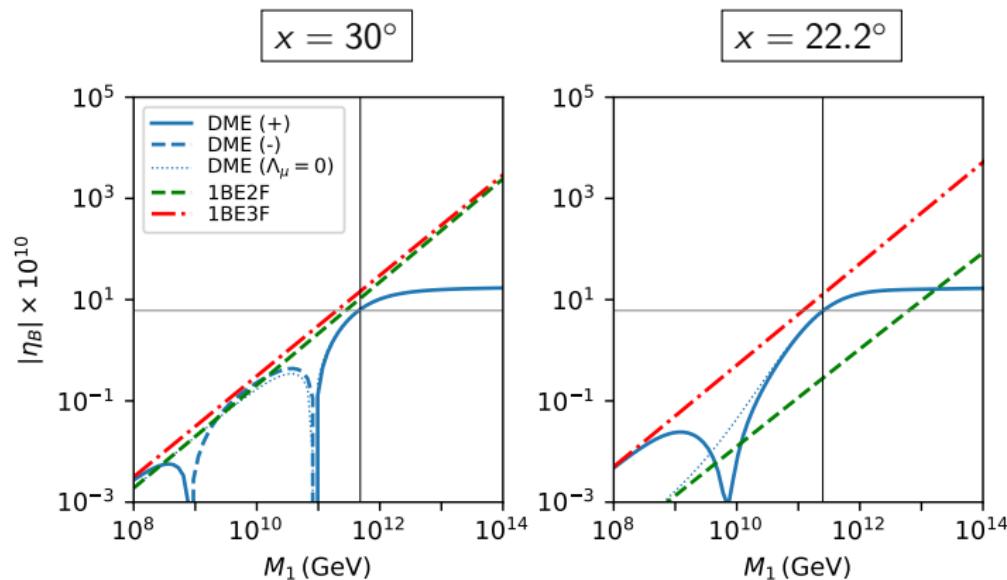
$$\eta_{B-L}^{1BE2F} \propto \epsilon_{\tau\tau}^{(1)}(p_{1\tau^\perp} - p_{1\tau})$$

$$\mathcal{I}_2(\kappa_1; zf) > 0$$

Sign change:

# High-Scale LG with Low-Energy Leptonic $\mathcal{CP}$

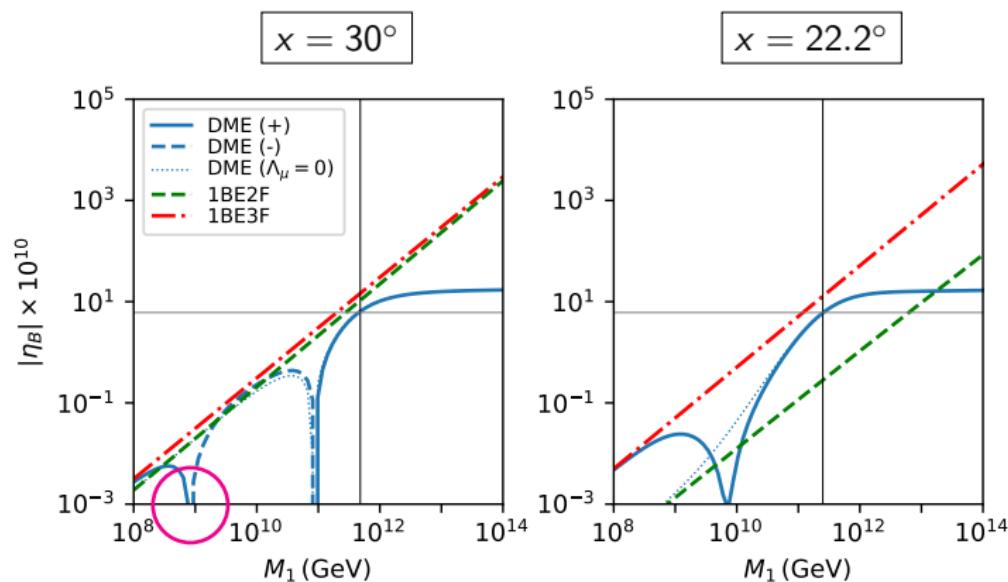
The case of decoupled  $N_3 - \mathcal{CP}$  from Dirac phase ( $\delta = 3\pi/2$ ,  $\alpha_{21} = \alpha_{31} = 0$ ), NH spectrum, real  $R$ -matrix ( $y = 0$ )



