FRED meeting

FRED for photon transport

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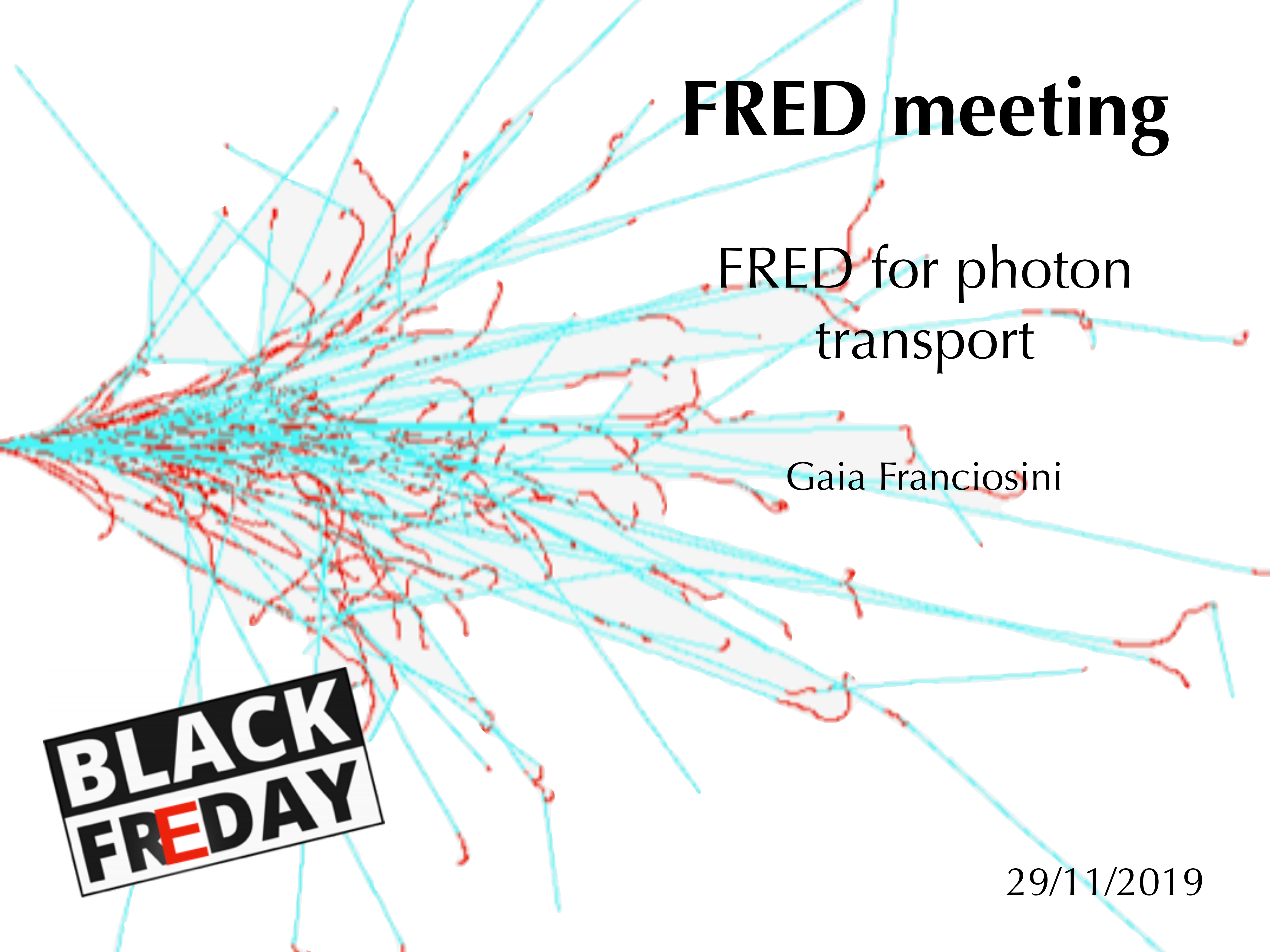


MC method for photon transport

Photons Interaction:

- **Photoelectric absorption**

  $$\gamma + A \rightarrow e^- + A^+$$

  Dominant at low energies (depending on Z, k < 100 keV for Si)

