Multi-differential resummation for the LHC.

Johannes Michel
DESY Hamburg

in collaboration with

G. Billis, M. Ebert, G. Lustermans, I. Stewart, F. Tackmann

[1908.00985, 1909.00811, and ongoing work]

University of Genoa – Phenomenology Seminar

04 December 2019

Motivation: $pp \to H, W, Z + X$

Transverse momentum $q_T \equiv p_T^H,\ p_T^W,\ p_T^Z$ spectra are key LHC observables.

p_T^Z

- $\frac{1}{\sigma}\mathrm{d}\sigma/\mathrm{d}p_T^Z$ measured to $\lesssim 0.5\%$ at the LHC
  - Precision test of QCD
- Bulk of the cross section at small $q_T \ll m_Z$
  - Focus of this talk

p_T^W

- Need precise modeling of p_T^W to measure m_W from position of Jacobian peak in p_T^ℓ spectrum
- $\simeq 2\%$ uncertainty on $\mathrm{d}\sigma/\mathrm{d}p_T^W$ translates to $\Delta m_W \simeq 10\,\mathrm{MeV}$
- Compare $m_W = 80370 \pm 19\,\mathrm{MeV}$ [ATLAS '17]
  - Strategy: use precisely measured Z spectrum

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T^W} = \left[\frac{\mathrm{d}\sigma}{\mathrm{d}p_T^Z}\right]_{\text{meas.}} \times \left[\frac{\mathrm{d}\sigma/\mathrm{d}p_T^W}{\mathrm{d}\sigma/\mathrm{d}p_T^Z}\right]_{\text{pred.}}$$

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- Need precise result for $gg \to H$ "background"
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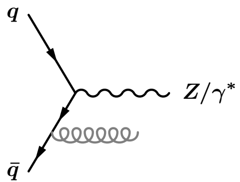

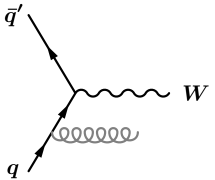

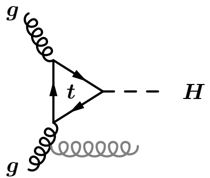

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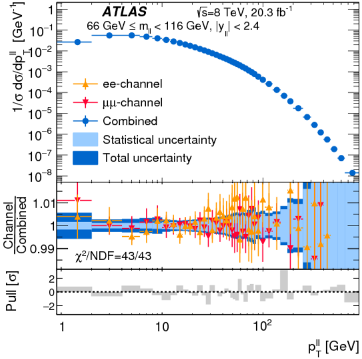

[ATLAS, 1512.02192]

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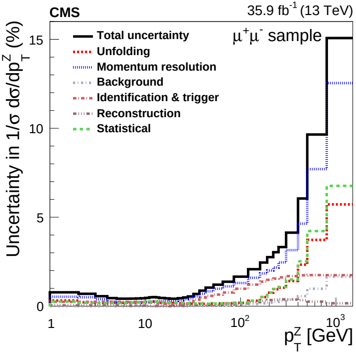

[CMS, 1909.04133]

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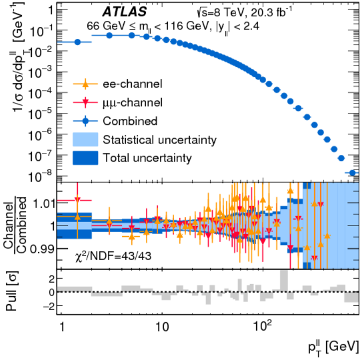

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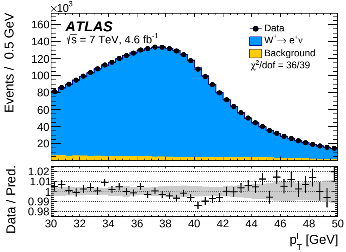

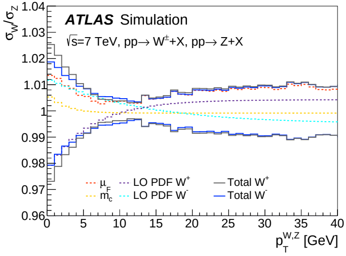

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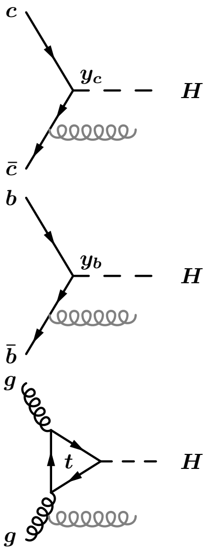


Outline.

1. Intro: Resummation for the inclusive q_T spectrum

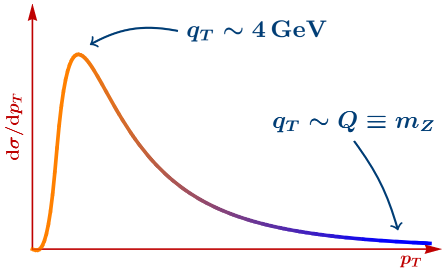
2. Resumming fiducial power corrections

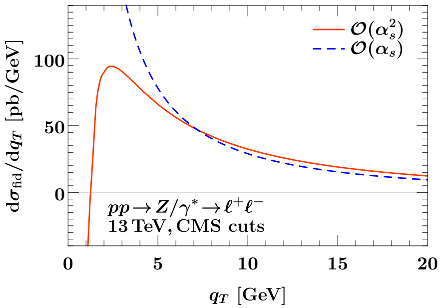
3. Application: q_T resummation effects in fiducial rapidity spectra

4. Generalized threshold factorization with full collinear dynamics

Intro: Resummation for the inclusive q_T spectrum.

Things going horribly wrong.



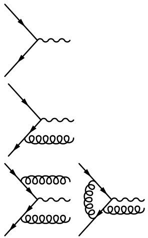


Perturbative series in the peak ($q_T \ll Q$) is dominated by terms $\mathcal{O}(1/q_T)$:

$$\frac{d\sigma}{dq_T} = \delta(q_T)$$

$$+ \alpha_s \Big[\frac{\ln(q_T/Q)}{q_T} + \frac{1}{q_T} + \delta(q_T) \qquad + \frac{\mathcal{O}(q_T^2/Q^2)}{q_T} \Big]$$

$$+ \alpha_s^2 \Big[\frac{\ln^3(q_T/Q)}{q_T} + \frac{\ln^2(q_T/Q)}{q_T} + \ldots + \frac{\mathcal{O}(q_T^2/Q^2)}{q_T} \Big]$$



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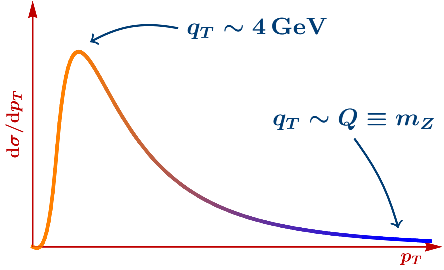

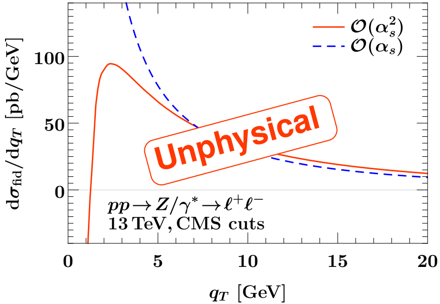

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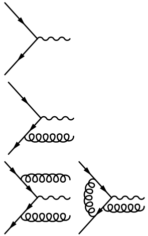
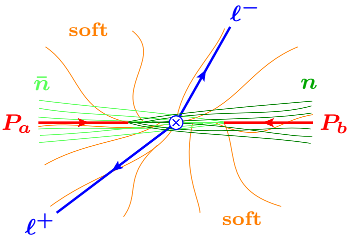


Factorization to the rescue.

