Istituto Nazionale di Fisica Nucleare
SEZIONE DI FIRENZE

Peccei-Quinn Phase Transition and Gravity Waves

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to appear with Delle Rose, Panico and Tesi

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Strong CP Problem:

$$\frac{\theta}{32\pi^2}\int d^4x\, G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$$

$$\theta + \mathrm{Arg}[\mathrm{Det}(y_u y_d)] < 10^{-10}$$

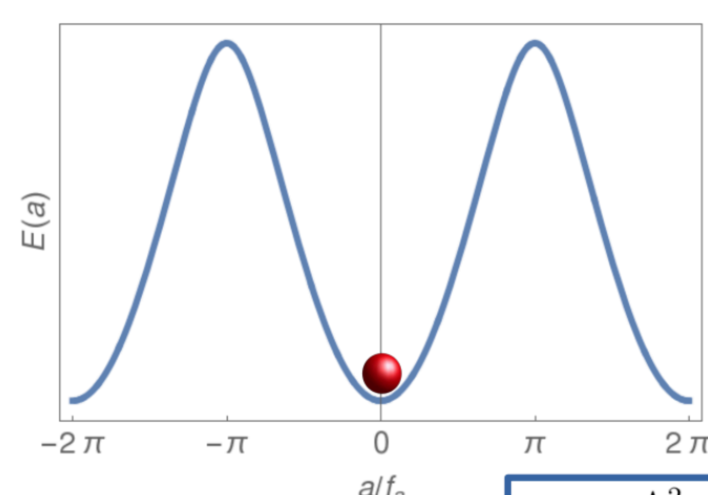
Axion solution:

$$\theta \to \frac{a(x)}{f}$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G}$$

[Peccei-Quinn ’77
Weinberg-Wilczek ’78]

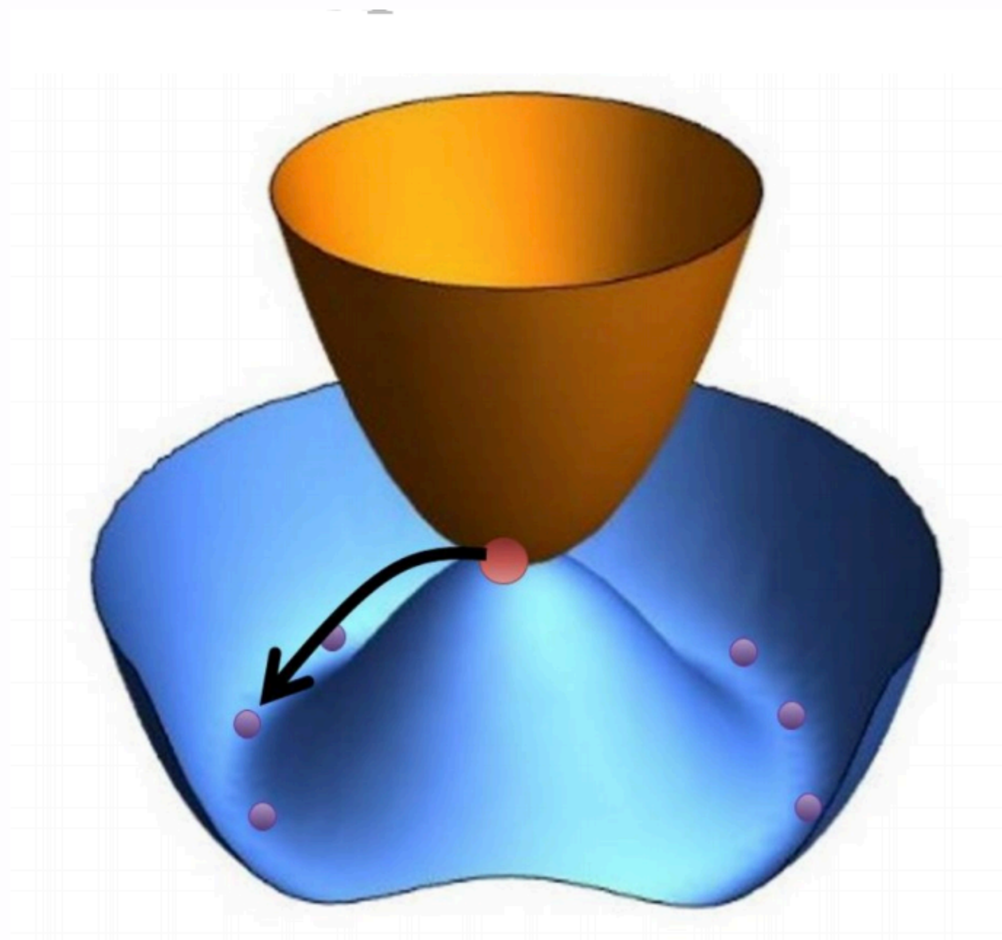
QCD dynamics aligns the vacuum to preserve CP:



Vafa Witten '84

$$m_a \approx \frac{\Lambda^2_{\mathrm{QCD}}}{f_a}$$

The axion is the Goldstone boson of a spontaneously broken U(1) global symmetry.
At T> f symmetry is restored:

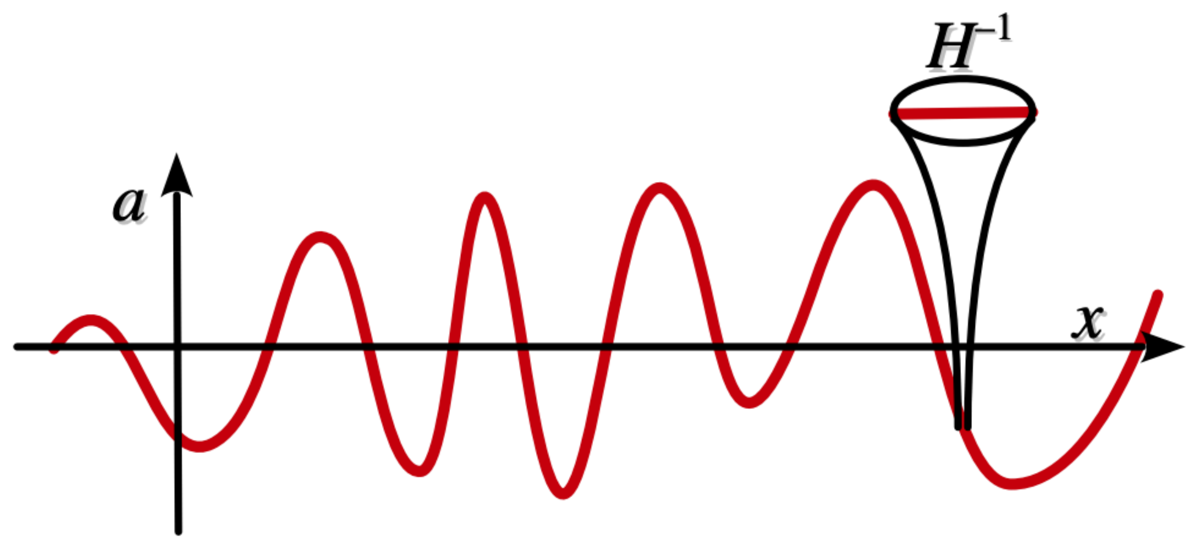


$$X = \frac{\rho}{\sqrt{2}} e^{i\frac{a}{f_a}}$$

As T decreases X relaxes to the bottom of the potential with a random phase.

Two scenarios:

PQ broken during inflation: $f_a > \mathrm{Max}[H_I, T_R]$

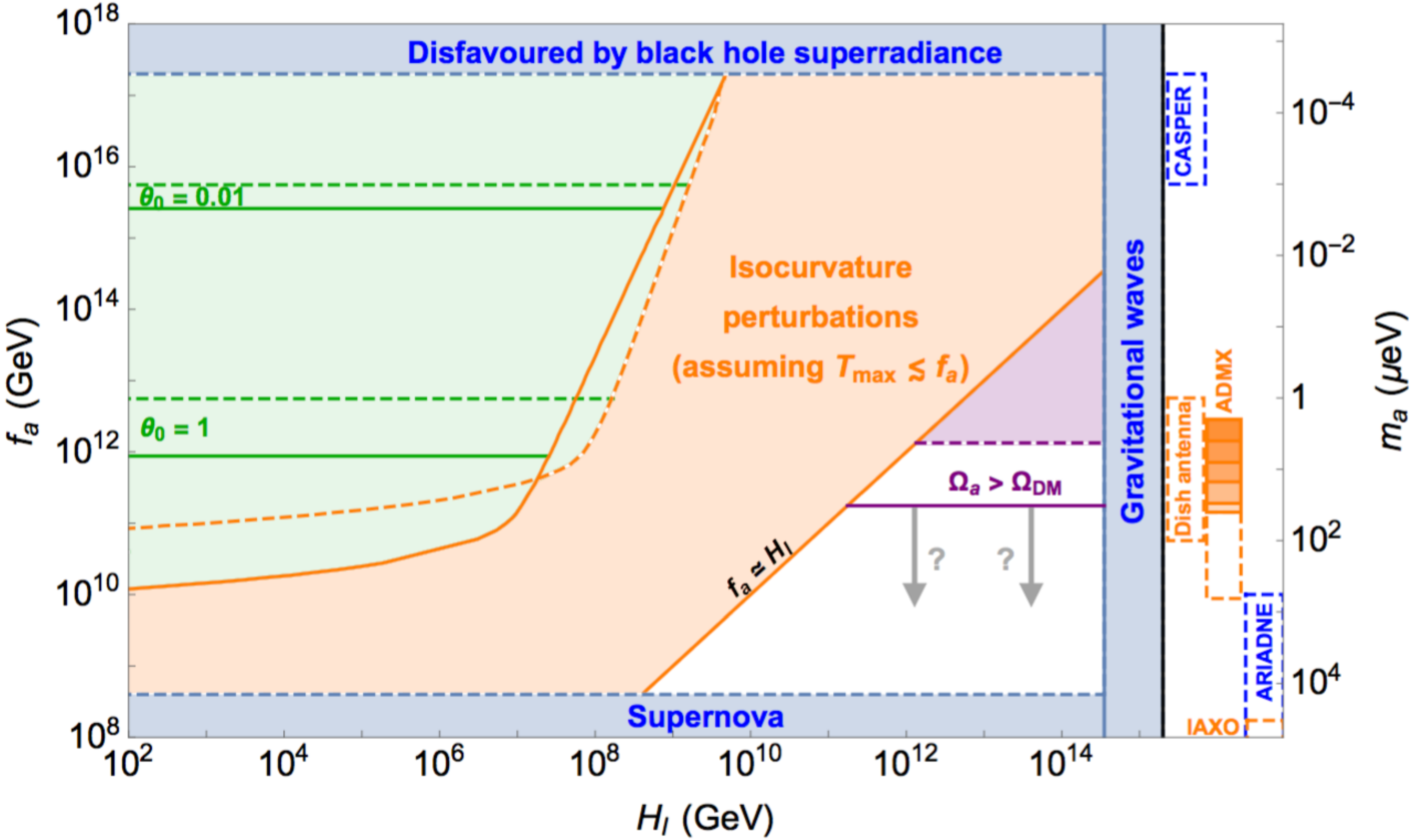


Initial misalignment is constant over visible universe.

