Thermal Axion Production across the QCD Phase Transition

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in collaboration with

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Particle era

HISTORY OF THE UNIVERSE

The concept for the above figure originated in a 1986 paper by Michael Turner.

Particle Data Group, LBNL © 2015

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Dark Radiation

- There is a discrepancy $\Delta N_{\rm eff}$ from the theoretical value of number of effective neutrinos: $N_{\rm eff} = 3.044$ and observed value from cosmic microwave background (CMB) and big bang nucleosynthesis (BBN).
- Extra degrees of freedom from dark sector can be responsible for dark radiation (DR).
- Beyond the standard model physics have some proposals for DR like sterile neutrinos, axion, etc. Moreover, gravitational wave background may have a contribution to $\Delta N_{\rm eff}$.
- Future CMB experiments can put stronger bound on light relics and $\Delta N_{\rm eff}$.
- Relativistic degrees of freedom depending on their nature can decouple at different temperatures. They may be connected to the new physics!
- To estimate the accurate amount of dark radiation contributed to the CMB from a typical DR candidate we require to consider all the possible production channels of DR and the precise thermal background in the early universe.

arXiv:1303.5379 arXiv:2011.13874

QCD Axion and Strong CP problem

- QCD axion is a solution for the strong CP problem and tiny value of neutron electric dipole moment!
- Connection to the UV completion physics through Peccei-Quinn symmetry restoration/breaking at high scales → giving mass to axions as pseudo Nambu-Goldstone bosons
- Possible contribution to the dark matter or dark radiation $(m_a \ll 1\ eV)$. The relation between axion mass and axion decay constant:

$$m_a \simeq 5.7 \times 10^{-6}\,\text{eV}\left(\frac{10^{12}\,\text{GeV}}{f_a}\right)$$

- There are some constraints from laboratory, astrophysical and cosmological experiments. The supernova constraint put a lower bound on axion decay constant f_a. The upper bound on that comes from the condition on the overclosure of the universe. Axion constraints can be model dependent …

arXiv:2003.01100   R. Pecci, H. Quinn, 1977

Axion Production in the Early Universe

Axions can have different couplings with the SM particles. We consider the KSVZ model.

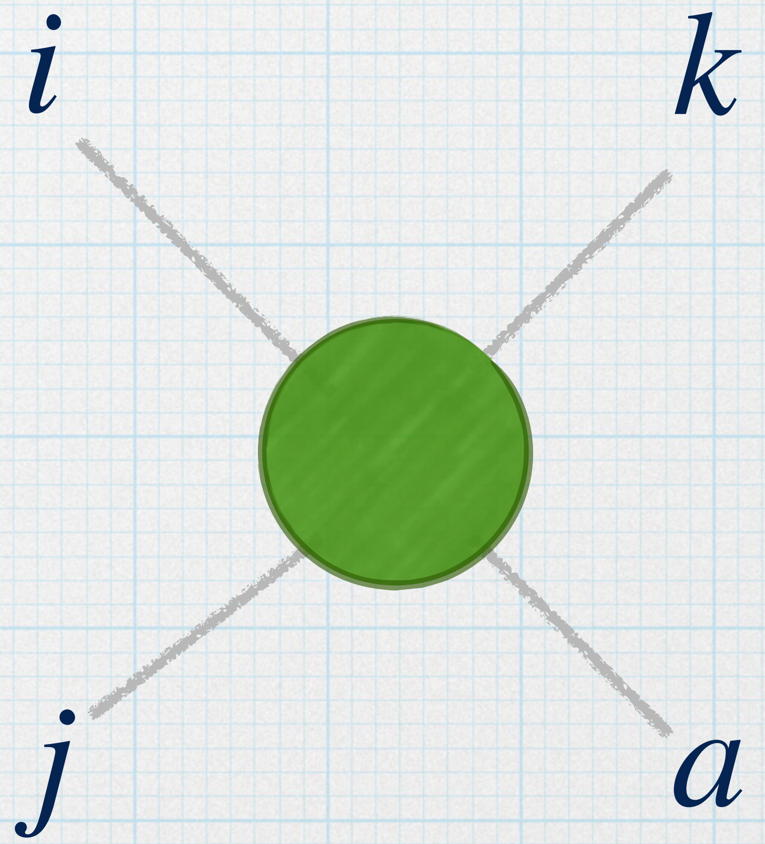
Lagrangian of KSVZ axion model:

$$\mathscr{L}_{aG} = \frac{\alpha_s}{8\pi}\frac{a}{f_a}G^A_{\mu\nu}\widetilde{G}^{A\mu\nu}$$

Coupling depends on light quark masses

$$\mathscr{L}_{a\pi} = \frac{\partial_\mu a}{f_a}\frac{c_{a\pi\pi\pi}}{f_\pi}\left[\pi^0\pi^+\partial^\mu\pi^- + \pi^0\pi^-\partial^\mu\pi^+ - 2\pi^+\pi^-\partial^\mu\pi^0\right]$$

Pion decay constant



Main interactions for KSVZ axion case above and below QCD transition:

$$T \gtrsim \Lambda_{\rm N} : \; q + \bar{q} \to g + a, \;\; q/\bar{q} + g \to q/\bar{q} + a, \;\; g + g \to g + a$$

$$T \lesssim T_{\rm QCD} : \pi^+ + \pi^0 \to \pi^+ + a, \;\; \pi^- + \pi^0 \to \pi^- + a, \;\; \pi^+ + \pi^- \to \pi^0 + a$$

The production rate from rest of hadrons e.g. kaons, neutrons, protons, etc. contribute up to 10 % at $T_{\rm QCD} \simeq 150$ MeV. It can safely be ignored!