At leading power in q_T/Q, can separate scales in the cross section e.g. using Soft-Collinear Effective Theory (SCET):

$$\frac{d\sigma}{d\vec{q}_T} = H(Q,\mu) \int d^2\vec{k}_a\, d^2\vec{k}_b\, d^2\vec{k}_s$$
$$\times\, B_a(x_a, \vec{k}_a, \mu, \nu)\, B_b(x_b, \vec{k}_b, \mu, \nu)$$
$$\times\, S(\vec{k}_s, \mu, \nu)\, \delta(\vec{q}_T - \vec{k}_a - \vec{k}_b - \vec{k}_s)$$



[Collins, Soper, Sterman '85; Chiu, Jain, Neill, Rothstein '12]

- Hard function describes underlying hard process $q\bar{q} \to Z/\gamma^* \to \ell^+\ell^-$
- Beam functions measure collinear radiation off incoming quarks a, b
  - Generalize PDFs $f_i(x,\mu)$ to include $\vec{k}_T$ dependence
  - Depend on $\vec{k}_T$, momentum fractions $x_{a,b} = Qe^{\pm Y}/E_{\rm cm}$, $\mu = \overline{\rm MS}$ scale, ν = rapidity or Collins-Soper scale
- Soft function measures recoil from isotropic soft radiation

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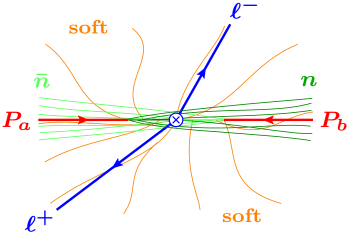

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- Convenient to take a Fourier transform $\vec{q}_T \to \vec{b}_T$:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\vec{q}_T} = H(Q,\mu) \int \mathrm{d}^2\vec{b}_T\, e^{\mathrm{i}\vec{b}_T\cdot\vec{q}_T}\, \tilde{B}_a(x_a,\vec{b}_T,\mu,\nu)\,\tilde{B}_b(x_b,\vec{b}_T,\mu,\nu)\,\tilde{S}(\vec{b}_T,\mu,\nu)$$

- Common definition of TMDPDFs includes parts of both beam and soft functions:

$$f_i^{\mathrm{TMD}}(x,\vec{b}_T,\mu,\zeta=x^2E_{\mathrm{cm}}^2) = \tilde{B}_i(x,\vec{b}_T,\mu,\nu)\sqrt{\tilde{S}(\vec{b}_T,\mu,\nu)}$$

Resummation from RG evolution.

Step 1 Factorize the cross section (here: toy example with only two terms, no ν)

$$\sigma(Q, p) = H(Q, \mu) \times F(p, \mu)$$

- Log gets split up: $\ln \frac{p}{Q} = \ln \frac{\mu}{Q} + \ln \frac{p}{\mu}$

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Step 2 Write down the renormalization group [cf. running coupling, $\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \alpha_s = \beta(\alpha_s)$]

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} H(Q, \mu) = \left[\Gamma(\alpha_s) \ln \frac{Q}{\mu} + \gamma(\alpha_s)\right] \times H(Q, \mu)$$

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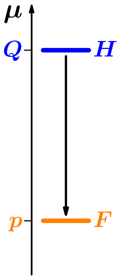


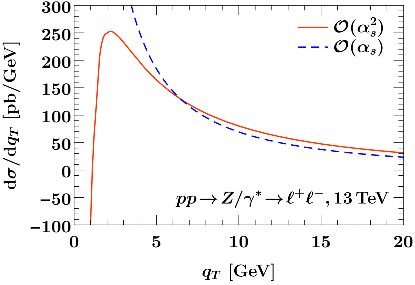
Step 3 Solve the RG between $\mu_H \equiv Q$ and $\mu_F \equiv p$ ("running")

$$\sigma_{\mathrm{res}}(Q,p) = H(Q,\mu_H) \times \exp\left\{\int_{\mu_H}^{\mu_F} \frac{\mathrm{d}\mu}{\mu} \left[\ldots\right]\right\} \times F(p,\mu_F)$$

- ▶ Logs resummed to all orders in the exponential
- LL, NLL, NLL′, NNLL, . . . correspond to loop orders of $H(Q,\mu_H)$, $F(p,\mu_F)$ and Γ, γ in the exponent

Result: Resummed inclusive q_T spectrum.

Fixed-order spectrum:



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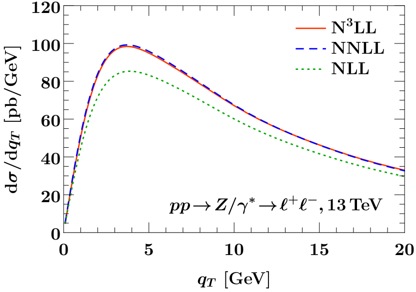

- Spectrum diverges as $q_T \to 0$
- Fixed-order series not convergent

- Exponential of large logs leads to physical Sudakov suppression as $q_T \to 0$
- Good convergence of resummed predictions

Resumming fiducial power corrections.

in collaboration with

M. Ebert, I. Stewart, F. Tackmann

[in preparation]

Leptonic/hadronic tensor for inclusive Drell-Yan.

Consider $p(P_a^\mu)p(P_b^\mu) \to Z/\gamma^*(q^\mu) \to \ell^-(k_1^\mu)\ell^+(k_2^\mu)$.

Want to measure (or cut on) decay products. How did we treat them earlier?

- Cross section factorizes into leptonic/hadronic tensor, $q^\mu L_{\mu\nu} = q^\mu W_{\mu\nu} = 0$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4 q} = L^{\mu\nu}(q^\mu) W_{\mu\nu}(q^\mu, P_a^\mu, P_b^\mu)$$

- Leptonic tensor is simple, by convention includes line shape (here: photon case)

$$L^{\mu\nu}(q^\mu) \equiv L(q^2)(q^2 g^{\mu\nu} - q^\mu q^\nu)\,, \qquad L(q^2) \sim \frac{\alpha_{\mathrm{em}}^2}{q^4}$$

- ▶ $\mathrm{d}\sigma/\mathrm{d}^4 q = L(q^2) W(q^\mu, P_a^\mu, P_b^\mu)$ proportional to trace $W \equiv g_{\mu\nu} W^{\mu\nu}$

- W is scalar, so only depends on $q^2 \equiv Q^2$ and $P_{a,b} \cdot q = E_{\mathrm{cm}} \sqrt{Q^2 + q_T^2}\, e^{\pm Y}$
- ▶ Resummation limit $q_T \to 0$ of W receives only quadratic power corrections:

$$W = \Big\{ \delta(q_T^2) + \alpha_s \Big[\frac{\ln q_T^2/Q^2}{q_T^2} + \frac{1}{q_T^2} + \delta(q_T^2) \Big] + \dots \Big\} \Big[1 + \mathcal{O}\Big(\frac{q_T^2}{Q^2}\Big) \Big]$$

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Leptonic observables & tensor decomposition.

Consider $p(P_a^\mu)p(P_b^\mu) \to Z/\gamma^*(q^\mu) \to \ell^-(k_1^\mu)\ell^+(k_2^\mu)$.

What happens if we go more differential and retain the dependence on $k_{1,2}^\mu$?

- Parametrize decay phase space $k_{1,2}^\mu$ by two rest-frame angles $\cos\theta, \varphi$:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4q\,\mathrm{d}\cos\theta\mathrm{d}\varphi} = L^{\mu\nu}(q^\mu, \cos\theta, \varphi)W_{\mu\nu}(q^\mu, P_a^\mu, P_b^\mu)$$

- Leptonic tensor carries dependence on lepton momenta (shown: P-even case)

$$L^{\mu\nu}(q^\mu, \cos\theta, \varphi) \sim \frac{\alpha_{\mathrm{em}}^2}{q^4}(k_1^\mu k_2^\nu + k_2^\mu k_1^\nu - g^{\mu\nu} k_1 \cdot k_2)$$

- Decompose $W^{\mu\nu}$ into nine allowed orthogonal tensor structures $K_i^{\mu\nu}$:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4q\,\mathrm{d}\cos\theta\mathrm{d}\varphi} = \sum_{i=-1}^{7}(L\cdot K_i)(K_i\cdot W) \equiv \sum_{i=-1}^{7} L_i W_i$$

Leptonic observables & tensor decomposition.

Consider $p(P_a^\mu)p(P_b^\mu) \to Z/\gamma^*(q^\mu) \to \ell^-(k_1^\mu)\ell^+(k_2^\mu)$.

What happens if we go more differential and retain the dependence on $k_{1,2}^\mu$?