PQ broken after inflation: $f_a < \mathrm{Max}[H_I, T_R]$

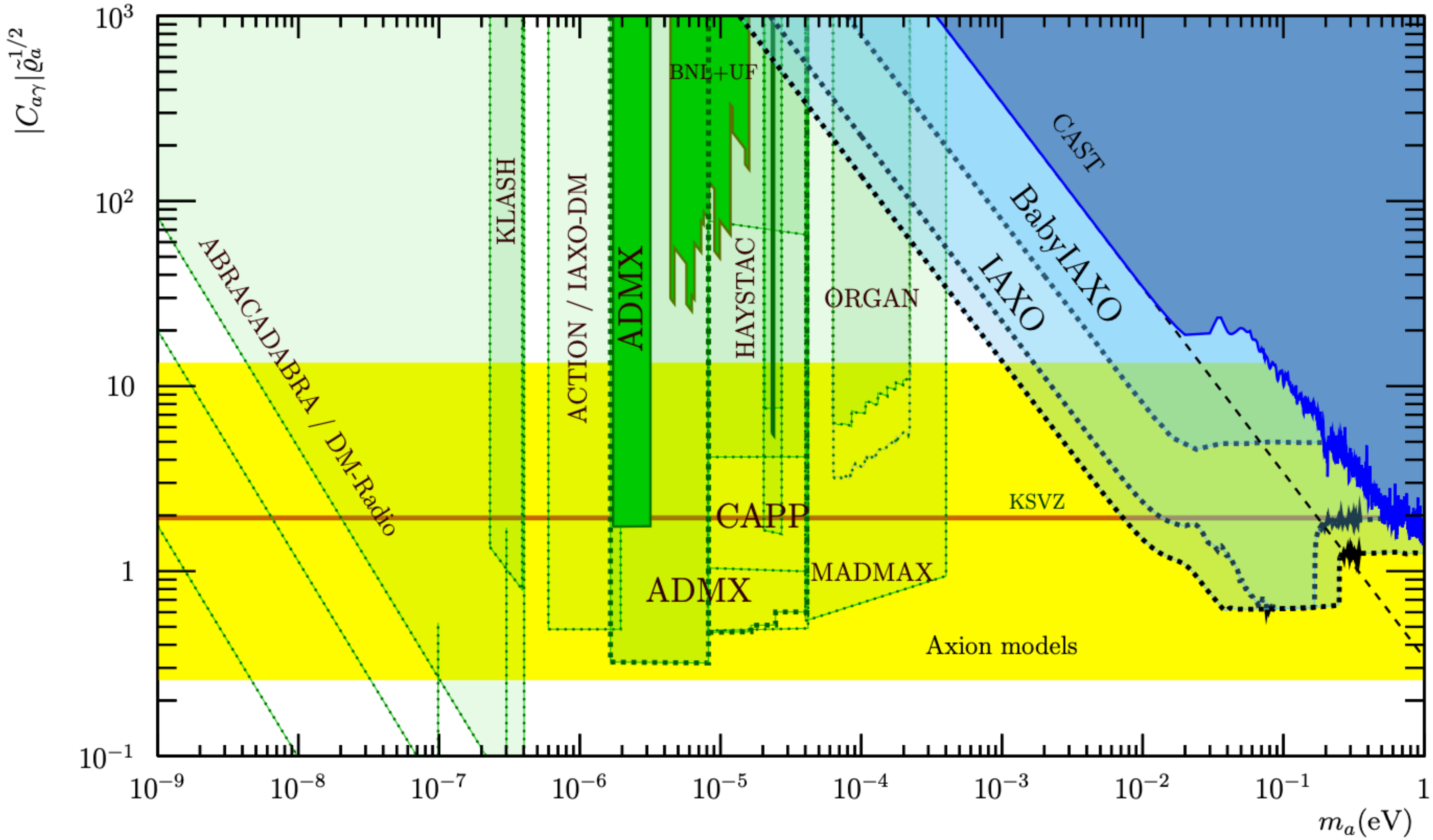
Initial misalignment scans over all values. Possible in principle to compute axion abundance.

[Gorghetto-Villadoro '18]



[Grilli Di Cortona et. al.'15]

Experiments:

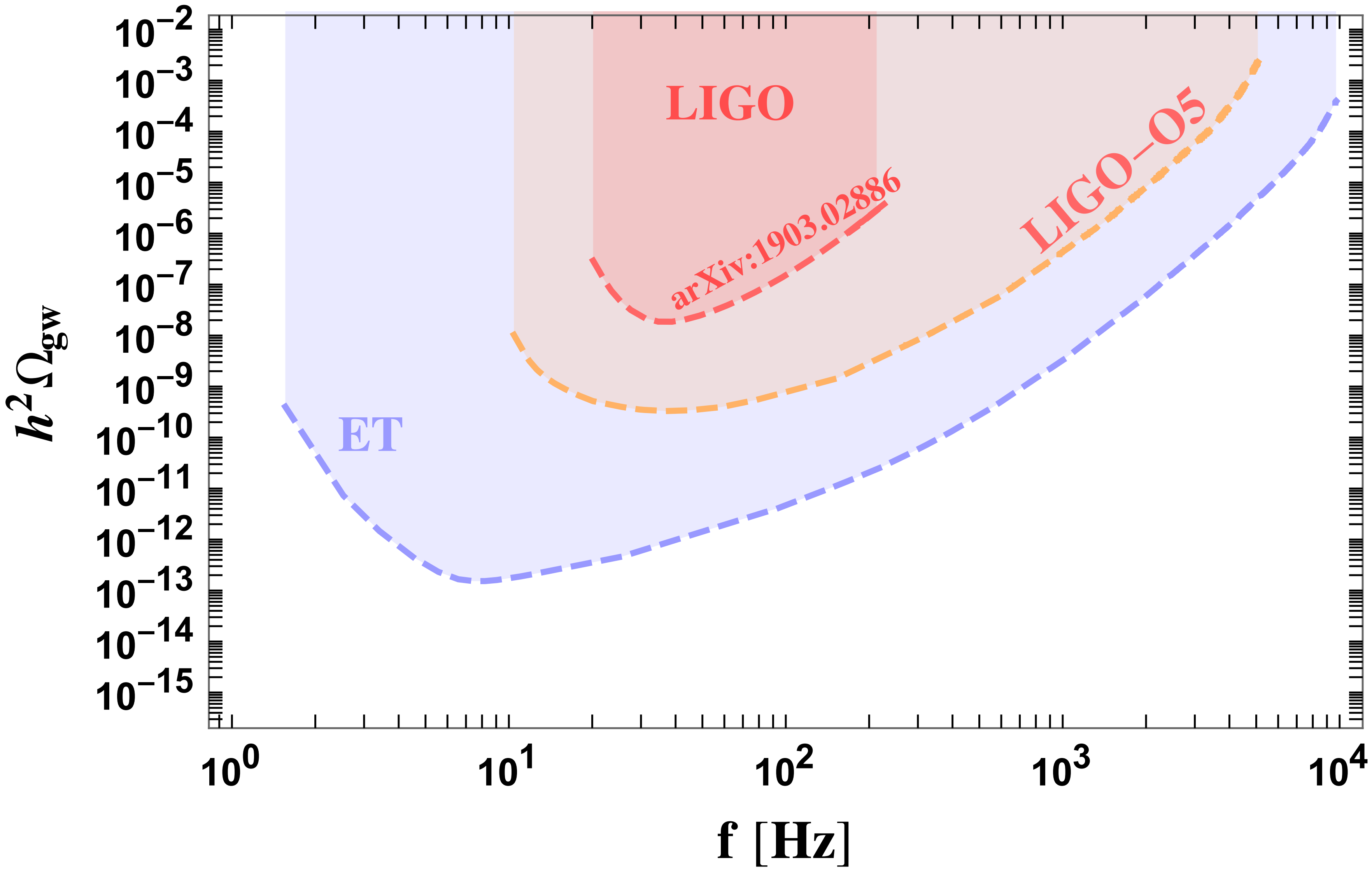


[Irastorza, Redondo '18]

If axion is DM will be likely found in the next decade!

All axion experiments are insensitive to UV completion.

1st order PQ phase transition produces gravity waves:



[see Azatov '19]

Supercooling:

$$h^2\Omega_{\rm gw}\big|_{\rm peak} \simeq 1.27\times 10^{-10}\left(\frac{100}{\beta/H}\right)^2 \qquad f_{\rm peak}\simeq 3.83\times 10^5\,{\rm Hz}\left(\frac{\beta/H}{100}\right)\left(\frac{T_*}{10^{11}{\rm GeV}}\right)$$

Is the PQ phase transition 1st or 2nd order?

* The question is only tangible if f< Hi. We assume: $f_a = 10^{11}\mathrm{GeV}$

- KSVZ axion:

$$U(1)_{\mathrm{PQ}}: \qquad X \to e^{i\alpha} X$$

$$\lambda_X (|X|^2 - f^2/2)^2 + (yXQQ^c + h.c.)$$

PQ phase transition is second order:

1. There are no massless bosonic states coupled to X where PQ is restored
2. Fermion contribution to 1-loop Coleman-Weinberg has wrong sign
3. Potential is always well approximated by $m^2(T)|X|^2 + \lambda(T)|X|^4$

PQ breaking needs to be non-minimal to realise a 1st order phase transition what about the Higgs portal?:

$$-\mu^2|H|^2+\lambda|H|^4+\lambda_{XH}|X|^2|H|^2+\lambda_X(|X|^2-f^2/2)^2$$

[Dev, Ferrer, Zhang,Zhang '19]

In the regime where $\lambda_{XH} \gg \lambda_X$ we can deviate from the previous case

However:

1. tuning of the electroweak VEV: $\mu_{eff}^2=\mu^2-\frac{\lambda_{XH}}{2}f_a^2=\mathcal{O}(100\,\mathrm{GeV})^2$
2. matching to the Higgs mass: $\lambda_{eff}=\lambda-\frac{\lambda_{XH}^2}{4\lambda_X}=.12$

Needs too large couplings!

expanding the potential in the limits $h, v \ll f,\ \lambda_{XH} \gg \lambda_X$

$$\frac{1}{2}\frac{\lambda_{XH}T^2}{6}s^2 + \frac{\lambda_X}{4}s^4 + \frac{\lambda_{XH}^2 s^4}{64\pi^2}\log\left(\frac{\lambda_{XH}}{2\bar{\mu}^2}|s^2|\right)$$

$s^2 \equiv |X|^2 - f^2$

the needed deviation from quadratic + quartic happens for

$$\lambda_{XH}^2 \sim 16\pi^2\lambda_X \longrightarrow \lambda \gtrsim 16\pi^2 \quad ☹$$

Necessary ingredients:

1. In order to generate a 1st order PT the Higgs should not be the Higgs.

2. Enhanced gravity waves are produced in supercooling regime. This requires the thermal barrier to last long enough.

$$\frac{S_3}{T} \approx \text{constant} \longrightarrow \text{scale invariance}$$

We will thus look for approximately conformal scalar theories.

Weak coupling

Radiative symmetry breaking:

$$\mathcal{L} = |\partial_\mu - ieA_\mu|^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \longrightarrow \frac{M_\phi^2}{M_A^2} = \frac{3\alpha}{2\pi}$$

[Coleman, Weinberg '73]

Radiative breaking is a common property of massless scalar theories.