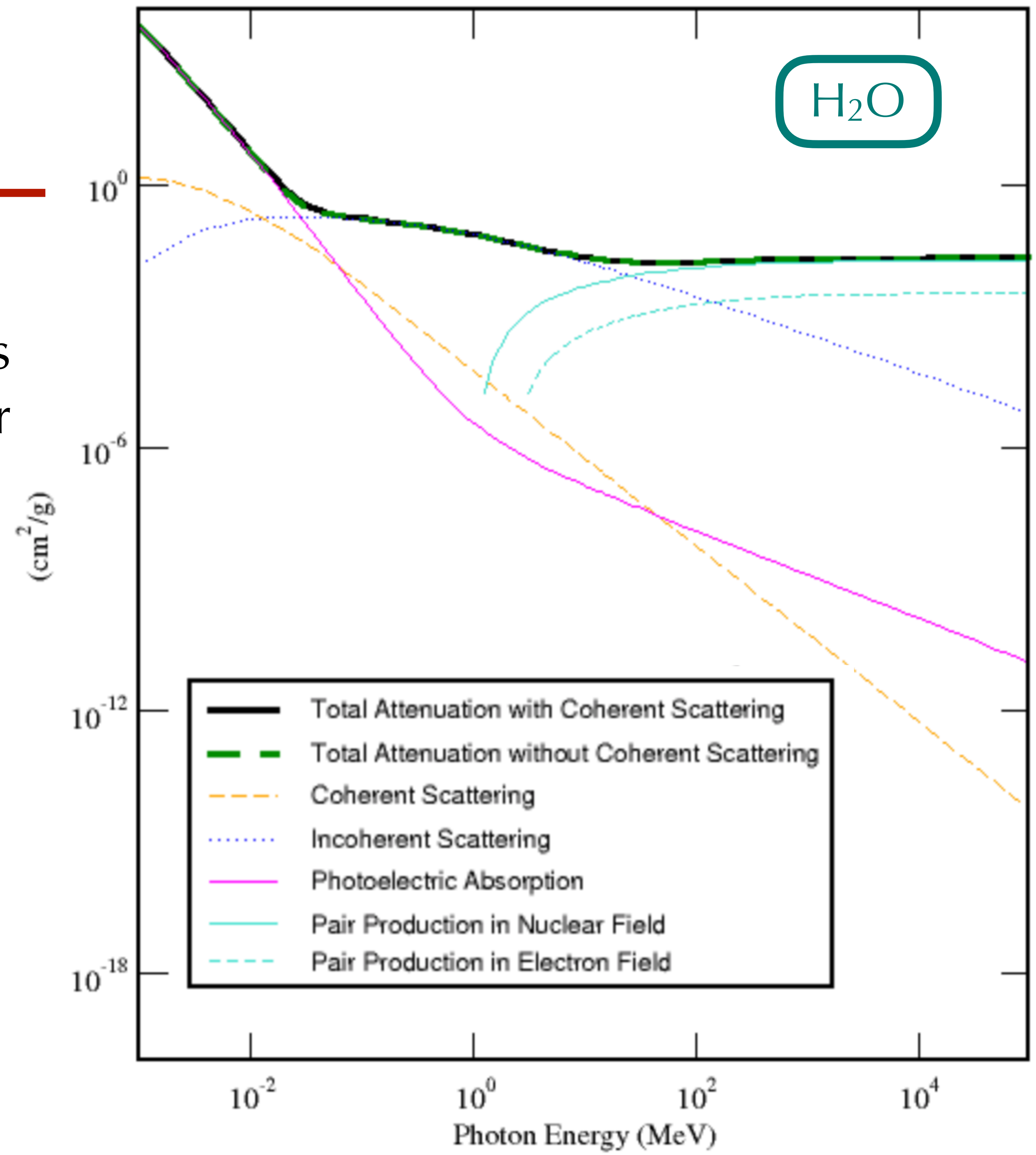
- **Compton Scatter**

  $$\gamma + e^- \rightarrow \gamma + e^-$$

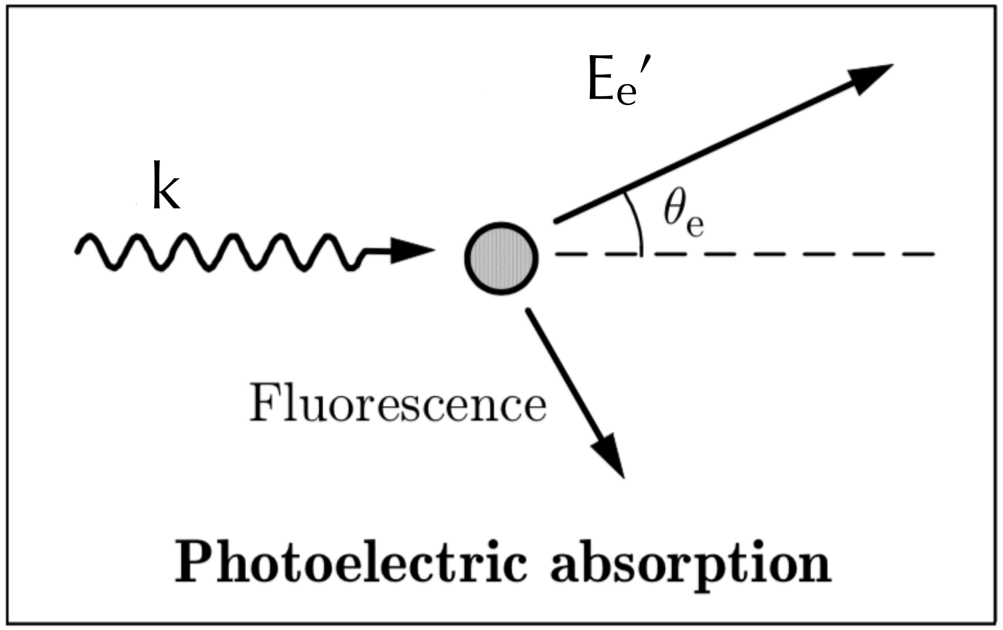
  Dominant at energy of few MeV

- **Pair Production**

$$e^+ + e^- \rightarrow \gamma$$



Photoelectric absorption



Key points:

- The photon energy must exceed the atomic binding energy:

$$k > E_B$$

- The ejected electron energy is: $E'_e = k - E_B$

- The emission of a **fluorescent** (characteristic) **X-ray** due to the filling of a K-orbital electron vacancy via the cascade of an L-orbital electron. The emitted photon has a definite energy given by the difference between the two electron binding energies.

