

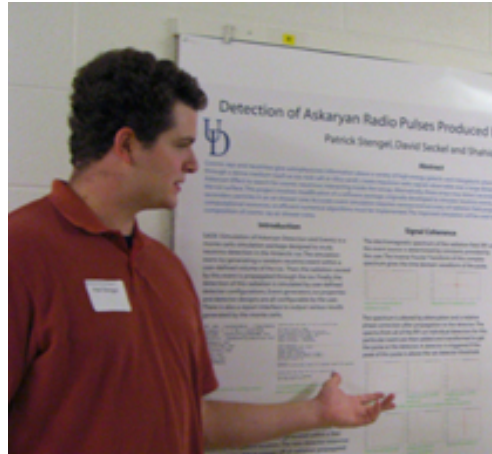
# A Simplified Dark Matter Model for Muon $g-2$ with Scalar Lepton Partners at the TeV Scale

*Jan Tristram Acuña*

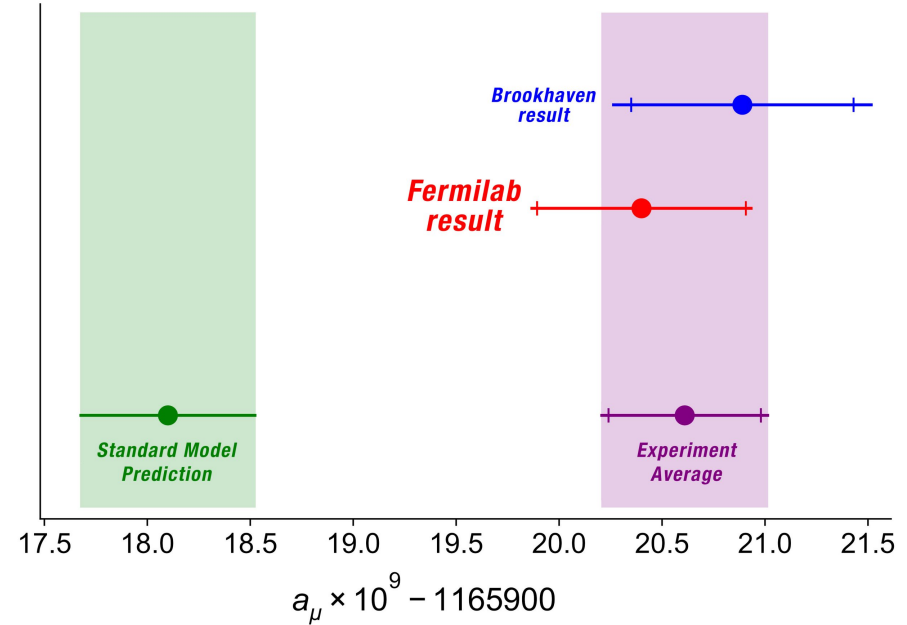
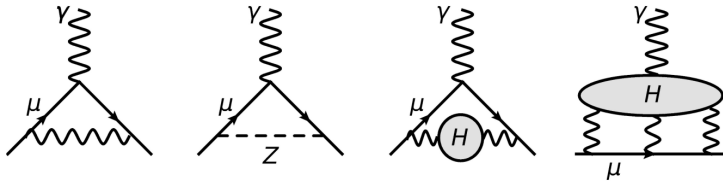
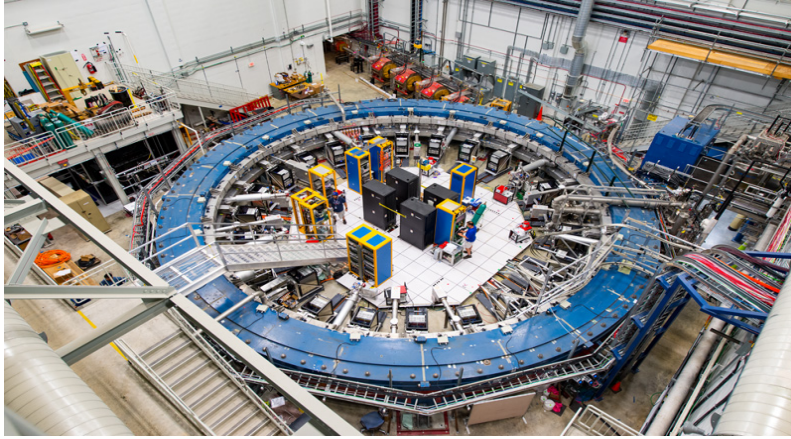
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Trieste, Italy*



- JTA, Patrick Stengel, and Piero Ullio, *arxiv:2109:xxxxx*



# Motivation: FNAL measurement

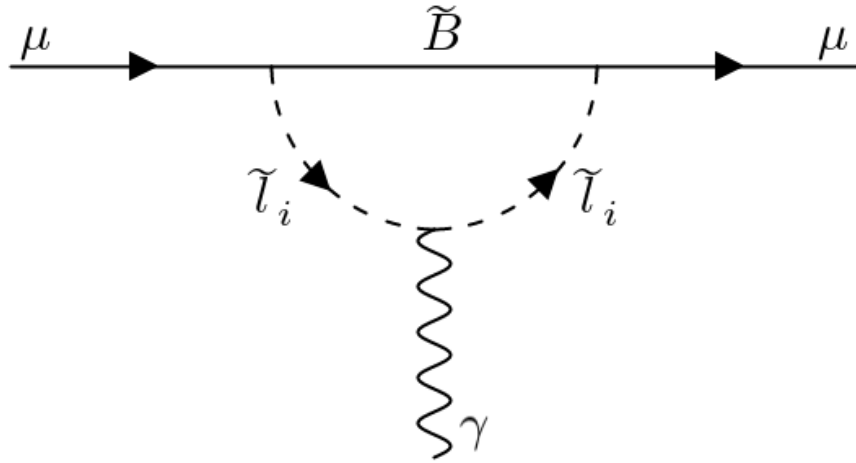


$$\begin{aligned}
 a_\mu(FNAL) &= 116592040(54) \times 10^{-11} (0.46 \text{ ppm}) \\
 a_\mu(BNL) &= 116592080(54)(33) \times 10^{-11} (0.54 \text{ ppm}) \\
 a_\mu(\text{exp}) &= 116592061(41) \times 10^{-11} (0.35 \text{ ppm}) \\
 a_\mu(SM) &= 116591810(43) \times 10^{-11} (0.37 \text{ ppm})
 \end{aligned}$$