[Gildener, Weinberg '76]

$$V = \frac{\lambda_{ijkl}}{4}\phi_i\phi_j\phi_k\phi_l + \text{gauge} \qquad \lambda_{ijkl} = \mathcal{O}(e^2)$$

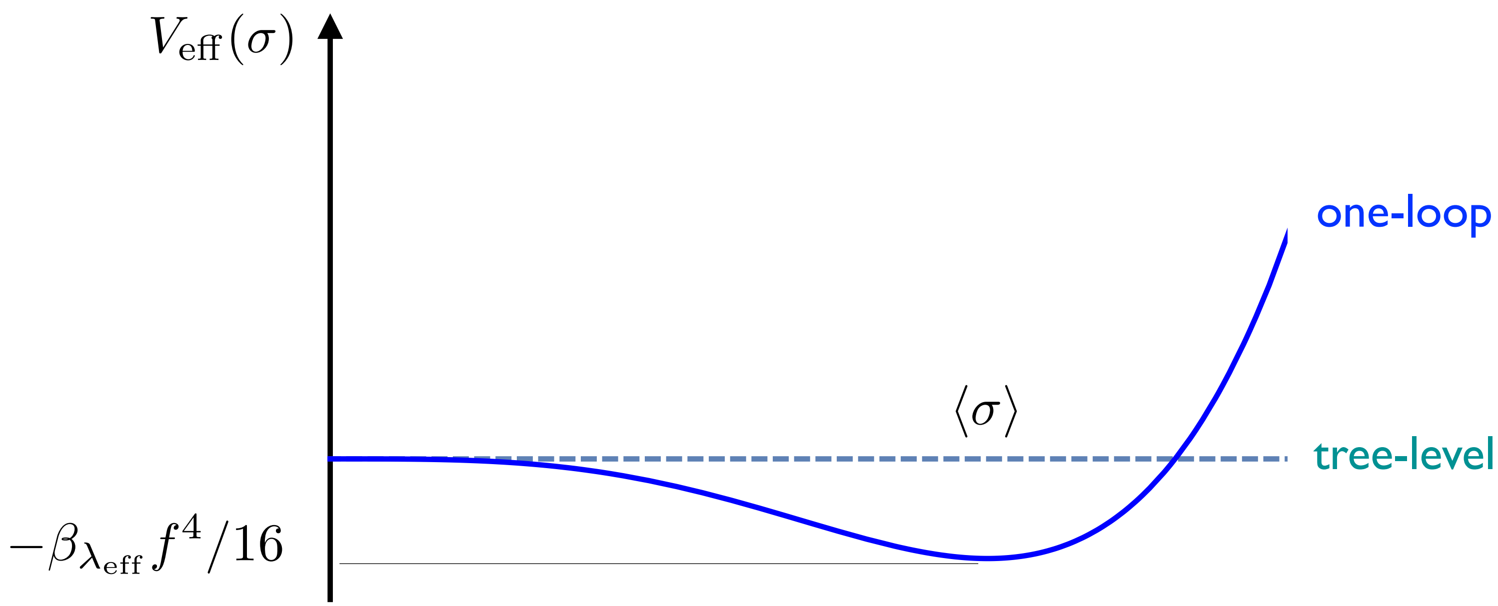
$$\lambda_{\text{eff}}(\mu) = \lambda_{ijkl}(\mu)n_in_jn_kn_l\,, \quad \lambda_{\text{eff}}(\Lambda) = 0, \quad \phi_i = n_i\sigma$$

Due to RG evolution we can choose the potential at some scale to vanish on a ray in field space.

Effective potential:

At 1-loop the flat direction is lifted

$$V_{\rm eff}(\sigma) \approx \frac{\beta_{\lambda_{\rm eff}}}{4}\sigma^4\left(\log\frac{\sigma}{f} - \frac{1}{4}\right) \qquad \langle\sigma\rangle \equiv f \approx \Lambda$$



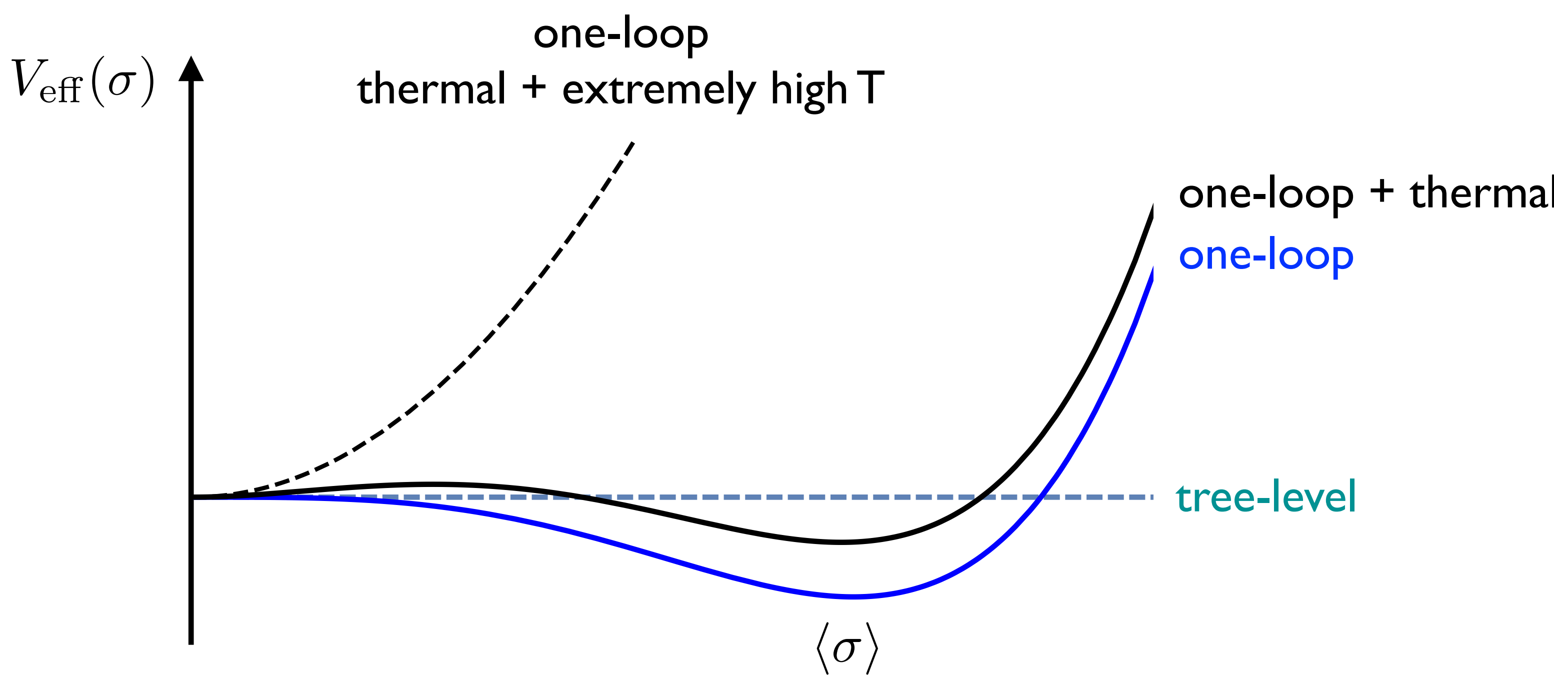
Finite temperature:

$$F(\sigma;T) = \frac{T^4}{2\pi^2} \sum_i^{N_B} J_B\left(\frac{m_i^2(\sigma)}{T^2}\right) + V_{\rm eff}(\sigma)$$

Around the origin the fields are massless so thermal corrections are always important:

$$F \approx -N\frac{\pi^2}{90}T^4 + \frac{\hat{g}^2}{24}\sigma^2 T^2 + \sum_i \frac{m_i^4}{64\pi^2}\log\frac{T^2}{m_i^2} \qquad\qquad m_i \sim \frac{\hat{g}}{\sqrt{N}}\sigma$$

A barrier is generated where the universe is trapped. The phase transition is 1st order and very slow because breaking of classical conformal invariance is small.



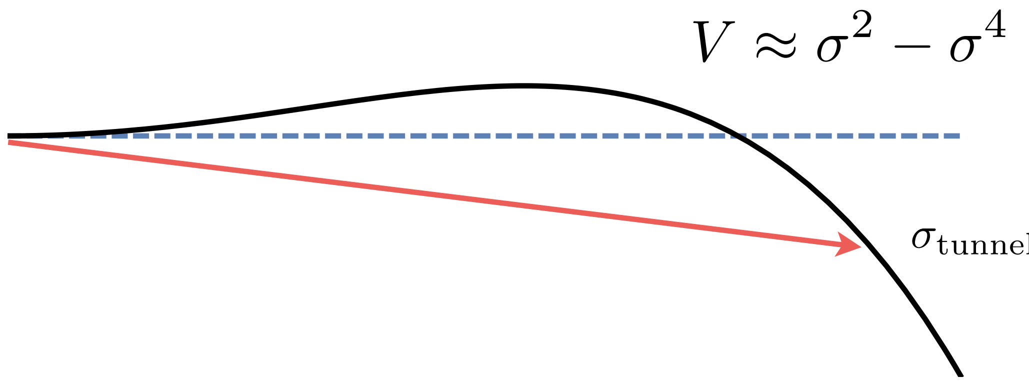
The tunnelling rate is:

$$\Gamma(T) \approx T^4 \left(\frac{S_3/T}{2\pi}\right)^{\frac{3}{2}} \exp(-S_3/T)$$

Nucleation temperature:

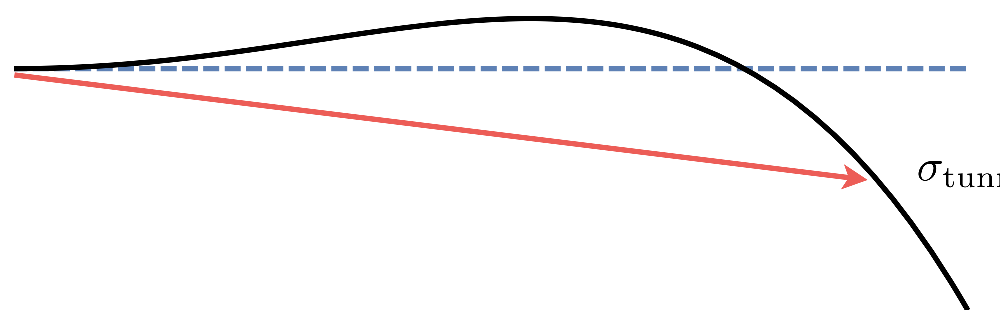
$$\Gamma(T_n) = H(T_n)^4 \qquad 3M_p^2 H^2 = \frac{\beta_{\lambda_{eff}}}{16} f^4 + g_* T^4$$

Approximation:



$$\log \frac{\sigma}{f} = \log \frac{\hat{g}\sigma}{T} + \log \frac{T}{\hat{g} f}$$

Approximation:



$$\log \frac{\sigma}{f} = \cancel{\log \frac{\hat{g}\sigma}{T}} + \log \frac{T}{\hat{g} f}$$

[Witten '80]

In the tunnelling region

$$F \approx \frac{\hat{g}^2}{24}\sigma^2 T^2 - \frac{\beta_{\lambda_{\text{eff}}}}{4}\sigma^4 \log \frac{M}{T}$$

Just a quartic potential!!

Bounce action can be computed analytically:

$$\frac{S_3}{T} \approx \frac{A_3}{\log(\frac{M}{T})} \qquad A_3 = 18.897 \frac{m(T)}{\beta_{\lambda_{eff}} T}$$

[Brezin, Parisi]

Supercooling:

$$T_n \approx \sqrt{M H_I} \exp\left(\frac{1}{2}\sqrt{-A_3(\hat{g}) + \log^2(M/H_I)}\right)$$

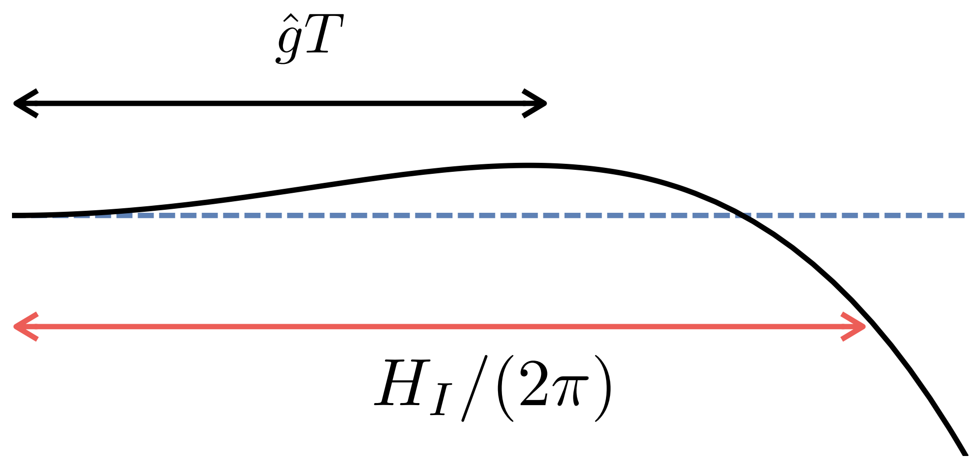
$$T_n \gtrsim \sqrt{M H_I} \sim 0.1 f \left(\frac{f}{M_{\rm Pl}}\right)^{\frac{1}{2}}$$

PT duration:

$$\frac{\beta}{H} = -4 + T \frac{\partial(S_3/T)}{\partial T}\bigg|_{T_n} = -4 + \frac{1}{\log(M/T_n)} \frac{S_3}{T}\bigg|_{T_n}$$

For strong supercooling gravity waves enhanced!