- Parametrize decay phase space $k_{1,2}^\mu$ by two rest-frame angles $\cos\theta, \varphi$:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4q\,\mathrm{d}\!\cos\theta\,\mathrm{d}\varphi} = L^{\mu\nu}(q^\mu, \cos\theta, \varphi)W_{\mu\nu}(q^\mu, P_a^\mu, P_b^\mu)$$

- Leptonic tensor carries dependence on lepton momenta (shown: P-even case)

$$L^{\mu\nu}(q^\mu, \cos\theta, \varphi) \sim \frac{\alpha_{\mathrm{em}}^2}{q^4}(k_1^\mu k_2^\nu + k_2^\mu k_1^\nu - g^{\mu\nu} k_1 \cdot k_2)$$

- Decompose $W^{\mu\nu}$ into nine allowed orthogonal tensor structures $K_i^{\mu\nu}$:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4q\,\mathrm{d}\!\cos\theta\,\mathrm{d}\varphi} = \sum_{i=-1}^{7}(L\cdot K_i)(K_i\cdot W) \equiv \sum_{i=-1}^{7} L_i W_i$$

$$= \frac{\mathrm{d}\sigma}{\mathrm{d}^4q}\Big[1 + \cos^2\theta + \sum_{i=0}^{7} A_i f_i(\cos\theta, \varphi)\Big]$$

▶ K_i map onto spherical harmonics $f_i(\cos\theta, \varphi)$ with angular coefficients A_i

Resummation in the strict $q_T \to 0$ limit.

What are the leading singular terms as $q_T \to 0$ that must be resummed?

- Expand hadronic structure functions as $W_i \to W_i^{\mathrm{res}}$
  - All* vanish except for $W_{-1}^{\mathrm{res}} \leftrightarrow \mathrm{d}\sigma^{\mathrm{res}}/\mathrm{d}q_T$ and $W_4^{\mathrm{res}} \leftrightarrow \mathrm{d}A_{FB}^{\mathrm{res}}/\mathrm{d}q_T$

*Up to some very cool intrinsically nonperturbative effects

- Expand leptonic tensor as $L_i(q_T, \cos\theta, \varphi) \to L_i(q_T = 0, \cos\theta, \varphi)$
  - Leptons are exactly back to back, $p_T^{\ell^-} = p_T^{\ell^+}$, no recoil
  - No distinguished transverse direction in the decay anymore
  - $L_i(\not{\varphi})$ is independent of the azimuth

- $$\frac{\mathrm{d}\sigma}{\mathrm{d}^4q\,\mathrm{d}\cos\theta\,\mathrm{d}\varphi} \sim \frac{1}{2\pi}\Big[(1+\cos^2\theta)W_{-1}^{\mathrm{res}} + \cos\theta\, W_4^{\mathrm{res}}\Big] \quad \text{as} \quad q_T \to 0$$

Resumming fiducial power corrections.

What are the power corrections to $\sum_i L_i(q_T = 0)\, W_i^{\mathrm{res}}$?

- Hadronic structure functions are scalar, so $W_i = W_i^{\mathrm{res}}\big[1 + \mathcal{O}(q_T^2/Q^2)\big]$
- L_i are also scalar, but $L_i(q_T) = L_i(0)\big[1 + \mathcal{O}(q_T/Q)\big]$ in the presence of any measurements on the decay products

WHY? Boosting back into lab frame from rest frame, we have

$$p^\mu_{\ell^-} = \frac{Q}{2}\Big(\gamma + \frac{q_T}{Q} s_\theta c_\varphi, \gamma s_\theta c_\varphi + \frac{q_T}{Q}, s_\theta s_\varphi, c_\theta\Big)\,, \quad \gamma = \frac{\sqrt{Q^2 + q_T^2}}{Q}$$

- ▶ Measuring p_T^ℓ and/or η_ℓ results in a linear dependence on q_T

- We can fix this – we can even exploit it!

  By restoring the exact q_T dependence of the L_i,
  we resum all linear power corrections to all orders in α_s:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4q\,\mathrm{d}\cos\theta\mathrm{d}\varphi} = \Big[L_{-1}(q_T) W_{-1}^{\mathrm{res}} + L_4(q_T) W_4^{\mathrm{res}}\Big]\Big[1 + \mathcal{O}(q_T^2/Q^2)\Big]$$

Resumming fiducial power corrections.

What are the power corrections to $\sum_i L_i(q_T = 0)\, W_i^{\rm res}$?

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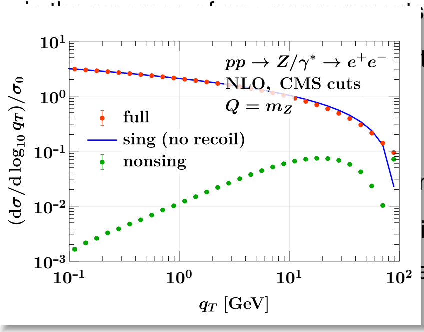
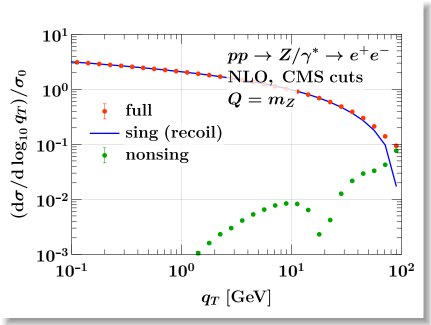




$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4 q\, \mathrm{d}\cos\theta\, \mathrm{d}\varphi} = \left[L_{-1}(q_T) W_{-1}^{\rm res} + L_4(q_T) W_4^{\rm res}\right]\left[1 + \mathcal{O}(q_T^2/Q^2)\right]$$

Relation to literature.

- Keeping $L_i(q_T)$ exact allows for nonzero recoil in the resummation
  ... see numerics on the next slides for examples of this

- Equivalent to evaluating tree-level matrix element in a (sensibly) boosted frame, "recoil prescription" in ResBos or DyRes

- One-parameter ambiguity in how to distribute p_T between annihilating partons
  [Catani, Ferrera, de Florian, Grazzini '15]

  - Corresponds to slightly rotated basis choice for the $K_i^{\mu\nu}$

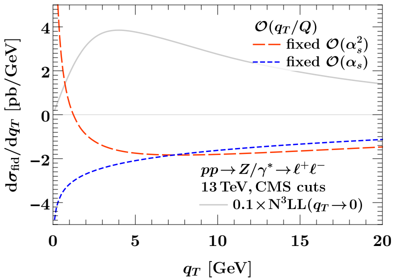
- Can show using $K_i^{\mu\nu}$ that the difference is strictly $\mathcal{O}(q_T^2/Q^2)$

  - Change of basis for P-even structure functions is known
    [Boer, Vogelsang '06]

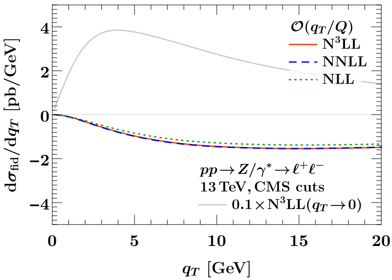
- Linear power corrections to fiducial q_T spectrum are uniquely predicted

Numerical impact: $\mathrm{d}\sigma_{\mathrm{fid}}/\mathrm{d}q_T$

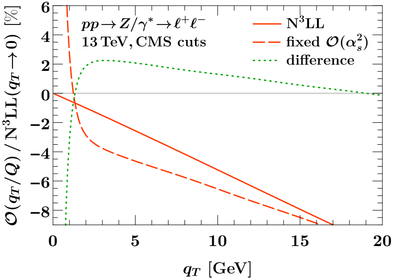
No recoil, $\mathcal{O}(q_T/Q)$ at fixed order:



With recoil, $\mathcal{O}(q_T/Q)$ resummed:



Normalize to N^{3}LL leading terms:



- Consider CMS cuts:

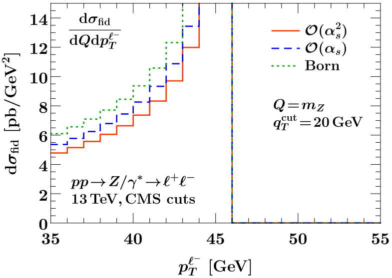
$$p_T^\ell \geq 25\,\mathrm{GeV}, |\eta_\ell| \leq 2.4$$

- Shorthand for fiducial power corrections:

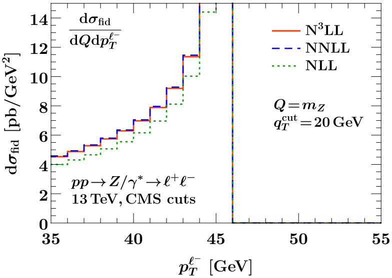
$$\mathcal{O}(q_T/Q) \equiv \sum_i \Big[L_i(q_T) - L_i(0) \Big] W_i^{\mathrm{res}}$$

