Going non-linear: toward a 1-loop model for the 3-point matter correlation function

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Three-point correlation function: the need of going non-linear

3PCF: a *non-Gaussianity* machine

Considering a non-gaussian density field opens the door to the **three-point correlation function**:

- **Gravitational** non-linear evolution of perturbations
- **Galaxy biasing**: how luminous tracers trace dark matter?

- **Primordial non-Gaussianity**: a possible source of NG encoded in the inflaton field

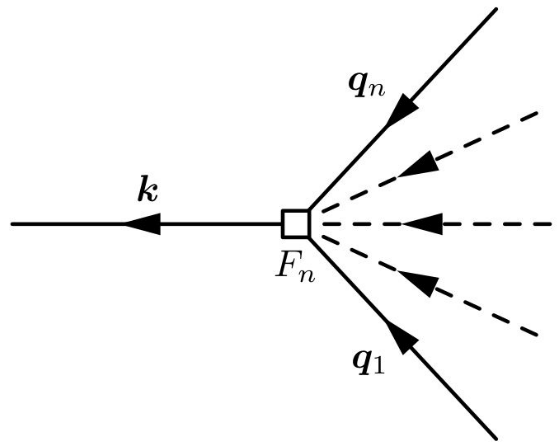
Non-linear phenomena —*generate*→ Non-gaussianity in the density field

The **3PCF** is the higher order correlation function with the high signal-to-noise ratio. It encodes **a lot of cosmological information**

Non-linearities: Cosmological Perturbation Theory

Conservation of momentum, mass and Poisson equation rule the **evolution of cosmological perturbations**. Fourier space is the most convenient space to deal with them: Non-linearity can be modelled by a **perturbation theory** approach.

- **Linear** regime: k- modes are **indipendent**
- **Non-linear** regime: **coupling** between different k-modes.
  - Tree-level: zero order of expansion
  - One-loop: first order of expansion
  - Higher orders

