Fast Simulations of Dark Matter

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arXiv:2012.14446

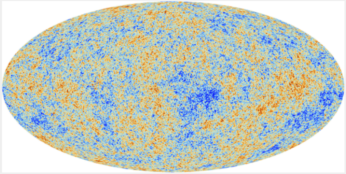
in collaboration with

Mark Neyrinck, Ben Granett,

Luigi Guzzo, Nicola Vittorio

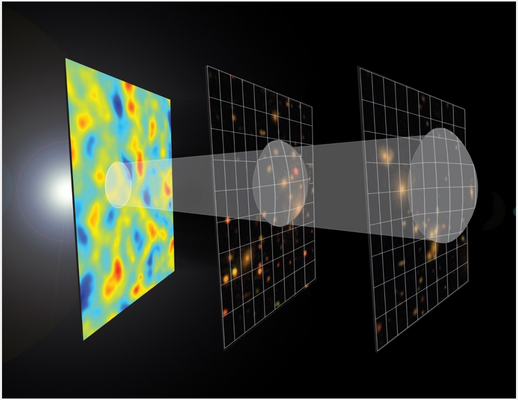
Barolo Astroparticle Meeting
09/09/2021

ΛCDM, a concordance model

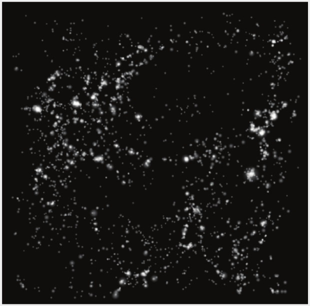


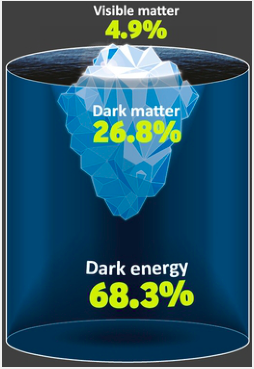
Planck 2018

Initial Gaussian distribution

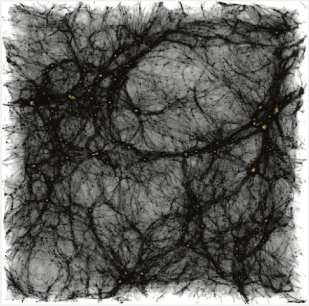


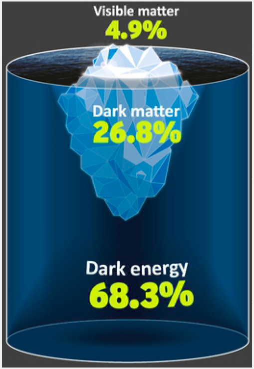
highly non-Gaussian distribution




Visible matter
4.9%
Dark matter
26.8%
Dark energy
68.3%

ΛCDM, A CONCORDANCE MODEL





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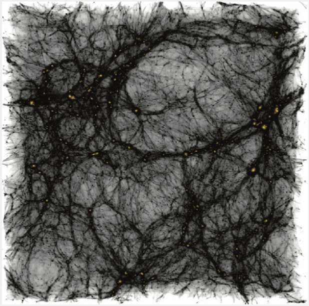
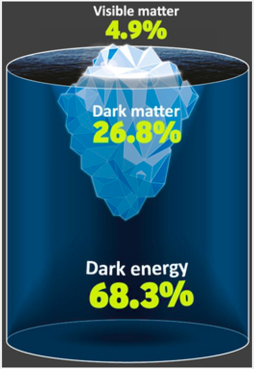
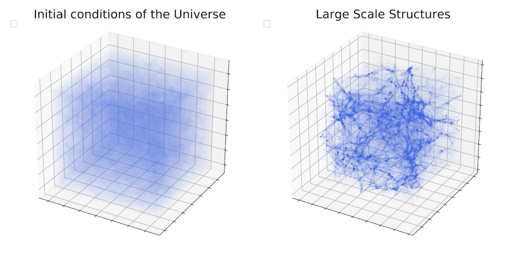




We need to compute CDM distribution

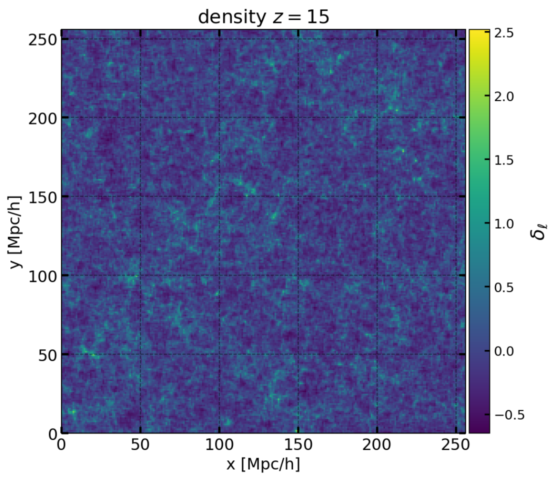
N-body approach

$$\begin{cases} \boldsymbol{u} = \frac{d\boldsymbol{x}}{d\tau} \\ m\frac{d\boldsymbol{u}}{d\tau} = -\nabla\phi \\ \nabla^2\phi = 4\pi G\rho(\boldsymbol{x}) \end{cases} \quad (1)$$



Credit: Chirag Modi

Computing Cold Dark Matter Distribution



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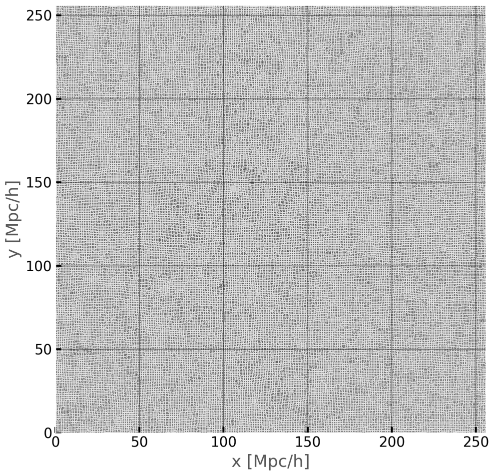
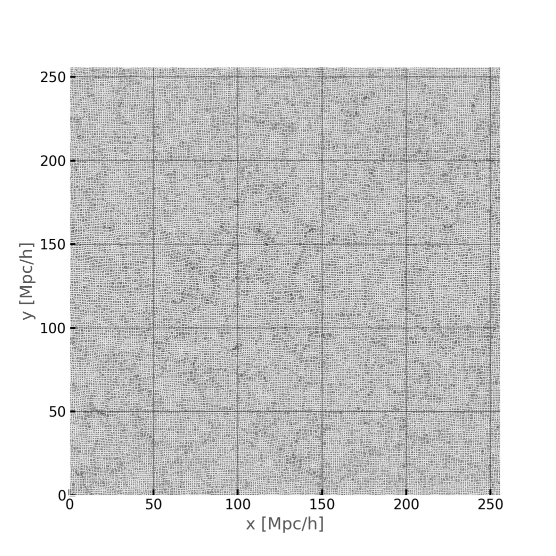