$$a_\mu \equiv \frac{g-2}{2}$$

$$\Delta a_\mu = (25.1 \pm 5.9) \times 10^{-10}$$

# Assume new physics

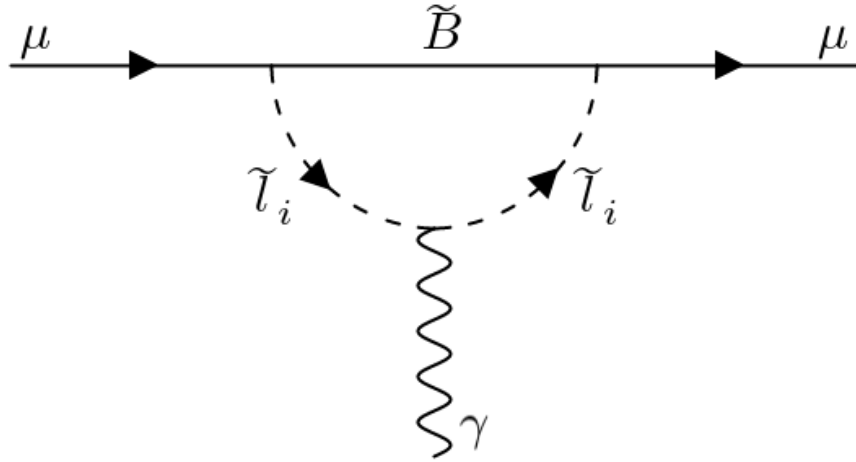


$$\mathcal{L} \supset -\lambda_L \tilde{l}_L^\dagger \bar{B}_R l_L - \lambda_R \tilde{l}_R^\dagger \bar{B}_L \mu_R + \text{h.c.}$$

$$\lambda_{L,R} = \sqrt{2} g_Y Y_{L,R}$$

| Particle                              | $\text{SU}(2)_L$ | $\text{U}(1)_Y$ | $\text{Z}_2$ |
|---------------------------------------|------------------|-----------------|--------------|
| $\sim l_L = (\sim \nu_L, \sim \mu_L)$ | 2                | -1/2            | -1           |
| $\sim \mu_R$                          | 1                | -1              | -1           |
| $l_L = (\nu_L, \mu_L)$                | 2                | -1/2            | 1            |
| $\mu_R$                               | 1                | -1              | 1            |
| $\sim B$                              | 1                | 1               | -1           |

# Additional terms



$$\mathcal{L} \supset - (\tilde{\mu}_L^\dagger \quad \tilde{\mu}_R^\dagger) \begin{pmatrix} m_{LL}^2 & m_{LR}^2 \\ m_{LR}^2 & m_{RR}^2 \end{pmatrix} \begin{pmatrix} \tilde{\mu}_L \\ \tilde{\mu}_R \end{pmatrix} - m_{LL}^2 \tilde{\nu}_L^\dagger \tilde{\nu}_L$$

$$\mathcal{L} \supset -m_{LL,soft}^2 \tilde{l}_L^\dagger \tilde{l}_L - m_{RR,soft}^2 \tilde{\mu}_R^\dagger \tilde{\mu}_R$$

$$\mathcal{L}_D = -\frac{g_Y^2}{2} \left| \sum_i Y_i \mathcal{S}_i^\dagger \mathcal{S}_i \right|^2 - \frac{g^2}{4} \left[ \text{tr} \{ M^2 \} - \frac{1}{2} (\text{tr} \{ M \})^2 \right]$$

$$M_{AB} \equiv \sum_i \mathcal{S}_{iA} \mathcal{S}_{iB}^\dagger$$

$$m_{LL}^2 = m_1^2 c_\theta^2 + m_2^2 s_\theta^2$$

$$m_{RR}^2 = m_1^2 s_\theta^2 + m_2^2 c_\theta^2$$

$$m_{LR}^2 = |m_2^2 - m_1^2| c_\theta s_\theta$$

Model parameters:

$M_B, m_1, m_2, \theta$

$$\tilde{\mu}_1 = \tilde{\mu}_L c_\theta - \tilde{\mu}_R s_\theta, \quad \tilde{\mu}_2 = \tilde{\mu}_L s_\theta + \tilde{\mu}_R c_\theta$$

# Contribution to g-2

$$\Delta a_\mu = -\frac{1}{32\pi^2} \lambda_L \lambda_R \sin(2\theta) \frac{m_\mu}{M_B} [L(x_1) - L(x_2)]$$

$$L(x) \equiv \frac{x}{(1-x)^2} \left[ 1 + x + \frac{2x \ln x}{(1-x)} \right], \quad x_i \equiv \frac{M_B^2}{m_i^2}$$

$$r \equiv \frac{m_1 - M_B}{M_B}, \quad y \equiv \frac{m_2^2 - m_1^2}{4m_W^2} |\sin(2\theta)|$$

$(M_B, r, y, \theta)$

$$m_{LL}^2 = m_1^2 c_\theta^2 + m_2^2 s_\theta^2$$

$$m_{RR}^2 = m_1^2 s_\theta^2 + m_2^2 c_\theta^2$$

$$m_{LR}^2 = |m_2^2 - m_1^2| |c_\theta s_\theta|$$

$$m_\nu^2 = m_1^2 + (m_2^2 - m_1^2) s_\theta^2 - m_W^2$$

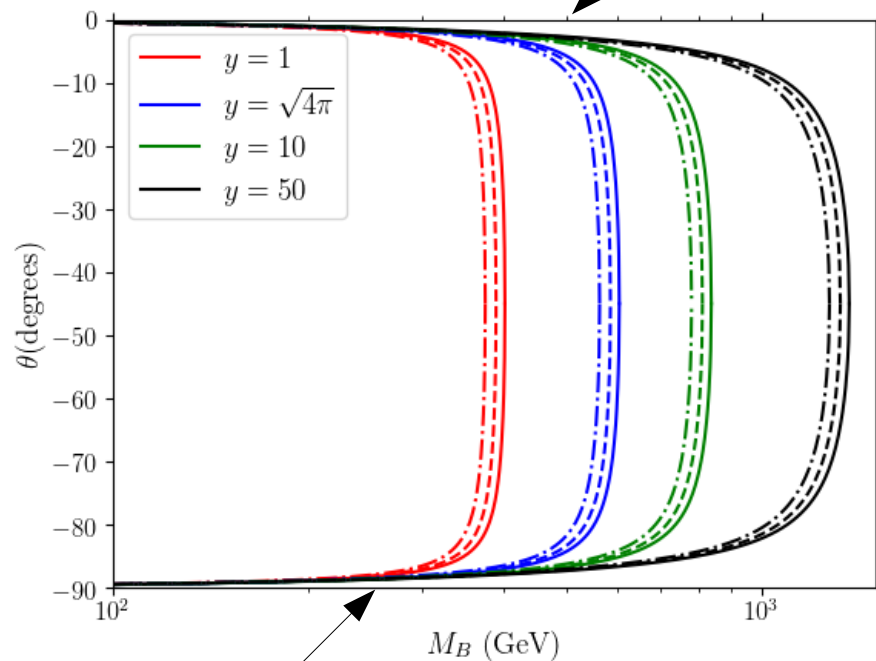
$$\tilde{\mu}_1 = \tilde{\mu}_L c_\theta - \tilde{\mu}_R s_\theta, \quad \tilde{\mu}_2 = \tilde{\mu}_L s_\theta + \tilde{\mu}_R c_\theta$$