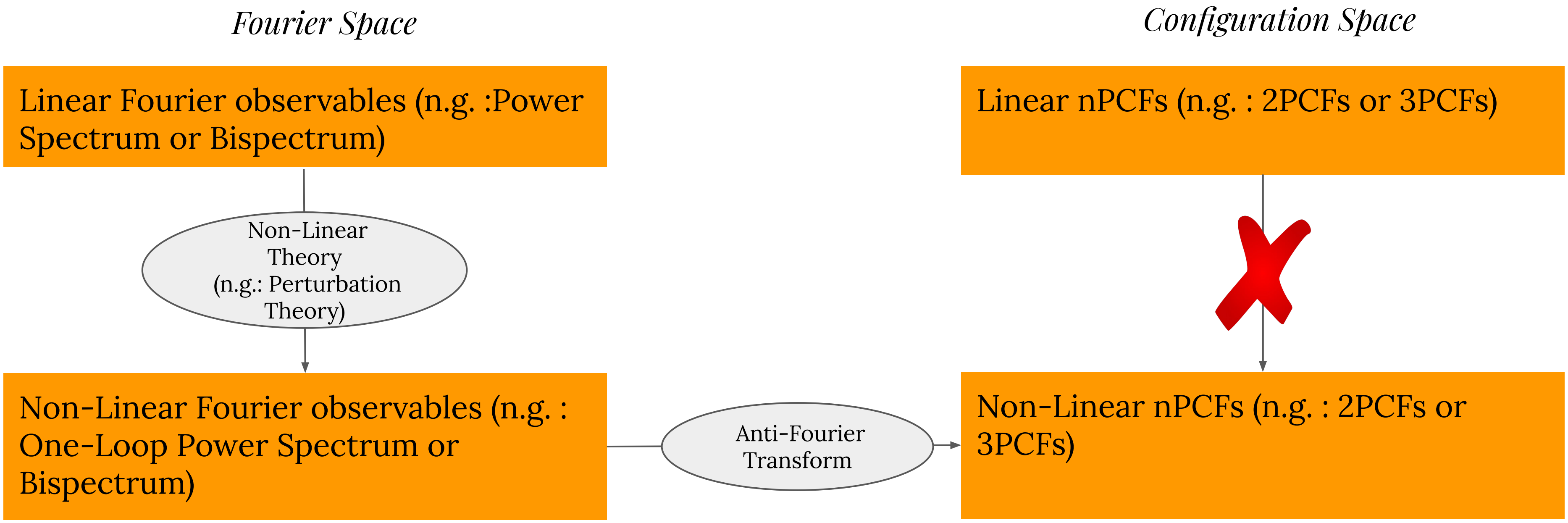


Picture from Baldauf's lectures on Advanced Cosmology, 2019

How modelling non-linear three-point correlation function: the method

Modelling non-linearity in Configuration Space

Perturbation theory cannot be **analytically** formalised in **configuration space** due to the complexity of mathematical operators involved. It needs to be **numerically** explored



Modelling non-linearity in Fourier Space

Advantages of Fouries Space modelling:

- k-modes are <u>indipendent</u> until we enter the non-<u>linear</u> regime
- It is possible to use a <u>Feynmann-loops</u>-like approach when we face <u>k-modes coupling</u>

Fourier Space

Linear Fourier observables (n.g. :Power Spectrum or Bispectrum)

↓

Non-Linear Theory (n.g.: Perturbation Theory)

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Non-Linear Fourier observables (n.g. : One-Loop Power Spectrum or Bispectrum)

Example of integrals involved in 1-loop matter bispectrum:

$$B_{222}^{1-\mathrm{loop}} = 8 \int_{\mathbf{q}} F_2(-\mathbf{q}, \mathbf{k}_3+\mathbf{q}) F_2(\mathbf{k}_3+\mathbf{q}, \mathbf{k}_2-\mathbf{q})\, F_2(\mathbf{k}_2-\mathbf{q}, \mathbf{q})\, P_L(q)\, P_L(|\mathbf{k}_2-\mathbf{q}|)\, P_L(|\mathbf{k}_3+\mathbf{q}|)$$

$$B_{321,I}^{1-\mathrm{loop}} = 6\, P_L(k_3) \int_{\mathbf{q}} F_3(-\mathbf{q}, -\mathbf{k}_2+\mathbf{q}, -\mathbf{k}_3)\, F_2(\mathbf{k}_2-\mathbf{q}, \mathbf{q})\, P_L(|\mathbf{k}_2-\mathbf{q}|)\, P_L(q) + 5 \text{ perm.}$$

$$B_{321,II}^{1-\mathrm{loop}} = 6\, P_L(k_2)\, P_L(k_3)\, F_2(\mathbf{k}_2, \mathbf{k}_3) \int_{\mathbf{q}} F_3(\mathbf{k}_3, \mathbf{q}, -\mathbf{q})\, P_L(q) + 5 \text{ perm.}$$

$$B_{411}^{1-\mathrm{loop}} = 12\, P_L(k_2)\, P_L(k_3) \int_{\mathbf{q}} F_4(\mathbf{q}, -\mathbf{q}, -\mathbf{k}_2, -\mathbf{k}_3)\, P_L(q) + 2 \text{ perm.}.$$

Modelling non-linear three-point correlation function

Multipoles from Fourier Space to Configuration Space are linked through n-dimensional Hankel transform. In the 2PCF case (1D Hankel transform):

$$\xi_l(r) = (-1)^l \int \frac{k^2}{2\pi^2} dk P_l(k) j_L(kr)$$

In the case of interest: **3PCF** and Bispectrum multipoles: **2D Hankel transform** (*Slepian et al*, 2016)

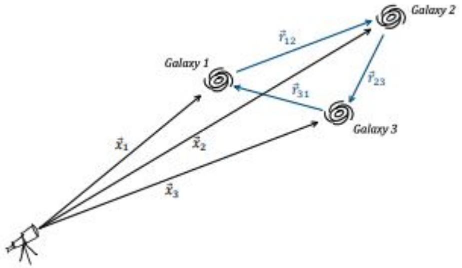
$$\boxed{\zeta_\ell(r_1, r_2) = (-1)^\ell \int \frac{k_1^2 k_2^2 dk_1 dk_2}{(2\pi^2)^2} B_{s,\ell}(k_1, k_2) j_\ell(k_1 r_1) j_\ell(k_2 r_2)}$$