J. E. Kim, 1979 M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, 1980 arxiv:2012.04736 arXiv:0203221

Computation of Axion Yield

We need precise equation of state for thermal bath of SM at different temperatures. Entropy and energy density of thermal bath are:

arXiv:1503.03513
arXiv:1803.01038

$$s = \frac{2\pi^2}{45} g_{*s}(T)\, T^3$$

Degrees of freedom

$$\rho = \frac{\pi^2}{30} g_*(T)\, T^4$$

Boltzmann equation for the evolution of the number density of axions: $\frac{d}{dt} n_a + 3 H n_a = \gamma_a \left(1 - \frac{n_a}{n_a^{\text{eq}}}\right)$

Number density

$$n_i^{\text{eq}} = \frac{m_i^2}{2\pi^2} T K_2\left(\frac{m_i}{T}\right) \propto T^3$$

For relativistic case

Modified Bessel function 2nd kind

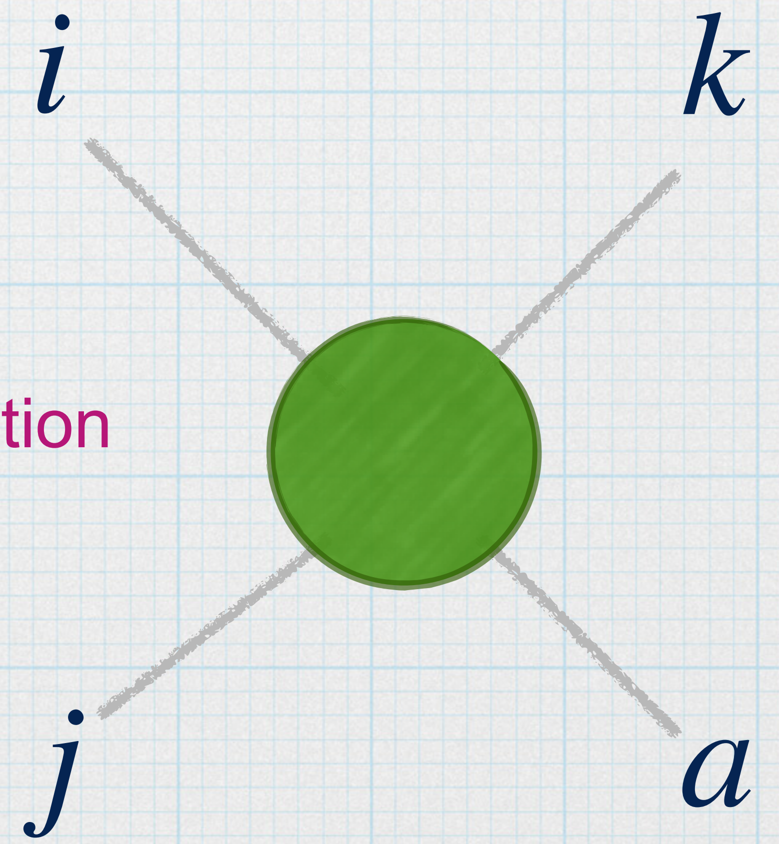
$$s\,H\,x \frac{dY_a}{dx} = \left(1 - \frac{1}{3}\frac{\ln g_{*s}}{\ln x}\right)\gamma_a(x)\left(1 - \frac{Y_a}{Y_a^{\text{eq}}}\right), \quad x = \frac{m}{T}, \quad Y_a = \frac{n_a}{s}, \quad \gamma_a \equiv n_a^{\text{eq}}\,\Gamma_a$$

Entropy degrees of freedom

Sum of all interaction rates

Axion yield

Axion production rate from two body scattering of SM particles in a general case:



Degrees of freedom of incoming particles

$$\Gamma_{a,S} = \frac{g_i g_j}{32\pi^4 n_a^{\text{eq}}} T \int_{s_{min}}^{\infty} ds\, \frac{\lambda\left(s, m_i, m_j\right)}{\sqrt{s}}\, \sigma_{ij\to ka}(s)\, K_1\left(\frac{\sqrt{s}}{T}\right)$$

Modified Bessel function 1st kind

Center of mass energy

Kaellen function

$$s_{min} = \text{Max}\left[\left(m_i + m_j\right)^2, m_k^2\right] \qquad \lambda\left(x, y, z\right) \equiv \left[x - \left(y + z\right)^2\right]\left[x - \left(y - z\right)^2\right]$$