If thermal transition too slow symmetry will be broken by de Sitter fluctuations:



$$N_{\rm max} \approx \log \frac{T_c}{H_I} \approx 15$$

Reheating:

$$T_{\rm RH} = T_I \min\left(1, \frac{\Gamma}{H_I}\right)^{1/2} \qquad T_I = \left(\frac{\beta_{\lambda_{\rm eff}}}{16\pi^2}\frac{30}{N_L}\right)^{1/4} f$$

TR is the temperature that enters the GW signal!

Example:

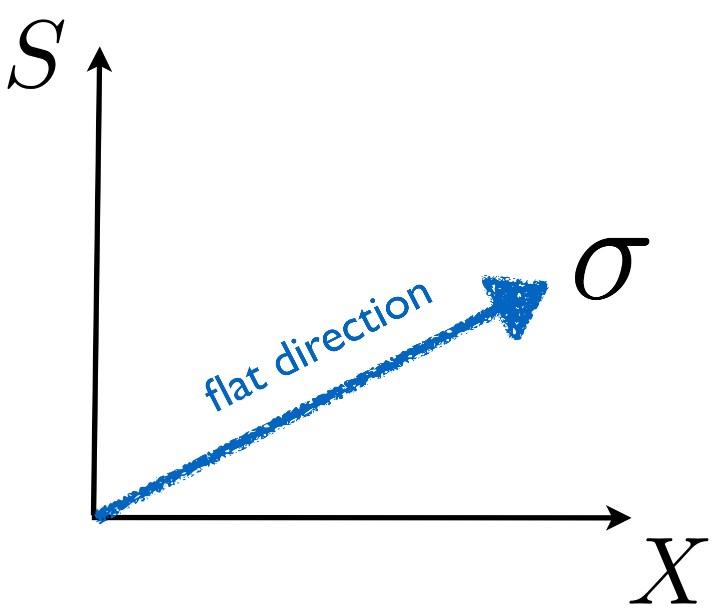
$$\mathcal{L} = -\frac{F^2}{4g^2} + |D_\mu S|^2 + |\partial_\mu X|^2 - V + (yXQQ^c + h.c)$$

$$V = \lambda_S |S|^4 + \lambda_X |X|^4 + \lambda_{XS} |S|^2 |X|^2$$

Flat direction:

[see Hambye, Strumia, Teresi '18]

$$\lambda_{XS} = -2\sqrt{\lambda_S \lambda_X}$$



$$(S\,, X) = (\sin\alpha\,, \cos\alpha)\frac{\sigma}{\sqrt{2}}\,, \quad \sin^2\alpha = \frac{\sqrt{\lambda_X}}{\sqrt{\lambda_X} + \sqrt{\lambda_S}}$$

Masses:

$$M_\tau = (4\lambda_X \lambda_S)^{1/4}\sigma\,, \qquad M_A = g\sin\alpha\,\sigma\,, \qquad M_Q = y\cos\alpha\,\frac{\sigma}{\sqrt{2}}$$

Effective potential:

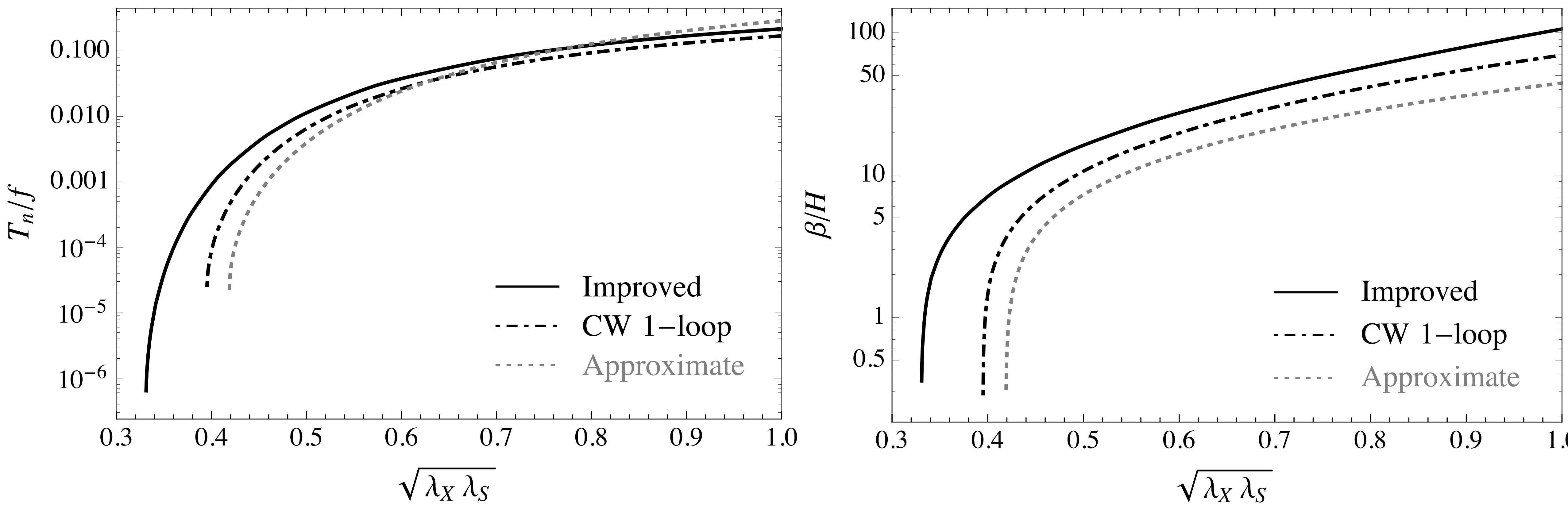
$$V_{\rm eff} = \frac{2\lambda_S\lambda_X + \frac{3}{2}g^4\sin^4\alpha - \frac{3}{2}y^4\cos^4\alpha}{16\pi^2}\,\sigma^4\left(\log\frac{\sigma}{f} - \frac{1}{4}\right)$$

Bounce action:

$$\frac{S_3}{T} \approx \frac{1}{\log(M_\tau/T)}\,{\rm Max}\left[\frac{150}{(\lambda_X\lambda_S)^{3/4}}\,,\,\frac{250}{g^3\sin^3\alpha}\right]$$

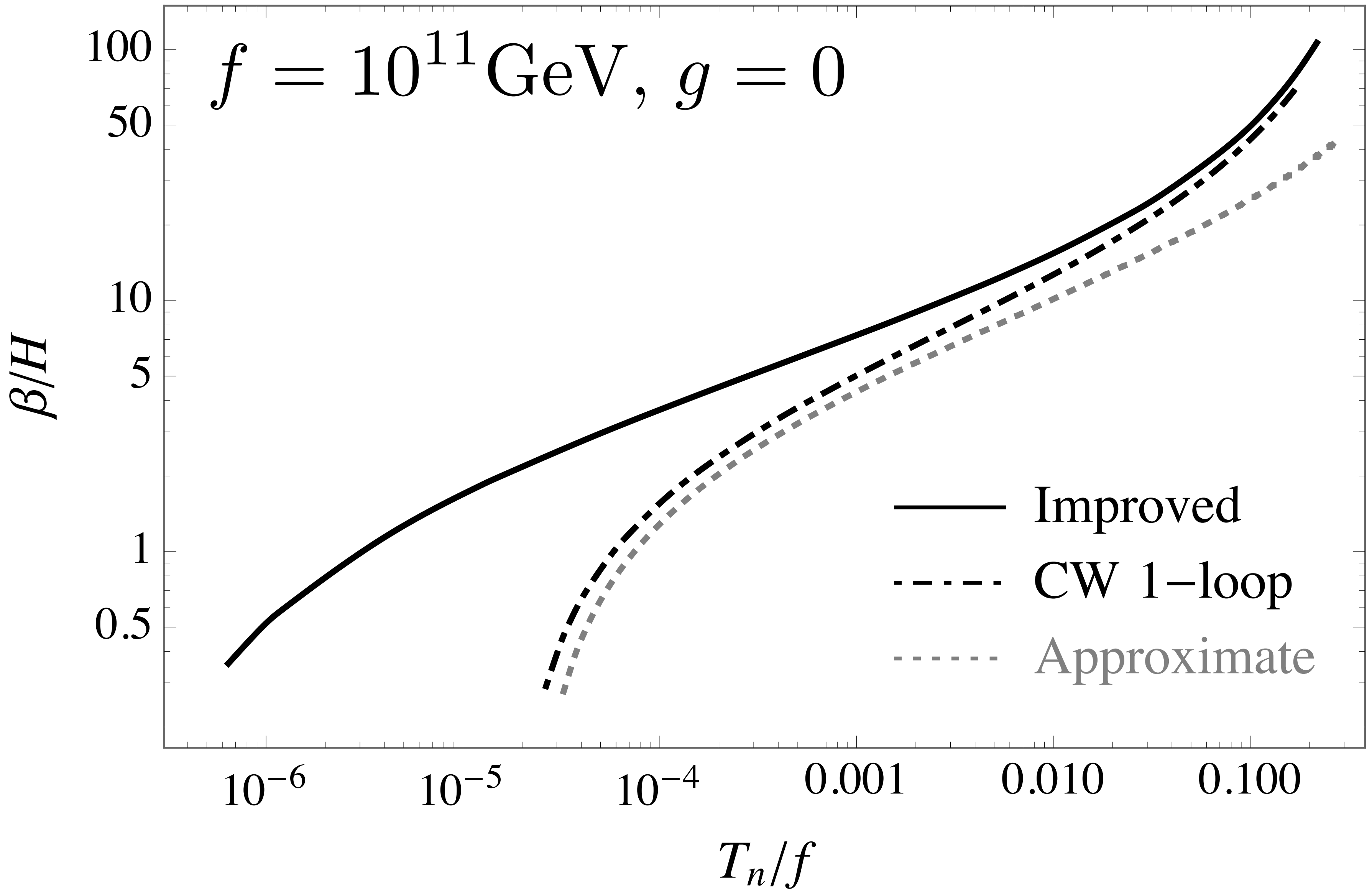
results: pure quartics

$f = 10^{11}\mathrm{GeV},\ g = 0$



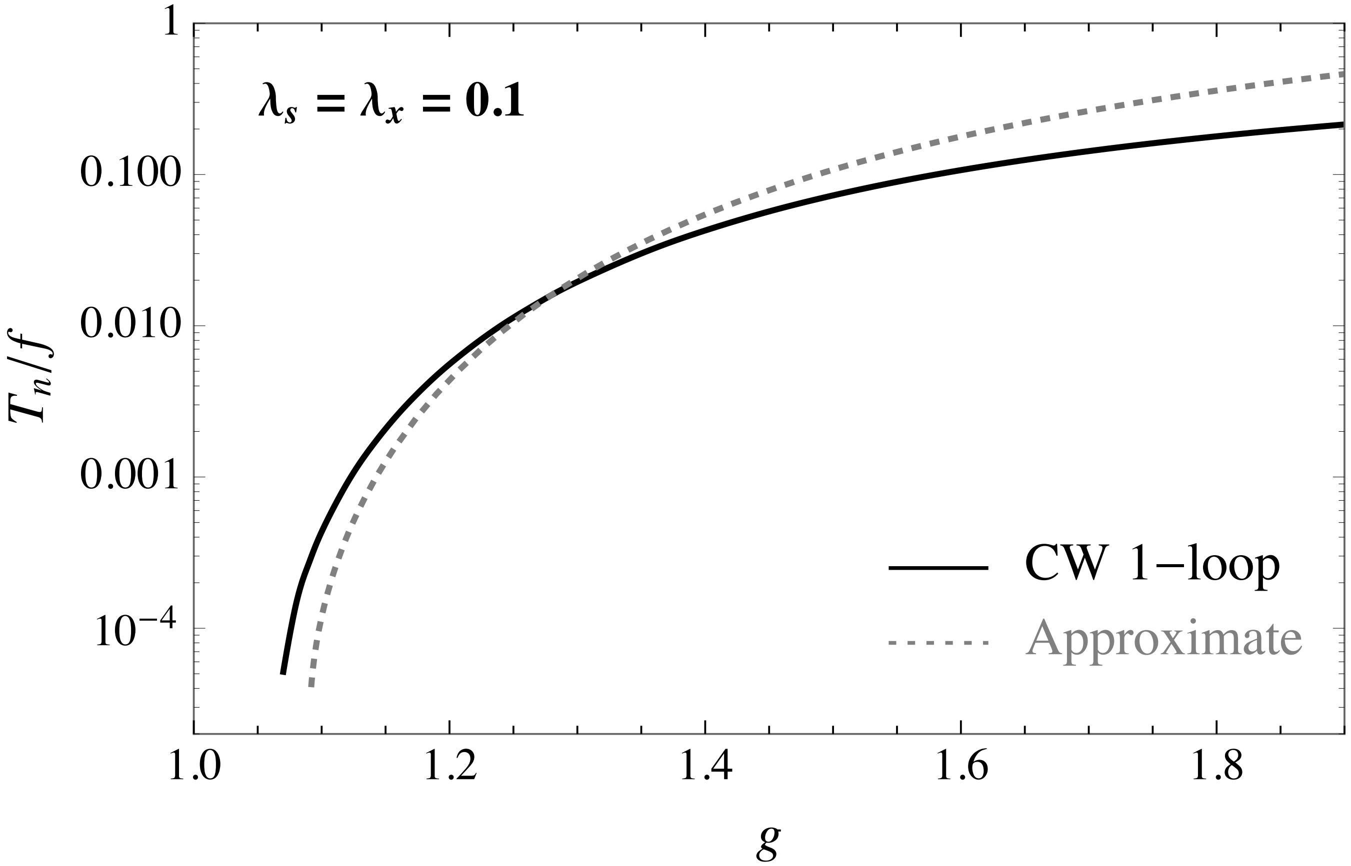
approximate analytic solution works remarkably well!