Photoelectric absorption in practical terms

```
pushParticle(stp,ELECTRON_ID,Ekin_electron,v_out);
```
← Electron generation

```
pushParticle(stp,PHOTON_ID,Ebinding[iZ][0],v_fluo);

setLocallyDepositedEnergy(stp,Ebinding[iZ][0]);
```
← Fluorescence photon generation or energy deposition depending on E

```
extinguishRay(stp);
```
← Photon absorption

Differential cross section

The electron angle is extracted according to the relativistic differential cross section:

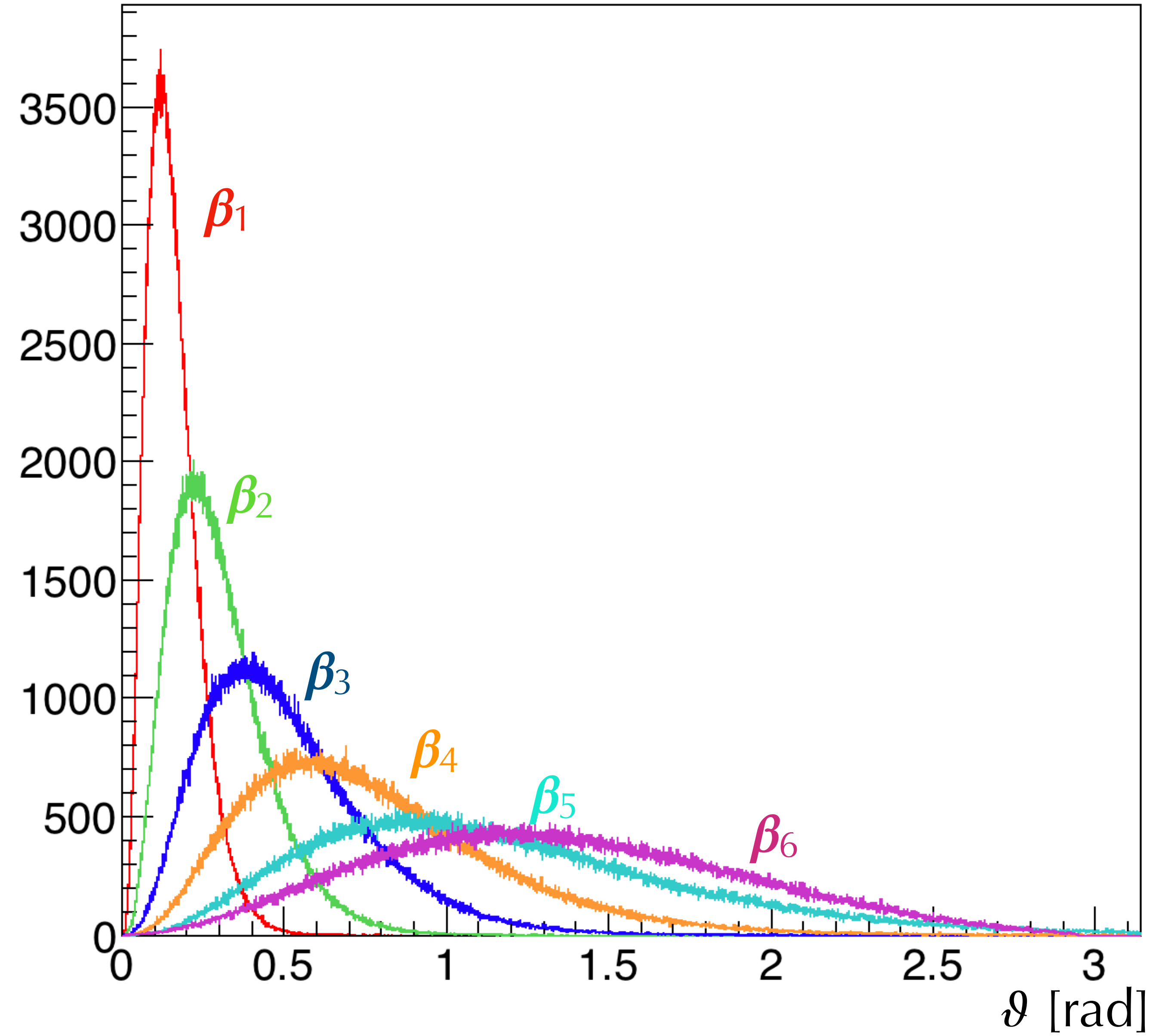
$$\frac{\mathrm{d}\sigma_{\mathrm{ph}}}{\mathrm{d}\Omega_{\mathrm{e}}} = \alpha^4 r_{\mathrm{e}}^2 \left(\frac{Z}{\kappa}\right)^5 \frac{\beta^3}{\gamma} \frac{\sin^2\theta_{\mathrm{e}}}{(1-\beta\cos\theta_{\mathrm{e}})^4} \left[1+\frac{1}{2}\gamma(\gamma-1)(\gamma-2)(1-\beta\cos\theta_{\mathrm{e}})\right]$$

For photon energies k << $m_e c^2$, the relativistic effects can be ignored and the differential cross section is reduced to the first term.