Numerical impact: $\mathrm{d}\sigma_{\mathrm{fid}}/\mathrm{d}p_T^{\ell^-}$

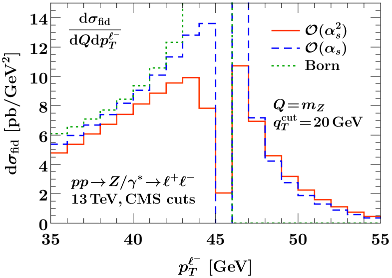
Fixed order, no recoil:



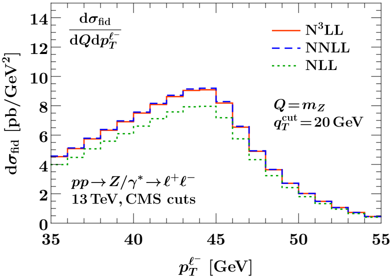
Resummed, no recoil:



Fixed order, with recoil:

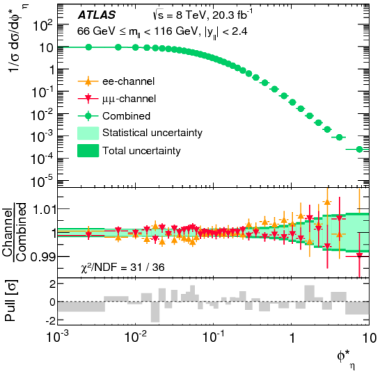


Resummed, with recoil:

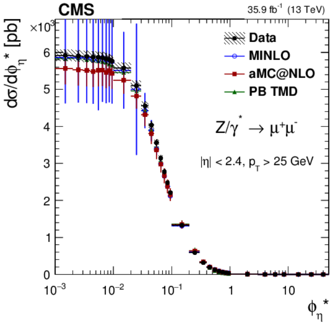


Numerical impact: $L_{-1}(\phi^*)$ and $L_{-1}^{\rm fid}(\phi^*)$

- $\phi^* \equiv \tan \frac{\pi - \Delta\varphi}{2} \sin\theta_\eta^* \ll 1$ is also sensitive to $q_T \ll Q$
  - Only depends on lepton angles $\Delta\varphi = \phi_1 - \phi_2$ and $\cos\theta_\eta^* = \tanh \frac{\eta^- - \eta^+}{2}$
  - Designed to maximize experimental resolution [Banfi et al. '10]
- Nice feature: $\phi^* \approx q_T^y/Q$, so can analytically integrate over q_T^x in resummation



[Banfi, Dasgupta, Marzani, Tomlinson '11-'12]



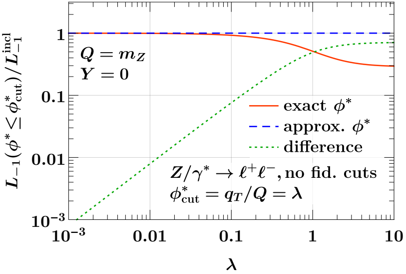
Numerical impact: $L_{-1}(\phi^*)$ and $L^{\rm fid}_{-1}(\phi^*)$

- $\phi^* \equiv \tan \dfrac{\pi - \Delta\varphi}{2} \sin\theta^*_\eta \ll 1$ is also sensitive to $q_T \ll Q$
  - Only depends on lepton angles $\Delta\varphi = \phi_1 - \phi_2$ and $\cos\theta^*_\eta = \tanh \dfrac{\eta^- - \eta^+}{2}$
  - Designed to maximize experimental resolution [Banfi et al. '10]
- Nice feature: $\phi^* \approx q_T^y/Q$, so can analytically integrate over q_T^x in resummation

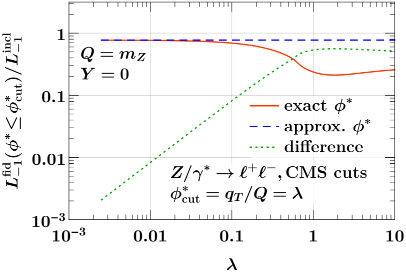
[Banfi, Dasgupta, Marzani, Tomlinson '11-'12]

- Power corrections to this approximation scale linearly as well!

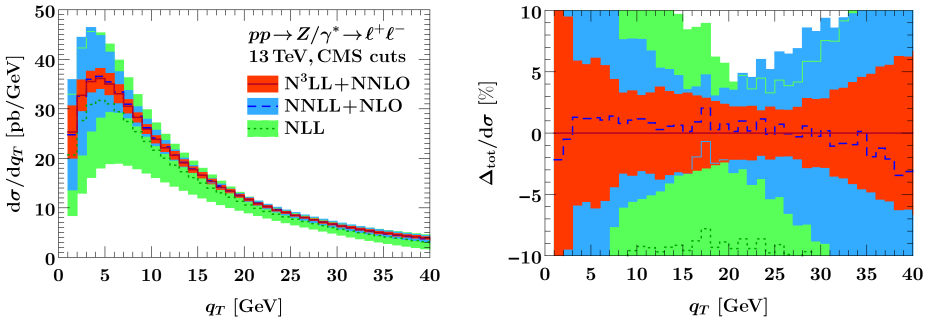
No fiducial cuts:



CMS cuts:



Sneak preview: $13\ \mathrm{TeV}$ predictions.



Application: q_T resummation effects in fiducial rapidity spectra.

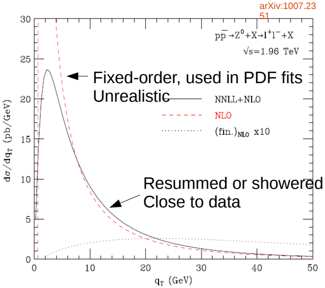
in collaboration with

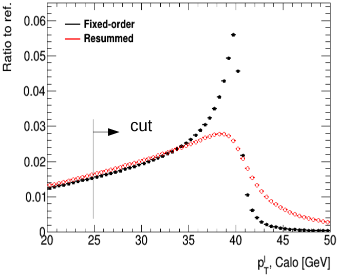
L. Aperio Bella, A. Apyan, M. Boonekamp, S. Camarda, F. Tackmann

[Les Houches study in preparation & more work in progress]

Application: q_T resummation effects in $\mathrm{d}\sigma_{\mathrm{fid}}/\mathrm{d}Y$

Motivation





Resummation effects affect the pT distributions, hence the acceptance of fiducial cuts

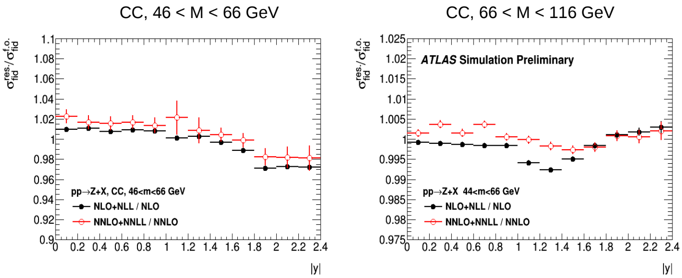
For same total cross section, fixed-order and resummed ***fiducial*** cross sections differ.

This leads to a small inconsistency when interpreting fiducial cross section measurements in terms of PDFs, which typically use fixed-order predictions

[Slides by M. Boonekamp at Durham EW Precision Sub-Group Workshop, April '19]

Application: q_T resummation effects in $\mathrm{d}\sigma_{\mathrm{fid}}/\mathrm{d}Y$

Effect of pT resummation – Z



3-4% drop towards high eta

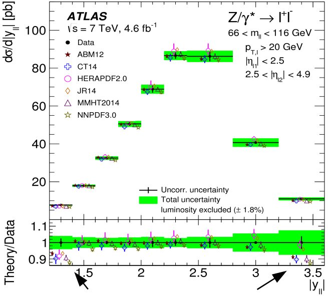
NLL : ~1% dip near |eta|=1.4
Reduced to .5% at NNLL

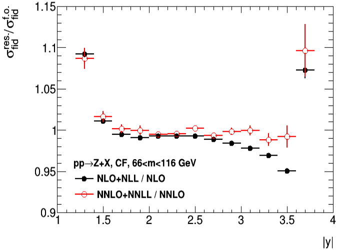
[Slides by M. Boonekamp at Durham EW Precision Sub-Group Workshop, April '19]

Application: q_T resummation effects in $d\sigma_{fid}/dY$

Comparison with data

- Left : ratios between measured cross sections and fixed-order predictions
- Right : ratios betweem resummed and fixed-order predictions