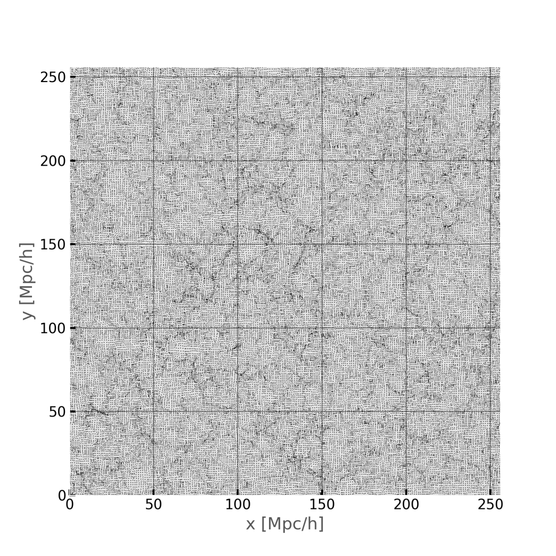
$z = 15$

Computing Cold Dark Matter Distribution



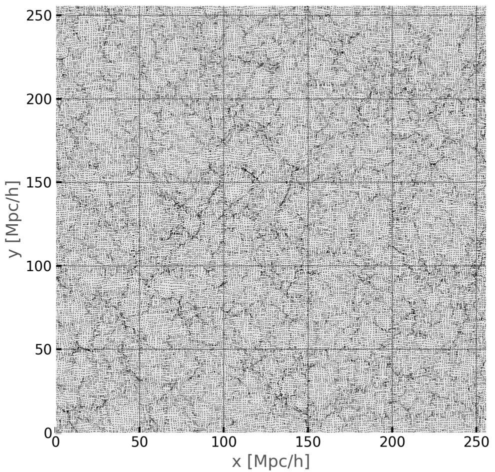
$z = 10$

Computing Cold Dark Matter Distribution



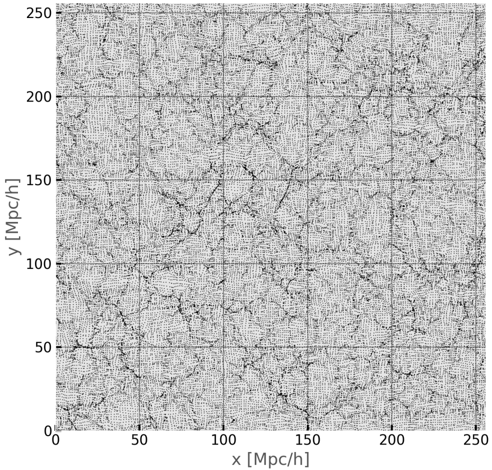
$z = 7$

Computing Cold Dark Matter Distribution



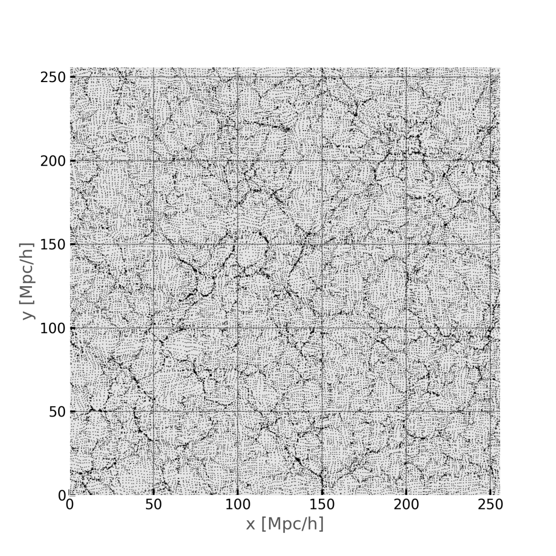
$z = 4.7$

Computing Cold Dark Matter Distribution



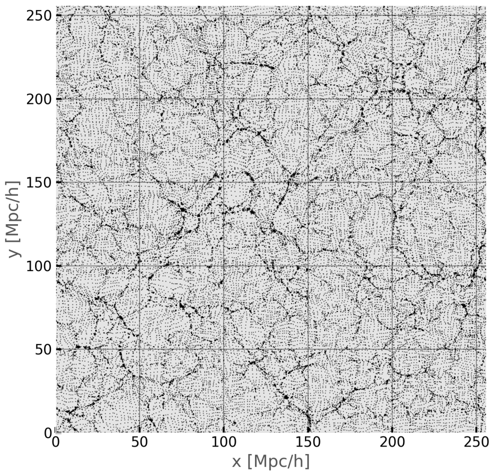
$z = 3$

Computing Cold Dark Matter Distribution



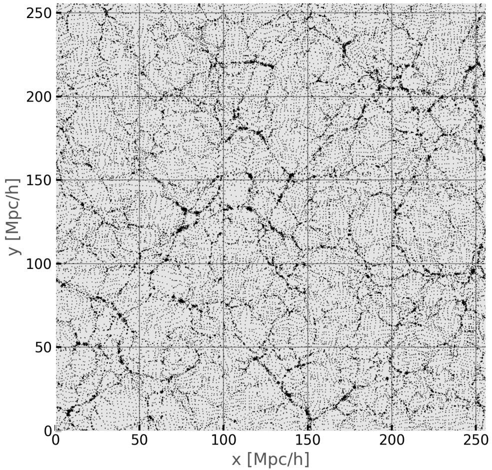
$z = 1.8$

Computing Cold Dark Matter Distribution



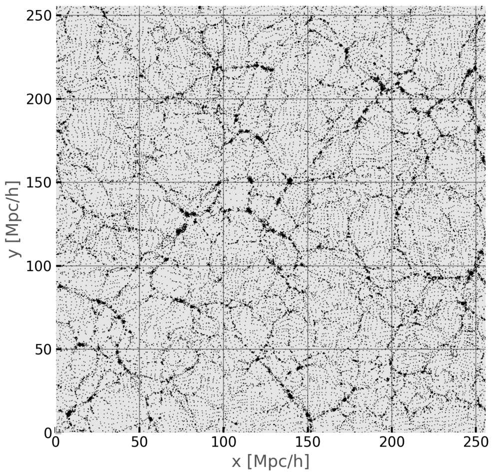
$z = 1$

Computing Cold Dark Matter Distribution



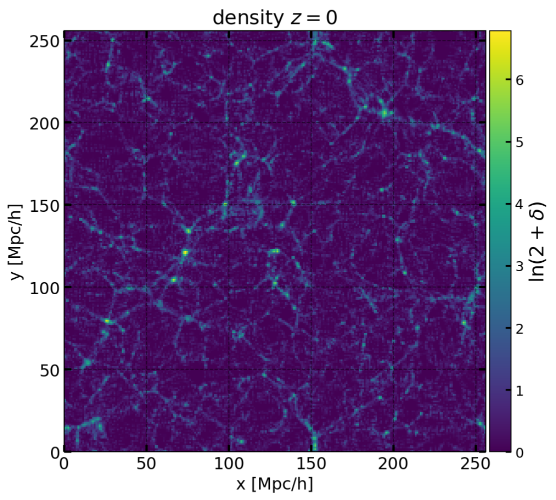
$z = 0.41$

Computing Cold Dark Matter Distribution



$z = 0$

Computing Cold Dark Matter Distribution



Why fast simulations?