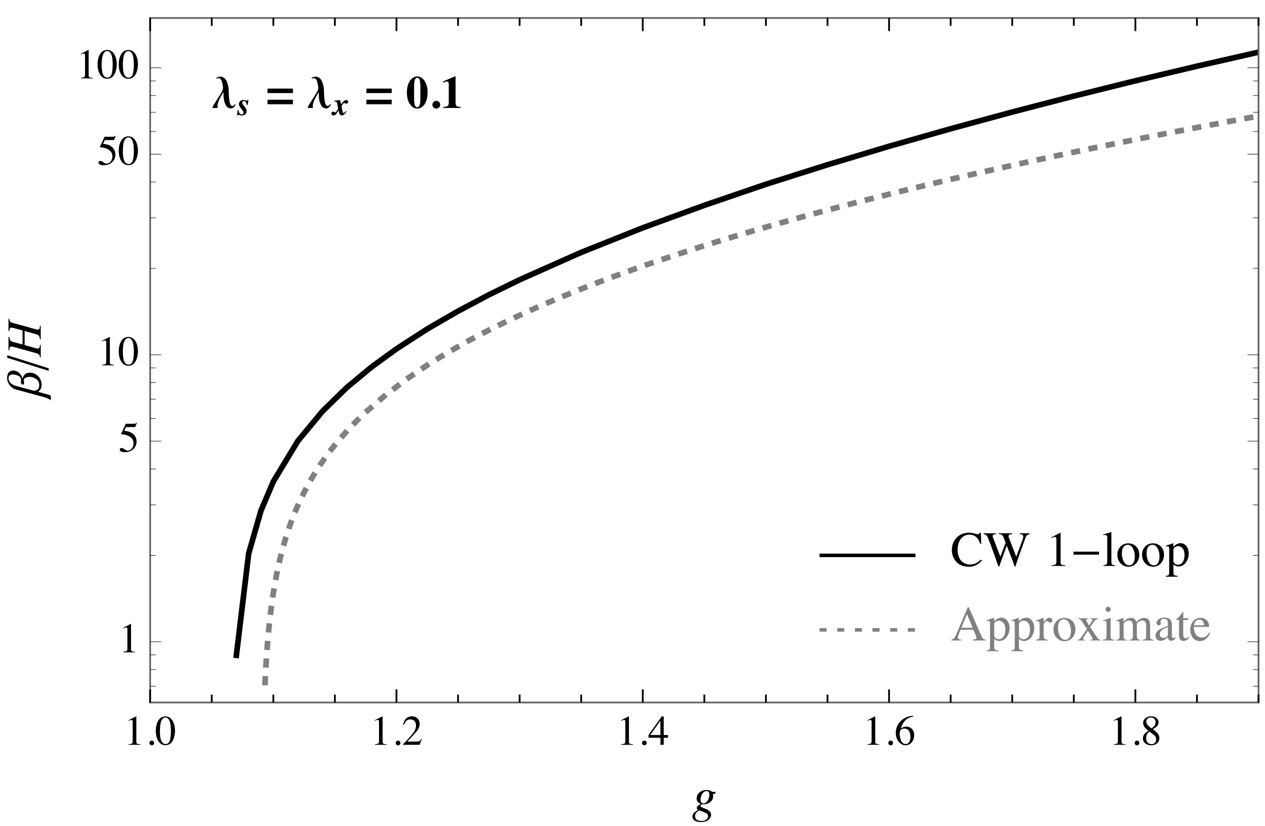
results: pure quartics



Supercooling enhances β/H

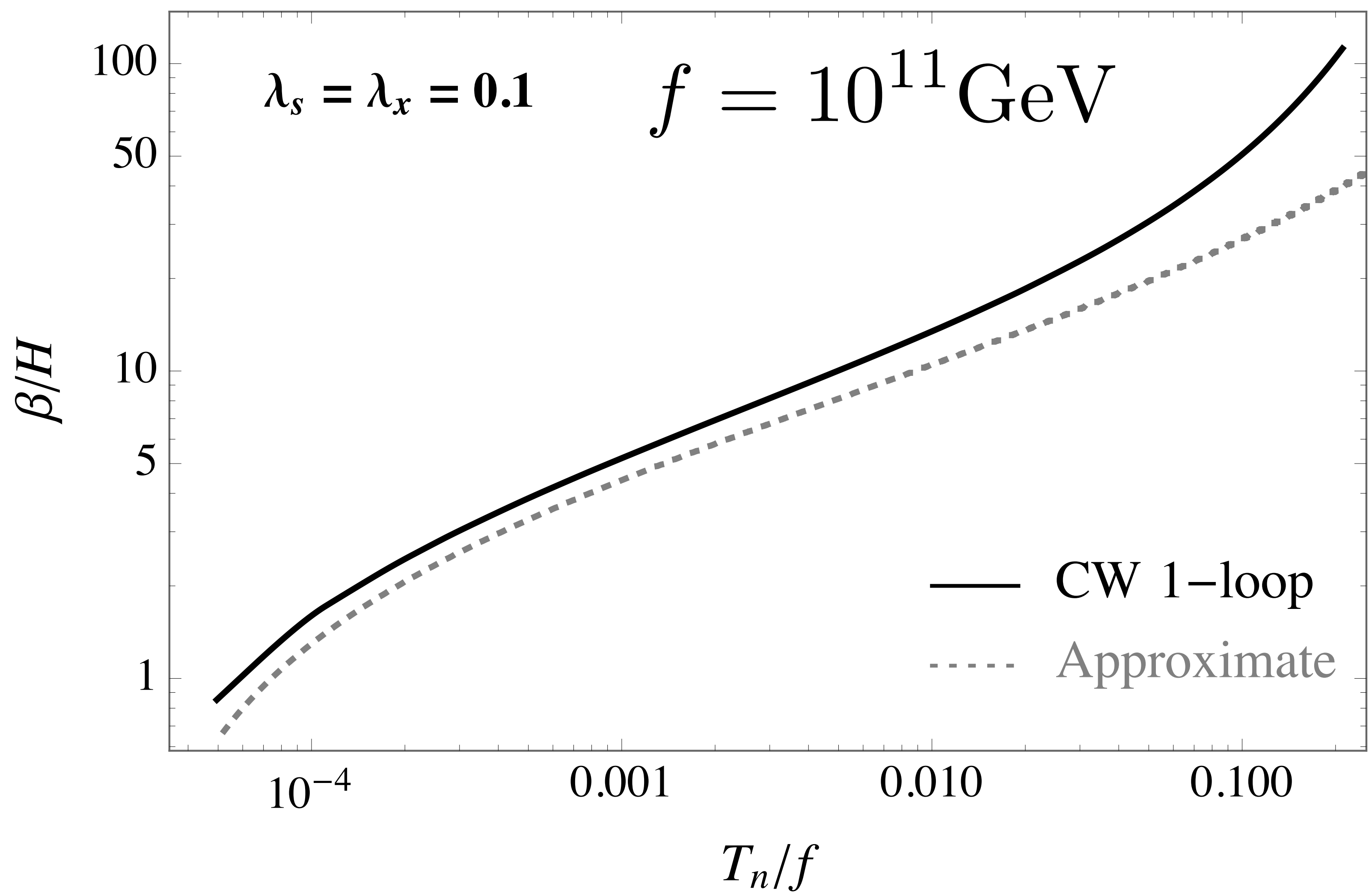
results: gauge coupling dominance





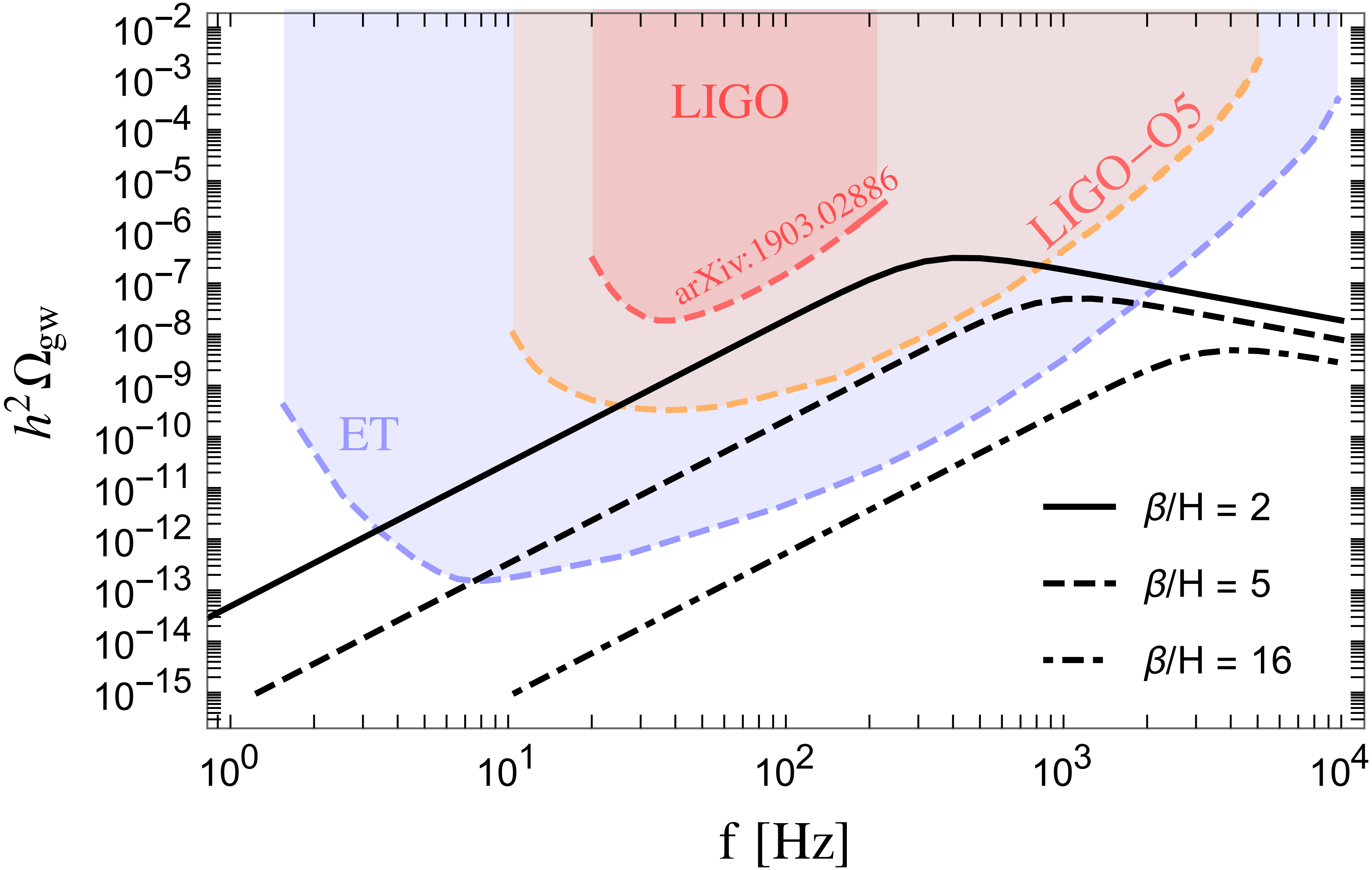
$$f = 10^{11}\text{GeV}$$

results: gauge coupling dominance



results insensitive to the improvement of the potential

Gravity Wave Signal



Strong Coupling

A beautiful class of models are composite axions based on SU(N) gauge theories with massless vector-like fermions:

$$(N_c, 3) + (\bar{N}_c, \bar{3}) + (N_c, 1) + (\bar{N}_c, 1)$$

Dynamics as in QCD:

$$\frac{SU(4) \times SU(4)}{SU(4)}$$

$$SU(3)_c \subset SU(4)$$

$$15 = 8 \oplus 3 \oplus \bar{3} \oplus 1$$

Singlet is the axion!