“Non-standard” behaviour of  $\eta_B$

# High-Scale LG with Low-Energy Leptonic $\mathcal{CP}$

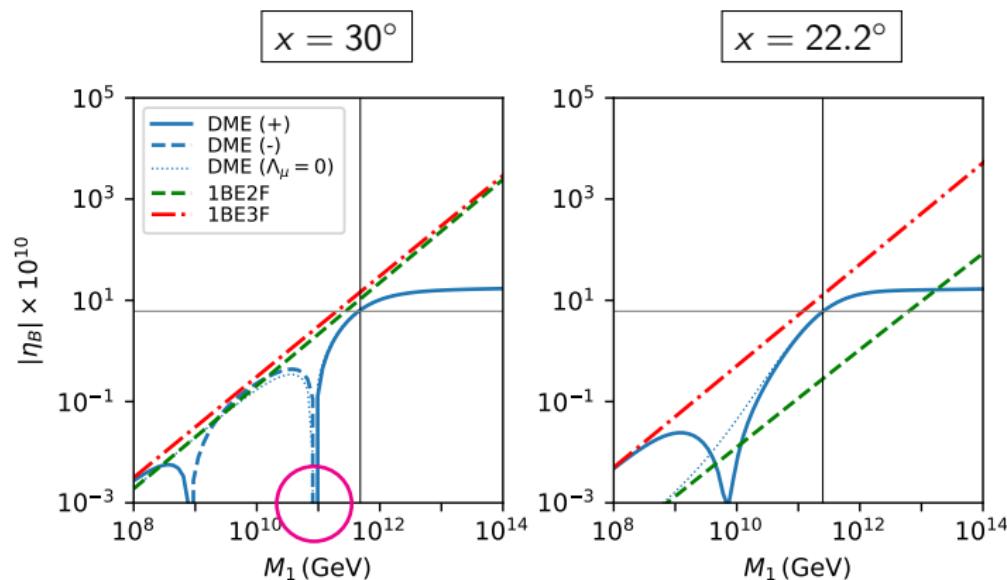
The case of decoupled  $N_3 - \mathcal{CP}$  from Dirac phase ( $\delta = 3\pi/2$ ,  $\alpha_{21} = \alpha_{31} = 0$ ), NH spectrum, real  $R$ -matrix ( $y = 0$ )



2-to-3-flavour transition with sign change

# High-Scale LG with Low-Energy Leptonic $\mathcal{CP}$

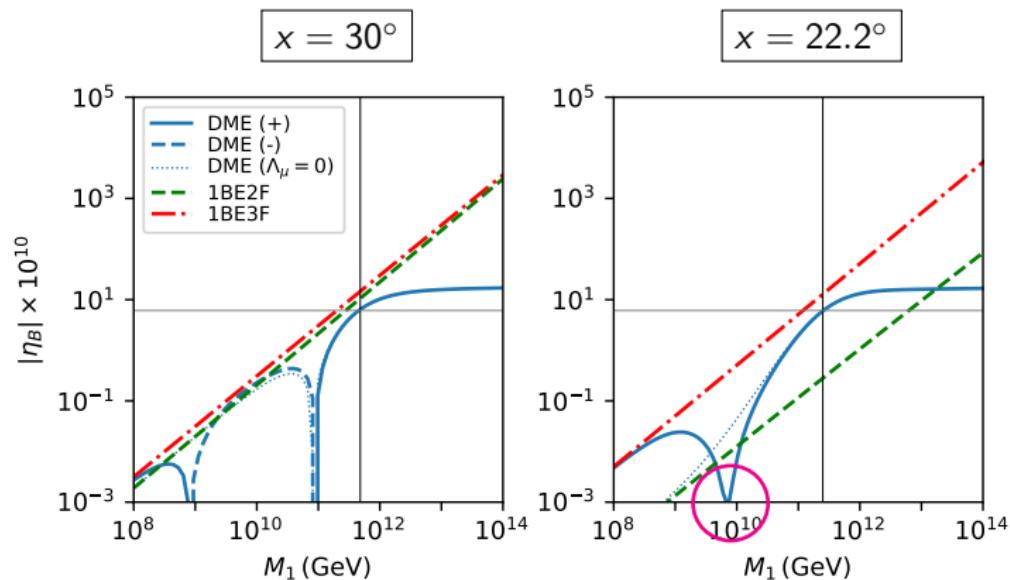
The case of decoupled  $N_3 - \mathcal{CP}$  from Dirac phase ( $\delta = 3\pi/2$ ,  $\alpha_{21} = \alpha_{31} = 0$ ), NH spectrum, real  $R$ -matrix ( $y = 0$ )