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If dark matter were a fluid, the EOM

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We define the displacement vector

$$\mathbf{x} = \mathbf{q} + \mathbf{\Psi}(\mathbf{q}), \tag{3}$$

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at first and second in perturbation theory, only the irrotational part matters

$$\nabla \cdot \boldsymbol{\Psi} \simeq \psi^{(1)} + \psi^{(2)} + \dots \tag{4}$$

$$\frac{d^2\psi^{(1)}}{d\tau^2} + \mathcal{H}\frac{d\psi^{(1)}}{d\tau} = \frac{3}{2}\mathcal{H}^2\Omega_m(\tau)\psi^{(1)} \tag{5}$$

1st order (Zel'dovich Approximation)

$$\psi^{(1)} \propto -\delta_\ell^{(1)}(\boldsymbol{q}, \tau_i) \tag{6}$$

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Second Order LPT (Bouchet+95)

$$\psi^{(2)}(\boldsymbol{q}, \tau) \propto \sum_{i \neq j} \left(\psi_{i,i}^{(1)} \psi_{j,j}^{(1)} - \psi_{i,j}^{(1)} \psi_{j,i}^{(1)} \right), \tag{7}$$

Spherical Collapse (Neyrinck13)

$$\psi_{\mathsf{sc}} = \begin{cases} -3, & \delta_\ell \geq \delta_{\mathsf{c}} \\ 3\left[\left(1 - \frac{\delta_\ell}{\delta_{\mathsf{c}}}\right)^{\delta_{\mathsf{c}}/3} - 1\right] & \delta_\ell < \delta_{\mathsf{c}} \end{cases} \tag{8}$$

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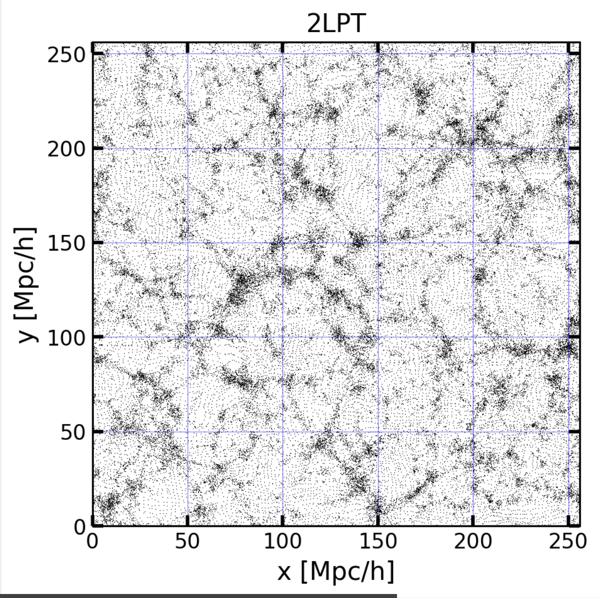
Augmented LPT

Combo of 2LPT and spherical collapse, (Kitaura+13)

$$\psi_{\mathrm{alpt}} = \underbrace{\psi_{\mathrm{2lpt}} \circledast \mathcal{G}(\sigma_{\mathcal{R}})}_{\text{large scales}} + \underbrace{\psi_{\mathrm{sc}} \circledast (1 - \mathcal{G}(\sigma_{\mathcal{R}}))}_{\text{small scales}}, \tag{9}$$

Particle positions



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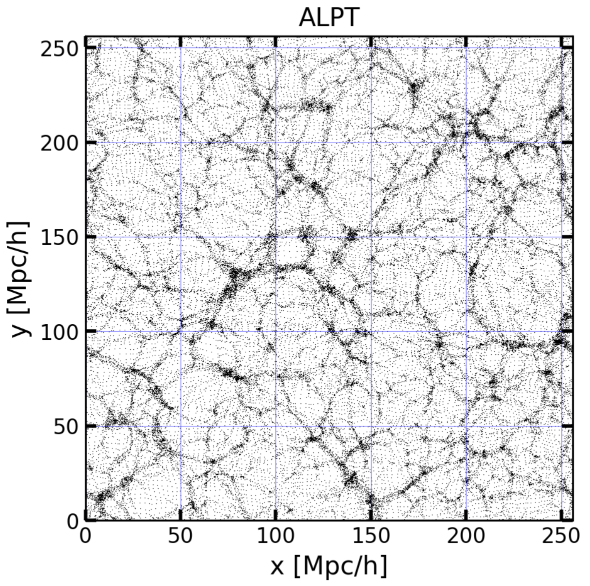

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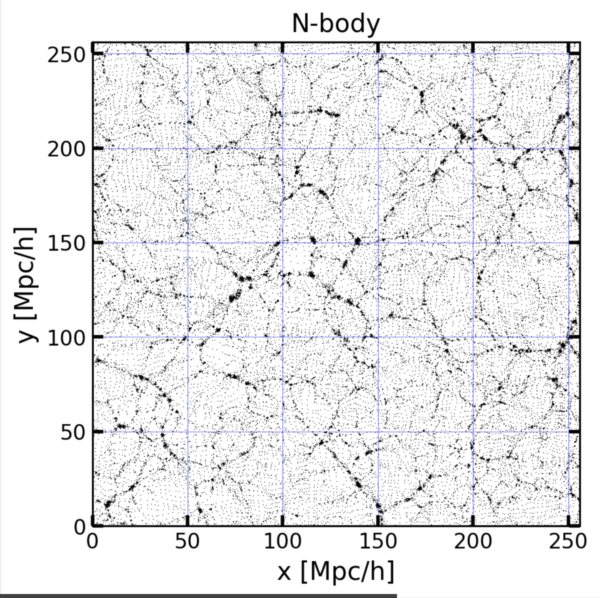


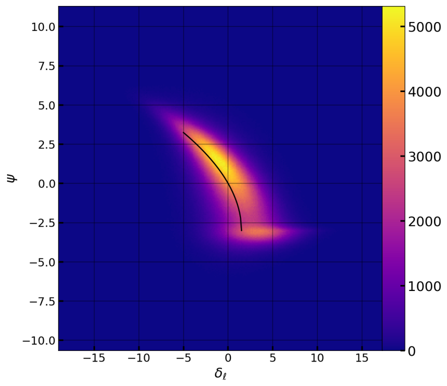
Ex. Press-Schechter

Proto-halo of radius *R* centered at ***x***

$$\delta_\ell(\boldsymbol{x}) \circledast \mathcal{G}(\mathcal{R}) \geq \delta_c \qquad (10)$$

For every particle in a proto-halo of radius *R* set

$$\psi = -3 \qquad (11)$$



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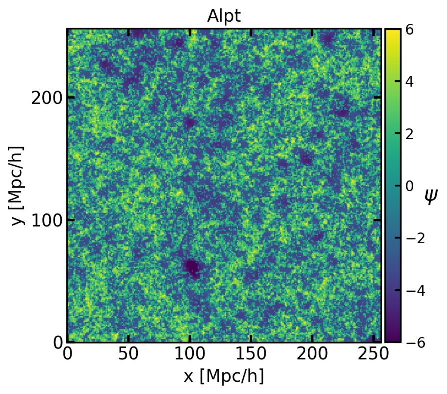