These theories undergo a 1st order phase transition:

$$3 \leq N_F < 4N \qquad N \geq 3$$

[Pisarski, Wilczek '84]

Transition expected to be rapid -> no supercooling.

Weakly coupled models can be understood as a combination of a dilaton and the axion.
The same can be realized at strong coupling:

$$\text{CFT} \xrightarrow[\text{breaking}]{\text{spontaneous}} \text{Dilaton}$$

Explicit breaking:

$$\text{CFT} + \frac{g}{\Lambda^\epsilon}\mathcal{O} \xrightarrow{\epsilon \ll 1} \text{shallow potential}$$

Dilaton EFT:

$$\mathcal{L} = \frac{N^2}{16\pi^2}[(\partial\varphi)^2 - \lambda(g(\varphi))\varphi^4]$$

$$g(\varphi) = \left(\frac{\varphi}{\Lambda}\right)^{\epsilon} g_0 \qquad\qquad \lambda = \lambda_0 + \lambda'(0)\, g\, (\varphi/\Lambda)^{\epsilon} + \ldots$$

$$V(\varphi) = \frac{N^2}{16\pi^2}\lambda_0 \varphi^4 \left[1 - \frac{4}{4+\epsilon}\left(\frac{\varphi}{f}\right)^{\epsilon}\right] + \mathcal{O}(\lambda_0^2)$$

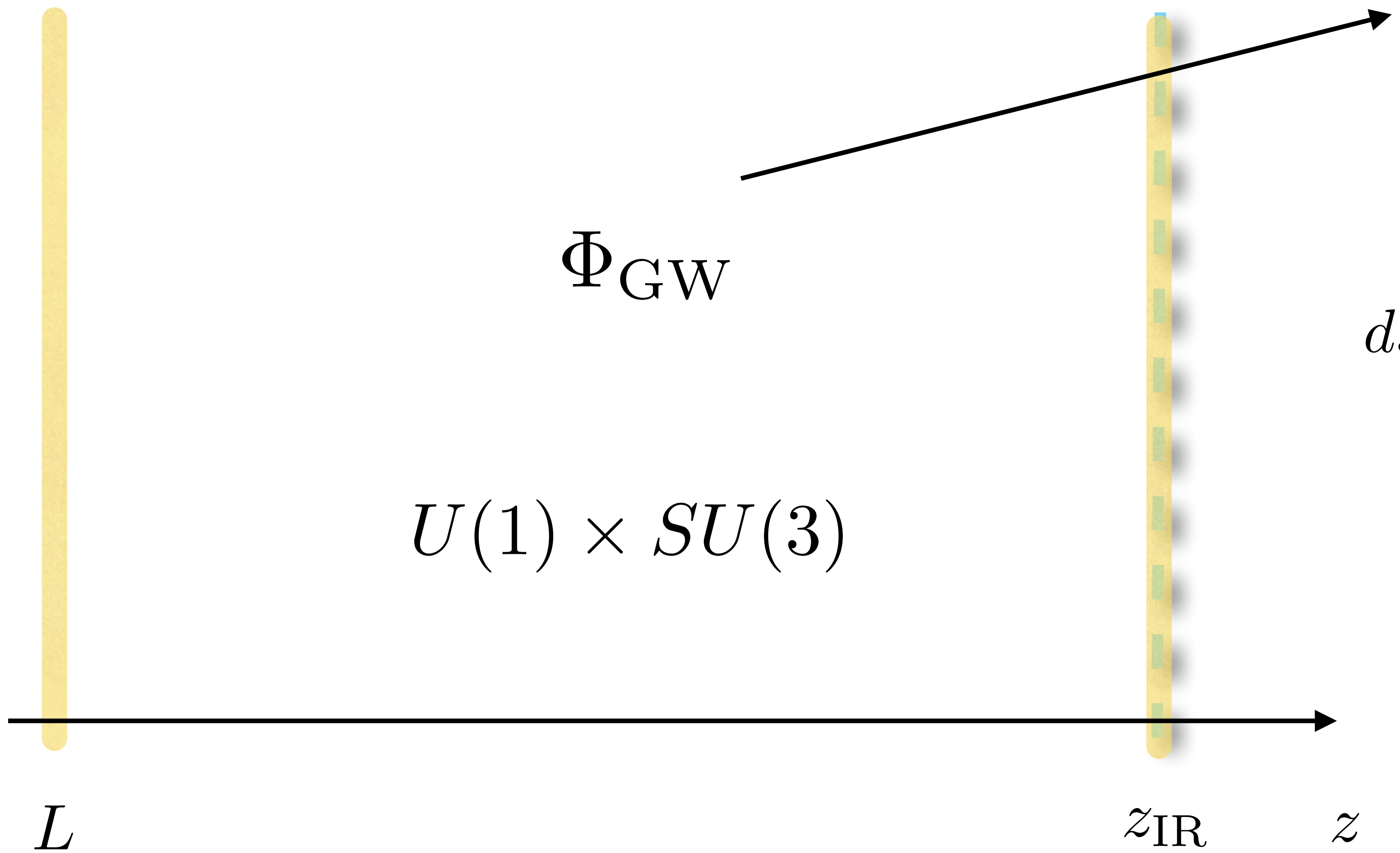
$$\epsilon \ll 1 \qquad\qquad \approx -\frac{N^2}{16\pi^2}\lambda_0 \epsilon \left(\log\frac{\varphi}{f} - \frac{1}{4}\right)$$

To realise the axion the CFT should have a global symmetry U(1)xSU(3) where U(1) is anomalous and spontaneously broken:

$$\langle 0 | j^{\mu}_{\rm PQ}(p) | a \rangle \sim \frac{N}{4\pi} f\, p^{\mu} \qquad\qquad \partial_{\mu} j^{\mu}_{\rm PQ} = \frac{K}{16\pi^2} G^a_{\mu\nu} \tilde{G}^{a\,\mu\nu}$$

Through the AdS/CFT correspondence this can be realised in 5D "Randall-Sundrum" type models

Goldberger-Wise scalar.
M<< 1/L^2



$$ds^2 = \frac{L^2}{z^2}(dx_\mu dx^\mu + dz^2)$$

$$\mathcal{L}_5 = -\frac{1}{4g_{PQ}^2}F_{MN}F^{MN} + \frac{1}{4g_3^2}G^a_{MN}G^{a\,MN} + \frac{K}{192\pi^2}\epsilon^{MNOPQ}A_M G^a_{NO}G^a_{PQ}$$

$$A_\mu|_{z=L} = A_\mu|_{z=z_{\rm IR}} = 0 \qquad G_{\mu 5}|_{z=L} = G_{\mu 5}|_{z=z_{\rm IR}} = 0$$

[flat space, Choi '03]

$$\text{Axion} = \int_0^L A_5^{PQ} dy$$

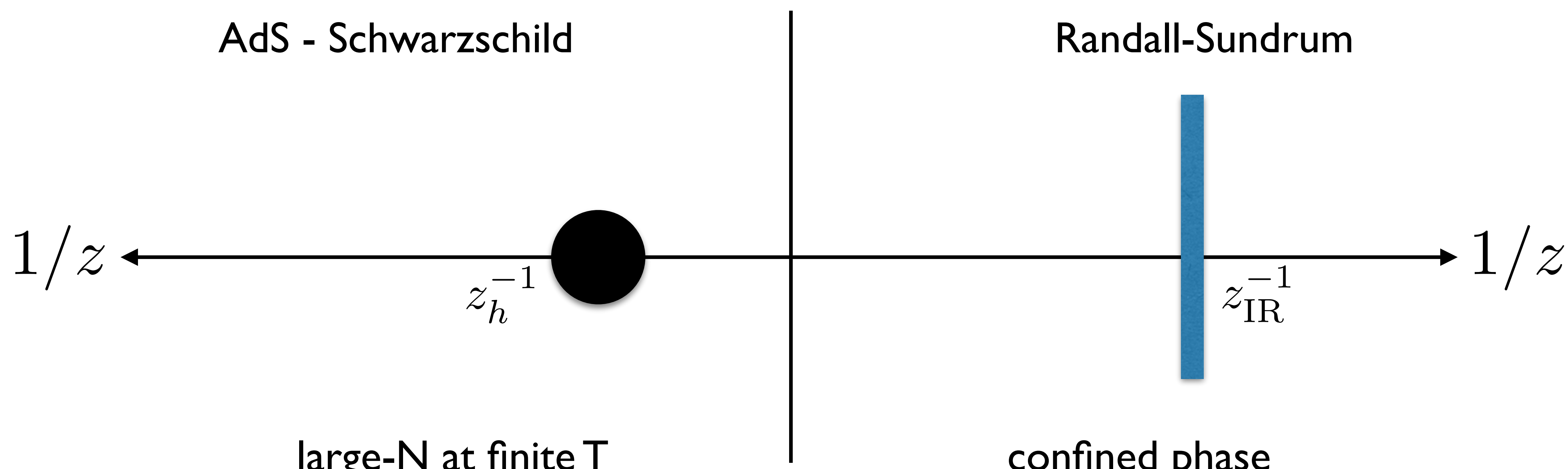
$$\frac{1}{g_s^2} = \frac{1}{g_0^2} + \frac{L}{g_3^2} \log \frac{z_{\rm IR}}{L}$$