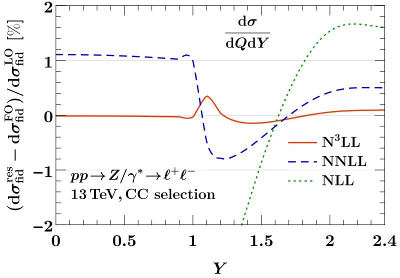


~10% theory deficit near acceptance boundaries (left) matches resummation correction

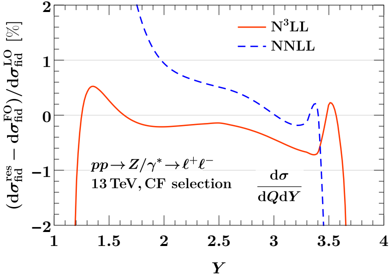
[Slides by M. Boonekamp at Durham EW Precision Sub-Group Workshop, April ’19]

Application: q_T resummation effects in $\mathrm{d}\sigma_{\mathrm{fid}}/\mathrm{d}Y$

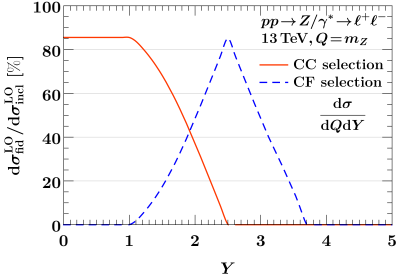
Res. effect, CC selection:



Res. effect, CF selection:



Born-level CC/CF acceptance:



- ✔ Presence of effect confirmed by independent codes [DYTurbo & SCETlib]
- Currently doing quantitative validation
- Challenge: ensuring "unitarity"

$$\int \mathrm{d}q_T \left[\sigma_{\mathrm{res}}^{\mathrm{incl}} - \sigma_{\mathrm{sing}}^{\mathrm{incl}}\right] = 0$$

Generalized threshold factorization with full collinear dynamics.

in collaboration with

G. Lustermans, F. Tackmann

[1908.00985]

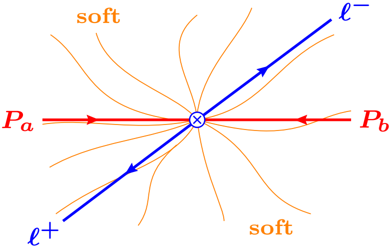
Motivation: Soft Threshold Factorization.

Drell-Yan production at invariant mass Q near threshold, $\tau \equiv Q^2/E_{\rm cm}^2 \to 1$:

$$\frac{d\sigma}{dQ} = \int dz\, \hat\sigma_{ij}(z)\, [f_i \otimes f_j]\Big(\frac{\tau}{z}\Big)$$

$$= H_{ij}(Q) \int dk^0\, S(k^0)\, [f_i^{\rm thr} \otimes f_j^{\rm thr}]\Big(\tau + \frac{k^0}{Q}\Big) \times \Big[1 + \mathcal{O}\big(1-\tau\big)\Big]$$

[Collins, Soper, Sterman '85-'88; Sterman '86]



- Cannot actually reach hadronic limit $\tau \to 1$ because PDFs are suppressed at large x
- But for steep PDFs, the integral is dominated by $z \sim 1$ even if $\tau \ll 1$ at the LHC
- Useful approximation at partonic level:
  $\hat\sigma_{ij} = H_{ij} \times S \times [1 + \mathcal{O}(1 - z)]$
- Approximate fixed-order cross sections, resum large logs to all orders, . . .

Motivation: Soft Threshold Factorization.

What if we measure the rapidity Y of the Z boson in addition?

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}Y} = H_{ij}(Q) \int \mathrm{d}k_a \mathrm{d}k_b\, S(k_a, k_b) \times f_i^{\mathrm{thr}}\Big(x_a + \frac{k_a}{Qe^{+Y}}\Big) f_i^{\mathrm{thr}}\Big(x_b + \frac{k_b}{Qe^{-Y}}\Big) \times \Big[1 + \mathcal{O}\big(1-\tau\big)\Big]$$

[Catani, Trentadue '89]

[Ahmed, Banerjee, Das, Dhani, Ravindran, Smith, van Neerven '07-'18; Owens, Westmark '17]

[see also: Bolzoni '06; Mukherjee, Vogelsang '06; Becher, Neubert, Xu '07; Bonvini, Forte, Ridolfi '10]

- Measurement sets two momentum fractions $x_{a,b} = \dfrac{Q}{E_{\mathrm{cm}}} e^{\pm Y}$
- $\tau = x_a x_b \to 1$ is equivalent to $x_a \to 1$ and $x_b \to 1$

QUESTION: What happens if only $x_a \to 1$ at generic x_b, i.e., $Y \to Y_{\max}$?

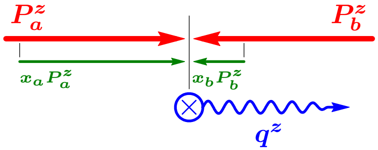
- One large-x PDF suppression instead of two
  - This hadronic limit is actually accessible at colliders!

Born kinematics at large Y.

Consider $p(P_a^\mu)p(P_b^\mu) \to q(x_a P_a^\mu)\bar{q}(x_b P_b^\mu) \to Z(q^\mu) \to \ell^-\ell^+$.

- Use light-cone coordinates $p^+ \equiv p^0 - p^z$ and $p^- \equiv p^0 + p^z$

lab frame



$$P_a^- = E_{\mathrm{cm}} \qquad P_b^+ = E_{\mathrm{cm}}$$

$$q^- = Qe^{+Y} \qquad q^+ = Qe^{-Y}$$

$$x_a = \frac{q^-}{P_a^-} = \frac{Qe^Y}{E_{\mathrm{cm}}}$$

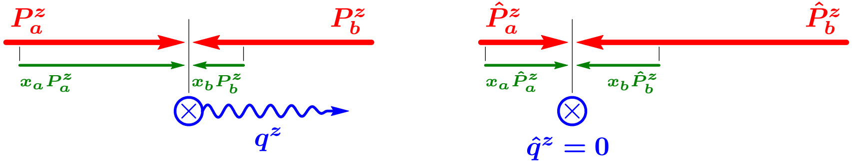
- Kinematic range: $x_a \leq 1 \Rightarrow Y \leq Y_{\max} = \ln \frac{E_{\mathrm{cm}}}{Q}$

Born kinematics at large Y.

Consider $p(P_a^\mu)p(P_b^\mu) \to q(x_a P_a^\mu)\bar{q}(x_b P_b^\mu) \to Z(q^\mu) \to \ell^-\ell^+$.

- Use light-cone coordinates $p^+ \equiv p^0 - p^z$ and $p^- \equiv p^0 + p^z$

lab frame | Z boson rest frame



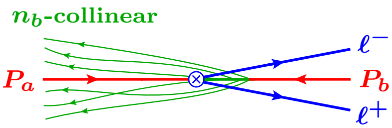
$$P_a^- = E_{\rm cm} \qquad P_b^+ = E_{\rm cm} \qquad \hat{P}_a^- = E_{\rm cm} e^{-Y} \qquad \hat{P}_b^+ = E_{\rm cm} e^{+Y}$$

$$q^- = Q e^{+Y} \qquad q^+ = Q e^{-Y} \qquad \hat{q}^- = Q \qquad \hat{q}^+ = Q$$

$$x_a = \frac{q^-}{P_a^-} = \frac{Q e^Y}{E_{\rm cm}} \qquad\qquad x_a = \frac{\hat{q}^-}{\hat{P}_a^-} = \frac{Q}{E_{\rm cm} e^{-Y}}$$

- Kinematic range: $x_a \leq 1 \Rightarrow Y \leq Y_{\rm max} = \ln \frac{E_{\rm cm}}{Q}$

Factorization at large Y



- Define a power counting parameter:

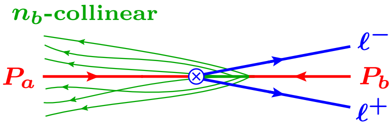
$$\lambda^2 \sim 1 - \frac{q^-}{P_a^-} \sim 1 - x_a \ll 1$$

- Keep q^+ and x_b generic

- Hadronic final state X becomes n_b-collinear near endpoint

$$p_X^\mu = (p_X^-, p_X^+, p_{X,T}) = (P_a^- - q^-, P_b^+ - q^+, p_{X,T}) \sim Q(\lambda^2, 1, \lambda)$$

Factorization at large Y.