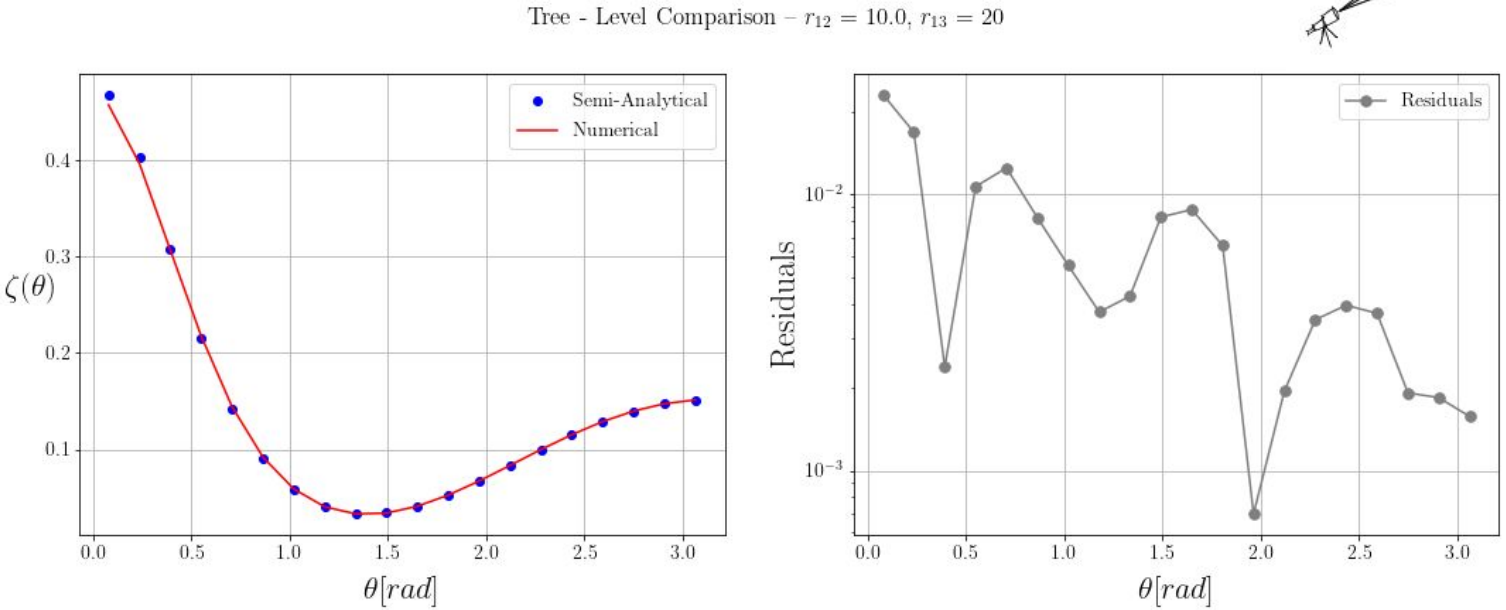
- **Tree level**: a semianalytical formula (*Slepian et al*, 2016)
- **Beyond tree level**: numerical approach by 2D-FFTLog (*Fang et al*, 2020), (*Guidi et al*, in prep)

Testing non-linear modelling of three-point correlation function

Testing the alghoritm

- **Leading order**: well established model, obtainable by a semianalytical approach
- **Next-to-leading order**: new model, numerical