Differential cross section distribution for different $\boldsymbol{\beta}$ values:

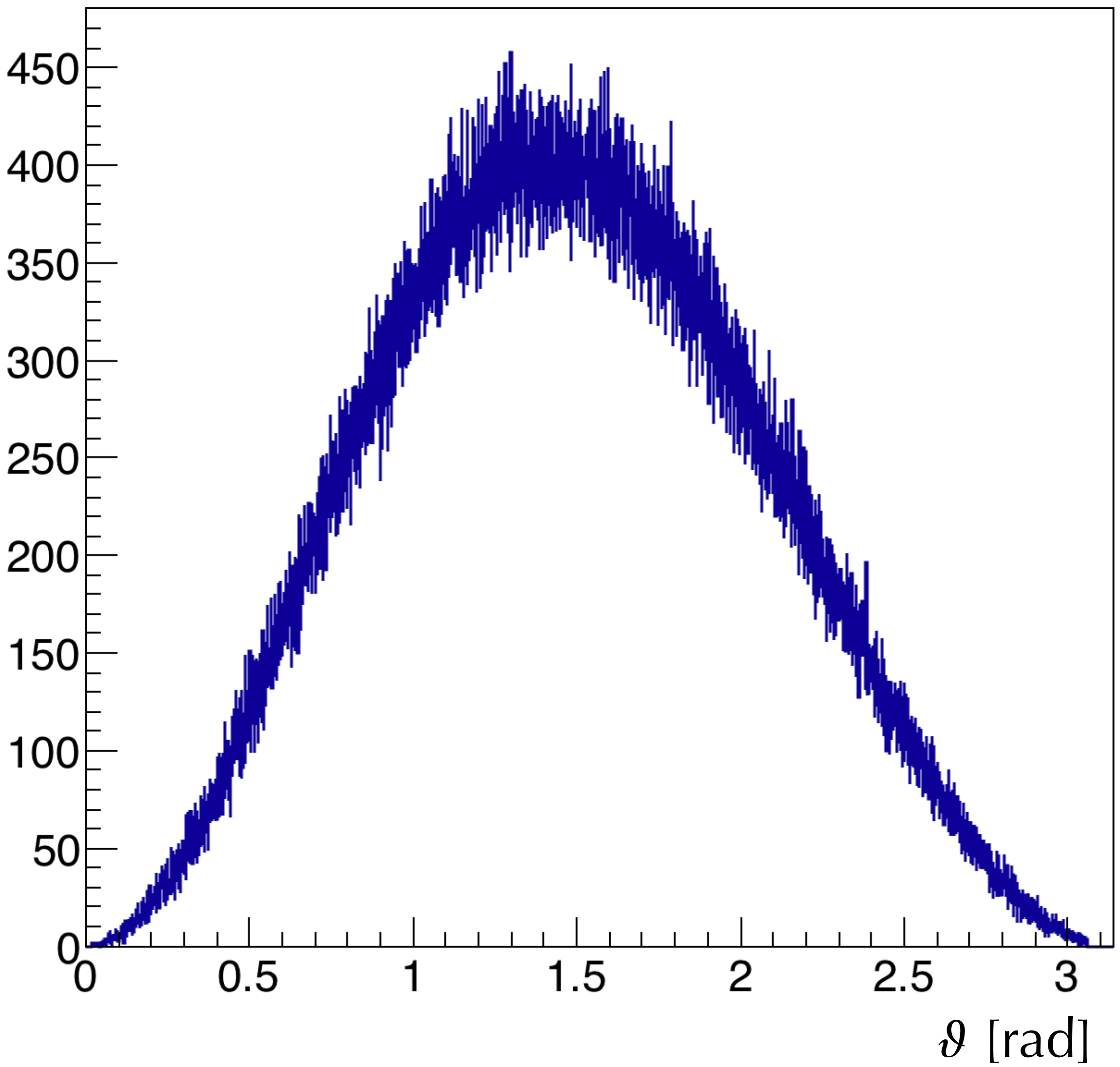
$$\frac{d\sigma}{d\Omega} \simeq \frac{sin^2\theta}{(1-\beta cos\theta)^4}[1-\frac{1}{2}\gamma(\gamma-1)(\gamma-2)(1-\beta cos\theta)]$$