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$$p_X^\mu = (p_X^-, p_X^+, p_{X,T}) = (P_a^- - q^-, P_b^+ - q^+, p_{X,T}) \sim Q(\lambda^2, 1, \lambda)$$

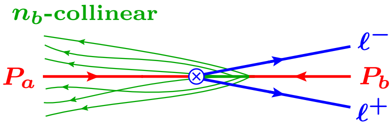
⇒ Resulting factorization theorem at leading power in λ:

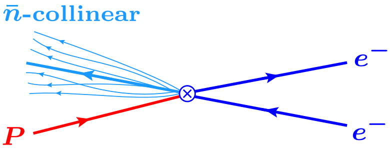
$$\frac{d\sigma}{dq^+ dq^-} = H_{ij}(q^+ q^-, \mu) \int dt\, B_j\Big(t, \frac{q^+}{E_{cm}}, \mu\Big)\, f_i^{thr}\Big(\frac{q^-}{E_{cm}} + \frac{t}{q^+ q^-}, \mu\Big)$$

- $B_j(t, x, \mu)$ is the same beam function as in exclusive N-jettiness factorization
- Key step in derivation: power counting in overall momentum conservation

$$\delta\Big[\underbrace{(\omega_a^- - q^-)}_{\mathcal{O}(\lambda^2)} + \underbrace{k_b^-}_{\mathcal{O}(\lambda^2)}\Big]\, \delta\Big[\underbrace{(\omega_b^+ - q^+)}_{\mathcal{O}(1)} + \underbrace{\cancel{k_a^+}}_{\mathcal{O}(\lambda^2)}\Big]$$

Connection to endpoint DIS.





- Degrees of freedom & convolution structure are the same as for endpoint DIS
- $x_a \sim q^-/E_\mathrm{cm} \to 1$ takes the role of $x_\mathrm{Bjorken} \to 1$, compare:

$$\frac{\mathrm{d}\sigma_\mathrm{DY}}{\mathrm{d}q^+\mathrm{d}q^-} = H_{ij}(q^+q^-,\mu) \int \mathrm{d}t\, B_j\Big(t, \frac{q^+}{E_\mathrm{cm}},\mu\Big)\; f_i^\mathrm{thr}\Big(\frac{q^-}{E_\mathrm{cm}} + \frac{t}{q^+q^-},\mu\Big)$$

$$\frac{\mathrm{d}\sigma_\mathrm{DIS}}{\mathrm{d}x_B} = H_{ij}(Q^2,\mu) \int \mathrm{d}s\, J_j(s,\mu)\; f_i^\mathrm{thr}\Big(x_B + \frac{s}{Q^2},\mu\Big)$$

[Sterman '86; Korchemsky, Marchesini '92; Manohar '03; Becher, Neubert, Pecjak '06]

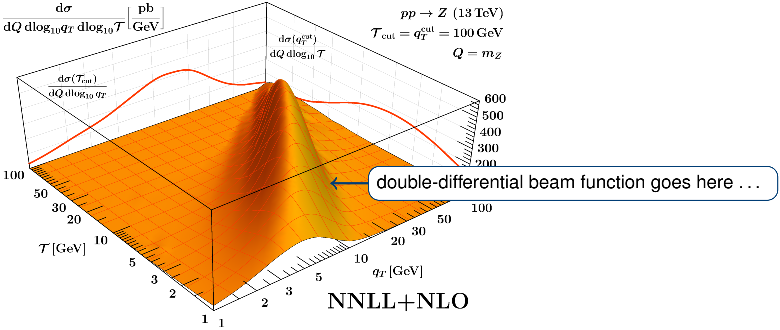
- Second, unconstrained Bjorken fraction $x_b \sim q^+/E_\mathrm{cm}$ is beam function argument

Measuring q_T in addition.

- Only $\bar{n}$-collinear radiation contributes recoil for $q_T \gtrsim \lambda Q$:

$$\frac{d\sigma}{dq^+ dq^- d\vec{q}_T} = H_{ij} \int dt\, B_j\Big(t, \frac{q^+}{E_{cm}}, \vec{q}_T\Big)\, f_i^{thr}\Big(\frac{q^-}{E_{cm}} + \frac{t}{q^+ q^-}\Big)$$

▶ Same double-differential beam function as in analytic 2D $(q_T, \mathcal{T}_0)$ Sudakov!
[Lustermans, JM, Tackmann, Waalewijn '19; see talk by Wouter Waalewijn in October]



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- Change variables from (q^+, q^-) back to $(Q, Y) \leftrightarrow (x_a, x_b)$:

$$x_{a,b} = \frac{Q}{E_{\rm cm}} e^{\pm Y} \quad \neq \quad \frac{q^\pm}{E_{\rm cm}} = \frac{\sqrt{Q^2 + q_T^2}}{E_{\rm cm}} e^{\pm Y}$$

- Power-counting parameter is now $\lambda^2 \sim 1 - x_a$. Reexpand:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\, \mathrm{d}Y\, \mathrm{d}\vec{q}_T} = H_{ij} \int \mathrm{d}t\, B_j(t, x_b, \vec{q}_T)\, f_i^{\rm thr}\Big(x_a + \frac{q_T^2}{2Q^2} + \frac{t}{Q^2}\Big)$$

▶ Results differ by a (relatively subtle) dependence on q_T in the PDF argument

CF. different 1-jettiness definitions in DIS [Kang, Lee, Stewart '13]

Back to the inclusive spectrum.

- Start from the triple-differential spectrum:

$$\frac{d\sigma}{dQ\,dY\,d\vec{q}_T} = H_{ij} \int dt\, B_j(t, x_b, \vec{q}_T)\, f_i^{\text{thr}}\Big(x_a + \frac{q_T^2}{2Q^2} + \frac{t}{Q^2}\Big)$$

Integrate over $\vec{q}_T$, shift $\tilde{t} \equiv t + \frac{q_T^2}{2}$ $\Rightarrow$ inclusive factorization theorem for (Q, Y):

$$\frac{d\sigma}{dQ\,dY} = H_{ij} \int d\tilde{t}\, \tilde{B}_j(\tilde{t}, x_b)\, f_i^{\text{thr}}\Big(x_a + \frac{\tilde{t}}{Q^2}\Big)$$

- Same form as $d\sigma/dq^+dq^-$, but with a new, modified beam function:

$$\tilde{B}_j(\tilde{t}, x) \equiv \int d^2\vec{k}_T\, B_j\Big(\tilde{t} - \frac{k_T^2}{2}, \vec{k}_T, x\Big)$$

  - Identical RGE as $B_j(t, x)$, but different boundary conditions
  - Calculated perturbative beam function matching coefficient $\tilde{\mathcal{I}}_{qk}(\tilde{t}, z)$ through $\mathcal{O}(\alpha_s^2)$ by projecting $\mathcal{I}_{qk}(t, z, \vec{k}_T)$ onto $\tilde{t}$ [two-loop inputs: Gaunt, Stahlhofen '14]

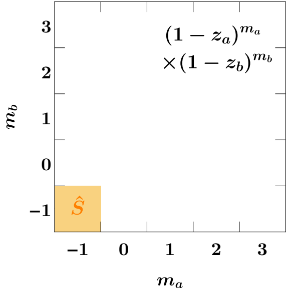
Perturbative checks.

- Parametrize partonic cross section as

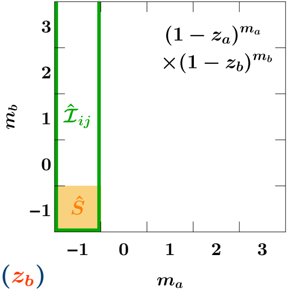
$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_a \mathrm{d}x_b} = \int \frac{\mathrm{d}z_a}{z_a} \frac{\mathrm{d}z_b}{z_b} \, \hat{\sigma}_{ij}(z_a, z_b) \, f_i\Big(\frac{x_a}{z_a}\Big) \, f_j\Big(\frac{x_b}{z_b}\Big)$$

- Soft threshold factorization only predicts terms

$$H_{q\bar{q}} \, \widehat{S}(z_a, z_b) \sim \frac{1}{1 - z_a} \frac{1}{1 - z_b}$$ in the $q\bar{q}$ channel



Perturbative checks.