1-to-2-flavour transition at  $M_1 \ll 10^{12}$  GeV

# High-Scale LG with Low-Energy Leptonic $\mathcal{CP}$

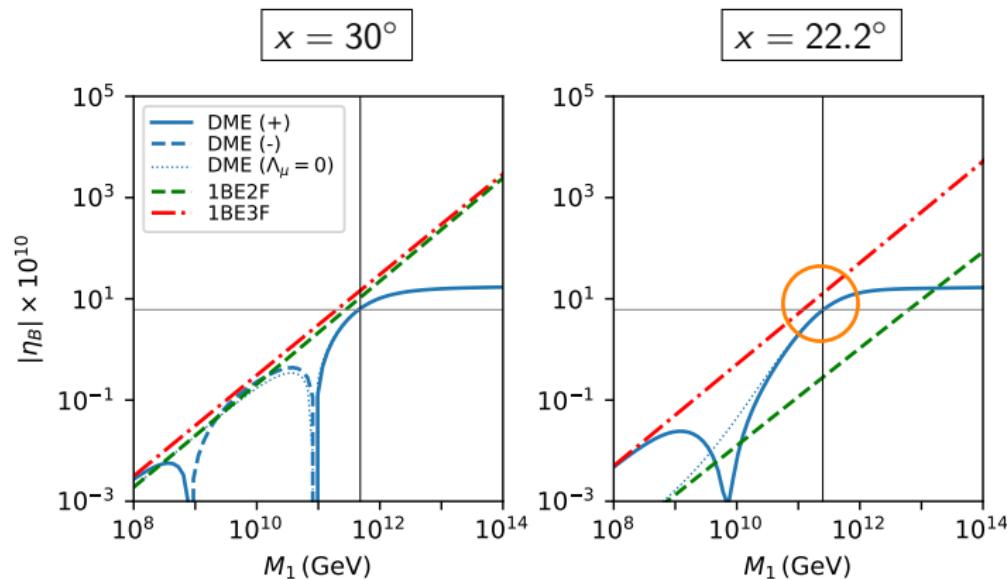
The case of decoupled  $N_3 - \mathcal{CP}$  from Dirac phase ( $\delta = 3\pi/2$ ,  $\alpha_{21} = \alpha_{31} = 0$ ), NH spectrum, real  $R$ -matrix ( $y = 0$ )



Overlap of the transitions at  $M \ll 10^{12} \text{ GeV}$

# High-Scale LG with Low-Energy Leptonic $\mathcal{CP}$

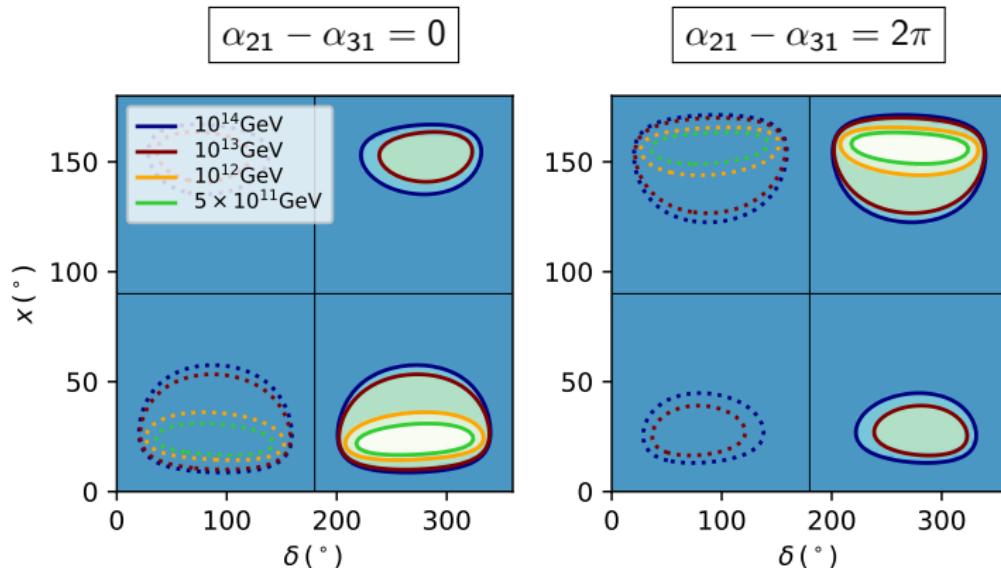
The case of decoupled  $N_3 - \mathcal{CP}$  from Dirac phase ( $\delta = 3\pi/2$ ,  $\alpha_{21} = \alpha_{31} = 0$ ), NH spectrum, real  $R$ -matrix ( $y = 0$ )



Successful LG at  $M_1 \simeq 2.55 \times 10^{11}$  GeV,  $\mathcal{O}(10)$  enhancement.

