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Overview of tau physcis at present and future e^+e^- colliders

Alberto Lusiani
Scuola Normale Superiore and INFN, sezione di Pisa

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INFN PISA

Introduction

The tau lepton is a fundamental particle of the Standard Model (SM) and in general it is useful to measure its properties precisely both to test the Standard Model and to search for evidence of physics beyond the Standart model (BSM, NP)

main Standard Model tests and parameters' measurements

- Lepton Flavour Universality (LFU), i.e. (mainly) charged weak coupling is equal for e, μ, τ
  - important NP model constraints for observed B anomalies and CKM 1st row unitarity violation
- SM-predicted Michel parameters, i.e. decay kinematics dictated by $V-A$ charged weak current
- measurement of $\alpha_s(m_\tau)$ and test of running of α_s from m_τ to m_Z
- measurement of $|V_{us}|$ (alternative to kaon decays, less precise)
- alternative measurement of HVP contribution to muon $g-2$ [muon $g-2$ anomaly]

main New Physics searches

- Lepton Flavour Violation (LFV) in tau decay
- CPV in tau decay, tau EDM, tau $g-2$

Tau pairs at past, present and future e^+e^- colliders

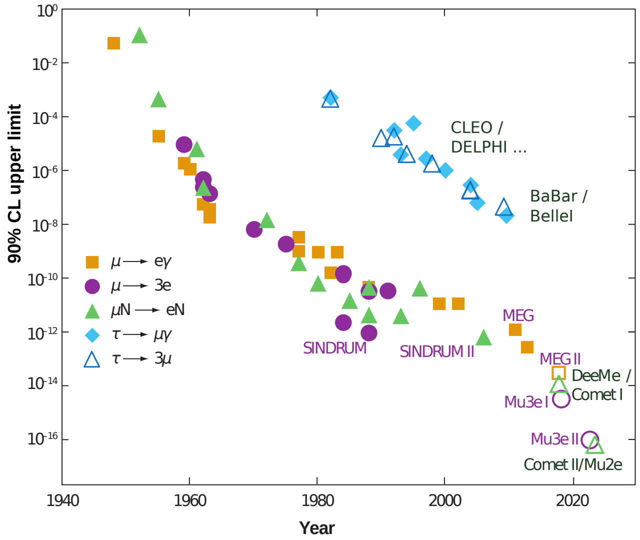
	CLEO, CLEOIII	LEP 100	Belle, BABAR	Belle II	SCT	STCF	CEPC(Z)	FCC-ee(Z)
E_{CM} [GeV]	$\sim$10.6	92	$\sim$10.6	$\sim$10.6	$2-6$	$2-7$	92	
$\int \mathcal{L} dt$ [ab^{-1}]	0.01		1.5	50	10			
tau pairs	$1 \cdot 10^7$	$0.8 \cdot 10^6$	$1.4 \cdot 10^9$	$46 \cdot 10^9$	$30 \cdot 10^9$		$30 \cdot 10^9$	$165 \cdot 10^9$

note: SCT & SCFT tau pairs estimate assuming 10 years of tau-pairs-optimized CM energies running

Conditions for tau physics measurements

- Z peak collisions best for most measurements
  - pure and efficient tau pair selection selecting on just one of the two taus
  - track multiplicity separates very well $\tau^+\tau^-$ from $q\bar{q}$
  - high momenta reduce multiple scattering uncertainty in impact parameter measurements
- threshold measurements at $E = 2m_\tau \sim 3.5$ GeV best for tau mass
  - threshold measurements help some LFV searches and tau BRs (super charm-tau factories)
- B-factories bested LEP with statistics on e.g. small branching fractionss, LFV searches, tau lifetime

LFV searches vigorously pursued

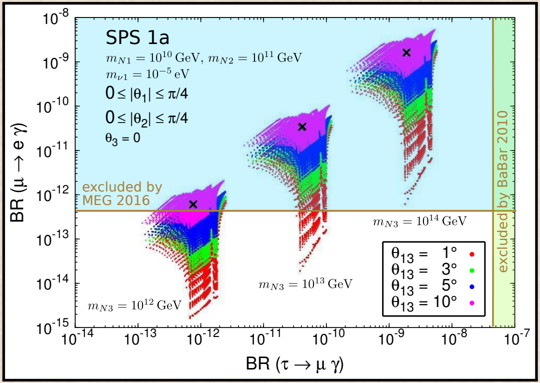


Updated from W.J. Marciano, T. Mori and J.M. Roney, Ann.Rev.Nucl.Part.Sci. 58, 315 (2008)

- NP effects usually scale as $\frac{m_\tau^2}{m_\mu^2}$
- muon LFV searches more powerful
- tau LFV has more channels
  $\Rightarrow$ discrimination on NP models
  $\Rightarrow$ more powerful for specific models

Tau LFV searches probe & constrain New Physics models

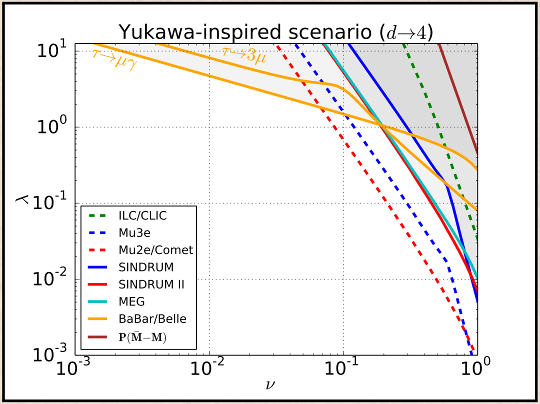
MSSM Seesaw
Antusch, Arganda, Herrero, Teixeira 2006



typical NP models

- $\mathcal{B}(\tau \to \mu\gamma) \sim 10{-}1000 \times \mathcal{B}(\mu \to e\gamma)$
- muon LFV searches more effective

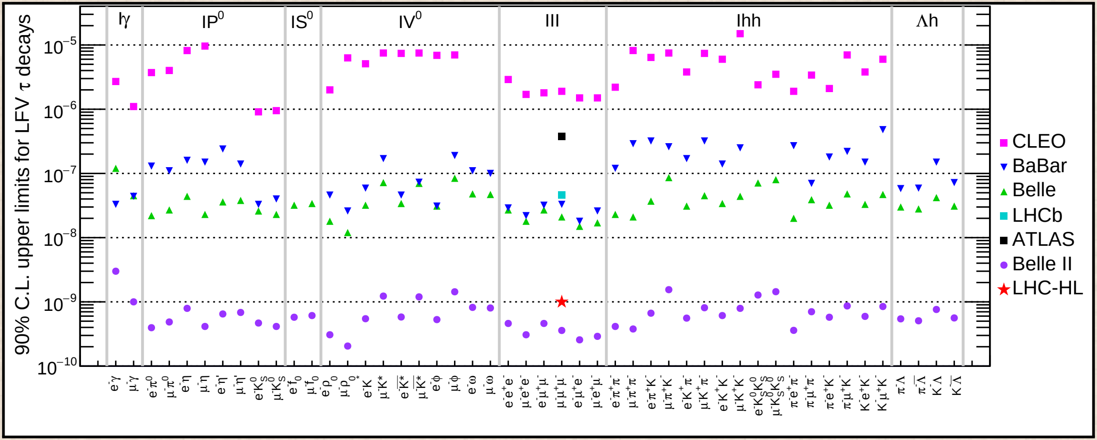
doubly charged scalar
Crivellin, Ghezzi, Panizzi, Pruna, Signer 2019



specific models / parameter space regions

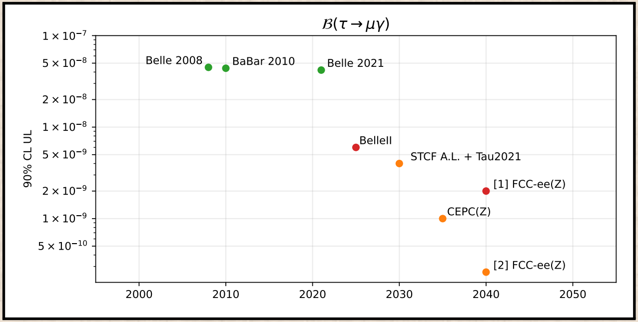
- part of plot only constrained by tau LFV limits