Ex. Press-Schechter

Proto-halo of radius *R* centered at $\boldsymbol{x}$

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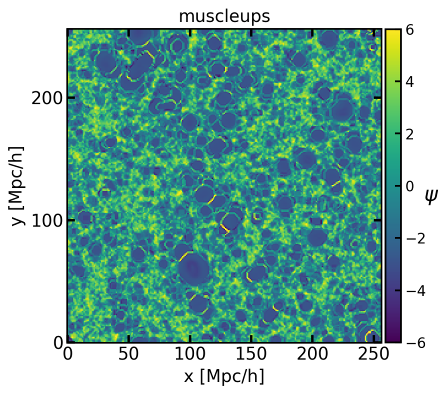

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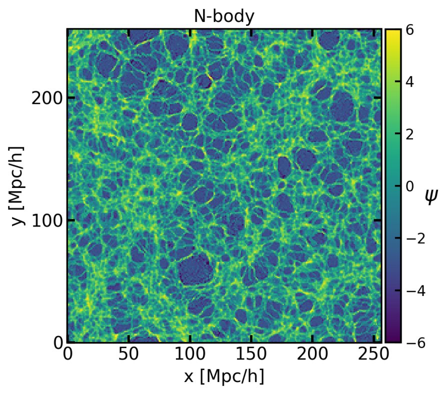
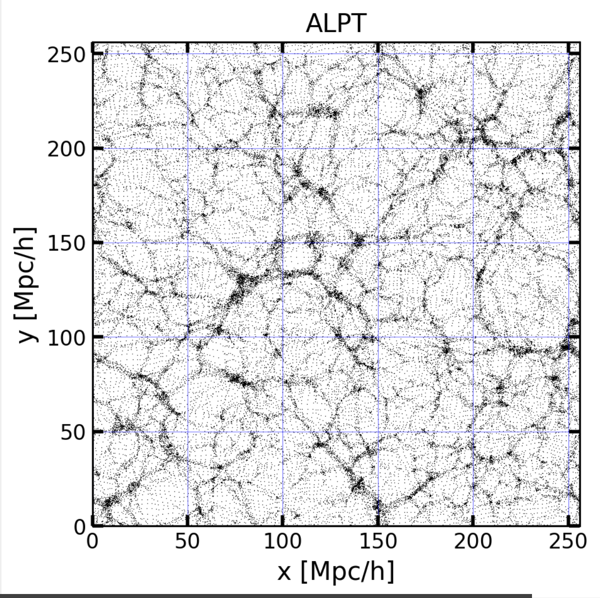


PARTICLE POSITIONS



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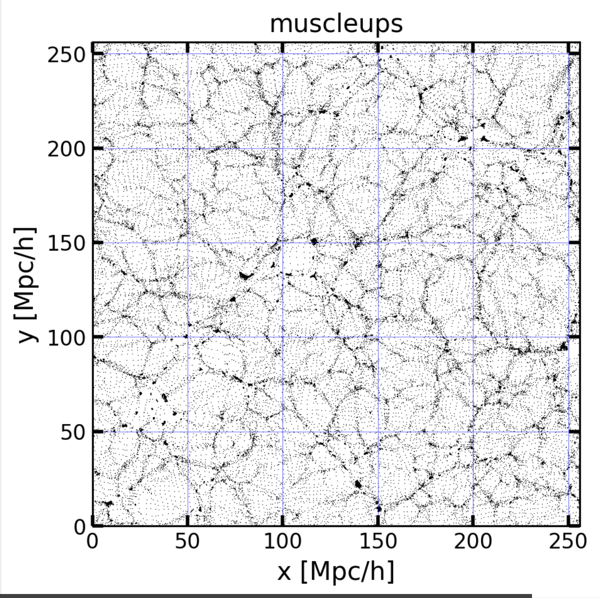

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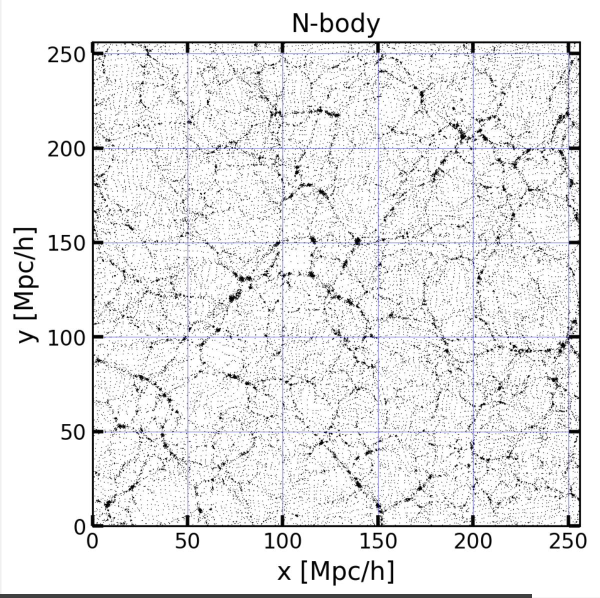
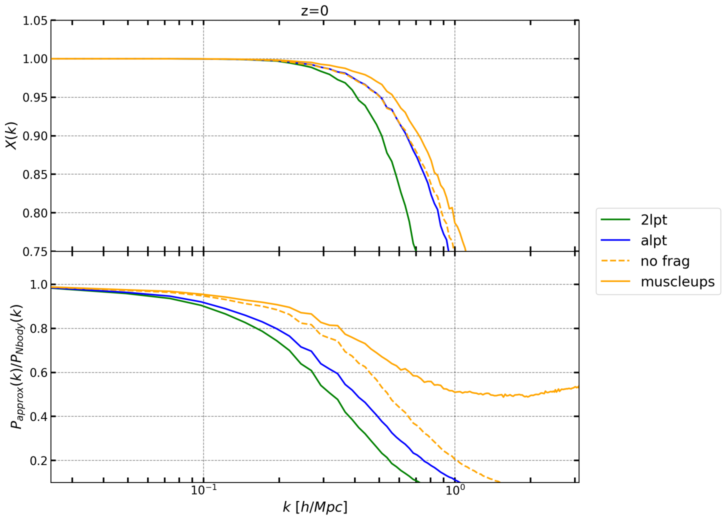


Power and Cross Spectra



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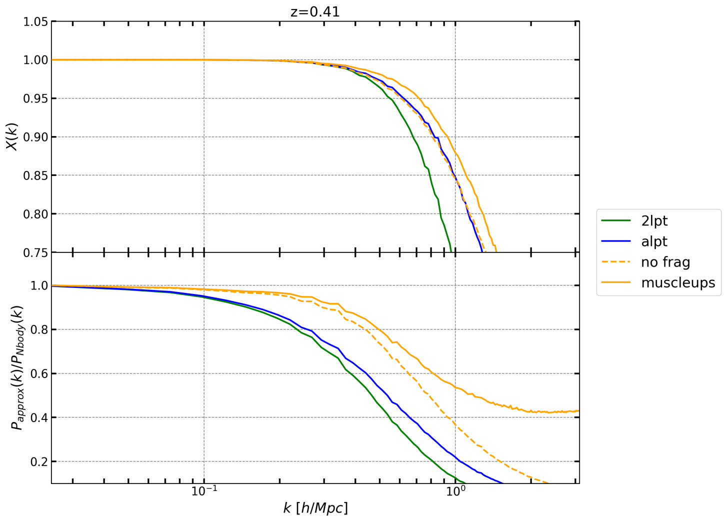