arxiv:2012.04736 arXiv:2003.01100

Thermal Effects on Gluon and Quark Scatterings

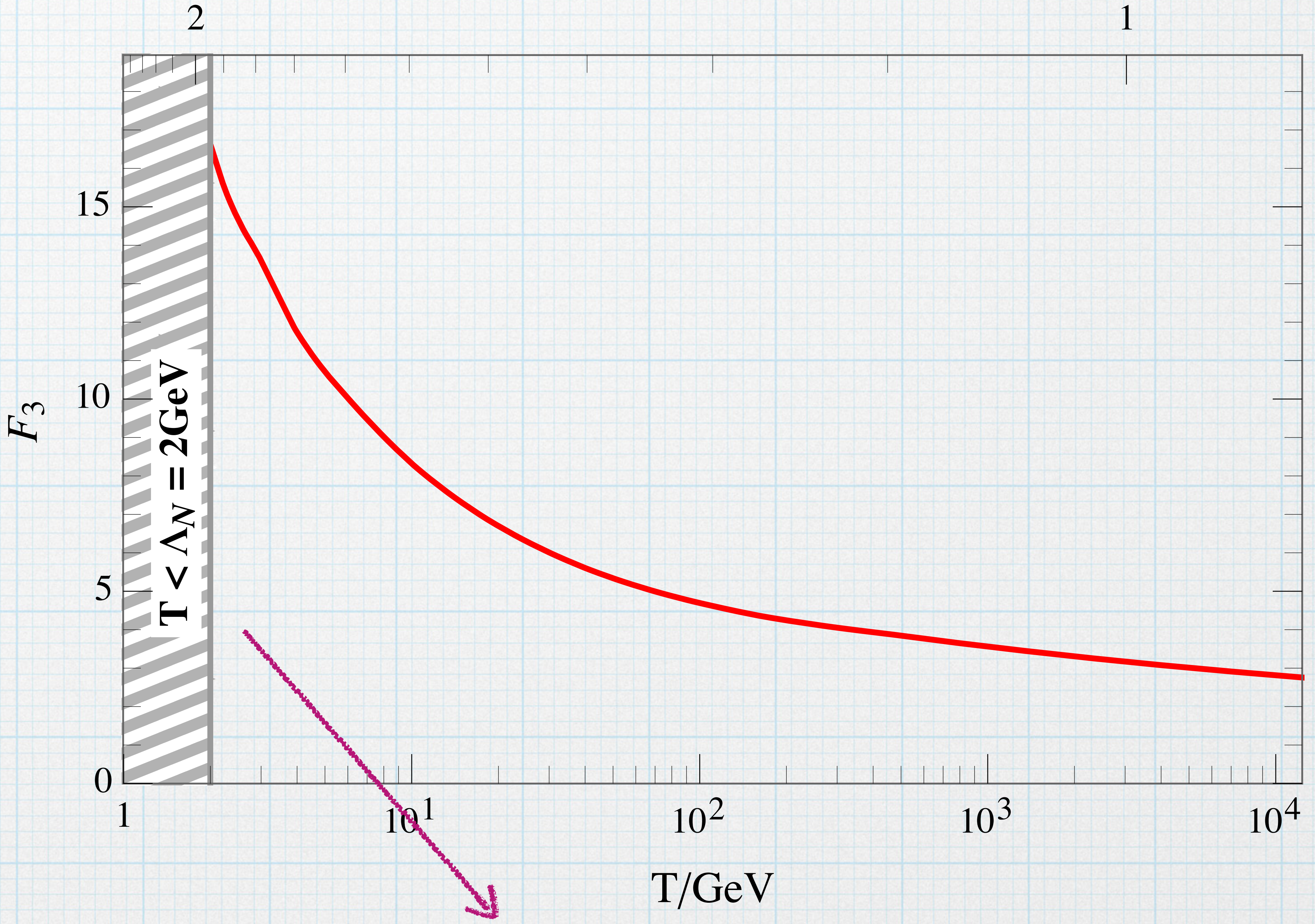
We consider the running of strong coupling g_s up to four loop orders for axion production from gluon and quark scatterings using `RunDec` package. Taking care of IR divergences due to massless gluons:

$$\gamma_{gg} = \frac{2\zeta(3)d_g}{\pi^3}\left(\frac{\alpha_s}{8\pi f_a}\right)^2 F_3\,(T)\ T^6$$

The only diagram contributing to the rate is the one-loop axion two-point function with virtual gluons exchanged, and with the tree-level gluon propagator replaced with the resummed thermal one.

Using continuum and pole approximations outside and inside the light cone for transverse and longitudinal parts of gluon propagator we can numerically calculate the function $F_3(T)$.

g_s Strong coupling



Perturbative QCD scale $T \geq \Lambda_N$

This calculation is also important for DFSZ model and contributes to the rate!