Tau LFV limits: present and future with Belle II and LHCb-HL



HL-LHC and HE-LHC opportunities, arXiv:1812.07638 [hep-ph]

LFV $\tau \to \mu\gamma$ measured / expected upper limits



FCC estimate for $\tau \to \mu\gamma$

[1] M. Dam simulation with 2% of full FCC statistics

[2] M. Dam 2021, guestimate with improved longitudinally segmented crystal EM calorimeter

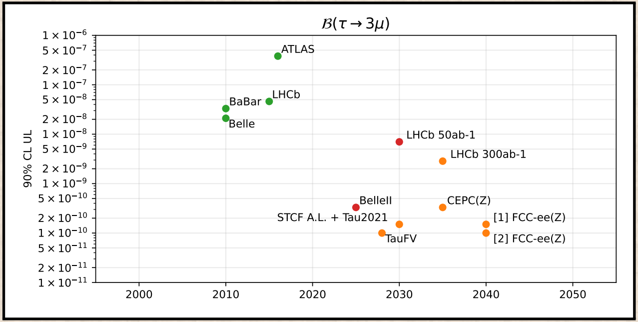
Other estimates

- ESG 2019 docs
- my extrapolation to 10y of SCTF limits presented at Tau2021

Plot notes

- Red more solid estimates
- Orange less solid estimates
- dates of future results are arbitrary, for plotting convenience

LFV $\tau \to 3\mu$ measured / expected upper limits



FCC estimate for $\tau \to \mu\mu\mu$

[1] my guestimate

[2] M. Dam, Tau2021

Other estimates

- ESG 2019 doc
- my extrapolation to 10y of SCTF limits presented at Tau2021

Guestimate of FCC expected 90% upper limit on $\tau \to \mu\mu\mu$

- $2.1{\cdot}10^{-8}$ published Belle limit at 0.782 ab^{-1}
- $/(50\ \text{ab}^{-1}/0.782\,\text{ab}^{-1}) = 3.3{\cdot}10^{-10}$, BelleII expected upper limit assuming background-free search
- FCC: $5{\cdot}10^{12}$ Z^0, 3.3% tau pair decays, $165{\cdot}10^{9}$ tau pairs, $\sim 3.6 \times 46{\cdot}10^{9}$ BelleII tau pairs
- estimate 4× better efficiency at FCC vs. BelleII
  - from DELPHI Phys.Lett. B359 (1995) 411-421 vs. *BaBar* Phys.Rev.Lett. 104 (2010) 021802
- muon PID efficiency and purity expected to be better for FCC
- in the improbable assumption that search remains backgroound free
  - $3.3{\cdot}10^{-10}/3.6/4.0 = 0.23{\cdot}10^{-10}$ estimated FCC 90% upper limit
- estimate / assume that
  - m_τ resolution comparable with B-factories
  - E resolution worse (850 MeV in M. Dam $\tau \to \mu\gamma$ study vs. 50-100 MeV ≈ 75 MeV in *BaBar*)
  - therefore search remains background free until $N^{\text{BelleII}}_{\tau^+\tau^-}/(850\,\text{MeV}/75\,\text{MeV})$
  - additional tau pairs improve upper limit proportionally to the square root (estimated bkg uncertainty)
- $3.3{\cdot}10^{-10} \cdot (850\,\text{MeV}/75\,\text{MeV})/\sqrt{[3.6 \cdot (850\,\text{MeV}/75\,\text{MeV})]}/4.0 \simeq 1.5{\cdot}10^{-10}$ FCC upper limit

Notes for tau LFV searches at FCC

- $\tau \to \mu\gamma$ reach improves with
  - energy resolution of EM calorimeter
  - angular precision (granularity) of EM calorimeter
  - efficiency & purity of muon PID
- $\tau \to 3\mu$ reach improves with
  - momentum resolution and tracking reconstruction accuracy
  - efficiency & purity of muon PID
  - other LFV searches profit from electron, pion, kaon PID
- existing Monte Carlo simulation technology seems sufficient

Lepton universality tests

from HFLAV Tau winter 2022 report

$$\left(\frac{g_\tau}{g_\mu}\right) = \sqrt{\frac{\mathcal{B}_{\tau e}}{\mathcal{B}_{\mu e}} \frac{\tau_\mu m_\mu^5 f_{\mu e} R_\gamma^\mu R_W^\mu}{\tau_\tau m_\tau^5 f_{\tau e} R_\gamma^\tau R_W^\tau}} = 1.0009 \pm 0.0014 = \sqrt{\frac{\mathcal{B}_{\tau e}}{\mathcal{B}_{\tau e}^{\text{SM}}}}$$

$$\left(\frac{g_\tau}{g_e}\right) = \sqrt{\frac{\mathcal{B}_{\tau \mu}}{\mathcal{B}_{\mu e}} \frac{\tau_\mu m_\mu^5 f_{\mu e} R_\gamma^\mu R_W^\mu}{\tau_\tau m_\tau^5 f_{\tau \mu} R_\gamma^\tau R_W^\tau}} = 1.0027 \pm 0.0014 = \sqrt{\frac{\mathcal{B}_{\tau \mu}}{\mathcal{B}_{\tau \mu}^{\text{SM}}}}$$

$$\left(\frac{g_\mu}{g_e}\right) = \sqrt{\frac{\mathcal{B}_{\tau \mu}}{\mathcal{B}_{\tau e}} \frac{f_{\tau e}}{f_{\tau \mu}}} = 1.0019 \pm 0.0014$$

using Standard Model predictions for leptons λ, $\rho = e, \mu, \tau$ (Marciano 1988)

$$\Gamma[\lambda \to \nu_\lambda \rho \bar{\nu}_\rho(\gamma)] = \Gamma_{\lambda\rho} = \Gamma_\lambda \mathcal{B}_{\lambda\rho} = \frac{\mathcal{B}_{\lambda\rho}}{\tau_\lambda} = \frac{G_\lambda G_\rho m_\lambda^5}{192\pi^3} f\left(m_\rho^2/m_\lambda^2\right) R_W^\lambda R_\gamma^\lambda$$

$$G_\lambda = \frac{g_\lambda^2}{4\sqrt{2} M_W^2} \; ; \qquad f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x \; ; \qquad f_{\lambda\rho} = f\left(m_\rho^2/m_\lambda^2\right)$$