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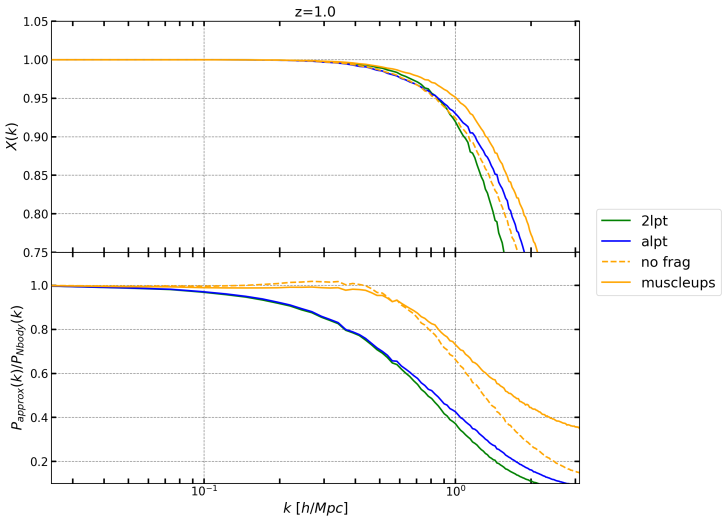

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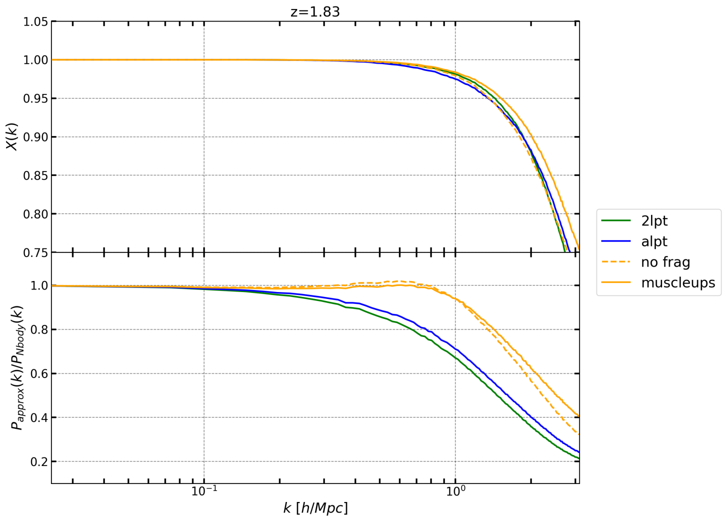
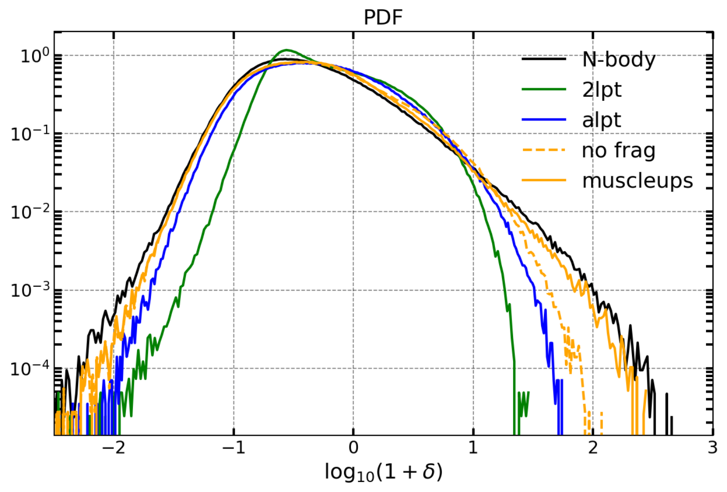


PDF


PDF
N-body
2lpt
alpt
no frag
muscleups
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
−2
−1
0
1
2
3
$\log_{10}(1+\delta)$

Summary

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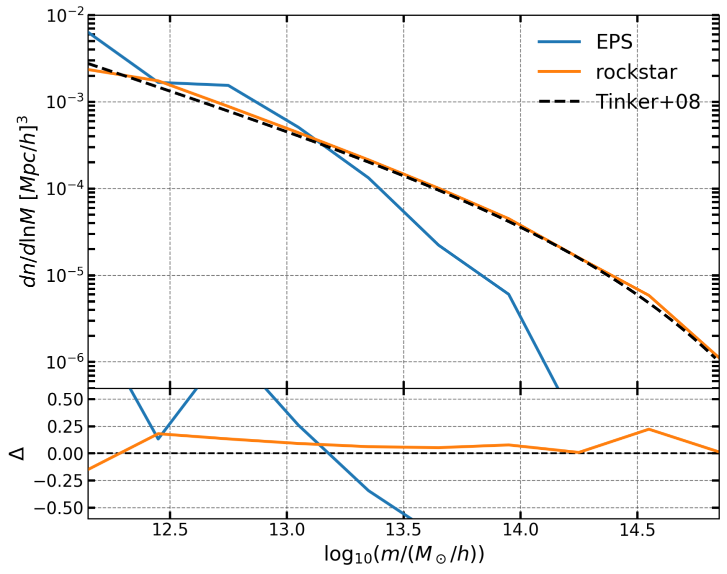
Thanks for the attention

Halo Mass Function



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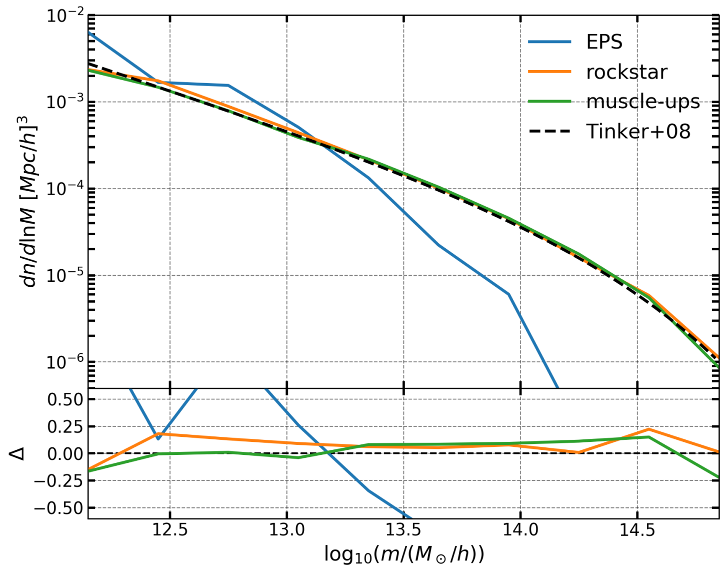
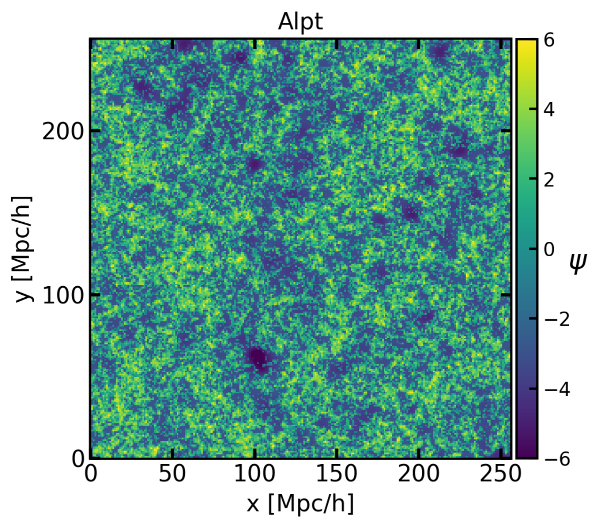