$\beta_1 > \beta_2 > \beta_3 > \beta_4 > \beta_5 > \beta_6$

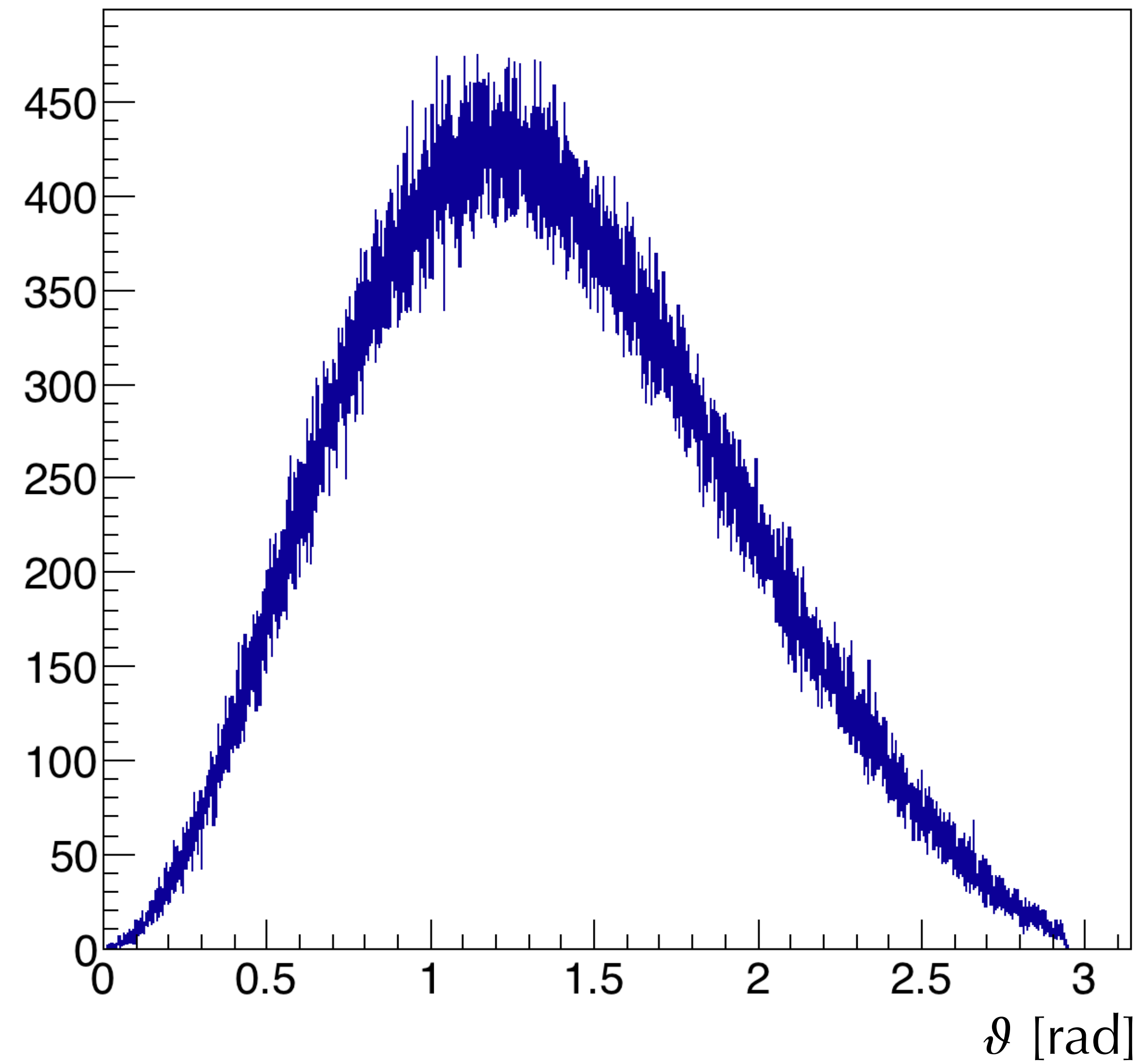


Electron angle extraction

k = 0.003 MeV $\beta = 0.10$



k = 0.010 MeV $\beta = 0.20$



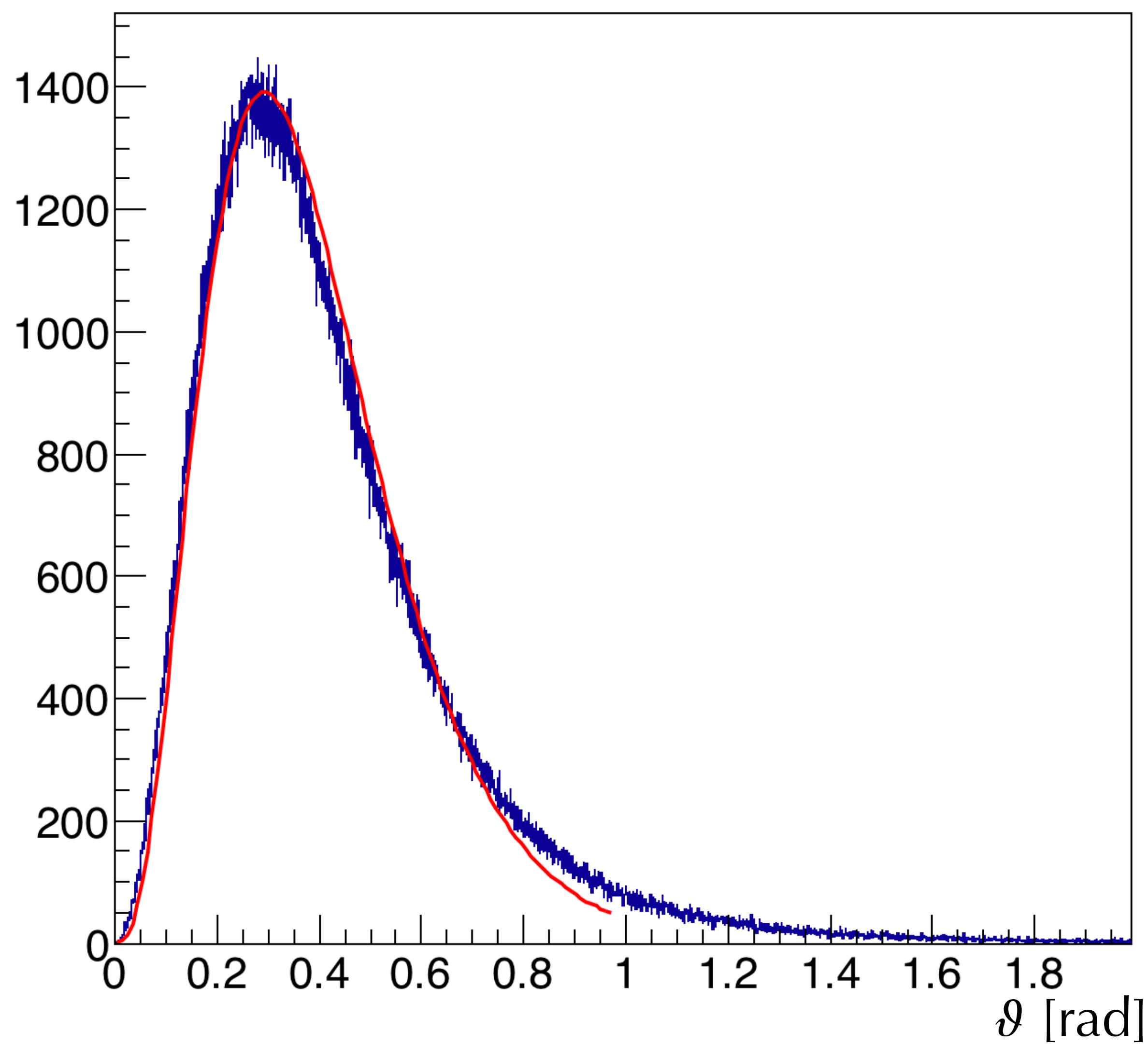
If $\beta < 0.25$ ($k \ll m_e c^2$) the differential cross section is maximized by $g(\vartheta)$:

$$\frac{d\sigma}{d\Omega} \simeq \frac{sin^2(\theta)}{(1-\beta_e cos(\theta))^4} \longrightarrow g(\theta) = \frac{sin(\theta+\beta_e)}{(1-\beta_e)}$$

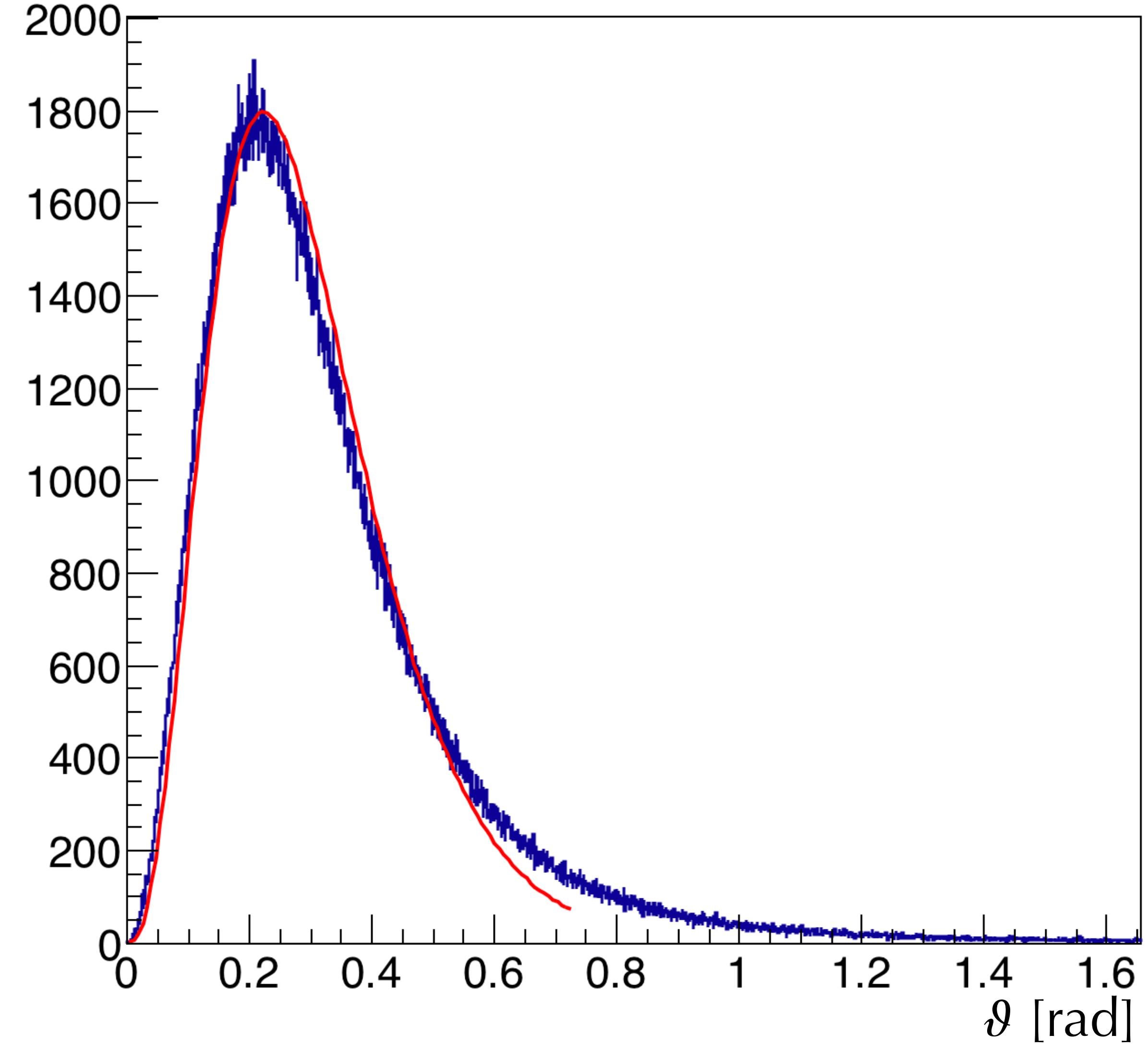
The electron angles are extracted using the HIT or MISS method.