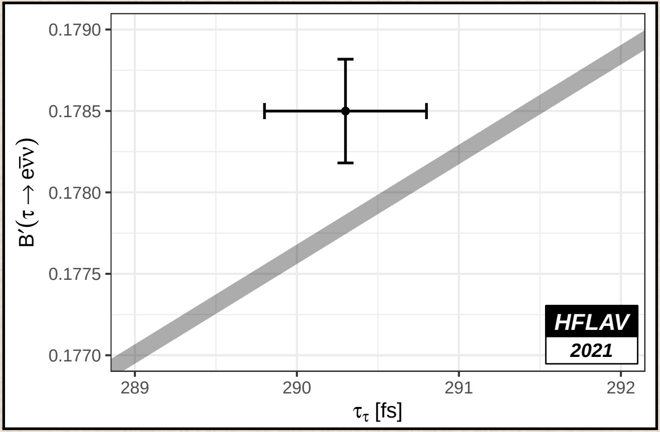
$$R_W^\lambda = 1 + \frac{3}{5}\frac{m_\lambda^2}{M_W^2} + \frac{9}{5}\frac{m_\rho^2}{M_W^2} \; ; \qquad R_\gamma^\lambda = 1 + \frac{\alpha(m_\lambda)}{2\pi}\left(\frac{25}{4} - \pi^2\right) \; ;$$

all statistical correlations included

LFU tests with hadronic tau decays

- are possible and performed, but less precise

Canonical tau lepton universality test plot



$(g_\tau/g_{e\mu}) = 1.0018 \pm 0.0013$
$[g_{e\mu} = g_e = g_\mu$ assuming $g_e = g_\mu]$

$\Delta(g_\tau/g_{e\mu})$ contributions

input	Δinput	$\Delta(g_\tau/g_{e\mu})$
$\mathcal{B}'_{\tau\to e}$	0.180%	0.090%
τ_τ	0.172%	0.086%
m_τ	0.007%	0.017%
total		0.126%

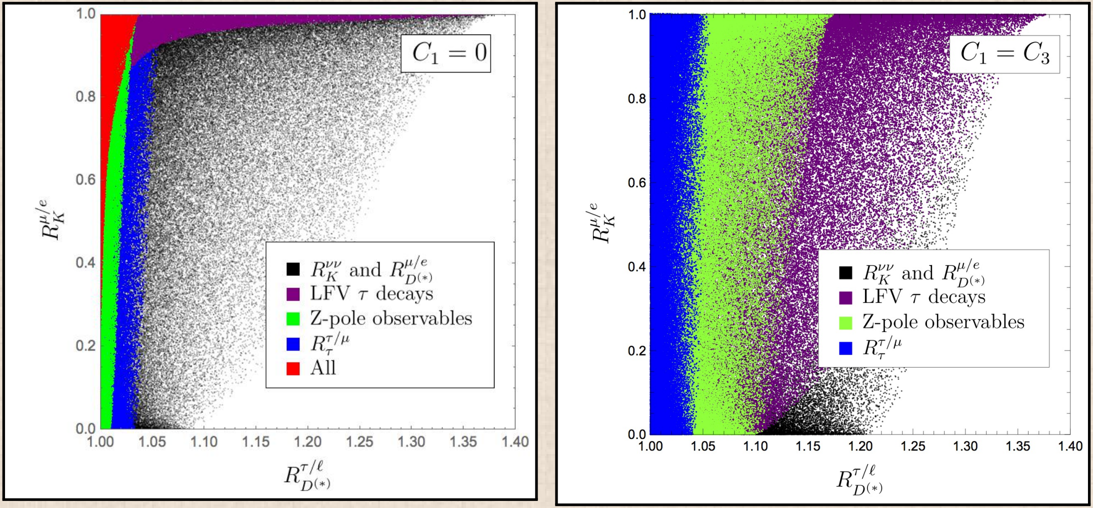
best measurements

$\mathcal{B}'_{\tau\to e}$	ALEPH
τ_τ	Belle
m_τ	BES III

- $\mathcal{B}'(\tau \to e\bar\nu\nu) =$ average of $\begin{cases} \mathcal{B}(\tau \to e\bar\nu\nu) \\ \mathcal{B}(\tau \to \mu\bar\nu\nu)\cdot f_{\tau e}/f_{\tau\mu} \end{cases}$
- $\dfrac{\mathcal{B}'(\tau \to e\bar\nu\nu)\tau_\mu}{\mathcal{B}(\mu \to e\bar\nu\nu)\tau_\tau} = \dfrac{g_\tau^2}{g_{e\mu}^2}\,\dfrac{m_\tau^5 f_{\tau e} R_\gamma^\tau R_W^\tau}{m_\mu^5 f_{\mu e} R_\gamma^\mu R_W^\mu}$
- $\left(\dfrac{g_\tau}{g_{e\mu}}\right)^2 = \dfrac{\mathcal{B}'(\tau \to e\bar\nu\nu)}{\mathcal{B}(\mu \to e\bar\nu\nu)}\,\dfrac{\tau_\mu}{\tau_\tau}\,\dfrac{m_\mu^5}{m_\tau^5}\,\dfrac{f_{\mu e} R_\gamma^\mu R_W^\mu}{f_{\tau e} R_\gamma^\tau R_W^\tau}$

Tau Lepton universality constrains models for B $R^{\tau/\ell}_{D^{(*)}}$- $R^{\mu/e}_K$ anomalies

Feruglio, Paradisi, Pattori JHEP 09 (2017) 061
blue points correspond to parameter space region allowed by tau lepton universality



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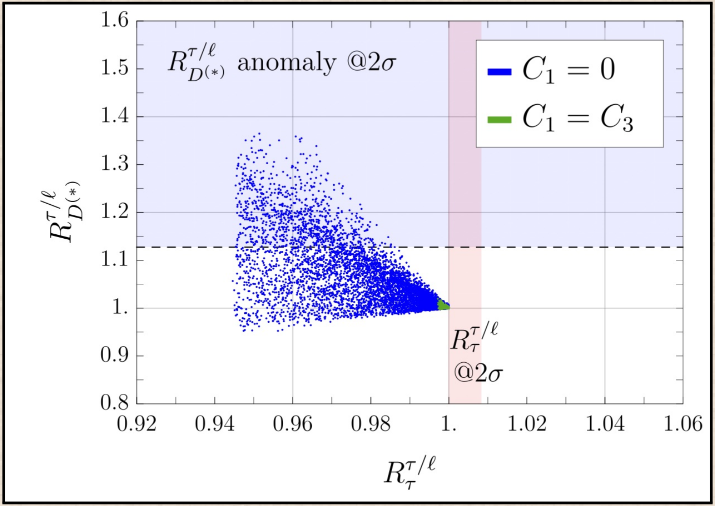
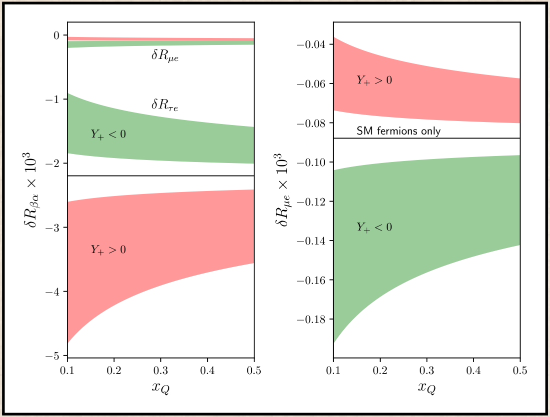
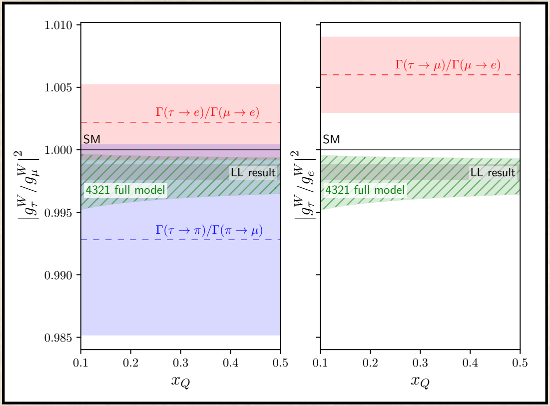