Left handed branch (LHB)

$$[M_B < m_1 \leq m_\nu \ll m_2]$$

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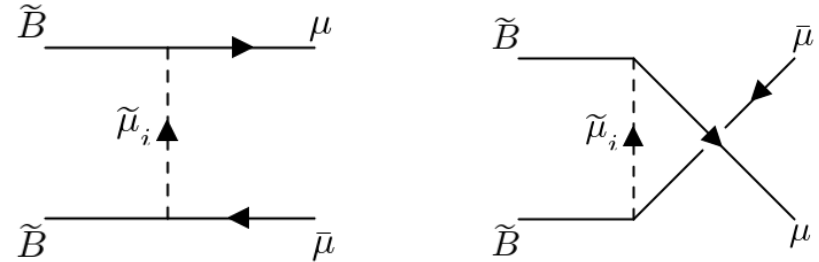


Right handed branch (RHB)

$$[M_B < m_1 \ll m_\nu \leq m_2]$$

# DM implications

| Particle                                       | $SU(2)_L$ | $U(1)_Y$ | $Z_2$     |
|------------------------------------------------|-----------|----------|-----------|
| $\tilde{l}_L = (\tilde{\nu}_L, \tilde{\mu}_L)$ | 2         | -1/2     | -1        |
| $\tilde{\mu}_R$                                | 1         | -1       | -1        |
| $l_L = (\nu_L, \mu_L)$                         | 2         | -1/2     | 1         |
| $\mu_R$                                        | 1         | -1       | 1         |
| <b><math>\tilde{B}</math></b>                  | <b>1</b>  | <b>1</b> | <b>-1</b> |



$$\langle \sigma v \rangle_0 = \frac{M_B^2}{32\pi} [\lambda_L \lambda_R \sin(2\theta)]^2 \left( \frac{1}{m_1^2 + M_B^2} - \frac{1}{m_2^2 + M_B^2} \right)^2$$

$$\begin{aligned} \langle \sigma v \rangle_0 &= \frac{32\pi^3}{m_\mu^2} (\Delta a_\mu)^2 \mathcal{F} \\ &= (2.25 \times 10^{-4} \text{ pb}) \left( \frac{\Delta a_\mu}{25 \times 10^{-10}} \right)^2 \mathcal{F}, \end{aligned}$$

$$\Omega h^2 \sim 0.1 \left( \frac{1 \text{ pb}}{\langle \sigma v \rangle(T_f)} \right)$$

$$\mathcal{F} \equiv \frac{1}{[L(M_B^2/m_1^2) - L(M_B^2/m_2^2)]^2} \left( \frac{1}{1 + m_1^2/M_B^2} - \frac{1}{1 + m_2^2/M_B^2} \right)^2$$

# Coannihilations

| Initial states                     | Final states                                                         |
|------------------------------------|----------------------------------------------------------------------|
| $\tilde{B}\tilde{B}$               | $\mu^-\mu^+, \nu_\mu\bar{\nu}_\mu$                                   |
| $\tilde{\mu}_i\tilde{\mu}_j^*$     | $f\bar{f}, \gamma\gamma, W^+W^-, ZZ, hh, Z\gamma, h\gamma, hZ$       |
| $\tilde{\mu}_i\tilde{\mu}_j$       | $\mu^-\mu^-$                                                         |
| $\tilde{\nu}_\mu\tilde{\nu}_\mu^*$ | $f\bar{f}, \nu_\mu\bar{\nu}_\mu, \nu_\mu\nu_\mu, W^+W^-, ZZ, hh, hZ$ |
| $\tilde{B}\tilde{\mu}_i$           | $\mu^-\gamma, \mu^-Z, \mu^-h, \nu_\mu W^-$                           |
| $\tilde{B}\tilde{\nu}_\mu$         | $\nu_\mu Z, \nu_\mu h, \mu^-W^+$                                     |
| $\tilde{\mu}_i^*\tilde{\nu}_\mu$   | $\mu^+\nu_\mu, W^+Z, W^+\gamma, W^+h, f_u\bar{f}_d$                  |
| $\tilde{\mu}_i\tilde{\nu}_\mu$     | $\mu^-\nu_\mu$                                                       |

$$\frac{dn}{dt} + 3Hn = -\langle\sigma v\rangle_{eff} (n^2 - n_{eq}^2)$$

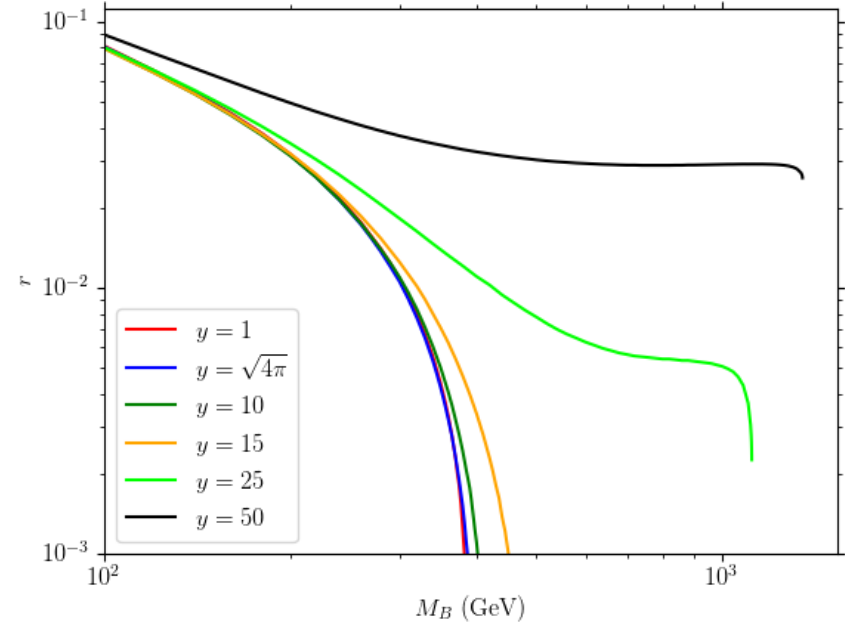
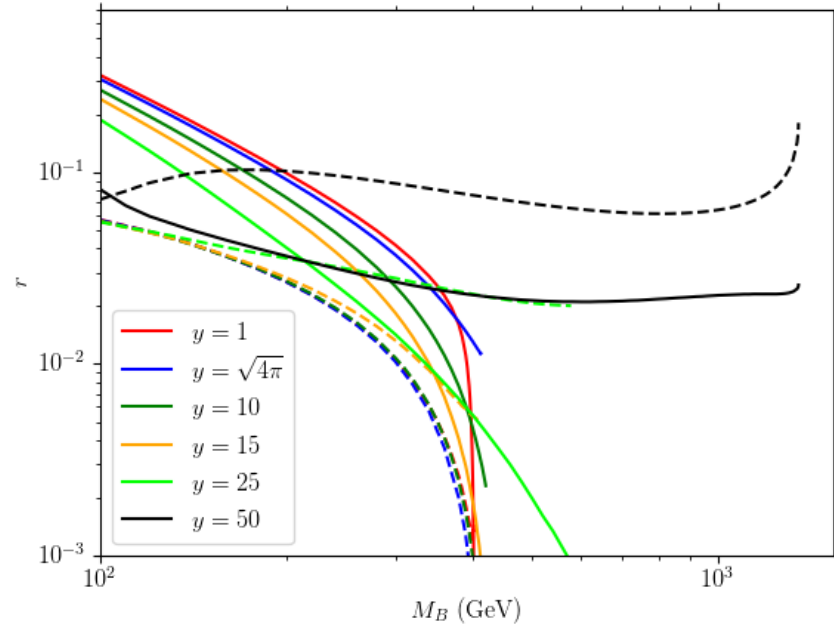
$$\langle\sigma v\rangle_{eff} = \sum_{ij} \langle\sigma v\rangle_{ij} r_i r_j,$$