Electron angle extraction

k = 0.600 MeV *β = 0.88*



k = 3 MeV *β = 0.99*

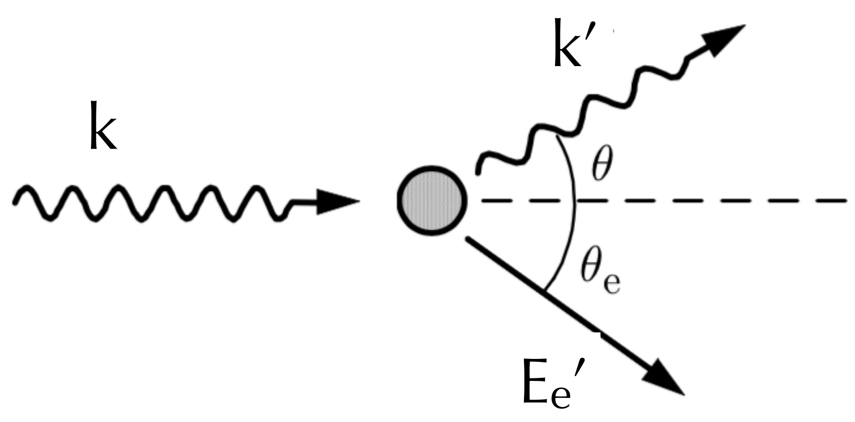


If $\beta > 0.25$ the relativistic differential cross section is well described by the Γ function (sampling from fit)

$$f(x; a, b) = a(ax)^{b-1} e^{-ax} / \Gamma(b)$$

The electron angles are extracted according to $\longrightarrow$ $x = -\ln(\xi_1 \cdot \xi_2 \cdot \ldots \cdot \xi_b)/a$

Compton scatter



Compton scattering

The photon is scattered through an angle ϑ with a reduced energy k' and the electron recoils at an angle ϑ_e with a total energy E_e'. From the conservations of momentum and energy respectively equal to:

$$\mathbf{k} = \mathbf{k}' + \mathbf{p}_e' \qquad k + m_e = k' + E_e'.$$

$$\rightarrow \quad k' = m_e \frac{k}{m_e + k(1 - \cos\theta_\gamma)}$$

$$\rightarrow \quad T_e' = k - k'$$

$$\rightarrow \quad \cot\theta_e = (1 + \alpha)\tan\frac{\theta_\gamma}{2}. \qquad \boldsymbol{\alpha} = k/m_e$$

If we know the ejected photon angle ϑ we are able to reconstruct the entire process!!

Compton scatter in practical terms…..

```
pushParticle(stp,PHOTON_ID,Ekin_photon,v_out);
```
← Photon generation

```
pushParticle(stp,ELECTRON_ID,Ekin_electron,v_out);
```
← Electron generation

```
extinguishRay(stp);
```
← Photon absorption

Klein-Nishina differential cross section

$$\frac{\mathrm{d}\sigma_{\mathrm{Co}}^{\mathrm{KN}}}{\mathrm{d}\Omega} = \frac{r_{\mathrm{e}}^2}{2}\left(\frac{E_{\mathrm{C}}}{E}\right)^2\left(\frac{E_{\mathrm{C}}}{E} + \frac{E}{E_{\mathrm{C}}} - \sin^2\theta\right)$$

Many Monte Carlo photon transport codes draw samples of the scattering cosine for Compton scatter from the KN differential cross section. For example FLUKA uses:

-Koblinger method above k = 1.4 MeV;

-Kahn algorithm below k= 1.4 MeV.

During these days I will try different methods to obtain the algorithm that maximizes the coding efficiency and the parsimony.