- Parametrize partonic cross section as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_a \mathrm{d}x_b} = \int \frac{\mathrm{d}z_a}{z_a} \frac{\mathrm{d}z_b}{z_b}\, \hat{\sigma}_{ij}(z_a, z_b)\, f_i\Big(\frac{x_a}{z_a}\Big)\, f_j\Big(\frac{x_b}{z_b}\Big)$$

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$$H_{q\bar{q}}\, \widehat{S}(z_a, z_b) \sim \frac{1}{1-z_a} \frac{1}{1-z_b}$$ in the $q\bar{q}$ channel

- Generalized threshold factorization predicts all terms $\sim \dfrac{F(z_b)}{1-z_a}$

$$\sigma_{qj}(z_a, z_b) = H_{q\bar{q}}(Q^2) \times \hat{\mathcal{I}}_{\bar{q}j}(z_a, z_b) \times \big[1 + \mathcal{O}(1-z_a)\big]$$

  ✔ Checked against known NNLO partonic cross sections
  [Anastasiou, Dixon, Melnikov, Petriello '02-'03]

- Corollary: Soft function captures singular terms within the beam function

$$\hat{\mathcal{I}}_{ij}(z_a, z_b) = \delta_{ij}\, \widehat{S}(z_a, z_b) \times \big[1 + \mathcal{O}(1-z_b)\big]$$ ✔ Checked through $\mathcal{O}(\alpha_s^2)$

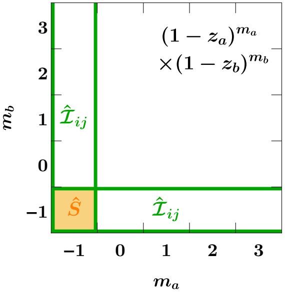
  ▶ Used this relation to extract leading terms in the beam function at $\mathcal{O}(\alpha_s^3)$
  [Billis, Ebert, JM, Tackmann '19]

Generalized threshold factorization theorem.

Combine $(x_a \to 1, x_b)$ with $(x_a, x_b \to 1)$ to describe all endpoints in pp collisions:

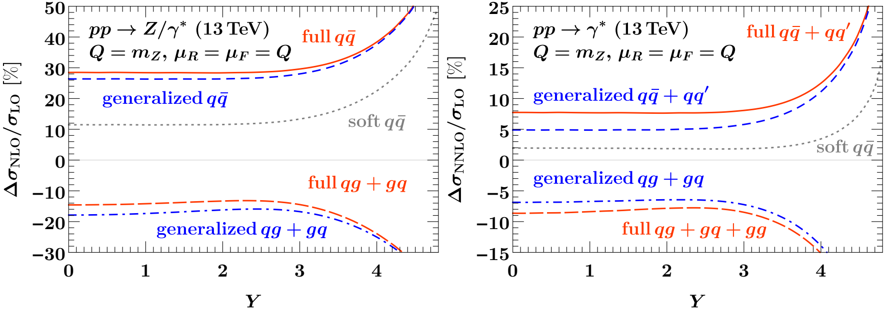
$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_a \mathrm{d}x_b} = H_{ij} \times \Big[f_i^{\rm thr} \tilde{B}_j + \tilde{B}_i f_j^{\rm thr} - S f_i^{\rm thr} f_j^{\rm thr} \Big] \times \Big\{ 1 + \mathcal{O}[(1-x_a)(1-x_b)] \Big\}$$

$$\hat{\sigma}_{k\ell}(z_a, z_b) = H_{ij} \times \Big[\delta_{ik}\, \hat{\mathcal{I}}_{j\ell}(z_a, z_b) + \hat{\mathcal{I}}_{ik}(z_b, z_a)\, \delta_{j\ell} - \widehat{S}(z_a, z_b)\, \delta_{ik} \delta_{j\ell} \Big] \times \Big\{ 1 + \mathcal{O}[(1-z_a)(1-z_b)] \Big\}$$



Application: Fixed-order approximants at (N)NLO.

- Use $\hat{\sigma}_{k\ell} = H_{ij}\big[\delta_{ik}\hat{\mathcal{I}}_{j\ell} + \hat{\mathcal{I}}_{ik}\delta_{j\ell} - \widehat{S}\delta_{ik}\delta_{j\ell}\big]$ to approximate full cross section
- Use physical PDFs $\Rightarrow$ singular limits get enhanced by steep PDFs



- Much better approximation than soft threshold factorization alone
  - Captures offdiagonal channels (like $qg \to Zq$)
  - Becomes exact at large Y

Drell-Yan rapidity spectrum at N^{3}LO.

- Generalized threshold factorization theorem + beam function RGE predict all terms $\sim \mathcal{L}_n(1-z_a)$ at N^{3}LO (much richer structure than just soft):

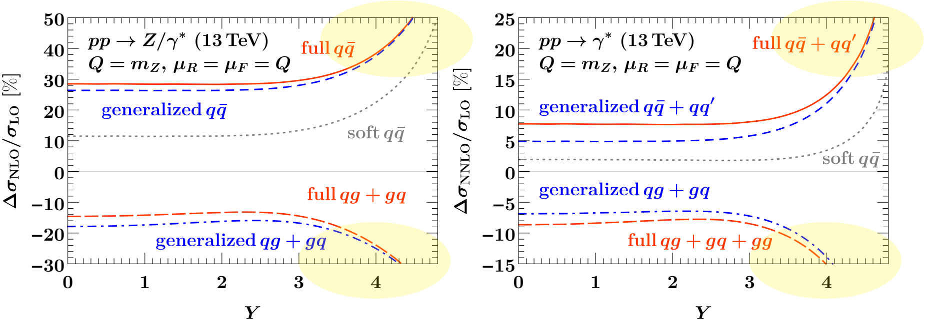
$$\hat{\sigma}_{ij}\Big|_{\alpha_s^3 \mathcal{L}_5} = H_{ij}^{(0)} \delta(1-z_b) \, \frac{(\Gamma_0^i)^3}{8}$$

$$\hat{\sigma}_{ij}\Big|_{\alpha_s^3 \mathcal{L}_4} = H_{ij}^{(0)} \delta(1-z_b) \Big[-\frac{5}{12}\beta_0 - \frac{5}{16}\gamma_{B\,0}^i\Big] (\Gamma_0^i)^2 + H_{ik}^{(0)} P_{kj}^{(0)}(z_b) \, \frac{5}{8} (\Gamma_0^i)^2$$

$$\begin{aligned}\hat{\sigma}_{ij}\Big|_{\alpha_s^3 \mathcal{L}_3} &= H_{ij}^{(1)} \delta(1-z_b) \, \frac{(\Gamma_0^i)^2}{2} + H_{ik}^{(0)} \tilde{I}_{kj}^{(1)}(z_b) \, \frac{(\Gamma_0^i)^2}{2} \\ &+ H_{ik}^{(0)} \big[P_{k\ell}^{(0)} \otimes P_{\ell j}^{(0)}\big](z_b) \, \Gamma_0^i \\ &+ H_{ik}^{(0)} P_{kj}^{(0)}(z_b) \Big[-\frac{5}{3}\beta_0 - \gamma_{B\,0}^i\Big] \Gamma_0^i \\ &+ H_{ij}^{(0)} \delta(1-z_b) \Big[-\frac{\pi^2}{6}(\Gamma_0^i)^2 + \frac{\beta_0^2}{3} + \frac{(\gamma_{B\,0}^i)^2}{4} \\ &\qquad + \Gamma_1^i + \frac{5}{6}\beta_0 \gamma_{B\,0}^i\Big] \Gamma_0^i\end{aligned}$$

- See [Billis, Ebert, JM, Tackmann '19] for $\mathcal{L}_2, \mathcal{L}_1, \mathcal{L}_0$ in the α_s^3 beam function

Outlook: Resummation at large Y.



- Fixed-order expansion of the cross section badly convergent at large Y where $\alpha_s \ln^2(1 - x_a)$ becomes $\mathcal{O}(1)$
- Can resum these terms using beam function RGE to get best perturbative prediction e.g. for PDF fits in this region
- All ingredients for N^3LL resummation already implemented in SCETlib
  [Ebert, JM, Tackmann; public version in preparation]

Outlook: Combining large and small x.

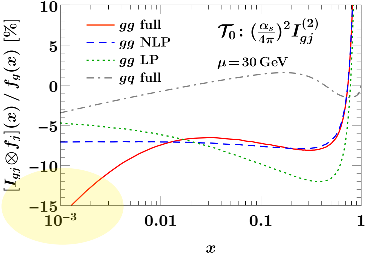
- High-energy (or small x or BFKL) limit of QCD also features divergent behavior at fixed order, requiring resummation
- Previous combination of small and large x effects includes them separately

$$\sigma = \sigma_{\mathrm{FO}} + \left[\sigma_{\mathrm{threshold}} - \sigma_{\mathrm{threshold}}^{\mathrm{FO}}\right] + \left[\sigma_{\mathrm{BFKL}} - \sigma_{\mathrm{BFKL}}^{\mathrm{FO}}\right]$$

[e.g. Bonvini, Marzani '18 for the $gg \to H$ inclusive cross section]

- Generalized threshold factorization implies a nontrivial overlap as $x_a \to 1$ and $x_b \ll 1$ (or vice-versa)

▶ Need to study BFKL dynamics within the beam function



Summary.