# High-Scale LG with Low-Energy Leptonic $\mathcal{CP}$

The case of decoupled  $N_3 - \mathcal{CP}$  from Dirac phase, NH spectrum, real  $R$ -matrix ( $y = 0$ )

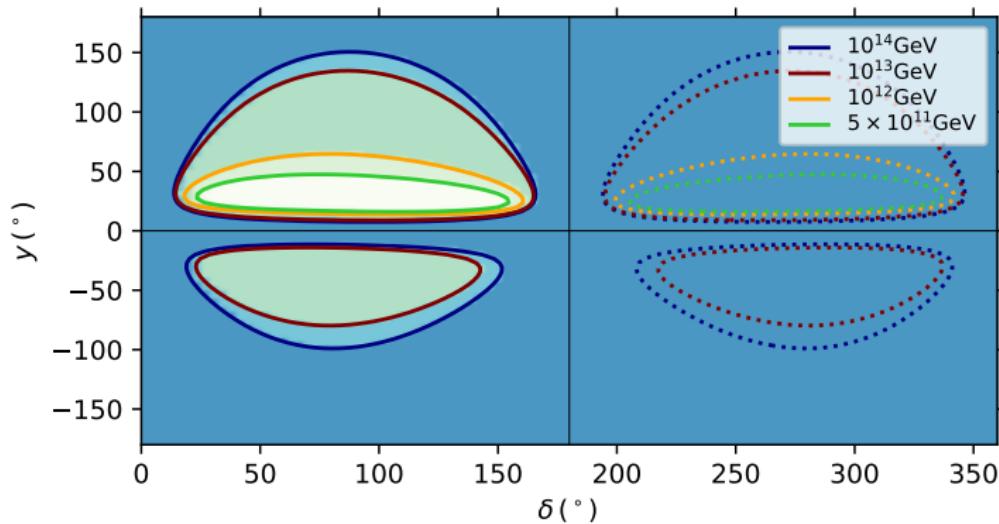


Regions of successful LG: viable only if  $\delta \in (\pi, 2\pi)$

Minimal mass scale  $M_1 \simeq 2.55 \times 10^{11} \text{ GeV}$ ,

# High-Scale LG with Low-Energy Leptonic $\mathcal{CP}$

The case of decoupled  $N_3$  — NH spectrum,  $\mathcal{CP}$  from Dirac phase ( $\alpha_{21} - \alpha_{31} = \pi$ ), purely imaginary  $R_{11}R_{12}$  ( $R_{12}R_{13}$ ) ( $x = 0, \pi, \dots$ )

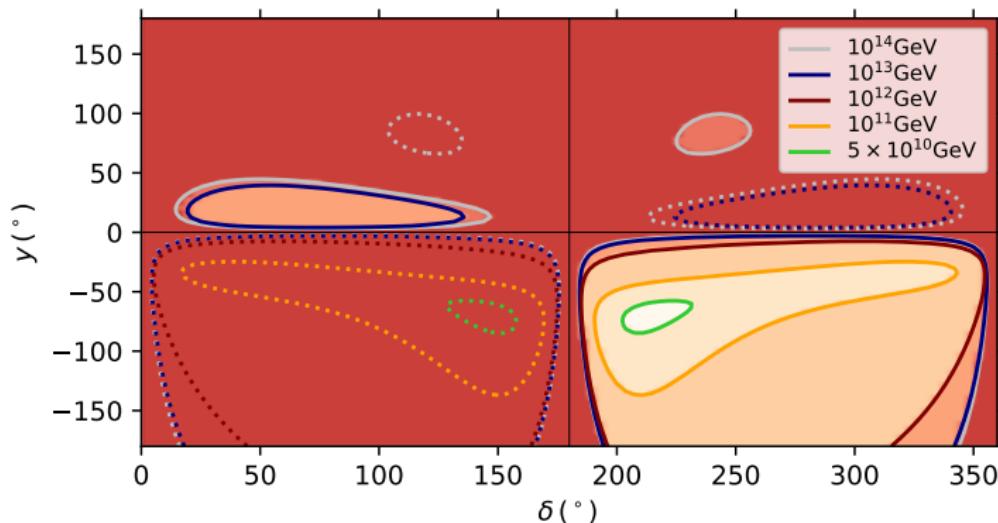


For NH, LG is viable only if  $\delta \in (0, \pi)$

Minimal mass scale for viable LG:  $M_1 \simeq 1.7 \times 10^{11}$  GeV,

# High-Scale LG with Low-Energy Leptonic $\mathcal{CP}$

The case of decoupled  $N_3$  — IH spectrum,  $\mathcal{CP}$  from Dirac phase ( $\alpha_{21} = \pi$ ), purely imaginary  $R_{11}R_{12}$  ( $R_{12}R_{13}$ ) ( $x = 0, \pi, \dots$ )