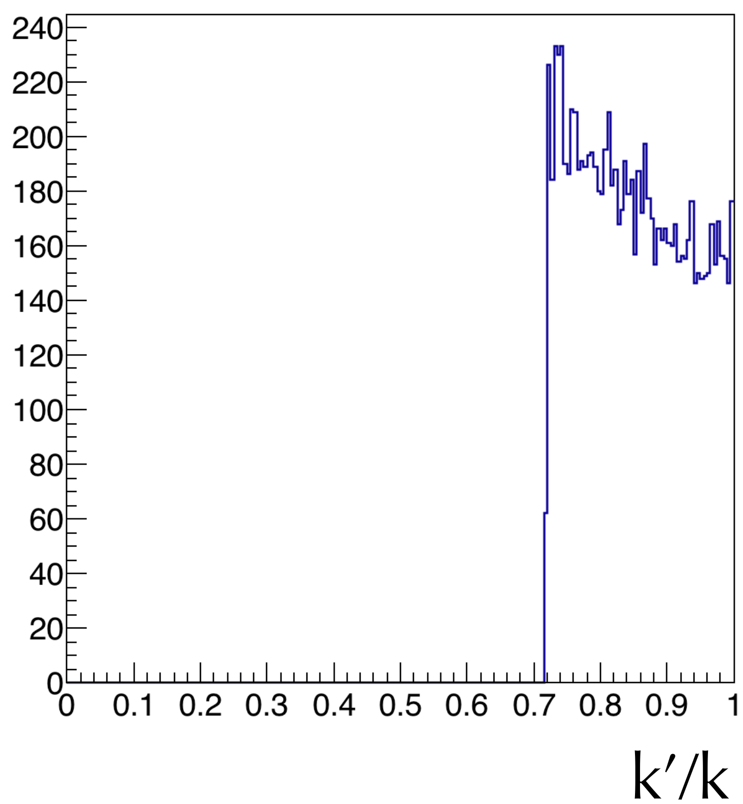
ϑ extraction: first method

PRELIMINARY

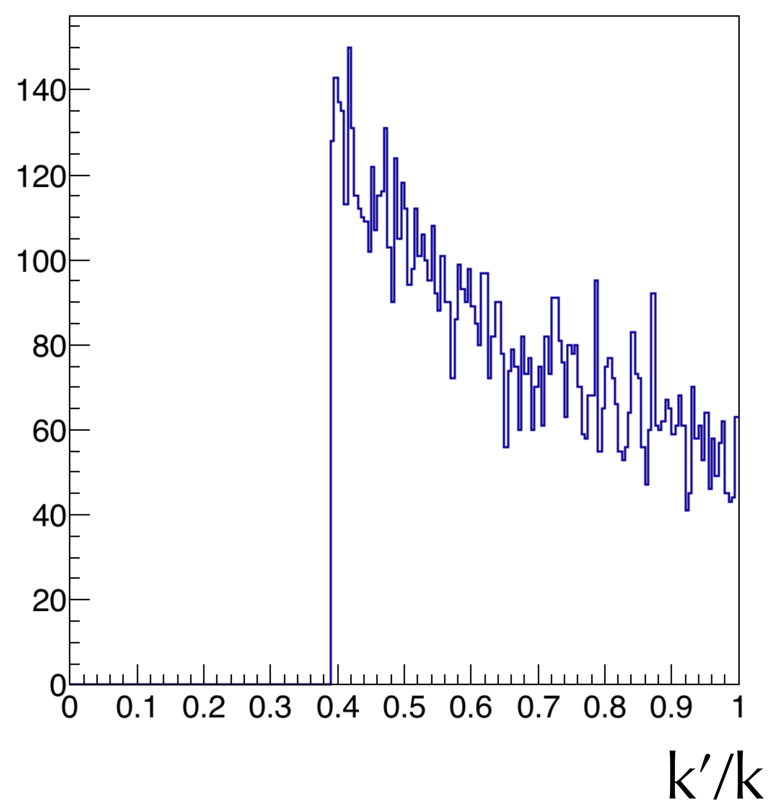
Sampling by rejection:

$$P_\theta(\cos\theta) = \left(\frac{E_C}{E}\right)^2 \left(\frac{E_C}{E} + \frac{E}{E_C} - \sin^2\theta\right) \boxed{S(E,\theta)}$$

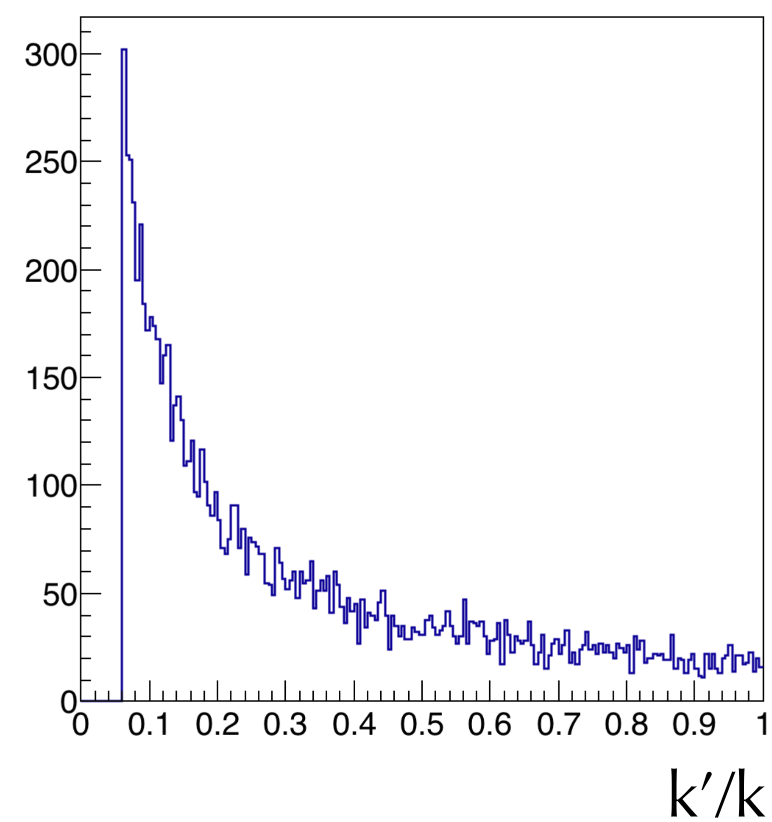
Set equal to 1 at the moment



E = 0.1 MeV



E = 0.5 MeV



E = 10 MeV