$$r_i \equiv \frac{\frac{g_i}{g_{LSP}} (1 + \Delta_i)^{3/2} \exp(-x\Delta_i)}{1 + \sum_{k \neq LSP} \frac{g_k}{g_{LSP}} (1 + \Delta_k) \exp(-x\Delta_k)}$$

$$\Delta_i \equiv \frac{m_i - m_{LSP}}{m_{LSP}}, \quad x \equiv \frac{m_{LSP}}{T}$$



# Coannihilations

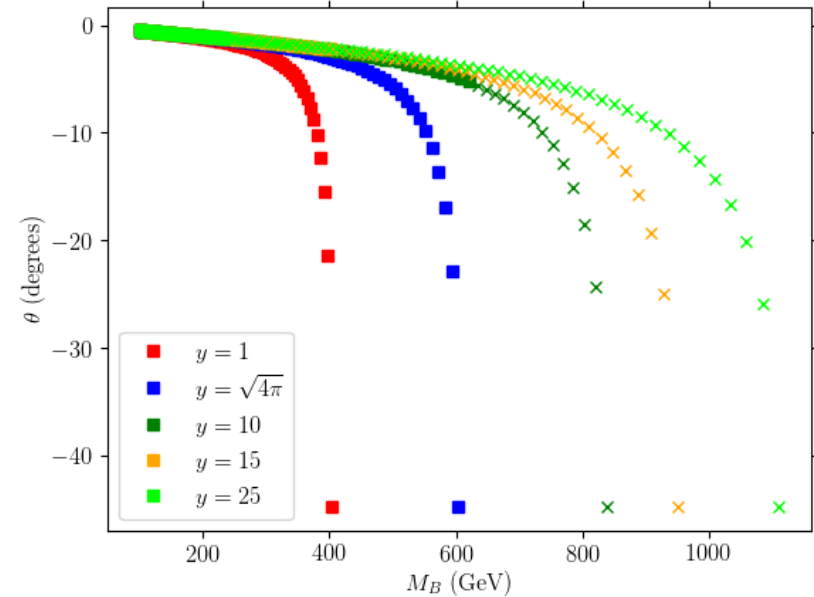
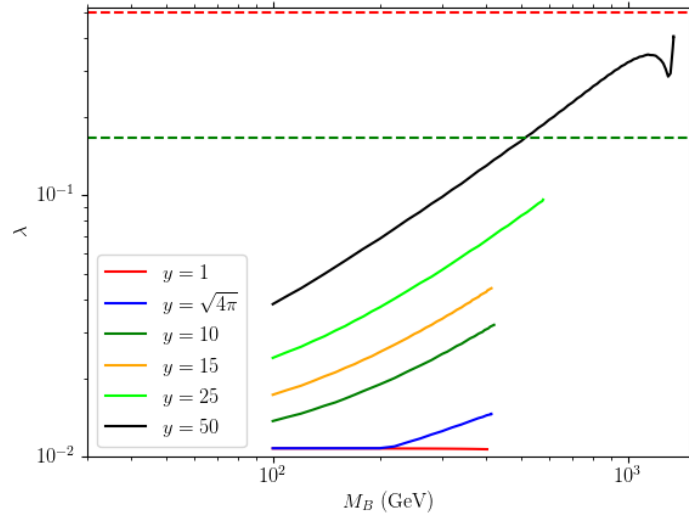


LHB  
(dashed curve:  
sneutrino-bino  
rel. mass split.)