$$f_a^2 = \frac{2}{g_{\rm PQ}^2} \frac{L}{z_{\rm IR}^2}$$

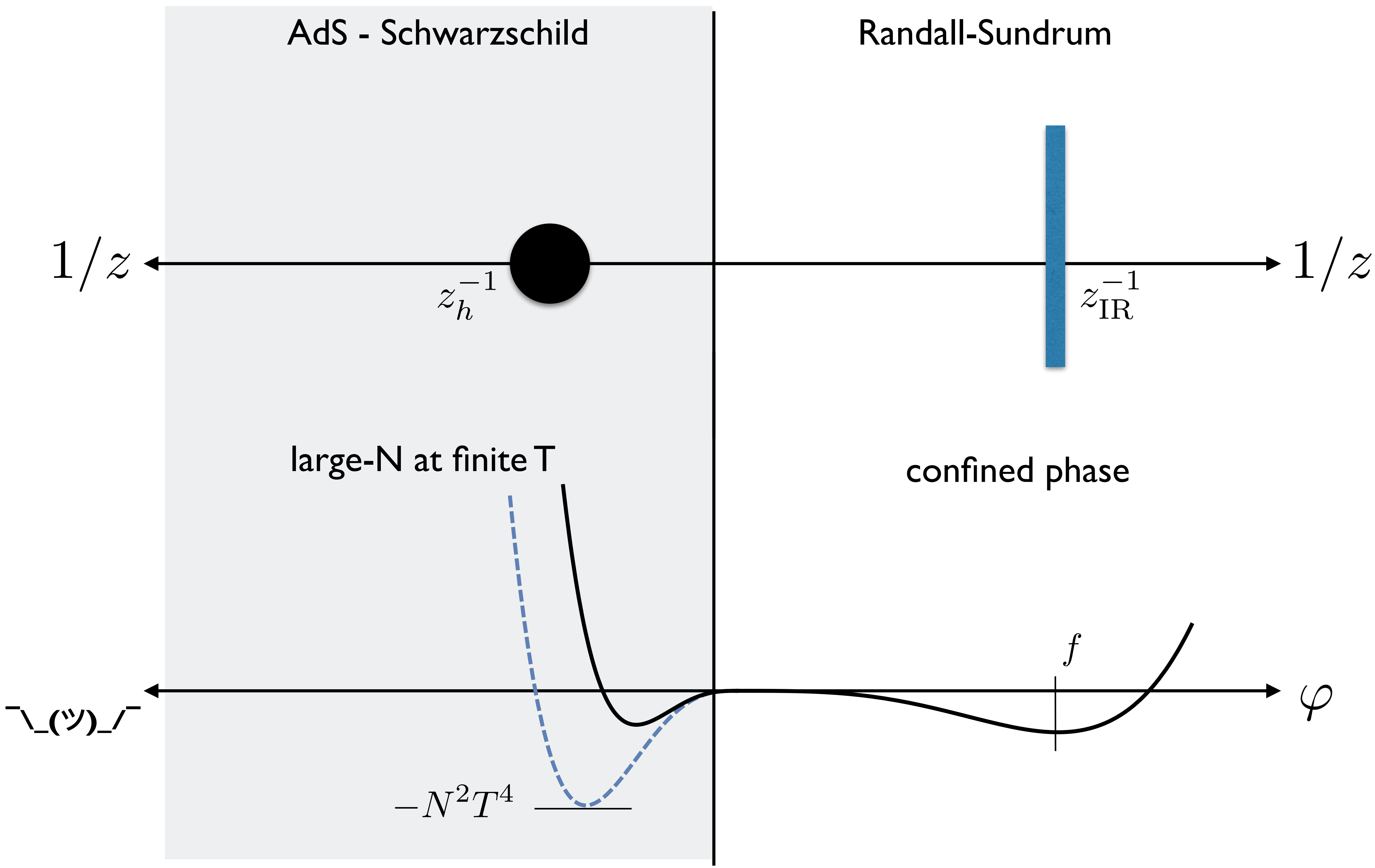
At T> 1/zIR the theory is in the deconfined phase

$$F \sim -N^2 T^4$$

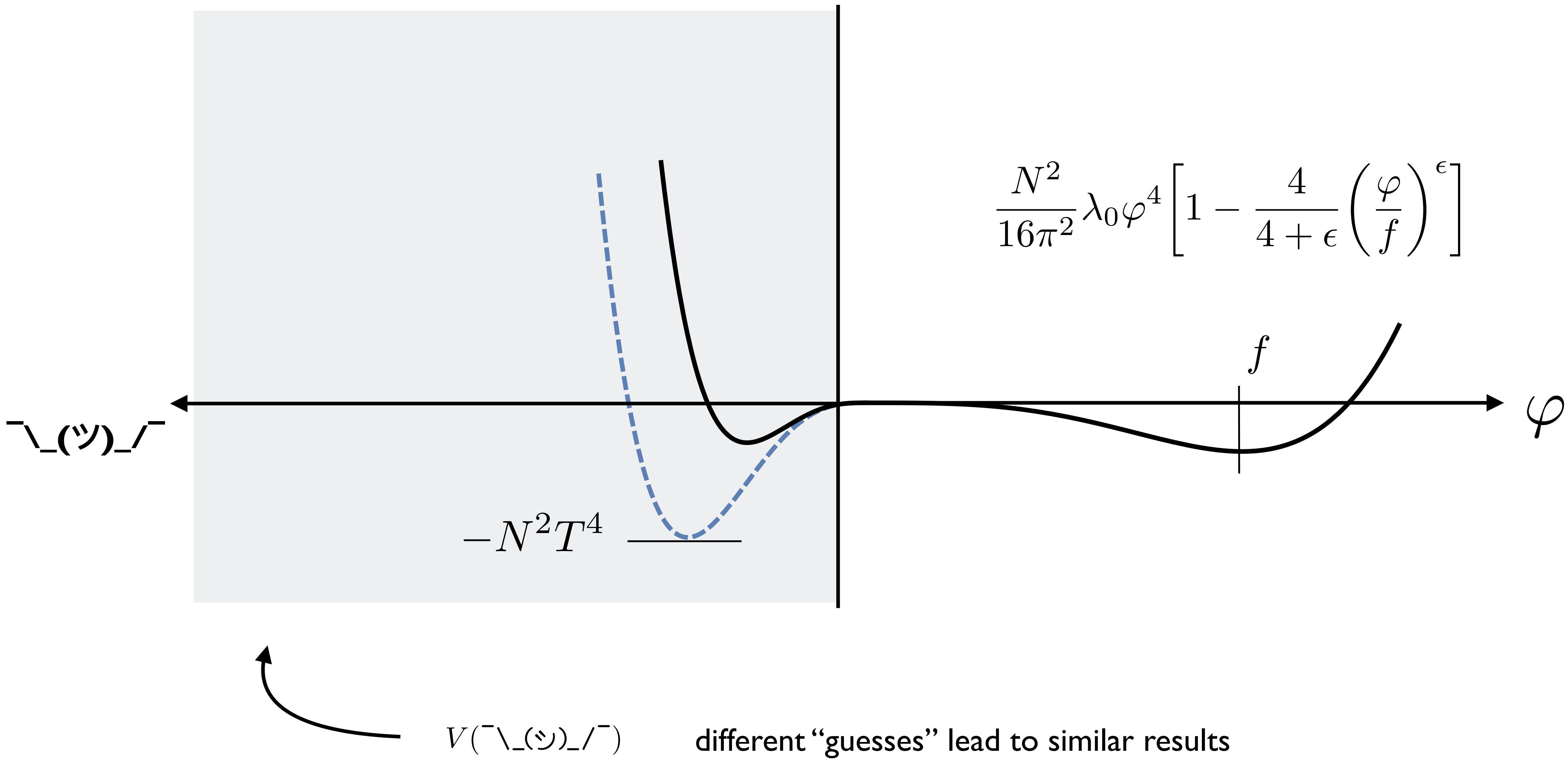
In 5D this corresponds to a black hole geometry: [Witten '98]



exploiting duality



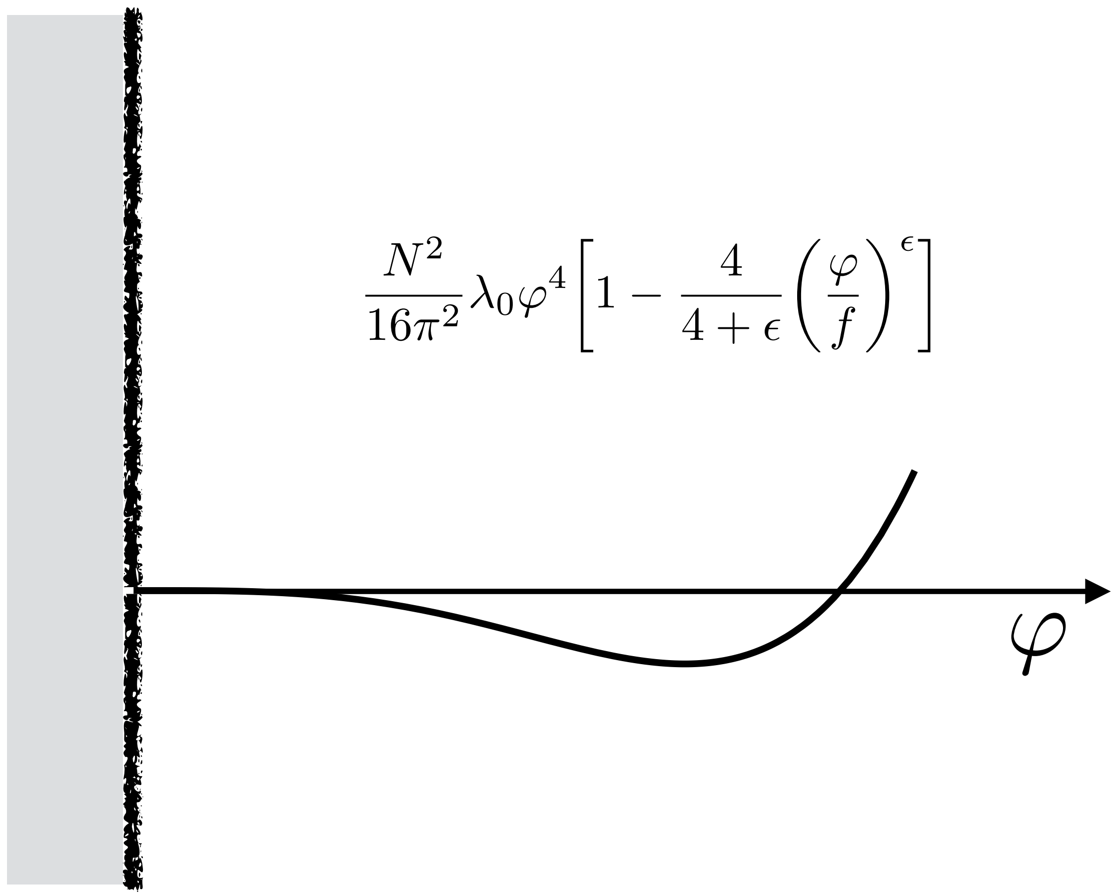
Bounce:



different "guesses" lead to similar results

[Creminelli, Nicolis, Rattazzi; Servant, Von Harling; Bruggisser et al; Baratella, Rompineve, Pomarol]

Bounce action can be estimated within dilaton EFT:



avoid discussing the CFT potential
by matching the gradient energy
to the free energy

$$\dot{\varphi}^2|_{\varphi=0} \sim 16\pi^2 T^4$$

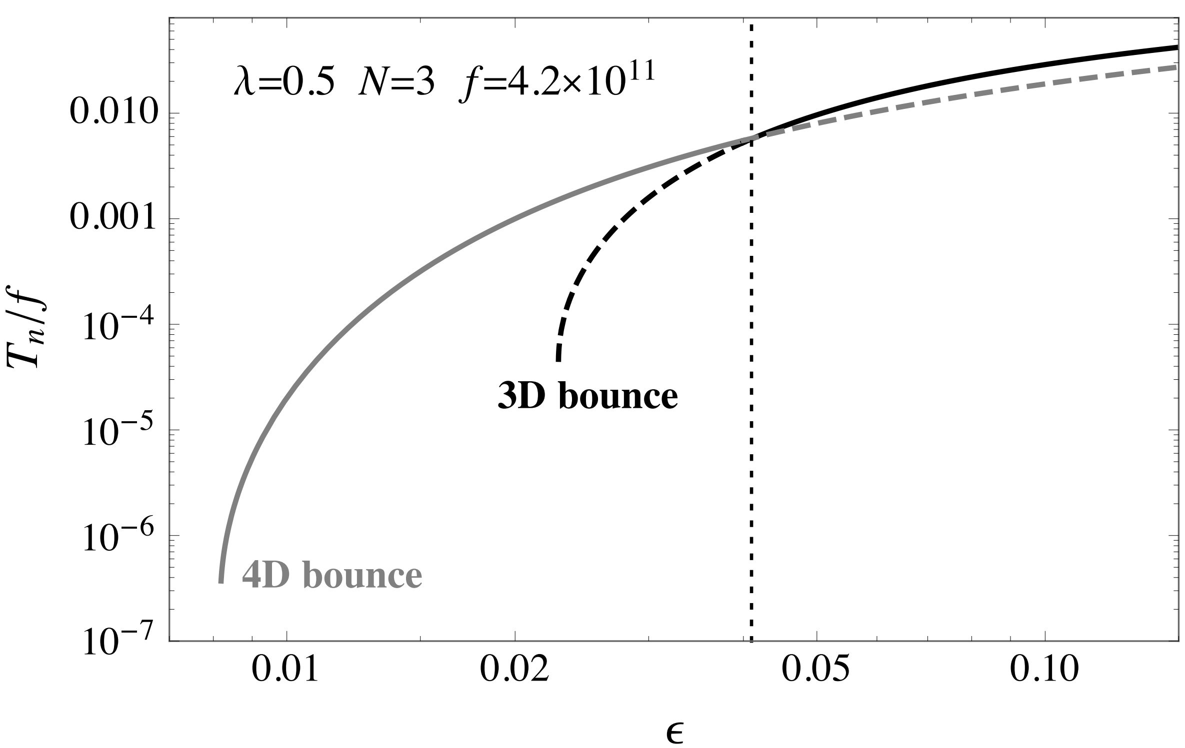
[Agashe et al '19]