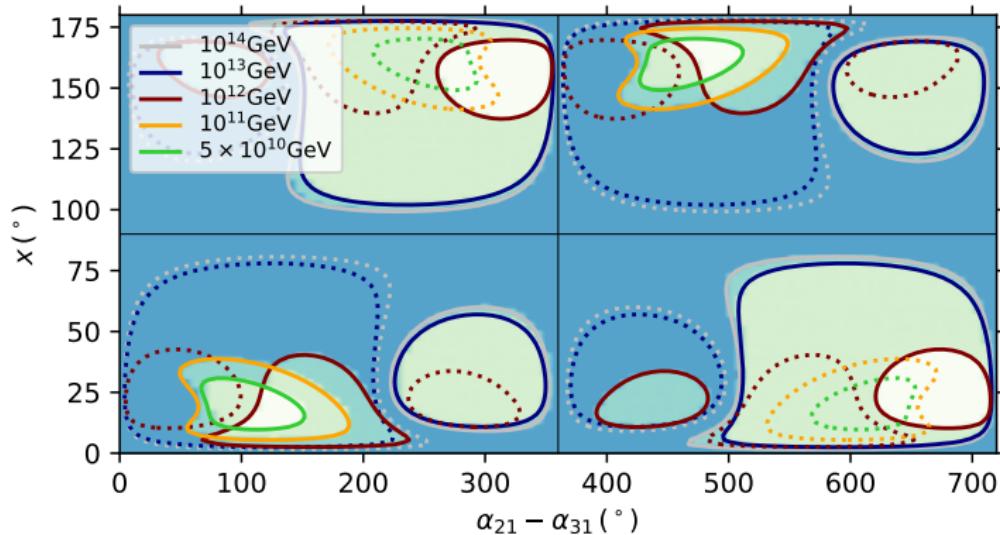


For IH, LG is viable for  $\delta \in (0, \pi)$  only if  $M \gtrsim 10^{13}$  GeV.

Otherwise, LG is viable only for  $\delta \in (\pi, 2\pi)$  and  $M_1 \gtrsim 4.6 \times 10^{10}$  GeV.

# High-Scale LG with Low-Energy Leptonic $\mathcal{CP}$

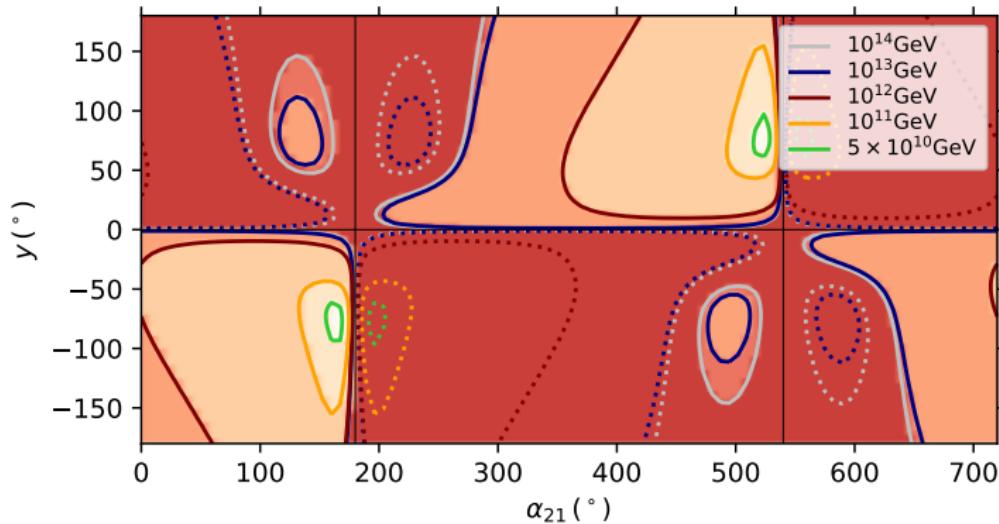
The case of decoupled  $N_3$  — NH spectrum,  $\mathcal{CP}$  from Majorana phases ( $\delta = \pi$ ), Real  $R$  ( $x = 0, \pi, \dots$ )



Minimal mass scale for viable LG:  $M_1 \simeq 3.3 \times 10^{10} \text{ GeV}$ ,

# High-Scale LG with Low-Energy Leptonic $\mathcal{CP}$

The case of decoupled  $N_3$  — IH spectrum,  $\mathcal{CP}$  from Majorana phases ( $\delta = \pi$ ), purely imaginary  $R_{11}R_{12}$  ( $R_{12}R_{13}$ ) ( $x = 0, \pi, \dots$ )



Minimal mass scale for viable LG:  $M_1 \simeq 4.3 \times 10^{10} \text{ GeV}$ ,

# Conclusions

LG for  $10^8 \lesssim M_1/\text{GeV} \lesssim 10^{14}$  and  $M_1 \ll M_2 \ll M_3$  solving BEs and DMEs.