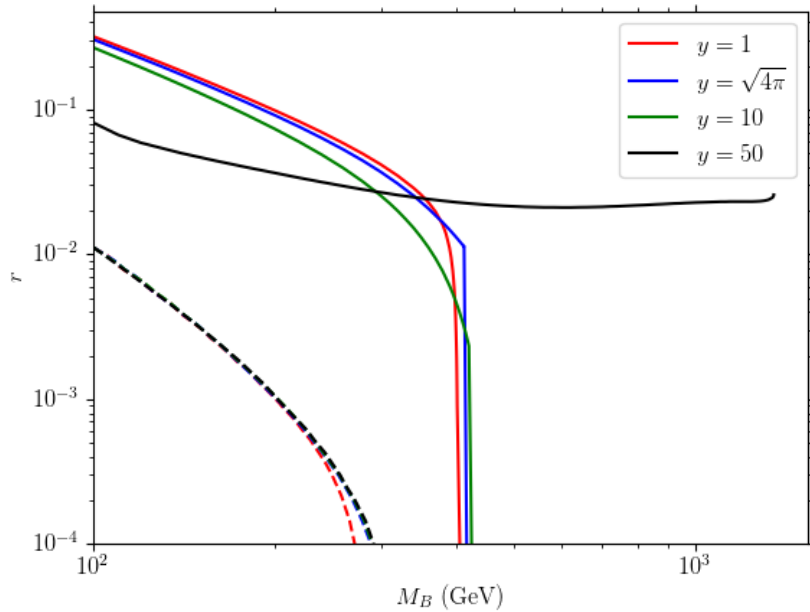
RHB

# Theoretical constraints

- Perturbative unitarity
- Vacuum stability



# Direct detection

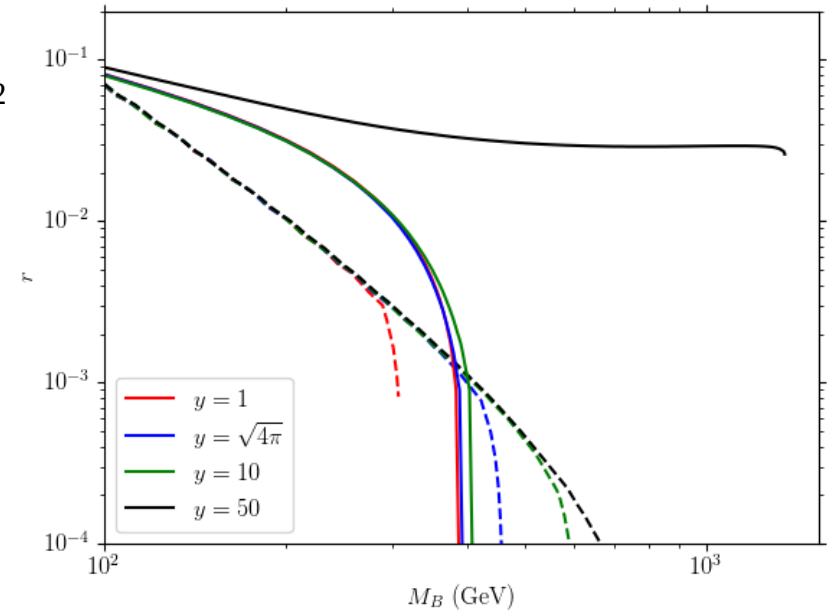


$$\mathcal{L}_{BN} = \sum_{N=p,n} \sum_{q=u,d} \left[ c_q^{(0)} \bar{B} B m_N f_{q,N}^{(0)} \bar{N} N + c_q^{(1)} \bar{B} \gamma_\mu \gamma^5 B f_{q,N}^{(1)} \bar{N} \gamma^\mu \gamma^5 N \right] + e c_A(q^2) \bar{B} \gamma_\mu \gamma^5 B \bar{p} \gamma^\mu p$$

$$\mathcal{A} \approx \frac{e}{48\pi^2} \sum_{i=1,2} \alpha_\mu^{(i)} \beta_\mu^{(i)} \int_0^1 dx \frac{3x-2}{x + (1-x)t_i - x(1-x)\eta_i}$$

$$t_i \equiv m_\mu^2 / m_i^2$$

$$\eta_i \equiv M_B^2 / m_i^2$$



# Summary

- Pheno constraints
  - $g-2$ +relic density, can reach as high as TeV
  - Direct detection
- Theoretical constraints
  - Perturbative unitarity
  - Vacuum stability



Grazie per la vostra attenzione!



Extra slides



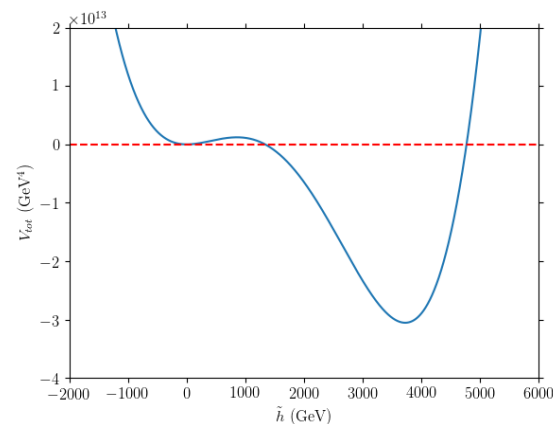
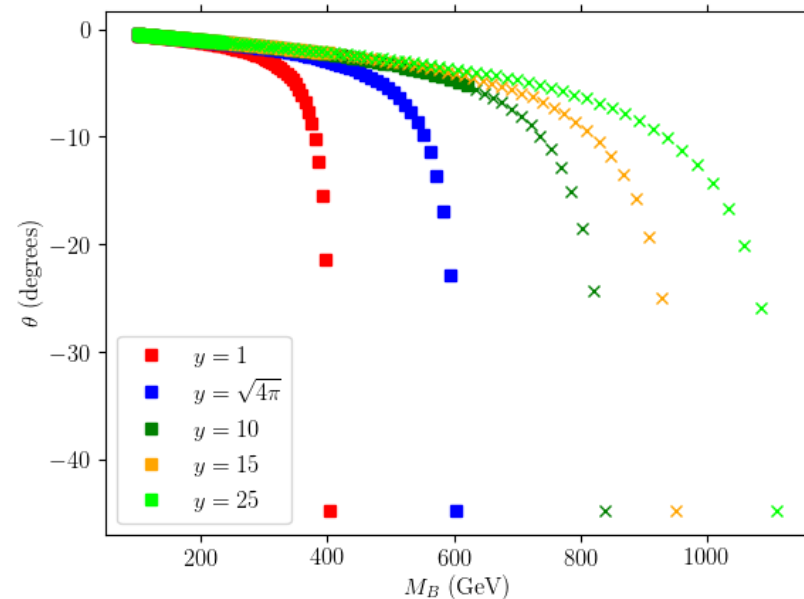
# Vacuum stability

$$h = v + \tilde{h} \quad \tilde{X} \equiv \frac{X}{\tilde{h}}, \quad \tilde{Y} \equiv \frac{Y}{\tilde{h}}, \quad \tilde{Z} \equiv \frac{Z}{\tilde{h}}$$

$$V_{tot}(\tilde{X}, \tilde{Y}, \tilde{Z}, \tilde{h}) = m^2(\tilde{X}, \tilde{Y}, \tilde{Z})\tilde{h}^2 - A(\tilde{X}, \tilde{Y}, \tilde{Z})\tilde{h}^3 + \lambda(\tilde{X}, \tilde{Y}, \tilde{Z})\tilde{h}^4$$

$$A(\tilde{X}, \tilde{Y}, \tilde{Z}) \equiv \frac{(g^2 + g_Y^2)v}{8}\tilde{X}^2 + \frac{(-g^2 + g_Y^2)v}{8}\tilde{Y}^2 - \frac{k_s}{\sqrt{2}}\tilde{Y}\tilde{Z} - \frac{g_Y^2 v Y_R}{4}\tilde{Z}^2 - \frac{(g^2 + g_Y^2)v}{8},$$

