Spin matching and Monte-Carlo simulation of radiative depolarization in electron-positron storage rings with Bmad and SLICKTRACK

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Recent developments in Bmad

Bmad (https://www.classe.cornell.edu/bmad/) is an object oriented, open source, subroutine library for charged-particle dynamics simulations in accelerators and storage rings. Bmad has been developed at Cornell University's Laboratory for Elementary Particle Physics and has been in use since the mid 1990s. Updates for spin tracking and spin matching include:

- **Linearized spin-orbit motion (SLIM):** introduced by Alex Chao, a way to represent the linearized orbital and spin transport as an 8×8 matrix which then can be analyzed using standard linear algebra techniques. Implemented in Tao program.
- **Spin-orbit Monte-Carlo tracking:** Implemented in multi-turn/long-term tracking program. Can use many spin-orbit tracking backend algorithms: PTC, Bmad, One-turn Map, Multi-map with sectioning.
- **Resonance strength calculation:** An energy scan procedure, calculating the spin-orbit resonance strengths (in presence of *fully coupled* orbital motion [*]). Implemented in Python/PyTao script.
- **Compatibility with SLICKTRACK:** Bmad lattice designs are compatible with SLICKTRACK code (developed by Desmond Barber). This is used for testing and benchmarking future Bmad developments.
- **Spin matching facilities:** are implemented as a part of spin-tracking framework.
- **Tracking and spin matching are self-consistently implemented in the same program.**

[*] G. Hoffstaetter, High energy polarized proton beams: a modern view (2009).

ISF

Invariant spin field (ISF) $\hat{n}(s, r)$ is the s-periodic, unit field over the phase space $(r = (x, p_x, y, p_y, z, p_z))$ that satisfies

$$R(s, s'; r)\hat{n}(s, r) = \hat{n}(s, M(s, s'; r))$$

where R is spin transport matrix from s to s', and M is orbital transport map from s to s'. ISF is a key QoI in polarization studies.

The SLIM formalism expresses the ISF using two right-hand coordinate systems

$$(l(s), n_0(s), m(s)), \quad (l_0(s), n_0(s), m_0(s)). \tag{1}$$

$l_0(s), n_0(s), m_0(s)$ are solutions to Thomas-BMT equation on the closed orbit. $l(s), m(s)$ are chosen to be one-turn periodic but can have an arbitrary s dependence. $l_0(s)$ and $m_0(s)$ are used for spin matching, $l(s)$ and $m(s)$ are used for calculating polarization and depolarization rate.

$$\hat{n}(s, r) = \sqrt{1 - \alpha(s, r)^2 + \beta(s, r)^2}\, n_0(s) + \alpha(s, r) l(s) + \beta(s, r) m(s) \approx n_0(s) + \alpha(s, r) l(s) + \beta(s, r) m(s),$$

With respect to these coordinate systems we write the spin vector and linearize it. With this the eight-dimensional spin-orbit phase space becomes used in SLIM is

$$(x, p_x, y, p_y, z, p_z, \alpha, \beta).$$

Linearized spin-orbit motion (SLIM formalism)

The first order map is

$$\widetilde{M}(s_1, s_2) = \begin{bmatrix} M_{6\times 6} & 0_{6\times 2} \\ G_{2\times 6} & D_{2\times 2} \end{bmatrix}.$$

M is the 6-by-6 orbital phase space transport matrix. G contains coupling of the spin coordinates to the orbital motion.

In contrast to previous approaches Bmad calculates G and D from the first order terms of Taylor expansion of quaternion spin transport map. From $\widetilde{M}$ the algorithm computes the eigenvalues and eigenvectors from which we compute (approximate) $\partial n/\partial\delta$ – the derivative of the invariant spin field (ISF).