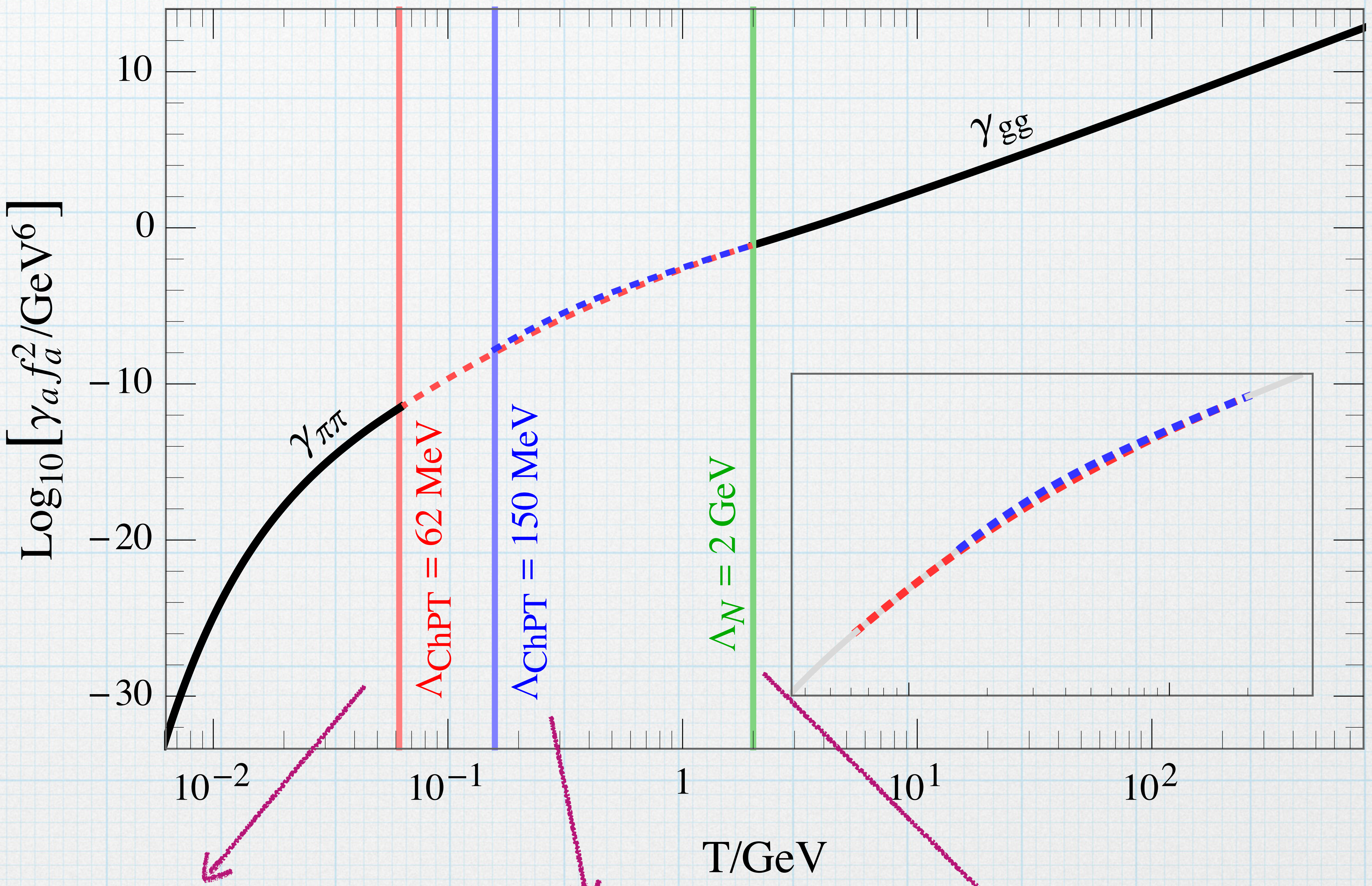
hep-ph/0004189 arXiv:1303.5379 arXiv:1008.4528 arXiv:2108.04259 arXiv:2108.05371

Total Rate of KSVZ Axion

Calculating the total rate of KSVZ axion at low temperatures:

Due to crossover nature of QCD transition in the early universe at vanishing baryon asymmetry we can interpolate between the two regimes that gives a reasonable rate in the intermediate QCD confinement region.

There is a small change in the rate due to different interpolation regime either choosing $\Lambda_{\text{ChPT}} =$ 62 or 150 MeV!



When NLO calculation of pion and axion scattering is important

QCD transition temperature

Perturbative QCD scale

arXiv:1808.07430 arXiv:2108.04259 arXiv:2108.05371

Effective Number of Relativistic Axion ΔN_{eff}

By calculating all axion interactions for any given model the number of effective extra degrees of freedom dominated by axions can be evaluated:

$$\Delta N_{\mathrm{eff}} \equiv N_{\mathrm{eff}} - N_{\mathrm{eff}}^{\mathrm{SM}} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3} \frac{\rho_a}{\rho_\gamma}$$