Displacement field



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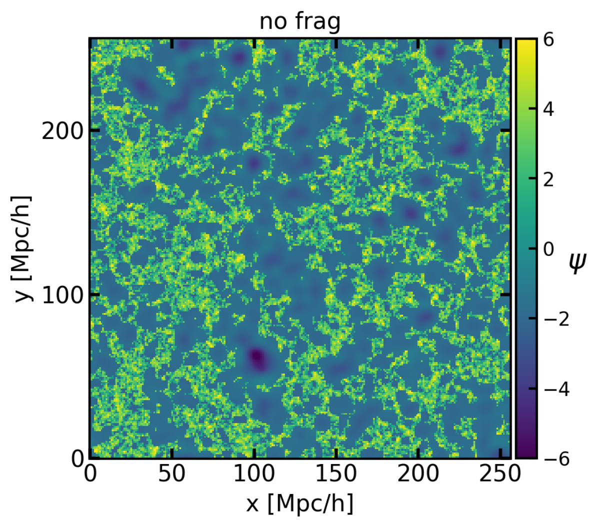

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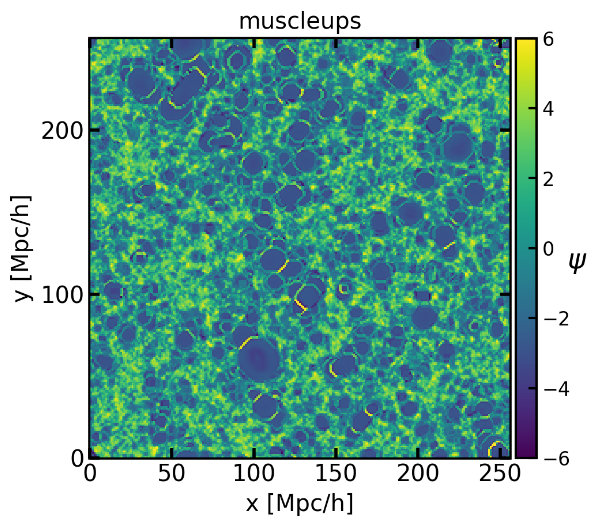

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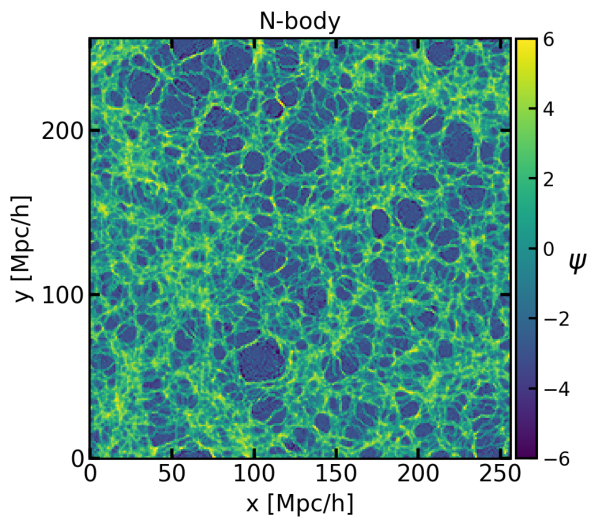
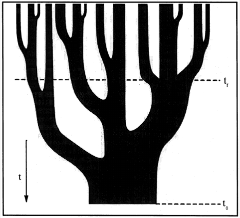
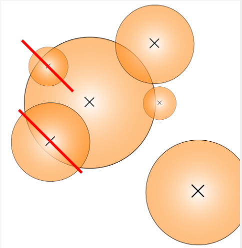


MUSCLEUPS

MUltiscal Spherical Collapse Lagrangian Evolution Using Press-Schechter

- list of halo candidates $\delta_\ell(\boldsymbol{x}, R)\ \forall\ \in \{R_0, \ldots, R_n\}, (\psi = -3)$.





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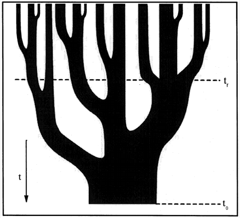

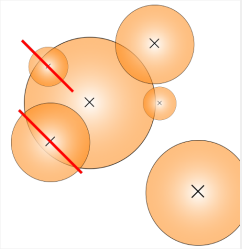

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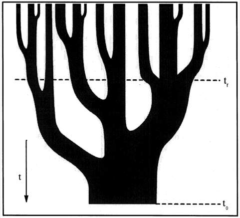

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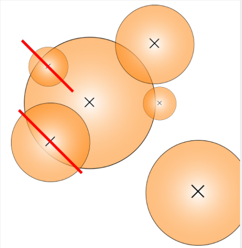

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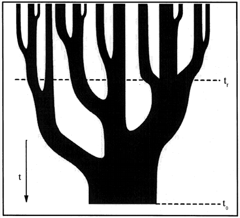

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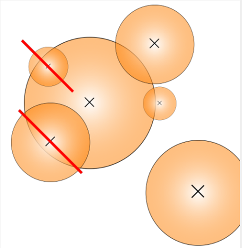
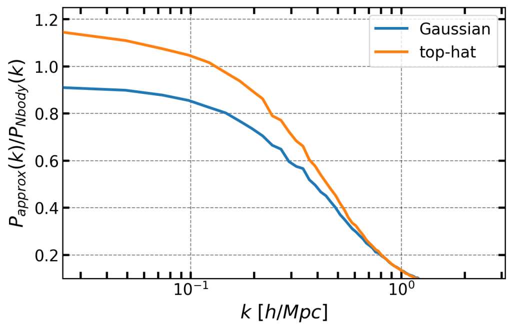
- collapse halo particles (halo model)



MUSCLE

The number of halo particles affect the linear power spectrum.

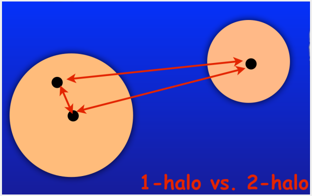
$$\psi_{\text{muscle}} = \begin{cases} 3\left[(1 - \frac{\delta_l}{\gamma})^{\gamma/3} - 1.\right], \; \delta_l < \gamma \\ -3, \qquad\qquad\qquad\qquad \delta_l(R) \geq \gamma, \forall R \geq R_{ip} \end{cases} \tag{12}$$



HALO MODEL

Ansatz (Peacock+00,Seljak00)