Tau Lepton universality constrains 4321 models for B $R^{\tau/\ell}_{D^{(*)}}$- $R^{\mu/e}_K$ anomalies

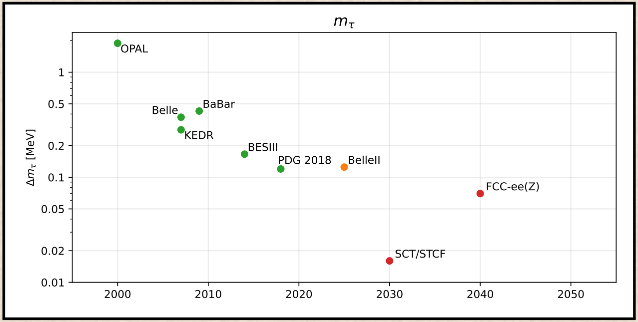
LFU violations in leptonic τ decays and B-physics anomalies

- Allwicher, Isidori, Selimovic, PLB 826 (2022) 136903
- finite 1-loop corrections for 4321 [$SU(4)\times SU(3)\times SU(2)\times U(1)$)] models from matching conditions at NP scale
- smaller impact on tau LU than "Effective Field Theory leading-log" calculations
  $\Rightarrow$ future precision measurements of leptonic τ decay widths important for testing 4321 models





m_τ experimental precision



FCC estimate

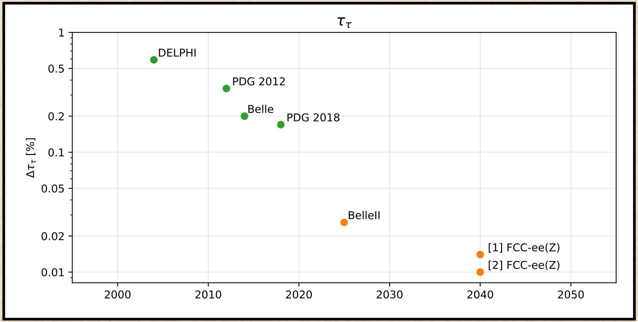
- M. Dam 11th FCC-ee Workshop 2019, limiting systematics of 0.1 MeV on pseudomass distribution modeling

Other estimates

- ESG 2019 docs

- best experimental facilities are e^+e^- at $\tau^+\tau^-$ threshold, then B-factories
- FCC
  - challenge is systematics from pseudomass distribution modeling
  - can use 5-prong decays (narrower pseudomass distribution drop)
  - attainable precision on momentum measurement scale appears not to be limiting

τ_τ experimental precision



FCC estimate

[1] M. Dam and CDR

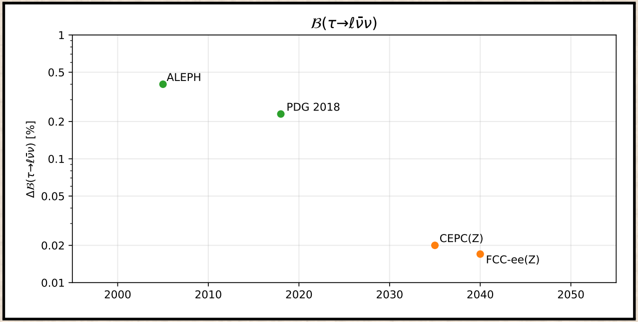
[2] A.L. FCC Workshop Jan 2020

Other estimates

- ESG 2019 docs

- best measurement by Belle on 3-prong vs. 3-prong tau pairs
- expect limiting systematics from absolute length scale calibration on minivertex detector, 100 ppm
- 68 ppm systematics from Δm_τ at current precision
- potential systematics from modeling of measurement bias subtraction
- potential systematics from accuracy of simulation of average radiation energy loss
  - would profit from improvements of tau pairs generators
- profits from high-resolution vertex detector close to interation region

FCC sensitivity for $\mathcal{B}(\tau \to \ell\bar{\nu}\nu)$



FCC estimate

- M. Dam Tau2018, Tau2021

Other estimates

- ESG 2019 docs

- sensitivity estimates very difficult, mostly guestimates
- best results from ALEPH global analysis of all tau decays
- important:
  - PID efficiency, purity, accurate PID modeling with control samples
  - efficiency, purity of π^0 reconstruction, accurate modeling with control samples
  - improve current poor simulation of high multiplicity inv. mass distributions
  - improvements on tau pairs Monte Carlo simulations highly desirable
- high statistics samples will help very much on first 3 points, but analyses will be very complex

Tau branching fractions notes

- world averages of large BRs still dominated by LEP
  - background separation from dileptons and hadrons much better
  - higher selection purity and efficiency
  - possible to tag single tau with good efficiency and purity and observe the other one
    $\Rightarrow$ wonderful base for reducing systematics using data, exploited in particular by ALEPH
- *B*-factories improved on small branching fractions using statistics
  $\Rightarrow$ FCC statistics $1300^2\times$ ALEPH, $175\times$ Belle, $3.5\times$ BelleII (& better efficiency w.r.t. *B*-factories)
- FCC is best imaginable context for tau BR measurements
- what are the limiting systematics?

Systematics of main ALEPH tau BR paper, Phys. Rept. 421 (2005) 191

systematics

Total systematic errors for branching ratios measured from the 1994–1995 data sample

Topology	π^0	sel	bkg	pid	int	trk	dyn	mcs	Total
e	0.011	0.021	0.029	0.019	0.009	0.000	0.000	0.015	0.045
μ	0.004	0.020	0.020	0.021	0.008	0.000	0.000	0.015	0.039
h	0.071	0.016	0.010	0.022	0.022	0.014	0.000	0.019	0.083
$h\pi^0$	0.063	0.027	0.019	0.011	0.045	0.009	0.000	0.027	0.090
$h2\pi^0$	0.089	0.021	0.014	0.004	0.007	0.003	0.040	0.028	0.105
$h3\pi^0$	0.056	0.012	0.015	0.000	0.008	0.001	0.008	0.030	0.068
$h4\pi^0$	0.029	0.005	0.011	0.000	0.015	0.000	0.000	0.019	0.040
$3h$	0.047	0.021	0.018	0.004	0.012	0.014	0.006	0.015	0.059
$3h\pi^0$	0.033	0.017	0.029	0.002	0.041	0.009	0.007	0.018	0.066
$3h2\pi^0$	0.027	0.008	0.015	0.000	0.009	0.003	0.012	0.014	0.038
$3h3\pi^0$	0.010	0.012	0.002	0.000	0.002	0.001	0.010	0.006	0.019
$5h$	0.002	0.000	0.002	0.000	0.000	0.001	0.000	0.003	0.004
$5h\pi^0$	0.002	0.000	0.006	0.000	0.000	0.000	0.000	0.002	0.007
Class 14	0.013	0.003	0.022	0.002	0.024	0.000	0.000	0.011	0.037

All numbers are absolute in per cent. The labels are defined as follows: photon and π^0 reconstruction (π^0), event selection efficiency (sel), non-τ background (bkg), charged particle identification (pid), secondary interactions (int), tracking (trk), Monte Carlo dynamics (dyn), Monte Carlo statistics (mcs), total systematic uncertainty (total).