$$u_k = \begin{pmatrix} v_k \\ w_k \end{pmatrix}, \quad k = 1, \ldots, 6, \quad u_k = \begin{pmatrix} 0_6 \\ w_k \end{pmatrix}, \quad k = 7, 8.$$

v_k are eigenvectors of M (orbital transport), first 6 w_kś are computed from

$$w_k = [\lambda_k I_2 - D]^{-1} G v_k, k = 1, \ldots, 6.$$

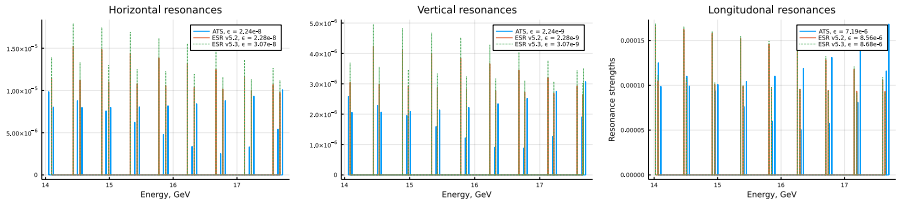
That give us

$$\frac{\partial n}{\partial \delta} = \sum A_k w_k, k = 1, \ldots, 6, \quad \text{where} \sum A_k v_k = (0, 0, 0, 0, 0, 1)^{\mathrm{T}}$$

Spin-orbit resonance strength calculations for ESR

G matrices and eigenvectors can be exploited to calculate resonance strengths

$$\xi_r = \frac{1}{2\pi}\left|\left(G(1,:) + iG(2,:)\right)\cdot v_k\right|, \quad r = 1,2,3, \quad k = 2r-1.$$

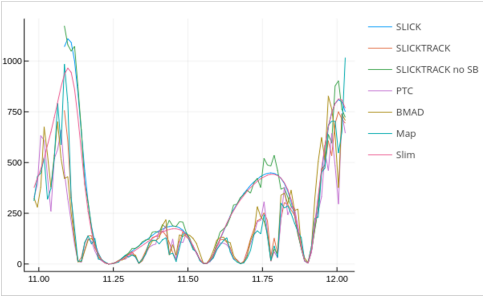


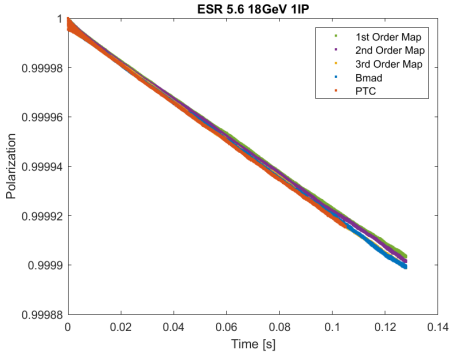
The rings include solenoidal spin rotators.

- Start with 14 GeV and 18 GeV lattices with 1 IR.
- Energy scan, varying solenoid strengths so that IP spin longitudinally polarized, to find energies where the true spin tune is in linear resonance with an orbital tune.
- At each energy where there is a spin/orbital linear resonance, vary the non-arc quadrupoles (except for quads near the IP) in the region so that:
  - Transfer matrices between solenoid pairs is decoupled.
  - Transfer matrix between IP and edges of arc region is the same as the baseline lattice

SLICKTRACK-Bmad comparison in energy scans

Bmad simulations were extensively benchmarked against SLICKTRACK results on various test lattices including ZDR eRHIC and ESR. It led to numerous adjustments and extensions in both codes.

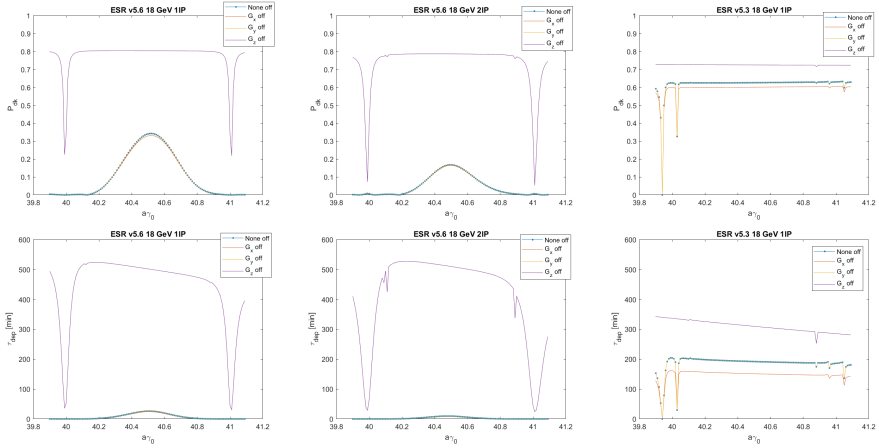




Left: Comparison of various spin-tracking and linear approximation codes in energy (GeV) scan benchmark for depolarization time (minutes)
Right: Polarization in ESR 5.6 from MC spin tracking (picture by M. Signorelli, Cornell)

Spin-matching diagnostics at ESR 5.6

In energy scans for ESR 5.6, to test the spin matching, separate components of G matrix were turned off in Bmad for diagnostics.