All the matter in the Universe is found into virialized haloes



$$P(k) = P_{2h}(k) + P_{1h}(k) \qquad (13)$$

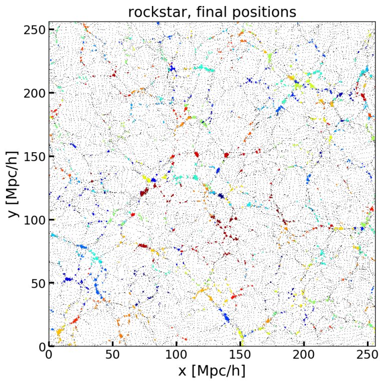
$$1 + \delta_{nfw}(x) = \frac{\Delta_v(z)}{\Omega_m(z)} \frac{c^3 f(c)}{x(1+x)^2}$$

$$x = c(m)\frac{r}{r_v} \qquad (14)$$

- $P_{2h}(k) \rightarrow P_{lin}(k)$
- $P_{1h}(k) = \int u(k,m) n^2(m) \frac{m^2}{\bar{\rho}^2} dm$
- $u(r,m) = \rho_s \left[\frac{r}{r_s} \left(1 + \frac{r}{r_s}\right)^2 \right]^{-1}$ (Navarro+96)

ROCKSTAR

We run the ROCKSTAR halo finder (Behroozi+11) to detect halo particles



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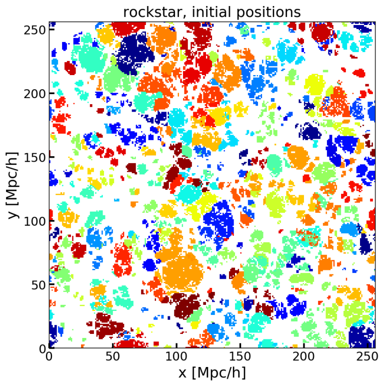
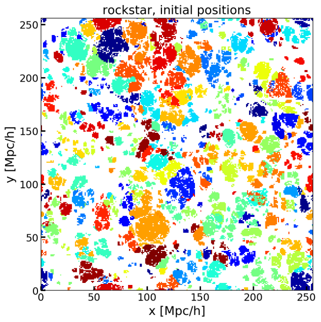
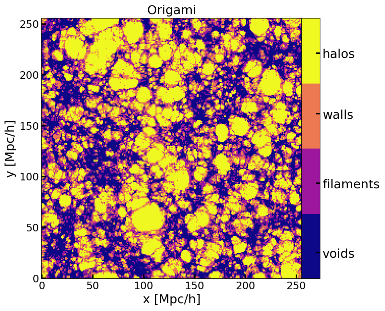


Origami vs Rockstar





Eulerian vs Lagrangian halo finder

Computing Cold Dark Matter Distribution

Vlasov approach

Gaussian distributions ρ and $\boldsymbol{u} \Rightarrow$ evolve the dark matter distribution function $f(\rho, \boldsymbol{u}, \tau)$

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$$\frac{df}{d\tau} = \frac{\partial f}{\partial \tau} + \boldsymbol{u} \cdot \frac{\partial f}{\partial \boldsymbol{x}} - \frac{\partial \phi}{\partial \boldsymbol{x}} \cdot \frac{\partial f}{\partial \boldsymbol{u}} = 0 \tag{15}$$

$$\nabla^2 \phi = 4\pi G \rho(\boldsymbol{x}) = 4\pi G \int f(\boldsymbol{x}, \boldsymbol{u}, \tau) d\boldsymbol{u} \tag{16}$$

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This is an approximation, when the stress tensor is zero: single stream approximation.

Lagrangian Perturbation Theory

We start from the Euler equation

$$\frac{\partial \boldsymbol{u}}{\partial \tau} + \mathcal{H}\boldsymbol{u} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla\Phi \tag{21}$$

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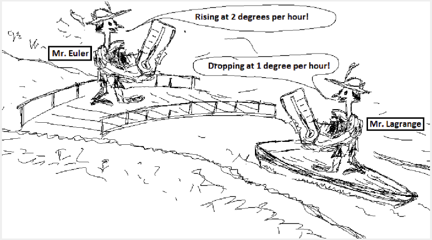
at first and second in perturbation theory, only the irrotational part matters

$$\nabla \cdot \boldsymbol{\Psi} \simeq \psi^{(1)} + \psi^{(2)} + \dots \tag{26}$$

Eluerian vs Lagrangian

$$\begin{cases} \frac{\partial \delta}{\partial \tau} + \nabla \cdot \{(1+\delta)\boldsymbol{u}\} = 0 \\ \frac{\partial \boldsymbol{u}}{\partial \tau} + \mathcal{H}\boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} = -\nabla\Phi \\ \nabla^2\Phi = \frac{3}{2}\Omega_m\mathcal{H}^2\delta \end{cases} \tag{27}$$



Define $\theta = \nabla \cdot \boldsymbol{u}$

$$\begin{aligned} \frac{\partial \delta}{\partial \tau} + \theta(\boldsymbol{x},\tau) = 0, \\ \frac{\partial \boldsymbol{u}}{\partial \tau} + \mathcal{H}\boldsymbol{u} = -\nabla\Phi, \end{aligned} \tag{28}$$

$$\frac{\partial \theta}{\partial \tau} + \mathcal{H}\theta(\tau) + \frac{3}{2}\Omega_m(\tau)\mathcal{H}^2(\tau)\delta(\boldsymbol{x},\tau) = 0, \tag{29}$$

$$\frac{\partial^2 \delta}{\partial \tau^2} + \mathcal{H}\frac{\partial \delta}{\partial \tau} - \frac{3}{2}\Omega_m(\tau)\mathcal{H}^2\delta(\boldsymbol{x},\tau) = 0, \tag{30}$$

How good are ψ approximations?

Interpolate density field from the particles positions

$$\delta(\boldsymbol{k}) = \int d\boldsymbol{x}\,\delta(\boldsymbol{x}) e^{-i\boldsymbol{k}\cdot\boldsymbol{x}} \tag{31}$$

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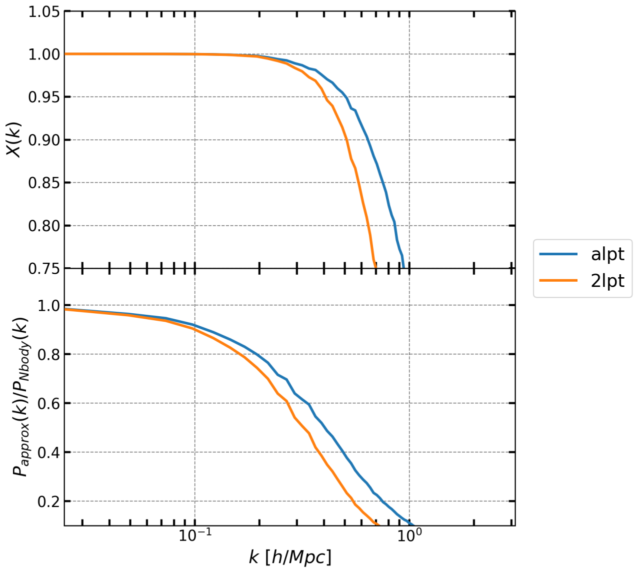
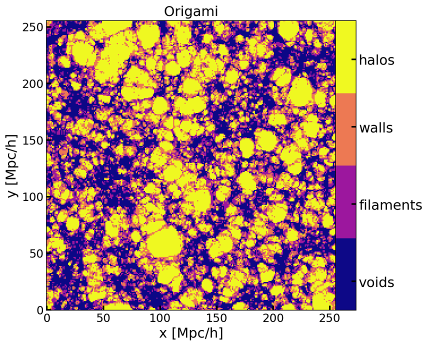


Add back the exact halo information

we use ORIGAMI to categorize particles into the cosmic web (Falck+12)



Let us use the exact information by setting $\psi = -3$ for halo particles.