π^0 systematics

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$5h$	0.002	0.000	0.002	0.000	0.000	0.001	0.000	0.003	0.004
$5h\pi^0$	0.002	0.000	0.006	0.000	0.000	0.000	0.000	0.002	0.007
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- many systematics but in general all limited only by data vs. MC comparisons
- non-trivial to extrapolate to 1300^2 more data

Main systematics of ALEPH tau BR paper, Phys. Rept. 421 (2005) 191

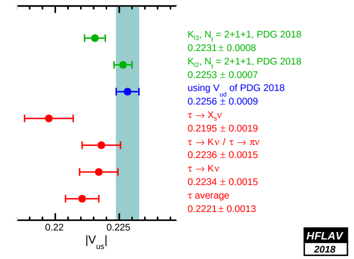
- non-tau backgrounds
  - estimated by varying MC estimate by 30%
  - does not trivially scale with luminosity, but can be improved
- tau pair selection
  - use break-mix method on data and MC, 0.1-0.2% uncertainties
    dominant systematics from data statistics of tau vs. hadron cut separation
  - scales with luminosity, but correlations between hemispheres limit how much
- PID
  - uncertainties from control samples studies
  - partially scales with luminosity, but limited by achievable purity of control samples
- photon efficiency
  - uncertainties from control samples studies data-MC comparisons
    - fit data using predicted MC fake and genuine photon distributions and compare number of genuine photons
    - compare photons $> 3\,\mathrm{GeV}$ as function of separation from tracks
    - compare converted photons
    - compare hadron to electron misidentification
    - compare photon identification efficiency
    - photon energy scale calibrated with momentum measurement on high-energy e from tau decay
    - compare fake photons

Main systematics of ALEPH tau BR paper, Phys. Rept. 421 (2005) 191

- π^0 efficiency
  - compare data and MC D_{ij} distributions (probability γ_i, γ_j) of π^0 mass fit
- efficiency for π^0 with unresolved photons
  - compare data and MC 2nd moment of transverse evergy in calorimeter cells
- radiative and bremsstrahlung photons
  - compare data and MC distributions
  - compare PHOTOS vs. exact calculation for $\tau \to \pi\pi^0\nu$ with radiative $E_\gamma > 12\,\text{MeV}$
- tracking
  - compare data and MC on same sign events events (two tracks missing in one hemisphere)
- tau decay dynamic
  - reduced because acceptances are large and flat
  - will become important with higher statistics
  - can be partially addressed with iterative concurrent measurements where also invariant mass distributions are fitted on data (complicate)

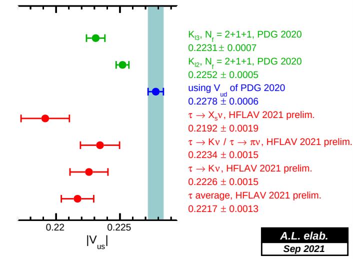
$|V_{us}|$-centric CKM matrix first row unitarity test

PDG 2018 review



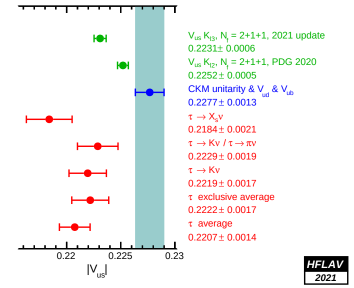
- CKM unitarity OK with kaons

PDG 2020 review



- new dispersive calculation of Δ_R^V inner or universal electroweak radiative corrections (RC) to super-allowed nuclear beta decays
  Seng, Gorchtein & Ramsey-Musolf, Phys. Rev. D 100, 013001 (2019)

2021 update



- J.C.Hardy & Ii.S.Towner, PRC 102, 045501 (2020)
  - inflated $|V_{us}|$ systematics
- Seng, Gorchtein & Ramsey-Musolf, 2021
- Seng, Galviz, Marciano, Meißner, 2021

- $\Delta\,|V_{us}|_\tau \approx \Delta\,|V_{us}|_{V\,ud}$ in 2021!

$|V_{us}|$ determinations using τ branching fractions measurements

Using tau measurements and OPE, no lattice QCD

- $$\frac{R(\tau \to X_{\text{strange}}\,\nu)}{|V_{us}|^2} = \frac{R(\tau \to X_{\text{non-strange}}\,\nu)}{|V_{ud}|^2} - \delta R_{\tau,\text{SU3 breaking}},$$

$\tau \to X_s \nu$

Using tau measurements and lattice QCD

- $$\frac{\Gamma(\tau^- \to K^- \nu_\tau)}{\Gamma(\tau^- \to \pi^- \nu_\tau)} = \frac{|V_{us}|^2}{|V_{ud}|^2}\left(\frac{f_{K\pm}}{f_{\pi\pm}}\right)^2 \frac{\left(1 - m_K^2/m_\tau^2\right)^2}{\left(1 - m_\pi^2/m_\tau^2\right)^2} \frac{R_{\tau/K}}{R_{\tau/\pi}} R_{K/\pi}$$

$\tau \to K / \tau \to \pi$

- $$\Gamma(\tau^- \to K^- \nu_\tau) = \frac{G_F^2}{16\pi\hbar} f_{K\pm}^2 |V_{us}|^2 m_\tau^3 \left(1 - \frac{m_K^2}{m_\tau^2}\right)^2 R_{\tau/K} R_{K\mu 2}$$

$\tau \to K$

Requirements

- Cabibbo-suppressed tau BRs
- tau spectral functions

α_s from tau decay measurements

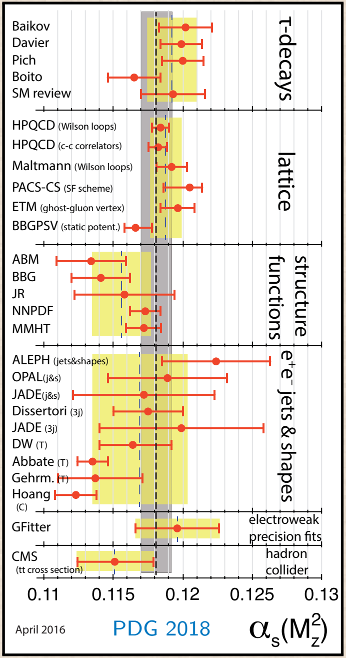
- $\alpha_s(m_\tau)$ from
  - $R_{VA} = \mathcal{B}(\tau \to X_d\nu)/\mathcal{B}(\tau \to e\bar{\nu}\nu)$
  - tau spectral functions
- tau data competitive
- $\alpha_s(m_\tau)$ confirms running of α_s
- best experimental inputs e^+e^- facilities at the Z peak
  - modest experimental progress since LEP times
  - statistics, clean data, non-trivial analysis needed
  - non-trivial exp. and theory systematics