$$k_s = \frac{\sqrt{2}m_{LR}^2}{v}$$



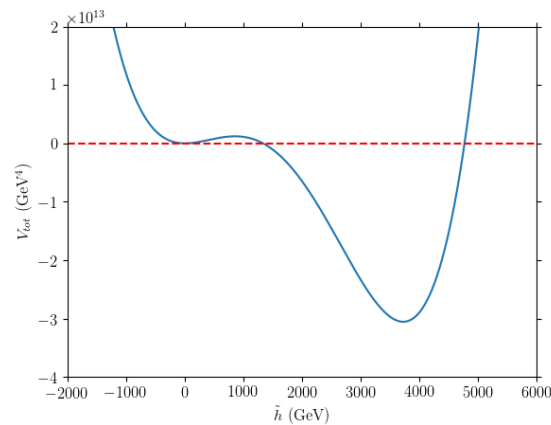
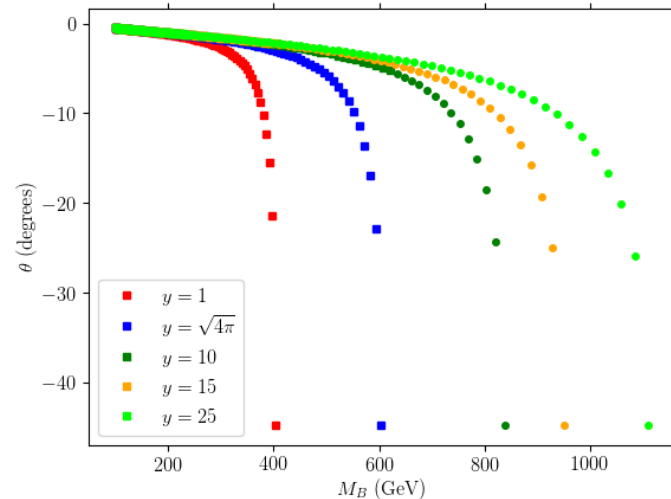
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$$k_s = \frac{\sqrt{2}m_{LR}^2}{v}$$





# Vacuum stability

$$V_H = \frac{\mu^2}{2} h^2, \quad V_{mix} = \frac{k_s}{\sqrt{2}} h Y Z$$

$$V_D^{(1)} = \frac{g_Y^2}{32} [h^2 - (X^2 + Y^2) + 2Y_R Z^2]^2$$

$$V_2 = \frac{m_{LL}^2}{2} (X^2 + Y^2) + \frac{m_{RR}^2}{2} Z^2$$

$$V_D^{(2)} = \frac{g^2}{32} [h^4 - 2h^2 (X^2 - Y^2) + (X^2 + Y^2)^2]$$

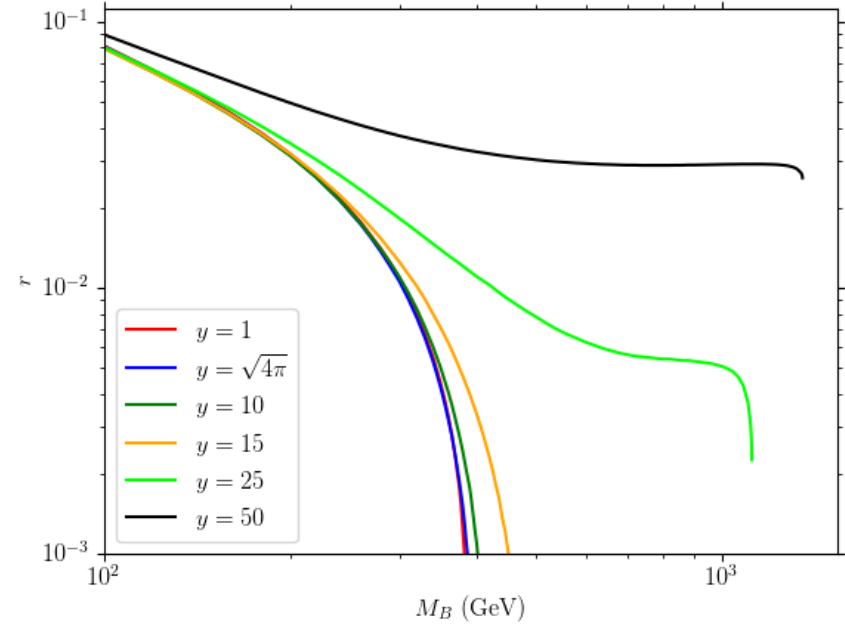
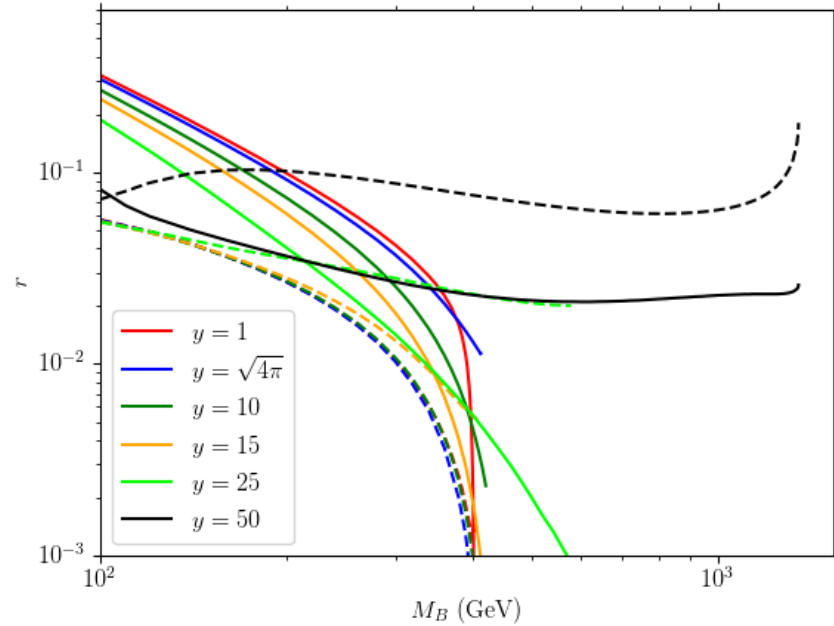
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$$k_s = \frac{\sqrt{2} m_{LR}^2}{v}$$