- ➊ Detailed analysis of the **1-to-2- and 2-to-3-flavour transitions**.
  - ▶  $\eta_B$  may **change its sign** at the transitions.
- ➋ The  **$\tau$ -decoherence effects** can still generate **BAU** in the **single-flavour regime** at  $M \gtrsim 10^{12}$  GeV with low-energy  **$\mathcal{CP}$** .  
Confirmation of K. Moffat, S. Pascoli, S. T. Petcov and J. Turner, arXiv:1809.08251.
- ➌ The **BE2F** can be very **inaccurate** even in the **two-flavour regime** for  $10^9 \lesssim M_1/\text{GeV} \lesssim 10^{12}$ .
- ➍ In the case of decoupled  $N_3$  (**NH** and **IH**) and low-energy  **$\mathcal{CP}$**  from the Dirac phase  $\delta$ , the **sign of**  $\eta_B$  is in **one-to-one correspondence** with the **sign of**  $\sin \delta$ .
- ➎ Revisited **ranges of masses and PMNS phases** for **viable LG** in the case of low-energy  **$\mathcal{CP}$**  and decoupled  $N_3$  (**NH** and **IH**).
- ➏ The considered different scenarios of LG are **testable** and **falsifiable** in low-energy neutrino experiments.

```
for your_attention in this_talk():
    print('Thanks!')
```

# Back-up Slides | ULYSSES Python package

A.G., K. Moffat, Y. Perez-Gonzalez, H. Schulz, J. Turner, *Computer Physics Communication* **262** (2021) 107813, [2007.09150].

ULYSSES is a Python package that calculates the BAU in LG within the type-I seesaw mechanism, solving semi-classical BEs.

## @properties

- ▶ **Exhaustive** It calculates the evolution of all LG ingredients (e.g.,  $N_i$ -distributions, wash-out factors, asymmetries in different flavours, BAU).
- ▶ **Universal** An “Odyssey” of more than 14 orders of magnitude with many different sets of eqs., e.g. BE1F, BE2F, BE3F, DME with 1,2 or 3 heavy neutrinos.
- ▶ **Precise** E.g., loop corrections, scattering contributions, flavour effects, decoherence effects.
- ▶ **Flexible** Easily modify-able or implement-able with new models.
- ▶ **Slim** Very fast and with small size ( $\sim 400$  kB).
- ▶ **Public** Freely available at <https://github.com/earlyuniverse/ulysses>.

**Still missing features:** thermal effects, spectator processes, next-to-leading-order corrections for the source term, LG via oscillations.

# Back-up Slides | Equations

## Single-flavoured Boltzmann Equations (1BE1F) <sup>a</sup>

<sup>a</sup>The equations are written in the hierarchical limit  $M_1 \ll M_2 \ll M_3$ .

$$\frac{dN_{N_1}}{dz} = -D_1 (N_{N_1} - N_{N_1}^{eq}) , \quad (1)$$

$$\frac{dN_{B-L}}{dz} = \epsilon^{(1)} D_1 (N_{N_1} - N_{N_1}^{eq}) - W_1 N_{B-L} . \quad (2)$$

$N_{N_1}$  ( $N_{N_1}^{eq}$ ): number of  $N_1$  in a comoving volume (if in thermal equilibrium).

# Back-up Slides | Equations

## Single-flavoured Boltzmann Equations (1BE1F) <sup>a</sup>

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$$\frac{dN_{B-L}}{dz} = \epsilon^{(1)} D_1 (N_{N_1} - N_{N_1}^{eq}) - W_1 N_{B-L} . \quad (2)$$

$N_{B-L}$ :  $B - L$  asymmetry in a comoving volume.

$$\eta_B \simeq \frac{28}{79} \frac{1}{27} N_{B-L} .$$

# Back-up Slides | Equations

## Single-flavoured Boltzmann Equations (1BE1F) <sup>a</sup>

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$$\frac{dN_{B-L}}{dz} = \epsilon^{(1)} D_1 (N_{N_1} - N_{N_1}^{eq}) - W_1 N_{B-L} . \quad (2)$$

$D_1 = \langle \Gamma_{N_1} \rangle / \text{Hz} = \kappa_1 z \frac{K_1(z)}{K_2(z)}$  and  $\kappa_1 = \Gamma_{N_1} / H(z=1)$ : decay parameters.

# Back-up Slides | Equations

## Single-flavoured Boltzmann Equations (1BE1F) <sup>a</sup>

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$$\frac{dN_{B-L}}{dz} = \epsilon^{(1)} D_1 (N_{N_1} - N_{N_1}^{eq}) - W_1 N_{B-L} . \quad (2)$$

$W_1 = \langle \Gamma_{N_1}^{\text{ID}} \rangle / 2\text{Hz}$ : wash-out factor.