Recent discussions

- different groups get somewhat inconsistent results
  disagreements on non-perturbative effects, duality violations
- Pich 2019
  Boito, Golterman, Maltman, Peris 2019
  Pich, Rojo, Sommer, Vairo 2018
  Boito, Golterman, Maltman, Peris 2017
  Pich, Rodríguez-Sánchez 2016

Requirements

- tau spectral functions, tau branching fractions



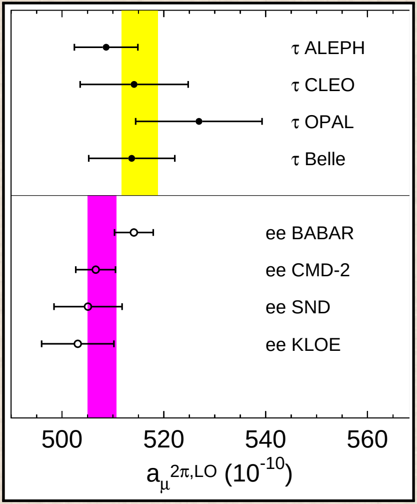
Muon $g-2$ hadronic contribution from tau

- $\alpha_\mu^{2\pi,\mathrm{LO}}$ from
  - $\tau \to \pi\pi^0\nu$ spectral function
  - normalization could come from $\mathcal{B}(\tau \to \pi\pi^0\nu)$, τ_τ
  - isospin rotation (associated theory systematics)
- best experimental inputs e^+e^- facilities at the Z peak
  - modest experimental progress since LEP times
  - statistics, clean data, non-trivial analysis needed
- tau data $\Rightarrow$ reduced discrepancy with exp.
- presently e^+e^- data more precise and complete

Requirements

- improved isospin-violating and EM corrections for $\tau \to \pi^0\pi\nu_\tau$
- tau spectral functions
- tau branching fractions

Davier 2013



Tau spectral functions

- reasonably complete sets only measured at LEP (ALEPH, OPAL)
- limited contributions from B-factories
- studies at the Z peak are by far the most favourable context
- significant improvements are possible at FCC especially for the poorly measurer rare modes
- analyses are complex and may be limited by manpower availability
- improvements on Monte Carlo simulation desirable

Other tau physics topics

- many additional tau physics topics have not been discussed
- recent Belle paper extends tau EDM measurement from 29.5fb^{-1}to 833fb^{-1}
- large analysis efforts on tau EDM and $g-2$ at Belle II
  - these measurements benefit significantly from beam polarization and precise vertexing
- large analysis effort on tau Michel parameters at Belle / Belle II and super charm-tau factories

Conclusions

- tau physics best done on e^+e^- colliders
- Z-peak conditions are best for most measurements
- threshold tau pair production best for tau mass
- useful experimental features
  - precise knowledge of beam energies
  - small luminous region
  - precise vertex detector close to luminous region
  - beams polarization

Thanks for your attention!

Backup Slides

Disconery of Tau Lepton

- in early 1960's, while digesting muon discovery, searches started for next heavy lepton
- photo-production: $e + \text{nucleus} \to \gamma + X, \quad \gamma + \text{nucleus} \to \ell^+\ell^- + X'$
  - upper limits on m_ℓ in $[0.5, 1.0]$ Gev (SLAC, 1968)
- e^+e^- colliders, $e^+e^- \to \gamma^* \to \ell^+\ell^-$
  - exclude $m_\ell < 1.15\,\text{GeV}$ (ADONE, Frascati, 1974)

MARK 1 at SLAC

- 1964 proposal of SPEAR e^+e^- collider, max CM energy 4.8 GeV, funded in 1970
- 1971 proposal of MARK 1 detector, 1971, Martin Perl *et al.*, proposal topics:
  - 1) Boson Form Factors, 2) Baryon Form Factors, 3) Inelastic Reactions
  - 4) Search for Heavy Leptons ($\tau_1 \to e\bar{\nu}\nu$, $\tau_2 \to \mu\bar{\nu}\nu$)
- theorists compute expected distributions (Tsai, Sakurai)
- 1974 beginning of data-taking
- 1975 evidence for "anomalous lepton production" (24 events)
- 1976 discovery "simplest hypothesis" ..."production of a pair of heavy leptons"
- 1995 Martin Perl gets Nobel prize

FCC tau physics references (non exhaustive)

- FCC CDR
- Mogens Dam: Tau 2018, FCC Jan 2019, Tau 2021
  - M. Dam, SciPost Phys. Proc. 1, 041 (2019)
  - M. Dam, Eur. Phys. J. Plus 136, 963 (2021) DOI:10.1140/epjp/s13360-021-01894-y
- A.L. ESG update 2019, FCC Jan 2020, Charm 2021, Tau 2021, FCC-Ita Dec 2021
  - European Strategy Update 2019, arXiv:1910.11775 [hep-ex]

Tau Lifetime

τ MEAN LIFE

PDG 2019

VALUE (10^{-15} s)	EVTS	DOCUMENT ID		TECN	COMMENT
290.3 ± 0.5	**OUR AVERAGE**				
290.17 ±0.53 ±0.33	1.1M	BELOUS	2014	BELL	711 fb^{-1} $E^{ee}_{\rm cm}$=10.6 GeV
290.9 ±1.4 ±1.0		ABDALLAH	2004T	DLPH	1991-1995 LEP runs
293.2 ±2.0 ±1.5		ACCIARRI	2000B	L3	1991--1995 LEP runs
290.1 ±1.5 ±1.1		BARATE	1997R	ALEP	1989--1994 LEP runs
289.2 ±1.7 ±1.2		ALEXANDER	1996E	OPAL	1990--1994 LEP runs
289.0 ±2.8 ±4.0	57.4k	BALEST	1996	CLEO	$E^{ee}_{\rm cm}$= 10.6 GeV

tau lifetime precision

precision (ppm)	
1700	PDG 2019
2100	Belle
5900	DELPHI
6400	ALEPH
7200	OPAL
260	Belle II guestimate, extrapolating from 0.711 ab^{-1} to 50 ab^{-1}
5	FCC, stat. only extrapolation from ALEPH (1e5) to FCC (1.65e11) tau pairs

⇒ what are the limiting systematics?