$$\Delta N_{\mathrm{eff}} \simeq 75.6\, Y_a^{4/3} \propto f_a^{-8/3}$$

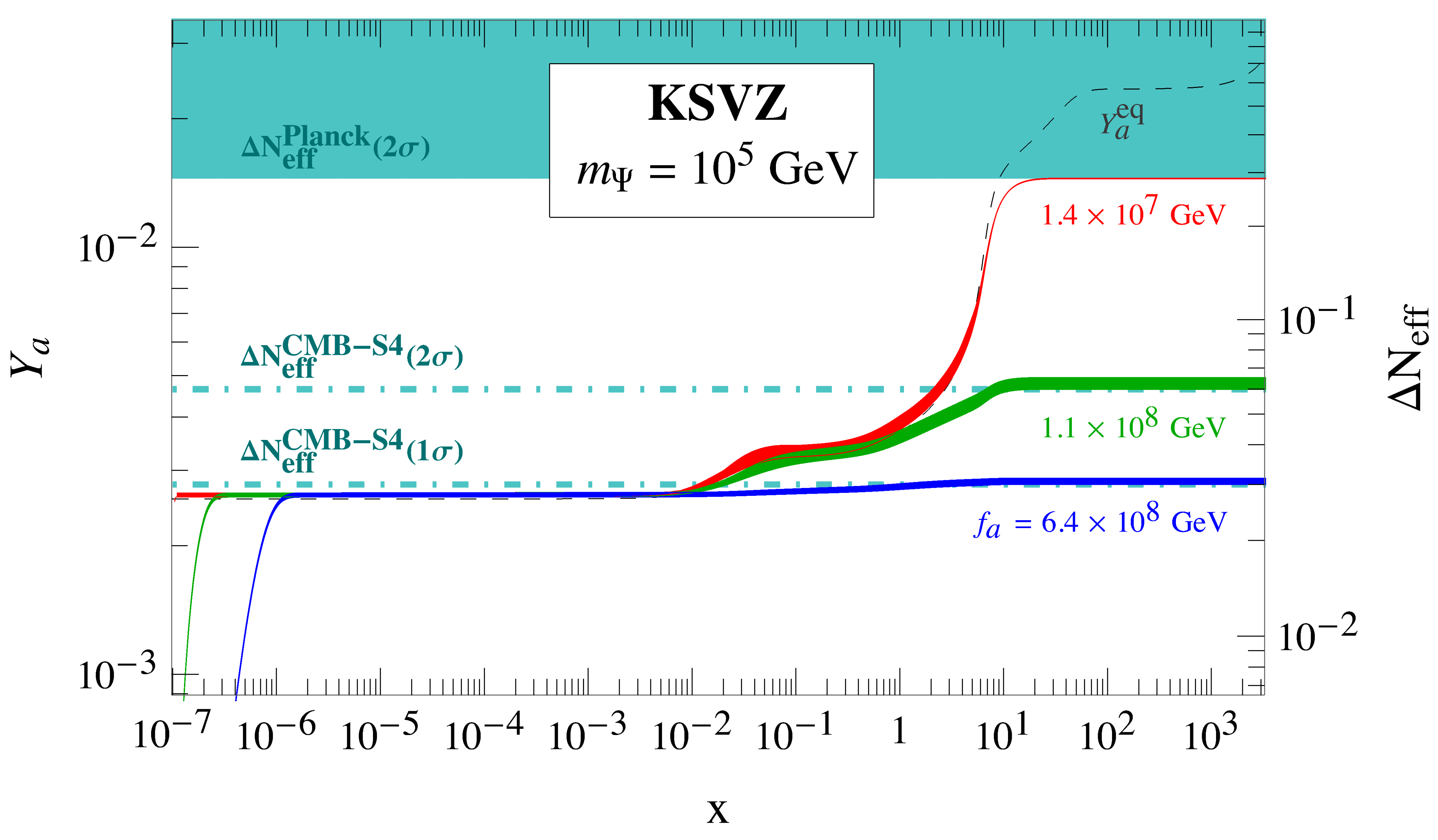
It can be constrained by CMB experiments which can falsify different axion models!

When axions thermalise $(\Gamma_a \gtrsim H)$ the value of ΔN_{eff} gets its maximum. If axions do not reach the thermal equilibrium the final value of ΔN_{eff} depends on the initial abundance of them!

arXiv:2003.01100 arxiv:2012.04736

Axion Yield in KSVZ Model

Axion yield evolution versus time or temperature:



Seokhoon Yun's Talk

J. E. Kim, 1979 M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, 1980

arXiv:2108.04259 arXiv:2108.05371

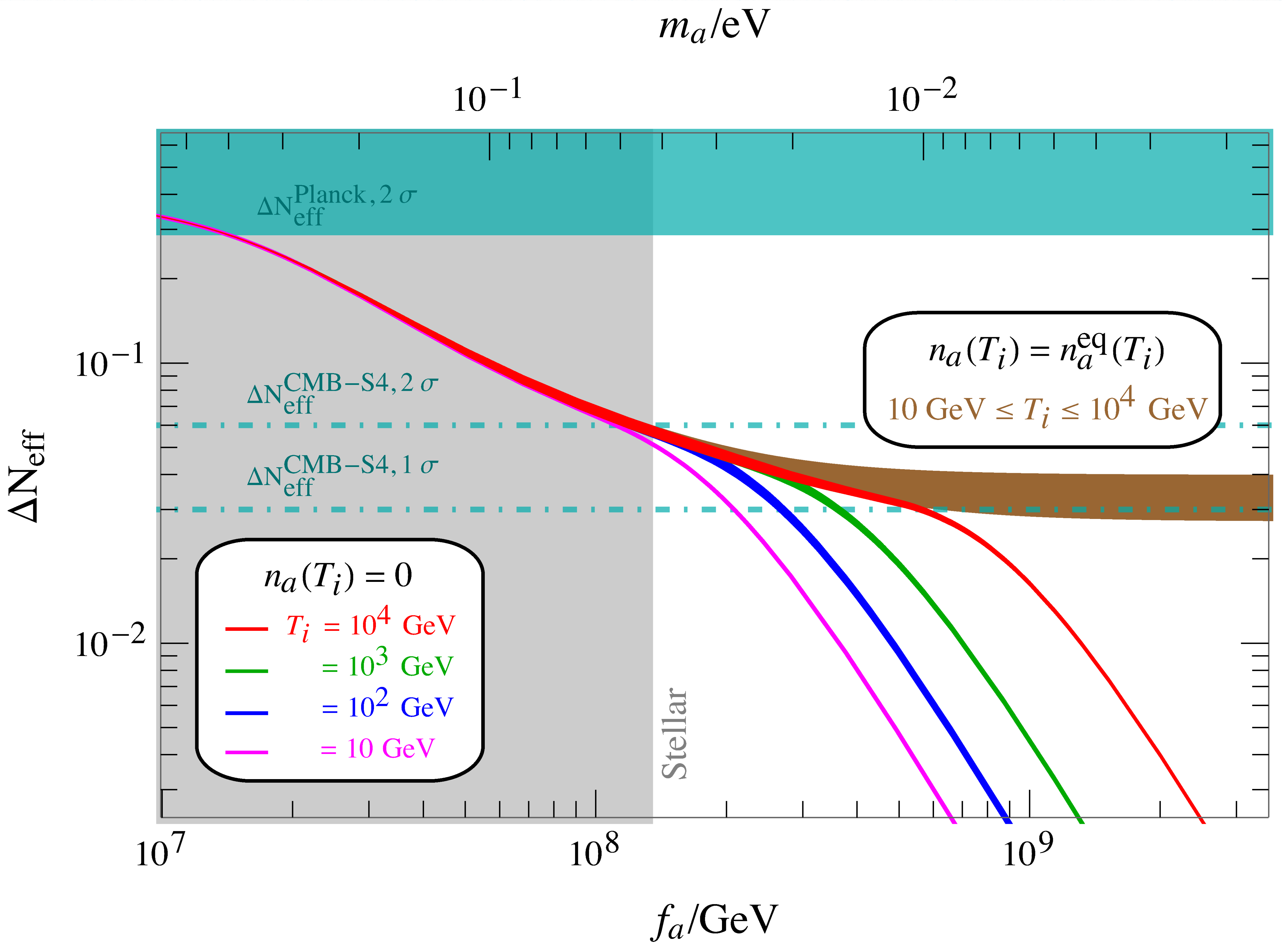
Detectability Perspective for Different Axion Decay Constants