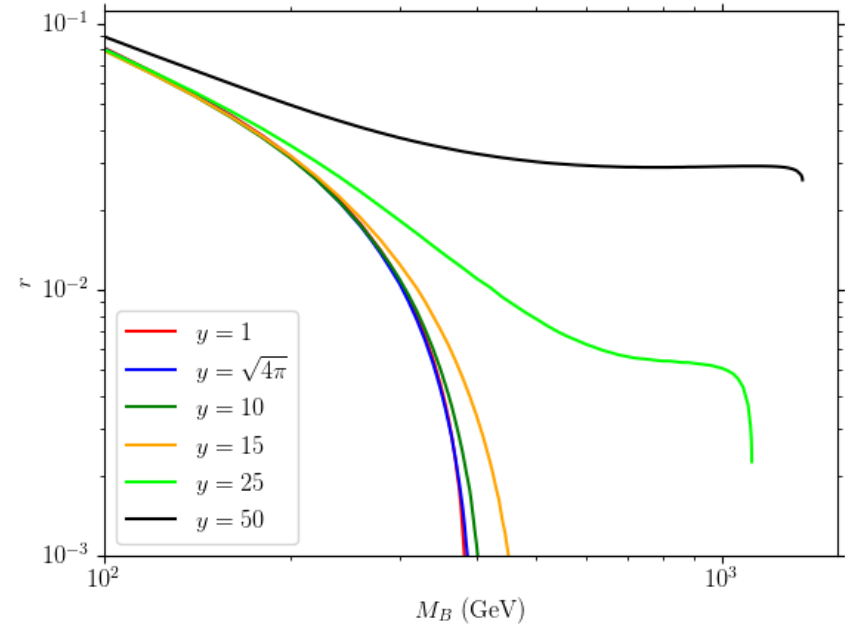
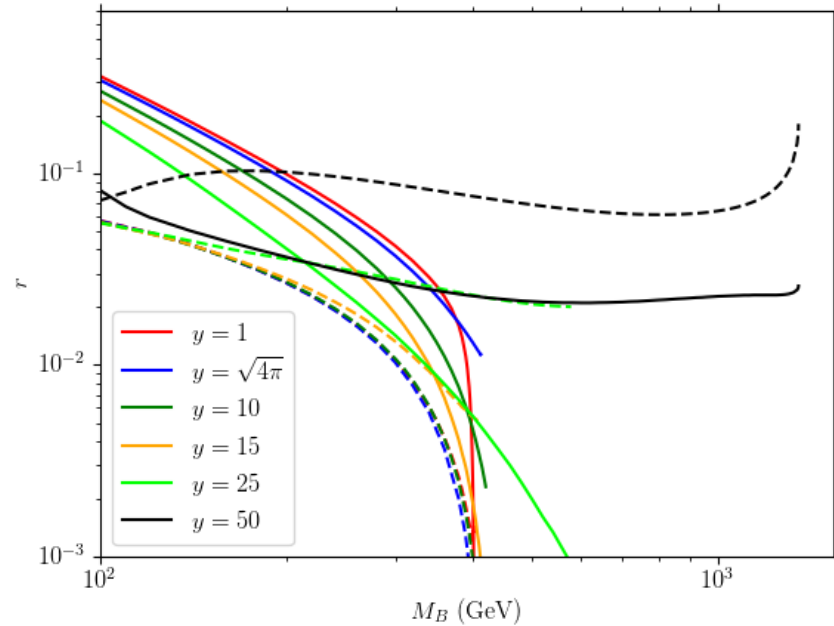
# Coannihilations



LHB  
(dashed curve:  
sneutrino-bino  
rel. mass split.)

RHB

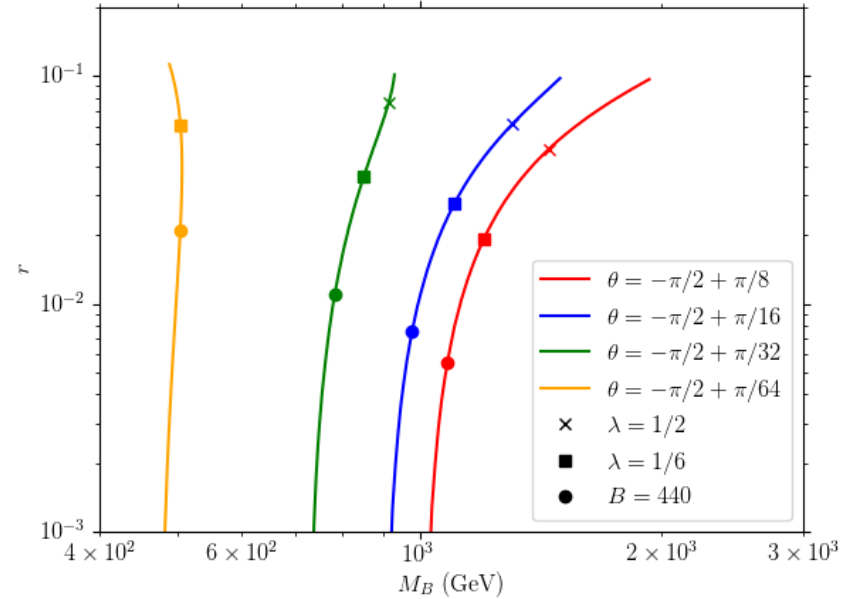
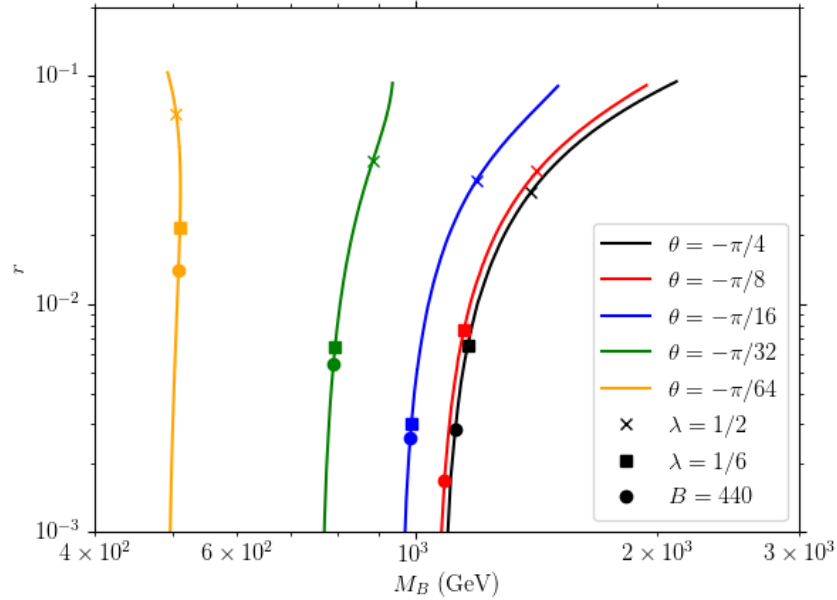
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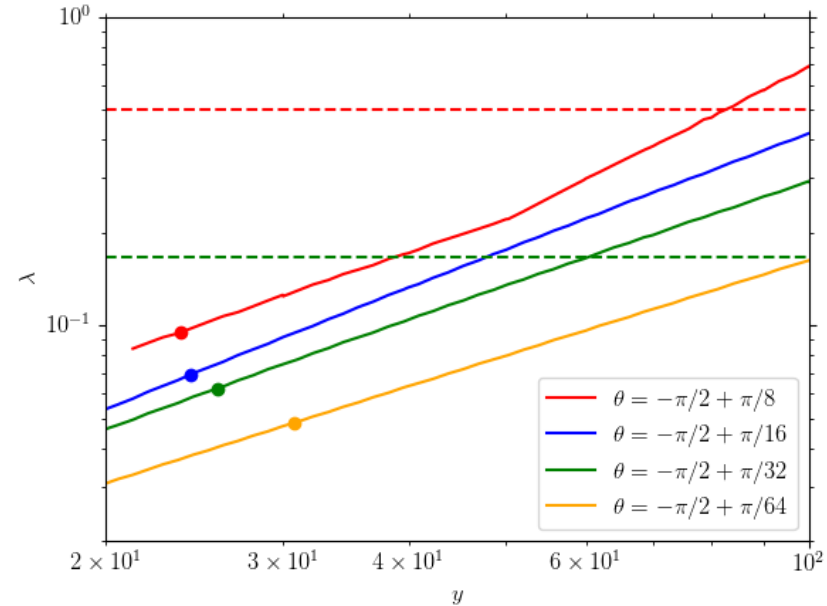
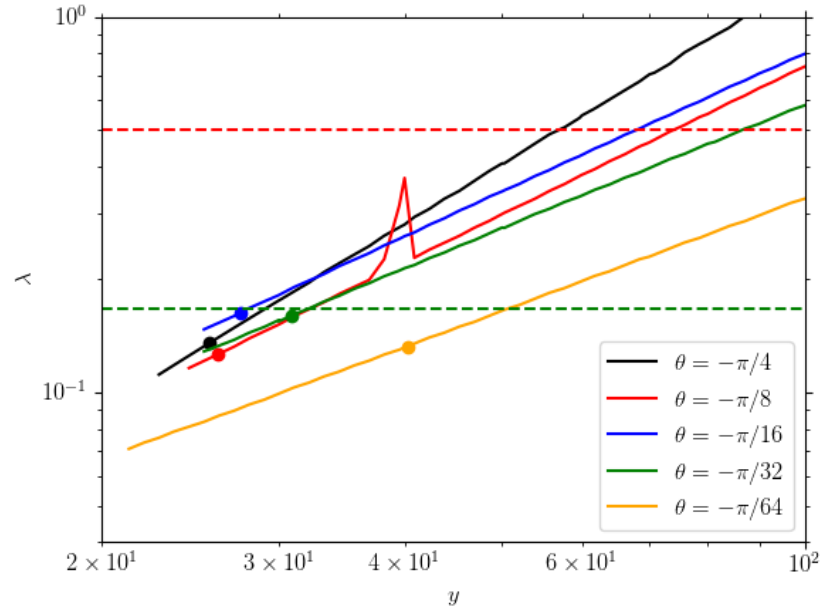
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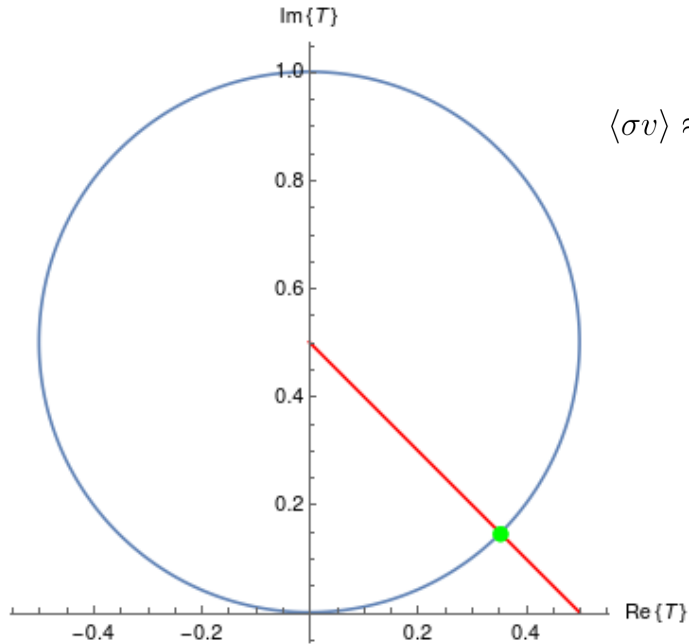
# Coannihilations