Tau Lifetime systematics at LEP

DELPHI main systematics, Eur.Phys.J.C36:283-296,200

- IP impact parameter difference on 1-1-prong tau pairs
  - trimming, backgrounds, impact parameter resolution, alignment
- MD miss-distance on 1-1-prong tau pairs
  - resolution on MD, bias, selection
- DL transverse decay length on 3-1 and 3-3 prong tau pairs
  - alignment

ALEPH main systematics, Phys.Lett.B414:362-372,1997

- MIPS, momentum-weighted impact parameter sum
  - resolution on impact parameter sum, bias (from MC)
- 3DIP 3D impact parameter, Z. Phys. C 74, 387–398 (1997)
  - bias (from MC), vertex chisq cut
- IPD, impact parameter difference
  - resolution and trimming of outliers
- DL, decay length
  - vertex chisq cut

expect that all these systematics scale with $1/\sqrt{N_{\text{events}}}$
including alignment systematics
although questionable if up to a factor $1/{\sim}1300$

Tau Lifetime systematics at FCC

Alignment systematic

- alignment calibration precision improves with statistics
- misalignment effects zero at first order for uniform azymuthal acceptance
  S.R.Wasserbaech, Nucl.Phys.Proc.Suppl. 76 (1999) 107-116
  - still, questionable how far this holds
- related systematic that does not scale
  absolute length scale of vertex detector average elements spacing, realiable to 10^{-4} or 100 ppm

Systematics from kinematics of tau decay

- $\tau_\tau = \lambda_\tau / \beta\gamma = \lambda_\tau / \frac{\sqrt{E_\tau^2 - m_\tau^2}}{m_\tau} = \lambda_\tau / \frac{\sqrt{(E_{\text{beam}} - E_{\text{rad}}^{\text{MC}})^2 - m_\tau^2}}{m_\tau}$

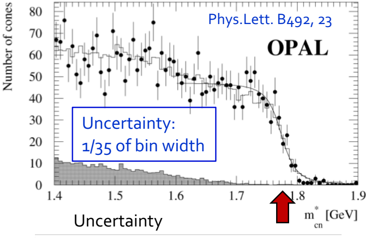
systematic [ppm]	
1	E_{beam}
68	m_τ PDG 2019
7	m_τ possible measurement at Super Charm-Tau Factories
?	MC accuracy on average radiation energy loss (*) (estimated 100 ppm for *BABAR*)

(*) depends on

- accuracy of generator, can be checked measuring momentum distribution of di-muon events
- accuracy of simulation of efficiency of selection procedure vs. E_τ (scales with luminosity)

Tau Mass (from M.Dam, FCC-ee Workshop Jan 2019)

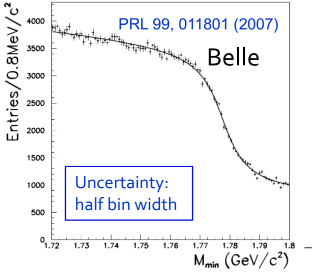
- **Current world average:** $m_\tau = 1776.86 \pm 0.12$ MeV
- Best in world: BES3 (threshold scan) $m_\tau = 1776.91 \pm 0.12\ (\text{stat.})\ ^{+0.10}_{-0.13}\ (\text{syst.})$ MeV
- Best at LEP: OPAL $m_\tau = 1775.1 \pm 1.6\ (\text{stat.}) \pm 1.0\ (\text{syst.})$ MeV
  - About factor 10 from world's best
  - Main result from endpoint of distribution of pseudo-mass in $\tau \to 3\pi^\pm(n\pi^0)\nu_\tau$
  - Dominant systematics:
    - Momentum scale: 0.9 MeV
    - Energy scale: 0.25 MeV (including also π^0 modes)
    - Dynamics of τ decay: 0.10 MeV



- Same method from Belle
  - Main systematics
    - Beam energy & tracking system calib.: 0.26 MeV
    - Parameterisation of the spectrum edge: 0.18 MeV

$m_\tau = 1776.61 \pm 0.13\ (\text{stat.}) \pm 0.35\ (\text{syst.})$ MeV

Pseudo-mass: $M_{min} = \sqrt{M_{3\pi}^2 + 2(E_{beam} - E_{3\pi})(E_{3\pi} - P_{3\pi})}$



Mogens Dam / NBI Copenhagen 11th FCC-ee Workshop, CERN 10/01/2019

Tau Mass (from M.Dam, FCC-ee Workshop Jan 2019)

- Prospects for FCC-ee:
  - 3 prong, 5 prongs, (perhaps even 7 prongs?)
  - Statistics 10^5 times OPAL: δ_{stat} = **0.004 MeV**
  - Systematics:
    - At FCC-ee, E_{BEAM} known to better than 0.1 MeV (~ 1 ppm) from resonant depolarisation
      - Negligible effect on m_τ
    - Likely dominant experimental contribution comes from understanding of the mass scale
      - Use high stats $e^+e^- \to \mu^+\mu^-$ sample to fix momentum scale. Extrapolate down to momenta typical for $\tau \to 3\pi$.
      - Use $D^0 \to K^-\pi^+$ / $K^-\pi^+\pi^-\pi^-$ and $D^+ \to K^-\pi^+\pi^+$ to fix mass scale (m_D known to 50 keV)
    - Reduce uncertainty from parametrisation of spectrum edge by use of theoretical spectrum checked against high statistics data
    - Cross checks using 5-prongs
  - Suggested overall systematics: δ_{syst} = **0.1 MeV**
    - Could potentially touch current precision but probably no substantial improvement ?

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Tau Physics plans of relevant facilities (as of 2019)

Belle II

- The Belle II experiment at SuperKEKB: input to the European Particle Physics Strategy
- The Belle II Physics Book arXiv:1808.10567 [hep-ex]
- 50 ab^{-1}, improved detector w.r.t. Belle/BaBar, 50× Belle statistics, $9{\cdot}10^{10}$ tau decays
- B-factories scored well on LFV, less well on precision measurements and spectral functions
- $\mathcal{B}(\tau \to \mu\gamma) < \sim 1{\cdot}10^{-9}$ 90% CL detailed study with BelleII sample, may be optimistic
- $\mathcal{B}(\tau \to 3\mu) < 3.3{\cdot}10^{-10}$ 90% CL extrap. from Belle assuming selection remains bkg-free
- similar improvements on many other tau LFV modes
- $\Delta m_\tau = \pm 0.10{-}0.15$ MeV "very optimistically" (BESIII ± 0.17 MeV)
- my personal statistics-only-driven estimate $\Delta\tau_\tau = 0.026\%$ (Belle 0.21%)
- improvements w.r.t. today WA expected on $\mathcal{B}(\tau \to \ell\bar{\nu}\nu)$ and τ_τ but non-trivial & non-assured
- significant improvements on Cabibbo-suppressed BRs and spectral functions, but non-trivial
- significant advances possible on many more measurements:
  Michel parameters, spectral functions, CPV, radiative decays, $g{-}2$, EDM...
- Belle III: luminosity upgrade of Belle II would advance the reach of the LFV searches

Tau Physics plans of relevant facilities (as of 2019)

HL-LHC and HE-LHC

- inputs to the European Particle Physics Strategy
- Opportunities in Flavour Physics at the HL-LHC and HE-LHC, arXiv:1812.07638 [hep-ph]
- $\mathcal{B}(\tau \to 3\mu)$ 90% CL

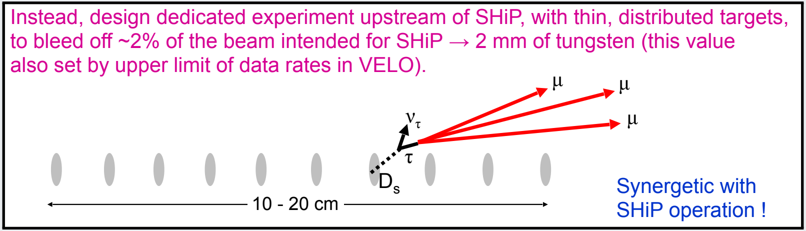
Table 23: Actual and expected limits on $\mathrm{BR}(\tau \to 3\mu)$ for different experiments and facilities. The ATLAS projections are given for the medium background scenario, see main text for further details.