Pictures by M. Signorelli.

Summary and ongoing work

Before we move on to more results

- Bmad became to be the one-stop-shop for large-scale Monte-Carlo spin tracking with radiation.
- It can generate maps to arbitrary order to do rapid Monte Carlo tracking with, or use "Bmad Tracking" or PTC tracking.
- Bmad implements 6D radiation kick which is found to be extremely important for accurate tracking when the energy loss in an element/per turn is large.
- One of the most useful features when doing Bmad tracking is the ability to specify how to track through certain elements.

In the few more slides I will show that

- Bmad generated data is useful for machine learning.
- ML learning algorithms predict non-linear spin motion very well with potential of calculating the ISF and speed up tracking using stochastic neural networks.

High order ISF calculations using ML

ISF

Invariant spin field (ISF) $\hat{n}(s, r)$ is the s-periodic, unit field over the phase space (r) that satisfies

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where R is spin transport matrix from s to s', and M is orbital transport map from s to s'. ISF is a key QoI in polarization studies.

- ISF dictates the polarization limit in proton circular machines.
- The study of the ISF is basis for Derbenev-Kondratenko approach to study polarization of electrons.
- Stroboscopic averaging and perturbation theory are currently two ways to calculate the ISF.
- Spin motion is highly nonlinear, even when the orbital motion is linearized.
- Neural networks are known to be a useful tool for non-linear problems in mutliple scales.
- We can use machine learning to calculate the ISF and have another tool for simulations.
- The data that Bmad can generate allows the use of machine learning in polarization studies.

From tracking data to artificial intelligence in finding the ISF

We provide the tracking data and make the neural network learn from the definition

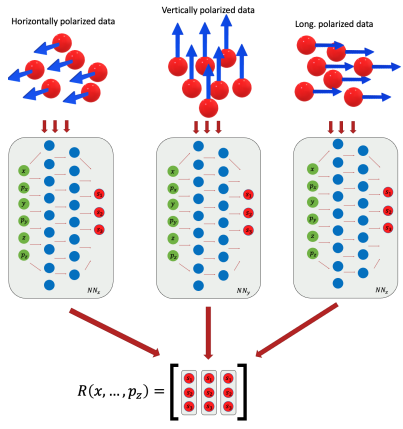
$$R(s, s'; r)\hat{n}(s, r) = \hat{n}(s, M(s, s'; r)),$$

Calculating the ISF using tracking data:

1. Track beams polarized in each direction and obtain spins after one turn.
2. Train 3 separate neural networks for each case that return the spin vector after one turn.
3. Combine 3 vector outputs into rotation matrix R.
4. Train the neural network representing the ISF using

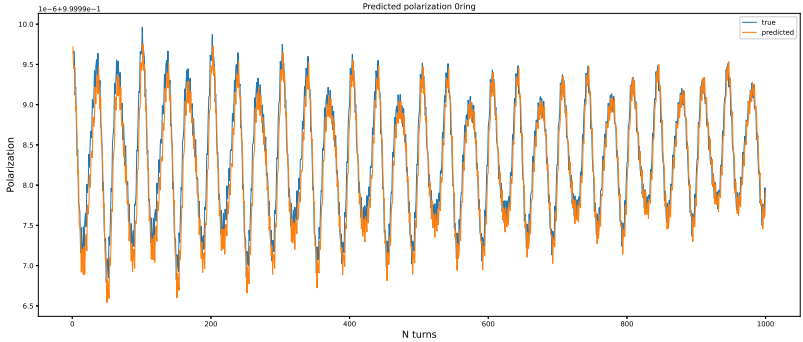
$$\|R(r) \cdot NN_{\mathrm{ISF}}(r) - NN_{\mathrm{ISF}}(M(r))\| \to 0$$

as the loss function (recall the definition of the ISF).



Results of polarization tracking with neural networks

Structure preserving neural networks predict the spin polarization very well, when trained on Bmad generated data gathered from just 1 turn. After the networks are trained 1000 turns take seconds on a laptop versus and hour with PTC tracking.



Thank you!