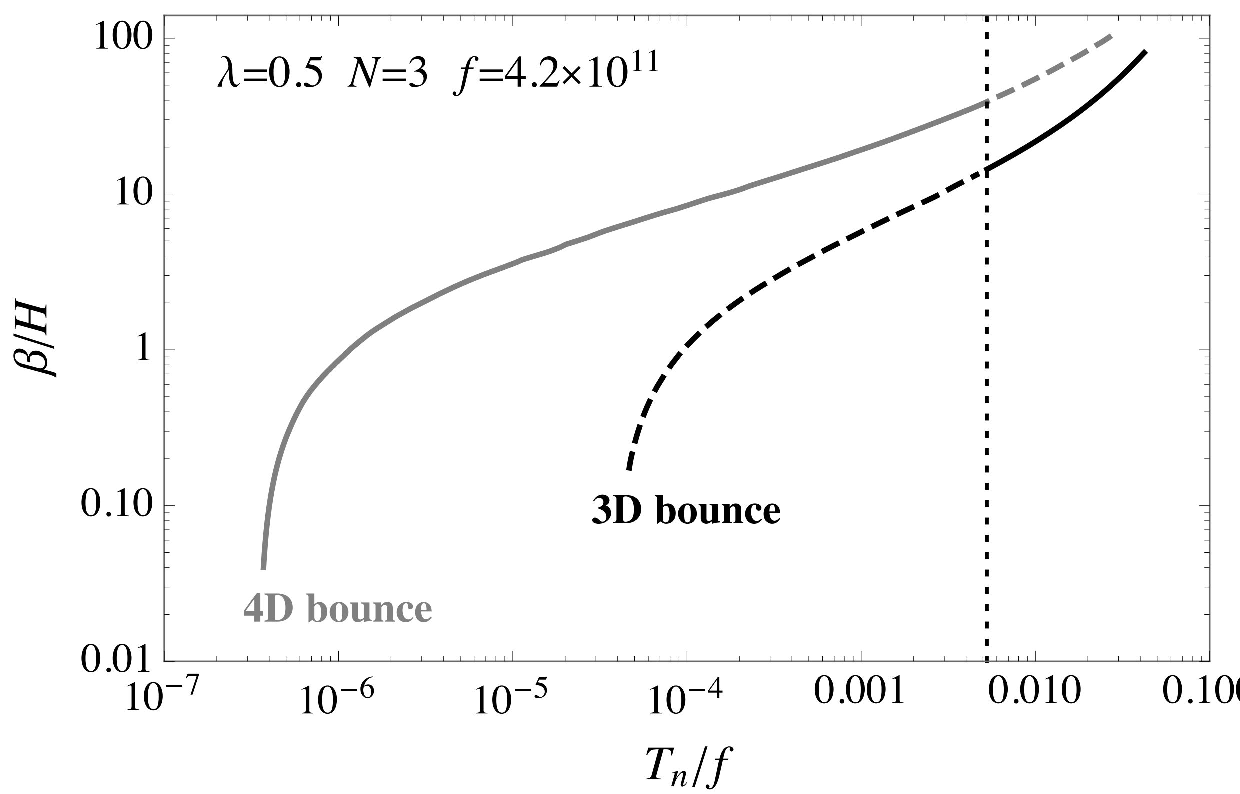
$$\frac{S_3}{T} = 28.5\,\frac{N^2}{16\pi^2} \times \frac{(16\pi^2 b)^{1/4}}{|\lambda_0 \epsilon \log(f/(cT))|^{3/4}}$$

O4 bounce also relevant.

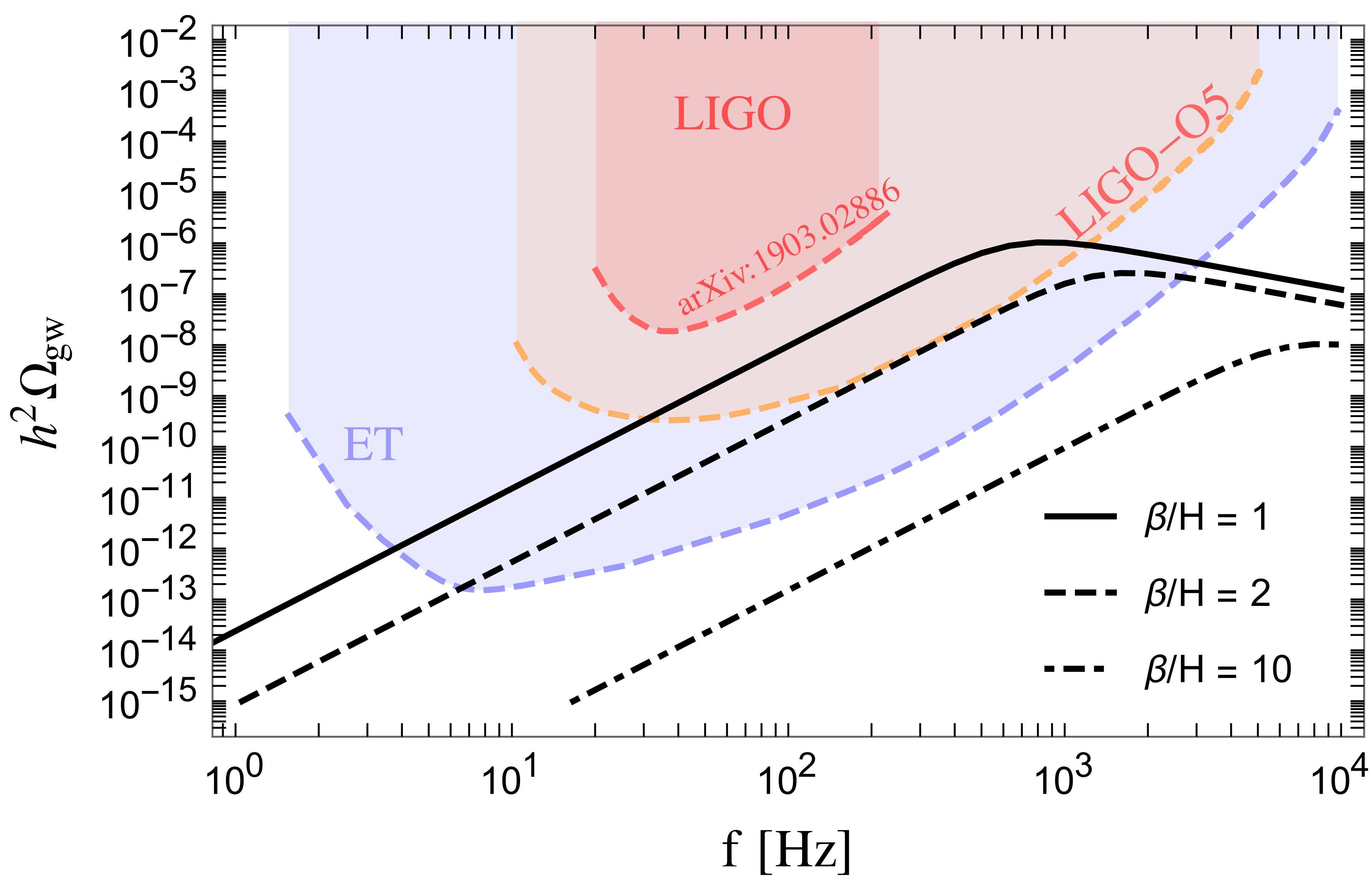
S3 and S4 bounces

most of the effects encoded in the size of the free-energy

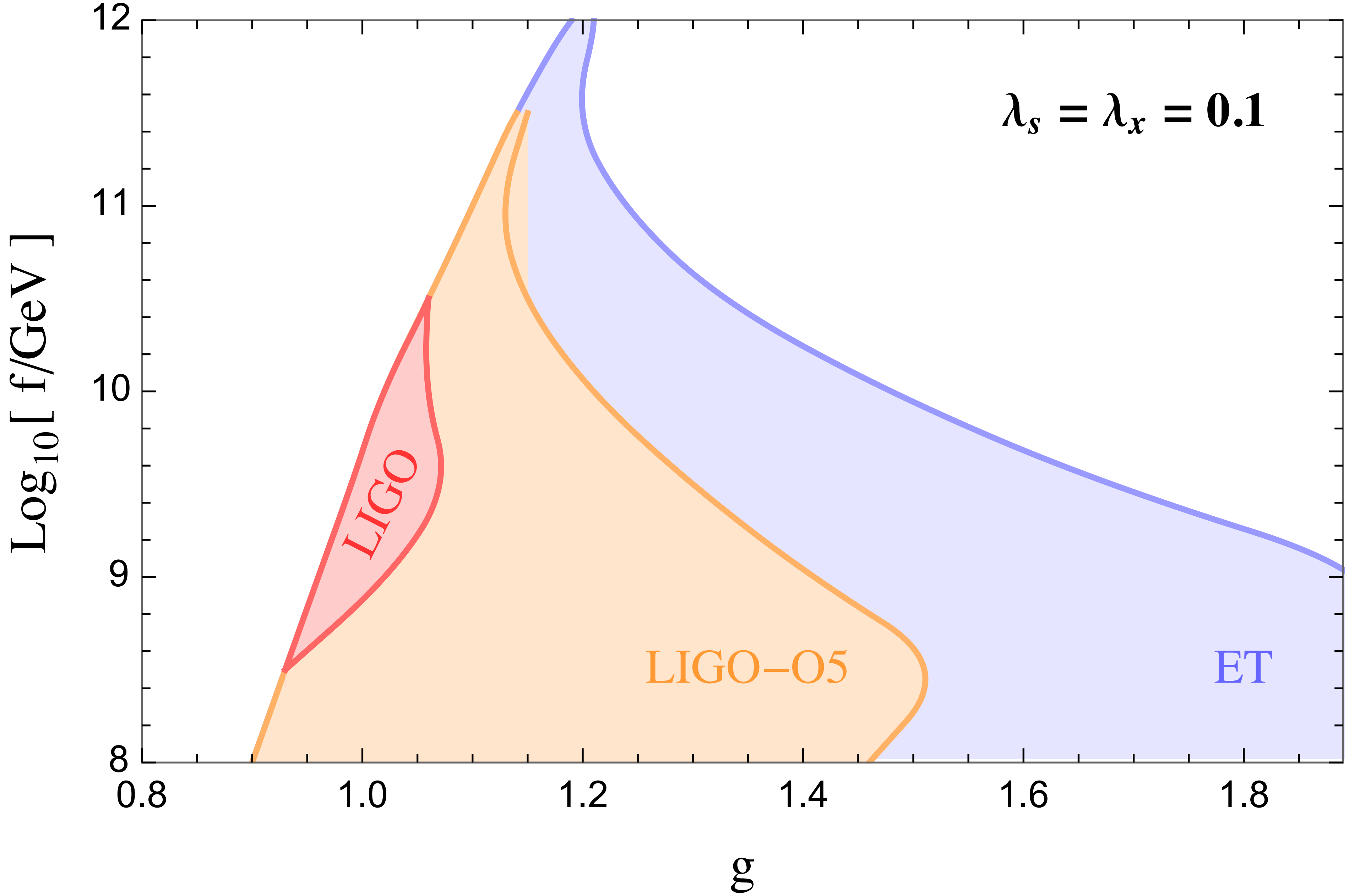




Gravity wave signal



Reach similar to weakly coupled models.



- Simplest axion models have 2nd order PT.
- First order PT realised in theories with no scales.
- Gravity wave-signal is within the reach of exp.