$\mathrm{BR}(\tau \to 3\mu)$ (90% CL limit)	Ref.	Comments
3.8×10^{-7}	ATLAS [429]	Actual limit (Run 1)
4.6×10^{-8}	LHCb [428]	Actual limit (Run 1)
3.3×10^{-8}	BaBar [417]	Actual limit
2.1×10^{-8}	Belle [423]	Actual limit
3.7×10^{-9}	CMS HF-channel at HL-LHC	Expected limit (3000 fb^{-1})
6×10^{-9}	ATLAS W-channel at HL-LHC	Expected limit (3000 fb^{-1})
2.3×10^{-9}	ATLAS HF-channel at HL-LHC	Expected limit (3000 fb^{-1})
$\mathcal{O}(10^{-9})$	LHCb at HL-LHC	Expected limit (300 fb^{-1})
3.3×10^{-10}	Belle-II [196]	Expected limit (50 ab^{-1})
7.9×10^{-9}	LHCb	M.Chrząszcz priv.comm. (50 fb^{-1})

Tau Physics plans of relevant facilities (as of 2019)

TauFV, project, SPS protons on fixed-targed, dedicated to tau LFV searches

- inputs to the European Particle Physics Strategy



- leverages on LHCb expertise, success and upgrade-related R&D, synergic with SHiP
- n. of tau decays: 900×BelleII, 60×LHCb(50 fb^{-1}), 10×LHCb(300 fb^{-1})
- target and detector optimized for tau LFV searches
- earliest date 2026-2027
- $\mathcal{B}(\tau \to 3\mu)$ 90% CL UL "down to 10^{-10}"
- also sensitive to other $\mathcal{B}(\tau \to \ell_1\ell_2\ell_3)$, one less order of magnitude for $e^+\mu^-\mu^-$
- promising enterprise, could match and improve on BelleII for $\mathcal{B}(\tau \to 3\mu)$

Tau Physics plans of relevant facilities (as of 2019)

Super Charm-Tau Factories: SCT (BINP, Novosibirsk) and STCF/HIEPA (China)

- SCT/Russia inputs to the European Particle Physics Strategy
- STCF/China Haiping Peng, priv.comm., $\tau \to \mu\gamma$ study arXiv:1511.07228 [hep-ex]
- very similar projects, common description
- $E = 2\text{–}6$ or $2\text{–}7\,\text{GeV}$, $\mathcal{L} = 10^{35}\,\text{cm}^{-2}\text{s}^{-1}$, polarized e^- beam
- begin datataking 2029-2030
- max $\tau^+\tau^-$ cross-section at 4.25 GeV (3.5 nb), unknown how many years at that CM energy
  - I rescaled estimates to 2 years at 4.25 GeV (each year $2{\cdot}10^7$ s and $7{\cdot}10^9$ tau pairs)
- Δm_τ from ± 0.166 MeV (BESIII) to ± 0.012 MeV [10× better systematic uncertainty]
- $\mathcal{B}(\tau \to \mu\gamma) < 5{\cdot}10^{-9}$ 90% CL
  - extrapolated from $3\,\text{fb}^{-1}$assuming search bkg free (my understanding)
  - note that background is significantly less than at B-factories energies
- $\mathcal{B}(\tau \to 3\mu) < 3.5{\cdot}10^{-10}$ 90% CL sensitive also to all other $\mathcal{B}(\tau \to \ell_1\ell_2\ell_3)$
- many LFV modes and other tau measurements possible, but little guiding past experience
  - both projects actively investigating/planning many tau Physics measurements

Tau Physics plans of relevant facilities (as of 2019)

CEPC at the Z peak

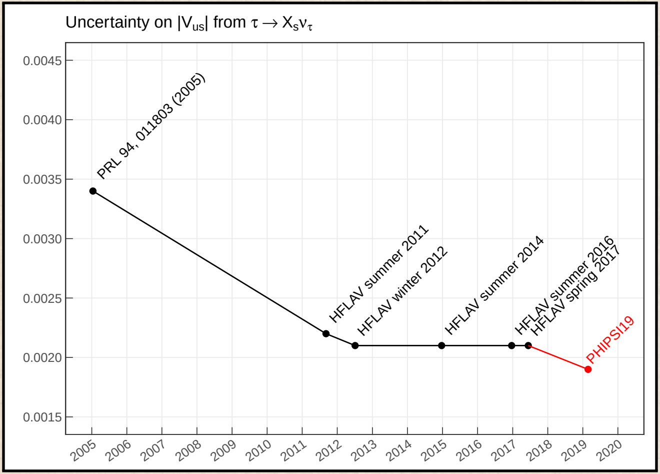
- inputs to the European Particle Physics Strategy
- The CEPC Conceptual Design Report, Vol II: Physics and Detector, arXiv:1811.10545 [hep-ex]
- could be approved in 2022!
- $1{\cdot}10^{12}$ Z, $3{\cdot}10^{10}$ tau pairs (comparable to $4.5{\cdot}10^{10}$ of BelleII)
- expect tau LFV sensitivities similar to BelleII
  - but historic LEP LFV limits are much better than B-factories, for the same number of tau
- stat. uncertainties $\mathcal{O}(450)\times$ better than LEP $\Rightarrow$ must estimate reasonable limiting systematics
- expect $\Delta\mathcal{B}(\tau \to \ell\bar{\nu}\nu) \sim 0.02\%$ (by improving $10\times$ ALEPH systematics 0.2%)
- expect $\Delta\tau_\tau \sim 0.02\%$ (by improving $10\times$ w.r.t. Belle total uncertainty of 0.2%)
- significant advances possible on about all measurements and LFV limits
- Z peak offers by far best conditions for about all tau Physics measurements

Tau Physics plans of relevant facilities (as of 2019)

FCC-ee at the Z peak

- inputs to the European Particle Physics Strategy
- Future Circular Collider, Vol. 1 : Physics opportunities (December 2018)
- Dam 2019 (Tau 2018 proc.)
- 8y preparation, 10y construction, 15y operation
- Z peak phase delivers $5{\cdot}10^{12}$ Zs, $15{\cdot}10^{10}$ tau pairs (BelleII $4.5{\cdot}10^{10}$)
- stat. uncertainties $\mathcal{O}(1000)\times$ better than LEP $\Rightarrow$ must estimate reasonable limiting systematics
- expect $\Delta\mathcal{B}(\tau \to \ell\bar{\nu}\nu) \sim 0.02\%$ (by improving $10\times$ ALEPH systematics 0.2%)
- expect $\Delta\tau_\tau \sim 0.01\%$ (by improving $9\times$ w.r.t. Belle detector alignment systematics of 0.1%)
- expect $\Delta m_\tau \sim 0.07\,\text{MeV}$ (by calibrating on m_{D^+}, PDG 2018 WA $\pm 0.12\,\text{MeV}$)
- $\mathcal{B}(\tau \to \mu\gamma) < 2{\cdot}10^{-9}$ 90% CL Monte Carlo study on 2% of full FCC-ee statistics
- $\mathcal{B}(\tau \to 3\mu) < [1{-}0.1]{\cdot}10^{-10}$ 90% CL guestimate
- significant advances possible on about all measurements and LFV limits
- Z peak offers by far best conditions for about all tau Physics measurements

Precision of $|V_{us}|$ from $\tau \to X_s \nu$ over time



Improvements

- *B*-factories measurements
- last improvement obtained with extra hard work on existing *B*-factories data
- further progress expected with Belle II data and hard work. . .