# Back-up Slides | Equations

## Single-flavoured Boltzmann Equations (1BE1F) <sup>a</sup>

<sup>a</sup>The equations are written in the hierarchical limit  $M_1 \ll M_2 \ll M_3$ .

$$\frac{dN_{N_1}}{dz} = -D_1 (N_{N_1} - N_{N_1}^{eq}) , \quad (1)$$

$$\frac{dN_{B-L}}{dz} = \epsilon^{(1)} D_1 (N_{N_1} - N_{N_1}^{eq}) - W_1 N_{B-L} . \quad (2)$$

$$\epsilon^{(1)} = - \left( \Gamma_{N_1 \rightarrow \Phi^+ \psi_1} - \Gamma_{N_1 \rightarrow \Phi^- \bar{\psi}_1} \right) / \Gamma_{N_1} : \text{CP-asymmetry parameter.}$$

# Back-up Slides | Equations

## Two-flavoured Boltzmann Equations (1BE2F)

$$\frac{dN_{N_1}}{dz} = -D_1 (N_{N_1} - N_{N_1}^{eq}) , \quad (3)$$

$$\frac{dN_{\tau\tau}}{dz} = \epsilon_{\tau\tau}^{(1)} D_1 (N_{N_1} - N_{N_1}^{eq}) - W_1 p_{1\tau} N_{\tau\tau} , \quad (4)$$

$$\frac{dN_{\tau^\perp\tau^\perp}}{dz} = \epsilon_{\tau^\perp\tau^\perp}^{(1)} D_1 (N_{N_1} - N_{N_1}^{eq}) - W_1 p_{1\tau^\perp} N_{\tau^\perp\tau^\perp} . \quad (5)$$

$N_{\alpha\alpha}$ : asymmetry  $\frac{1}{3}B - L_\alpha$  in a comoving volume,  $N_{\tau^\perp\tau^\perp} = N_{ee} + N_{\mu\mu}$ .

$$N_{B-L} = N_{\tau\tau} + N_{\tau^\perp\tau^\perp} .$$

# Back-up Slides | Equations

## Two-flavoured Boltzmann Equations (1BE2F)

$$\frac{dN_{N_1}}{dz} = -D_1 (N_{N_1} - N_{N_1}^{eq}) , \quad (3)$$

$$\frac{dN_{\tau\tau}}{dz} = \epsilon_{\tau\tau}^{(1)} D_1 (N_{N_1} - N_{N_1}^{eq}) - W_1 p_{1\tau} N_{\tau\tau} , \quad (4)$$

$$\frac{dN_{\tau^\perp\tau^\perp}}{dz} = \epsilon_{\tau^\perp\tau^\perp}^{(1)} D_1 (N_{N_1} - N_{N_1}^{eq}) - W_1 p_{1\tau^\perp} N_{\tau^\perp\tau^\perp} . \quad (5)$$

$p_{1\alpha} = |C_{1\alpha}|^2$  and  $p_{1\tau^\perp} = |C_{1e}|^2 + |C_{1\mu}|^2 = 1 - p_{1\tau}$  (*projection probabilities*).

# Back-up Slides | Equations

## Two-flavoured Boltzmann Equations (1BE2F)

$$\frac{dN_{N_1}}{dz} = -D_1 (N_{N_1} - N_{N_1}^{eq}) , \quad (3)$$

$$\frac{dN_{\tau\tau}}{dz} = \epsilon_{\tau\tau}^{(1)} D_1 (N_{N_1} - N_{N_1}^{eq}) - W_1 p_{1\tau} N_{\tau\tau} , \quad (4)$$

$$\frac{dN_{\tau^\perp\tau^\perp}}{dz} = \epsilon_{\tau^\perp\tau^\perp}^{(1)} D_1 (N_{N_1} - N_{N_1}^{eq}) - W_1 p_{1\tau^\perp} N_{\tau^\perp\tau^\perp} . \quad (5)$$

$$\epsilon_{\alpha\alpha}^{(1)} = - \left( p_{1\alpha} \Gamma_{N_1 \rightarrow \Phi^+ \psi_1} - p_{1\alpha} \Gamma_{N_1 \rightarrow \Phi^- \overline{\psi}_1} \right) / \Gamma_{N_1}, \quad \epsilon_{\tau^\perp\tau^\perp}^{(1)} = \epsilon_{ee}^{(1)} + \epsilon_{\mu\mu}^{(1)}$$