# Coannihilations



# Perturbative unitarity



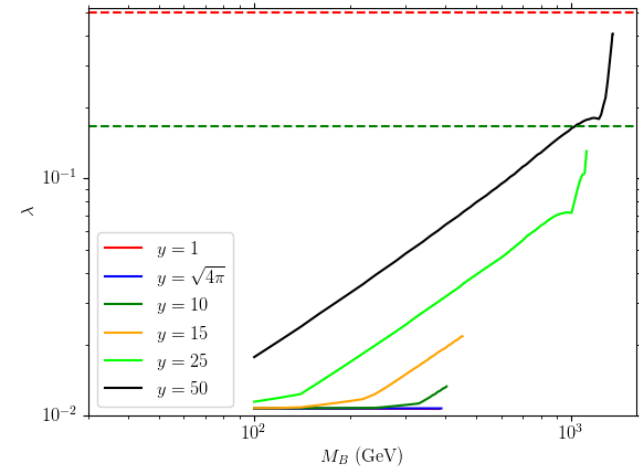
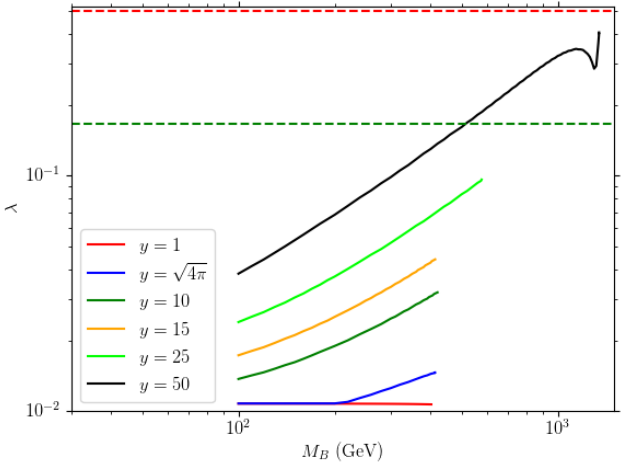
$$\langle \sigma v \rangle \approx (2.5 \times 10^{-9} \text{ GeV}^{-2}) \left( \frac{\alpha_\chi}{0.5 \times 10^{-2}} \right)^2 \left( \frac{100 \text{ GeV}}{m} \right)^2$$

WIMP unitarity  
limit: < 20 TeV

$$\left( \text{Re} \{ \hat{T}_i(s) \} \right)^2 + \left( \text{Im} \{ \hat{T}_i(s) \} - \frac{1}{2} \right)^2 = \frac{1}{4}$$

$$\text{Im} \{ \hat{T}_i(s) \} = |\hat{T}_i(s)|^2$$

$$\text{Re}\{T_i\} \leq 1/6, 1/2$$



# Vacuum stability

$$h = v + \tilde{h} \quad \tilde{X} \equiv \frac{X}{\tilde{h}}, \quad \tilde{Y} \equiv \frac{Y}{\tilde{h}}, \quad \tilde{Z} \equiv \frac{Z}{\tilde{h}}$$

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$$A(\tilde{X}, \tilde{Y}, \tilde{Z}) \equiv \frac{(g^2 + g_Y^2)v}{8}\tilde{X}^2 + \frac{(-g^2 + g_Y^2)v}{8}\tilde{Y}^2 - \frac{k_s}{\sqrt{2}}\tilde{Y}\tilde{Z} - \frac{g_Y^2 v Y_R}{4}\tilde{Z}^2 - \frac{(g^2 + g_Y^2)v}{8},$$

$$k_s = \frac{\sqrt{2}m_{LR}^2}{v}$$