Number of effective neutrinos ΔN_{eff} versus axion decay constant f_a :

Planck is not sensitive enough for large value of f_a that is not constrained by Supernova bound!

We consider different initial temperatures and initial axion densities.

CMB-S4 experiment can probe a large portion of this parameter space in case $\Delta N_{\text{eff}} \gtrsim 0.03$ for different f_a 's!



Stellar bound from supernova SN1987A $f_a \gtrsim 1.4 \times 10^8$ GeV !

arXiv:2108.04259 arXiv:2108.05371

Conclusions

- Computing the axion production rate below and above the QCD crossover transition and matching the two limits with the interpolation that is physically meaningful
- Considering precise interaction rates of axion including thermal effects at different scales that improves theoretical prediction for dark radiation from axion
- Accurate treatment of thermal bath at different temperatures is necessary to have a correct estimation of axion abundance
- Predicting the precise value of ΔN_{eff} for different axion models is theoretically motivated for future probes by experiments like CMB-S4

Thank you for your attention!

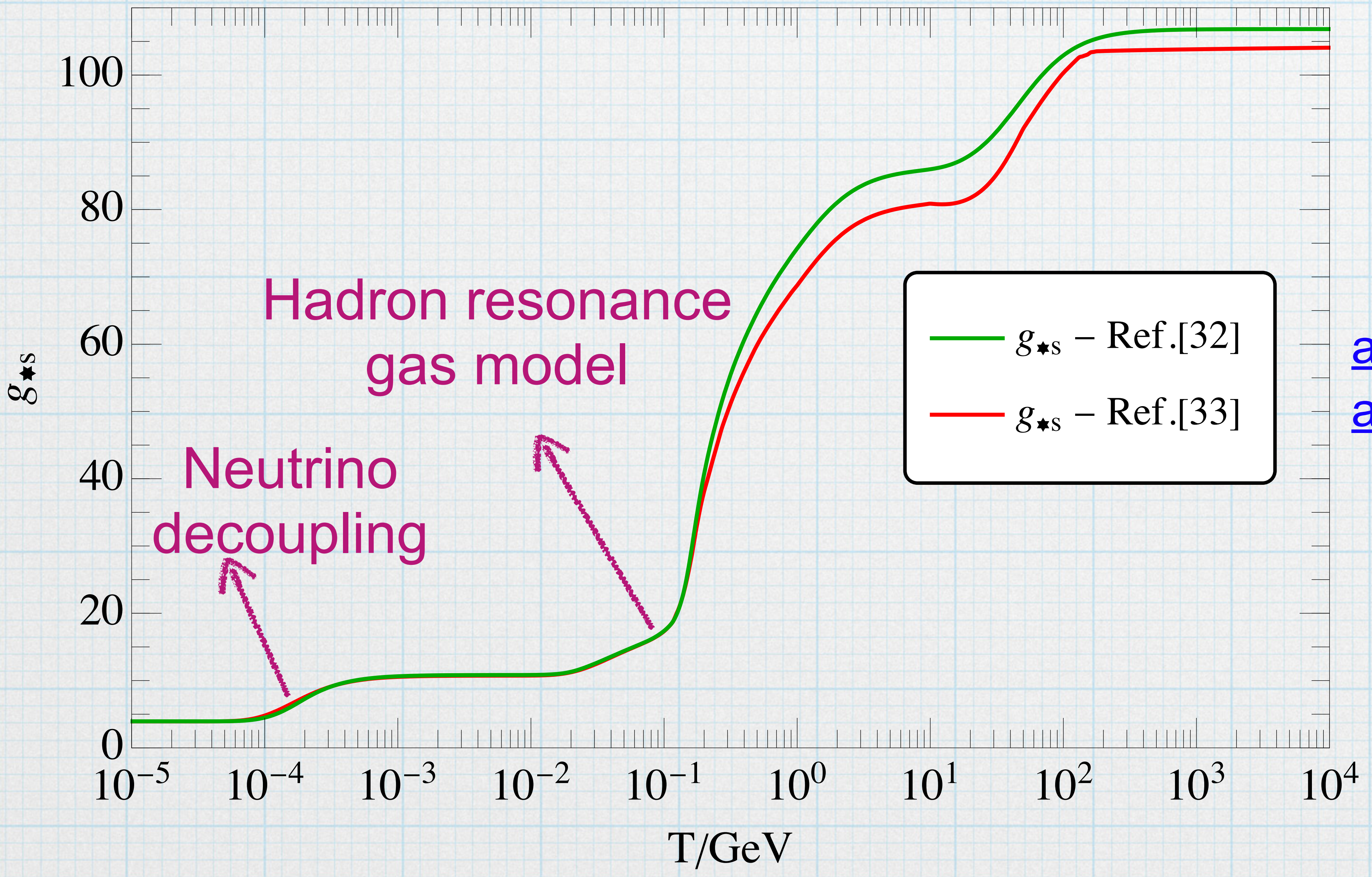
Thermal Background of SM Particles

We need precise equation of state for thermal bath of SM at different temperatures. Energy and entropy density of thermal bath:

$$s = \frac{2\pi^2}{45} g_{*s}(T)\, T^3 \qquad \rho = \frac{\pi^2}{30} g_{*}(T)\, T^4$$

Degrees of freedom

Entropy density degrees of freedom



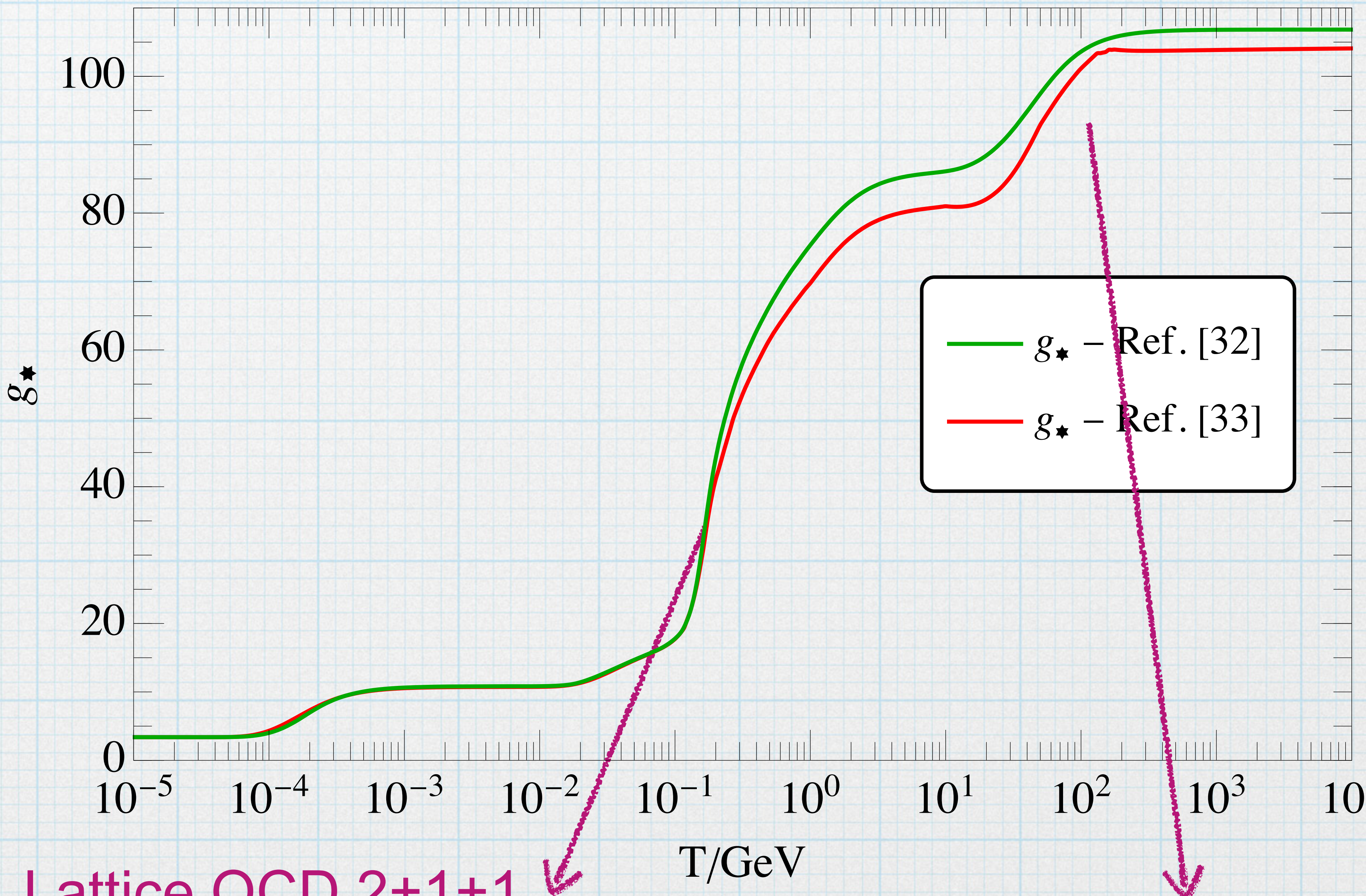
Hadron resonance gas model

Neutrino decoupling

arXiv:1503.03513
arXiv:1803.01038

The rest of SM particles considered free!