Add back the exact halo information

Before, ALPT

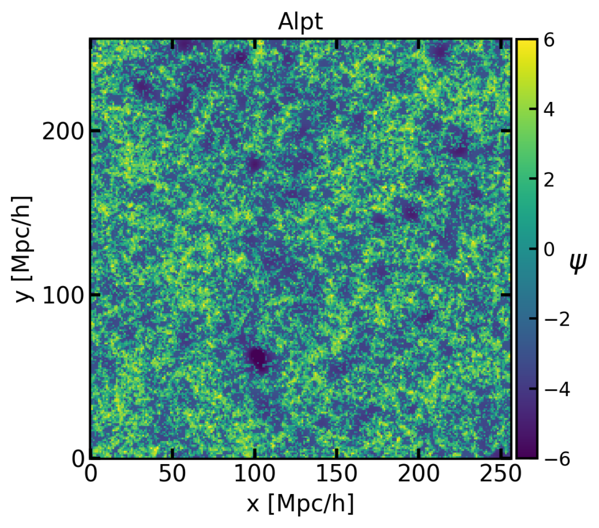
$$\psi_{\text{alpt}} = \underbrace{\psi_{\text{2lpt}} \circledast \mathcal{G}(\sigma_{\mathcal{R}})}_{\text{large scales}} + \underbrace{\psi_{\text{ss}} \circledast (1 - \mathcal{G}(\sigma_{\mathcal{R}}))}_{\text{small scales}}, \tag{31}$$

$$\psi_{\text{ss}} = \begin{cases} -3, & \delta_\ell > \gamma. \\ 3\left[\left(1 - \frac{\delta_\ell}{\gamma}\right)^{\gamma/3} - 1.\right] & \delta_\ell < \gamma, \end{cases} \tag{32}$$

Now we use the exact information (cheating)

$$\psi_{\text{ss}} = \begin{cases} -3, & \text{halo particles,} \\ 3\left[\left(1 - \frac{\delta_\ell}{\gamma}\right)^{\gamma/3} - 1\right] & \text{otherwise} \end{cases} \tag{33}$$

Add back the exact halo information



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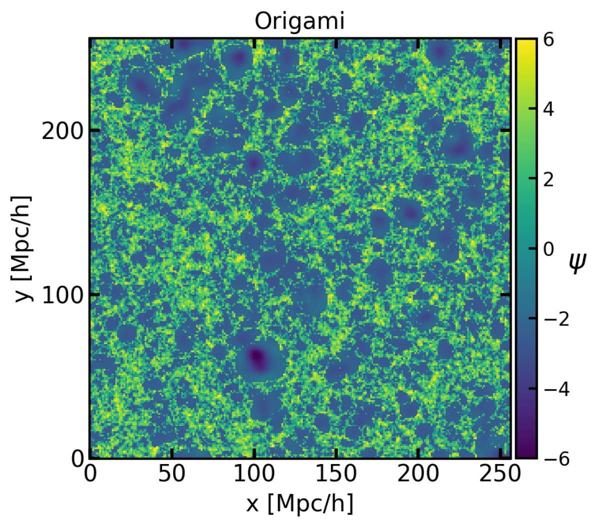

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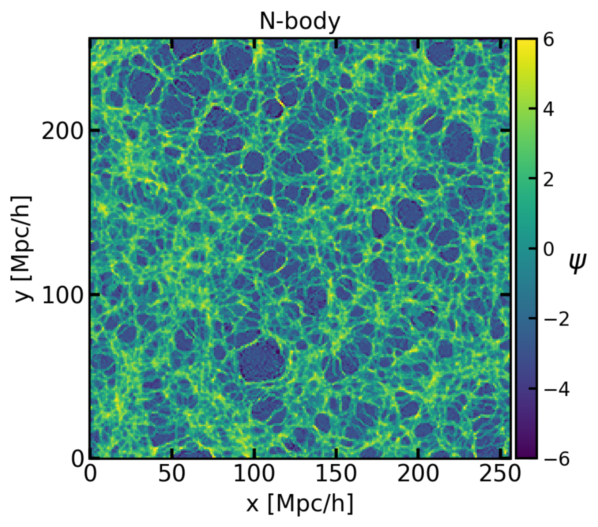

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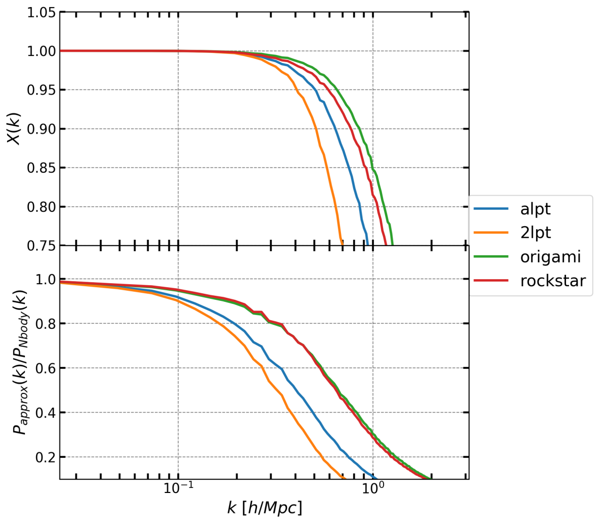

Add back the exact halo information

We need to add the halo information back, **without using halo finders**

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collapse condition for spherical patches

To detect haloes in the initial conditions

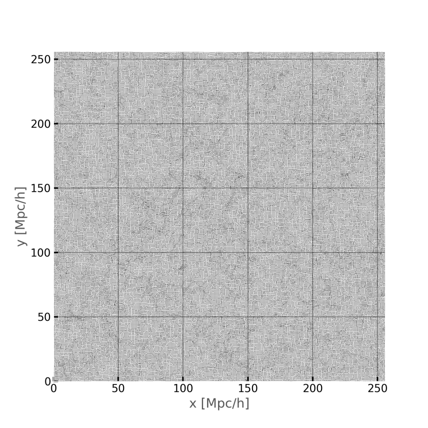
$$\delta_\ell(\boldsymbol{x}, R) = \int \frac{d^3k}{(2\pi)^3} \delta_\ell(\boldsymbol{k}) e^{-k^2R^2/2} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} > \delta_c \quad (34)$$

proto-halo of radius R centered at x (Extended Press and Schechter)

Non-perturbative Approach

The fundamental field of Lagrangian picture is

$$\psi = \nabla \cdot \boldsymbol{\Psi} = \nabla \cdot (\boldsymbol{x} - \boldsymbol{q}) \tag{35}$$



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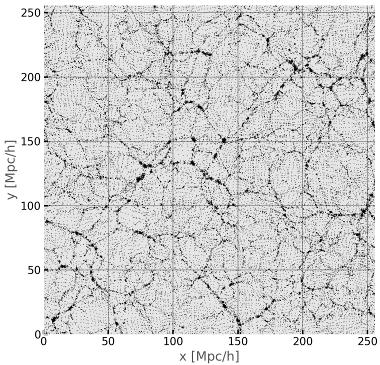

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