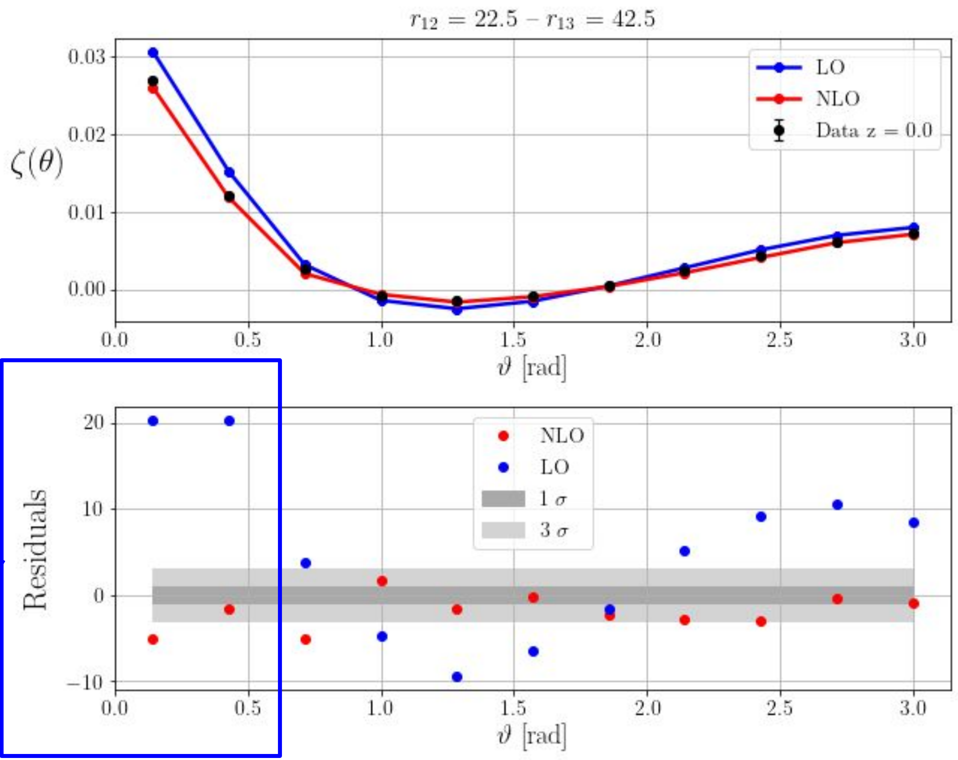




Non-linear 3PCF vs sim. data: *quasi-non* linear scales

- **Simulated Data**: DEMNUni (N-Body) simulation (21 *million particles (box lenght: 2Gpc/h), Standard* ΛCDM)
    - Measure: ~5 day (32 cores)

- **Leading order**: well established model, obtainable by a semianalytical approach
- **Next-to-leading order**: new model, fully numerical



Squeezed configuration!

Conclusion

Non-linear 3PCF: summary and perspectives

Summary:

- A new numerical method to take account of non-linearity in modelling 3PCFs
- Testing the algorithm and comparison between different models of non-linear 3PCF
- Comparison between non-linear 3PCF models against N-body simulation

Perspectives:

1. Non-linear model for <u>3PCF of galaxies</u> from bispectrum (Eggemeier et al, 2019): how distribution of galaxies traces dark matter?
2. Non-linear model for <u>3PCF in redshift space</u> (i.e. adding redshift space distortion):
3. Non-linear model of <u>3PCF with primordial non-gaussianity</u>
4. Analysis of real data (from Euclid):
    a. Expected launch: 2022
    b. Expected first release: 2025

2D-FFTLog algorithm

The **2D-FFTLog** algorithm allow us to perform a **fast** and **accurate** computation of **2D-Hankel transform:**

Step of 1D-FFTLog algorithm (Hamilton, 1999)

1. Suppose a periodic function in **log-space**
2. Standard 1D-FFT to obtain Fourier coefficient
3. Multiply by the analytically integrable part
4. 1D-FFT again to get the full Hankel transform

$$\tilde{a}(k) = \sum_m{}' c_m \int_0^\infty \mathrm{e}^{2\pi i m \ln(r/r_0)/L} \, (kr)^q J_\mu(kr) \, k \, \mathrm{d}r$$

$$\tilde{a}(k) = \sum_m{}' c_m u_m \mathrm{e}^{-2\pi i m \ln(k/k_0)/L}$$

Extension to **2D**-FFTLog by *Fang et al*, 2020. Originally proposed to compute 2PCF/3PCF covariances