Energy density degrees of freedom



arXiv:1503.03513
arXiv:1803.01038

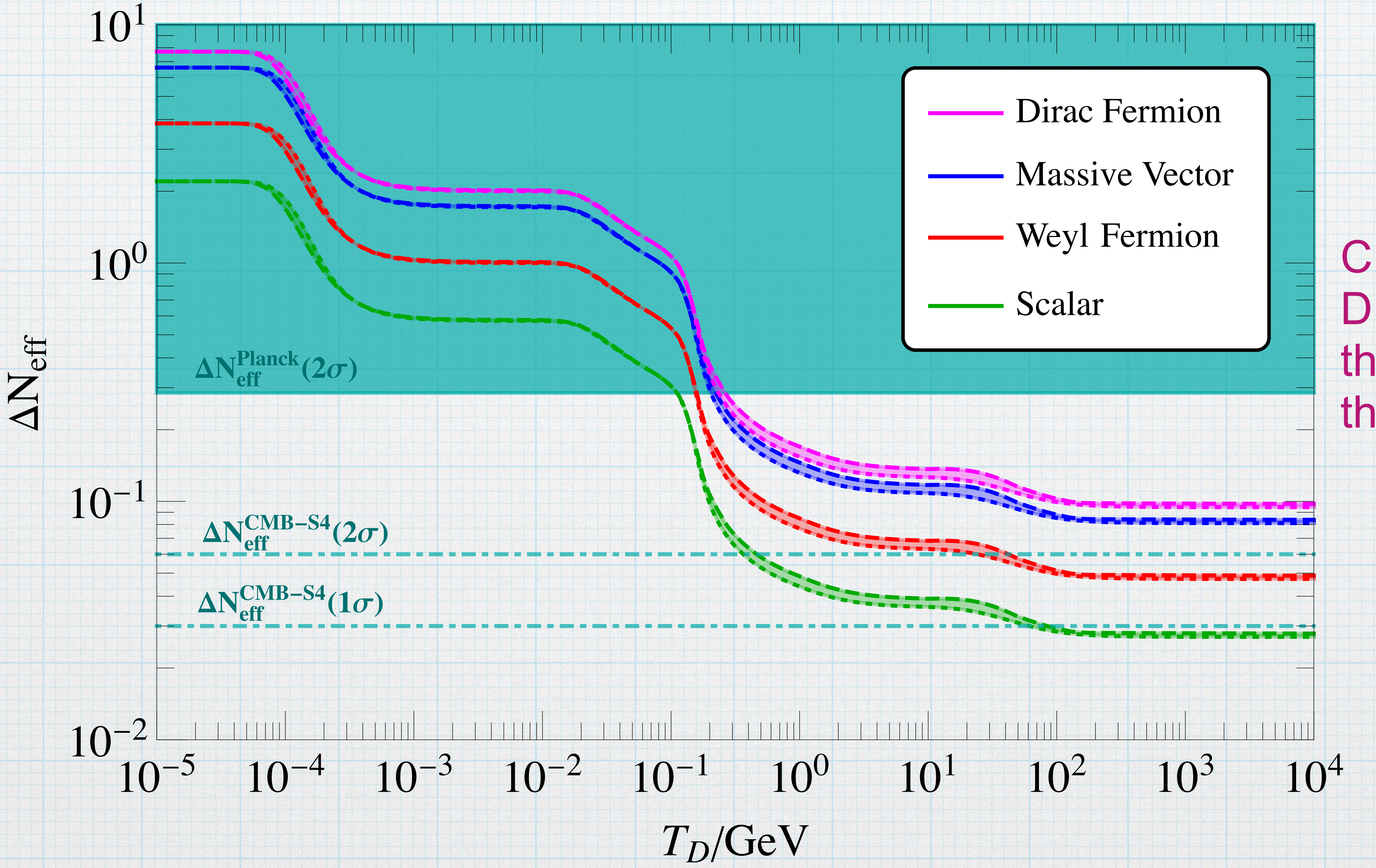
Lattice QCD 2+1+1
For QCD cross over transition

Electroweak crossover transition

Different Types of Dark Radiation

Number of effective neutrinos $\Delta N_{\rm eff}$ versus decoupling temperature $T_{\rm D}$ of a typical DR:

$$\Delta N_{\rm eff} \equiv N_{\rm eff} - N_{\rm eff}^{\rm SM} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3}\frac{\rho_{\rm DR}}{\rho_\gamma} \simeq g_{*\Phi} \times 13.69 \times g_{*s}^{\rm SM}(T_{\rm D})^{-4/3}$$



CMB-S4 experiment can probe DR candidates decoupled around the QCD transition and above that!

CMBS4, arXiv:1610.02743 CMBS4, arXiv:1907.04473 Planck, arXiv:1807.06209 arXiv:2011.13874