# Back-up Slides | Equations

## Three-flavoured Boltzmann Equations (1BE3F)

$$\frac{dN_{N_1}}{dz} = -D_1 (N_{N_1} - N_{N_1}^{eq}) , \quad (6)$$

$$\frac{dN_{ee}}{dz} = \epsilon_{ee}^{(1)} D_1 (N_{N_1} - N_{N_1}^{eq}) - W_1 p_{1e} N_{ee} , \quad (7)$$

$$\frac{dN_{\mu\mu}}{dz} = \epsilon_{\mu\mu}^{(1)} D_1 (N_{N_1} - N_{N_1}^{eq}) - W_1 p_{1\mu} N_{\mu\mu} , \quad (8)$$

$$\frac{dN_{\tau\tau}}{dz} = \epsilon_{\tau\tau}^{(1)} D_1 (N_{N_1} - N_{N_1}^{eq}) - W_1 p_{1\tau} N_{\tau\tau} . \quad (9)$$

$$N_{B-L} = \sum_{\alpha=e, \mu, \tau} N_{\alpha\alpha}.$$

# Back-up Slides | Equations

## Density Matrix Equations (DMEs)

$$\frac{dN_{N_1}}{dz} = -D_1(N_{N_1} - N_{N_1}^{\text{eq}}) \quad (10)$$

$$\begin{aligned} \frac{dN_{\alpha\beta}}{dz} = & \epsilon_{\alpha\beta}^{(1)} D_1 (N_{N_1} - N_{N_1}^{\text{eq}}) - \frac{1}{2} W_1 \left\{ P^{0(1)}, N \right\}_{\alpha\beta} \\ & - \frac{\Gamma_\tau}{\text{Hz}} [I_\tau, [I_\tau, N]]_{\alpha\beta} - \frac{\Gamma_\mu}{\text{Hz}} [I_\mu, [I_\mu, N]]_{\alpha\beta} . \end{aligned} \quad (11)$$

$N = \sum_{\alpha, \beta} N_{\alpha\beta} |\psi_\alpha\rangle\langle\psi_\beta|$ , density matrix;  $N_{B-L} = \text{Tr}(N) = \sum_{\alpha=e, \mu, \tau} N_{\alpha\alpha}$ .

# Back-up Slides | Equations

## Density Matrix Equations (DMEs)

$$\frac{dN_{N_1}}{dz} = -D_1(N_{N_1} - N_{N_1}^{\text{eq}}) \quad (10)$$

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$P_{\alpha\beta}^{0(1)} \equiv C_{1\alpha} C_{1\beta}^*$ : projection matrices.

# Back-up Slides | Equations

## Density Matrix Equations (DMEs)

$$\frac{dN_{N_1}}{dz} = -D_1(N_{N_1} - N_{N_1}^{\text{eq}}) \quad (10)$$

$$\begin{aligned} \frac{dN_{\alpha\beta}}{dz} = & \epsilon_{\alpha\beta}^{(1)} D_1(N_{N_1} - N_{N_1}^{\text{eq}}) - \frac{1}{2} W_1 \left\{ P^{0(1)}, N \right\}_{\alpha\beta} \\ & - \frac{\Gamma_\tau}{\text{Hz}} [I_\tau, [I_\tau, N]]_{\alpha\beta} - \frac{\Gamma_\mu}{\text{Hz}} [I_\mu, [I_\mu, N]]_{\alpha\beta} . \end{aligned} \quad (11)$$

$I_\tau$  and  $I_\mu$ :  $3 \times 3$  matrices such that  $(I_\tau)_{\alpha\beta} = \delta_{\alpha\tau}\delta_{\beta\tau}$  and  $(I_\mu)_{\alpha\beta} = \delta_{\alpha\mu}\delta_{\beta\mu}$

# Back-up Slides | Equations

## Density Matrix Equations (DMEs)

$$\frac{dN_{N_1}}{dz} = -D_1(N_{N_1} - N_{N_1}^{\text{eq}}) \quad (10)$$

$$\begin{aligned} \frac{dN_{\alpha\beta}}{dz} = & \epsilon_{\alpha\beta}^{(1)} D_1(N_{N_1} - N_{N_1}^{\text{eq}}) - \frac{1}{2} W_1 \left\{ P^{0(1)}, N \right\}_{\alpha\beta} \\ & - \frac{\Gamma_\tau}{\text{Hz}} [I_\tau, [I_\tau, N]]_{\alpha\beta} - \frac{\Gamma_\mu}{\text{Hz}} [I_\mu, [I_\mu, N]]_{\alpha\beta} . \end{aligned} \quad (11)$$

$\epsilon_{\alpha\beta}^{(1)}$ : CP-asymmetry projected into  $\alpha$ - and  $\beta$ -flavours.