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Resumming fiducial power corrections at N^{3}LL:

- Linear power corrections in $\mathrm{d}\sigma_{\mathrm{fid}}/\mathrm{d}q_T$ uniquely arise from lepton cuts
- Enables their resummation at N^{3}LL, including the physical effect of recoil
- Showed first results for the lepton p_T spectrum at N^{3}LL
- Confirmed presence of resummation effects in the fiducial Z rapidity spectrum – stay tuned!

Generalized threshold factorization for hadron-collider rapidity spectra:

- Extend soft threshold factorization to full collinear dynamics at large $|Y|$
- Predicts (or resums) a much richer set of terms than soft threshold alone
- Plenty of other interesting applications – stay tuned

Things I didn't have time to talk about:

- Two-dimensional resummation in $(q_T, \mathcal{T}_0)$ [but see Wouter's recent talk here!], N^{3}LO q_T and $\mathcal{T}_0$ subtractions, quark mass effects in q_T spectra, ...
- The toolbox for all of this: the SCETLIB C++ resummation library

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Thank you for your attention!

Backup.

Details of RG evolution for q_T

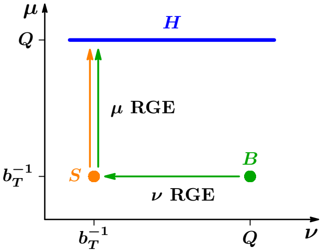
- Factorize:

$$\frac{d\sigma}{d^2\vec{q}_T} = H(Q,\mu) \int d^2\vec{b}_T\, e^{i\vec{b}_T\cdot\vec{q}_T} B_q(x_a,\vec{b}_T,\mu,\nu) B_{\bar{q}}(x_b,\vec{b}_T,\mu,\nu) S(\vec{b}_T,\mu,\nu)$$

- Factorization splits Sudakov logarithms as

$$\ln^2\frac{Q}{q_T} = \ln^2\frac{Q}{\mu} + 2\ln\frac{q_T}{\mu}\ln\frac{\nu}{Q} + \ln\frac{q_T}{\mu}\ln\frac{\mu\, q_T}{\nu^2}$$

- Derive RGEs governing the log structure in hard, beam and soft functions
- Combined solution to RGEs resums all large logs
  - Special for TMD resummation: two-scale evolution in virtuality μ and Collins-Soper scale ν



- Resummed cross section (simplified):

$$\frac{d\sigma}{d^2\vec{q}_T} = H(Q,\mu_H) \int d^2\vec{b}_T\, e^{i\vec{b}_T\cdot\vec{q}_T} B_a(x_a,\vec{b}_T,\mu_B,\nu_B) B_b(x_b,\vec{b}_T,\mu_B,\nu_B)$$

$$\times\, S(\vec{b}_T,\mu_S,\nu_S)\exp\left[\int_{\mu_B}^{\mu_H}\frac{d\mu'}{\mu'}\gamma(Q,\mu')\right]\exp\left[\ln\frac{\nu_B}{\nu_S}\gamma_\nu(b_T,\mu_B)\right]$$

Rapidity evolution.

So what happens inside $\otimes_{\vec{q}_T}$, really?

- Total $\vec{q}_T$ is the vectorial sum of (an all-order set of) soft and collinear emissions
- This is hard to solve: $\nu \frac{\mathrm{d}}{\mathrm{d}\nu} S(\vec{q}_T, \mu, \nu) = \gamma_\nu(\vec{q}_T, \mu) \otimes_{\vec{q}_T} S(\vec{q}_T, \mu, \nu)$

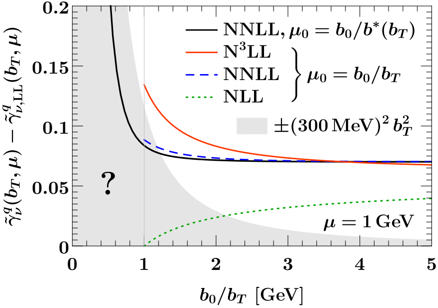
[... but can be done in principle; Ebert, Tackmann ’16]

► Take a Fourier transform $\vec{q}_T \mapsto \vec{b}_T$. Get something that is easy to solve:

$$\nu \frac{\mathrm{d}}{\mathrm{d}\nu} \tilde{S}(b_T, \mu, \nu) = \tilde{\gamma}_\nu(b_T, \mu)\, \tilde{S}(b_T, \mu, \nu)$$

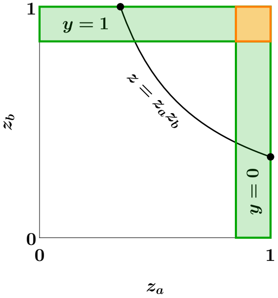
- Collins-Soper kernel $\tilde{\gamma}_\nu = 2\tilde{\gamma}_\zeta$ is known to three loops [Li, Zhu ’16]
- For large $b_T \leftrightarrow$ small $q_T \sim 1/b_T$, long-distance contributions become as important as 2 & 3-loop corrections

► Would be extremely interesting to constrain them from lattice

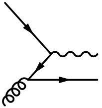


Analytic NLO check: qg

- NNLO Drell-Yan rapidity spectrum is known analytically
  [Anastasiou, Dixon, Melnikov, Petriello '02-'03]
  - ▶ Parametrized in terms of $z = z_a z_b$ and $y \in [0, 1]$
  - ✔ Analytically expand NLO results as $z_a \to 1$ with z_b generic $\to$ full agreement



- Instructive to look at some NLO terms explicitly:



$$= \sigma_B T_F \Big\{ \delta(y) \Big[2 P_{qg}(z) \ln \frac{(1-z)^2}{z} + 4z(1-z) \Big] + 2\, P_{qg}(z) \mathcal{L}_0(y) + 4z(1-z) + 2(1-z)^2 y \Big\}$$

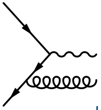
- Most nontrivial term, with $\mathcal{L}_0(x) \equiv \Big[\frac{1}{x}\Big]_+$:

$$\mathrm{d}z\, \mathrm{d}y\, P_{qg}(z)\, \mathcal{L}_0(y)$$

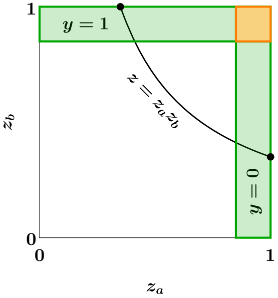
$$= \mathrm{d}z_a \mathrm{d}z_b\, P_{qg}(z_b) \Big\{ \mathcal{L}_0(1-z_a) + \delta(1-z_a) \Big[\ln \frac{2 z_b}{1+z_b} - \ln(1-z_b) \Big] \Big\} + \mathcal{O}[(1-z_a)^0]$$

Missing in $\mathcal{I}$, but captured by $\tilde{\mathcal{I}}$ ...

Analytic NLO check: $q\bar{q}$.



$$= \sigma_B C_F \Big\{ [\delta(y) + \delta(1-y)] \Big[\delta(1-z)(4\zeta_2 - 8) + 8(1+z^2)\mathcal{L}_1(1-z) - 2\frac{1+z^2}{1-z}\ln z + 2 - 2z \Big] + 2(1+z^2)\,\mathcal{L}_0(1-z)[\mathcal{L}_0(y) + \mathcal{L}_0(1-y)] - 2(1-z) \Big\}$$



- Most nontrivial term, with $\mathcal{L}_n(x) \equiv \Big[\frac{\ln^n x}{x}\Big]_+$

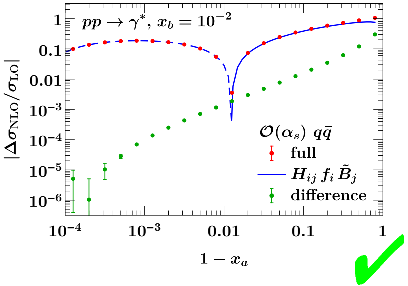
$$\mathrm{d}z\,\mathrm{d}y\;\; \mathcal{L}_0(1-z)[\mathcal{L}_0(y) + \mathcal{L}_0(1-y)]$$

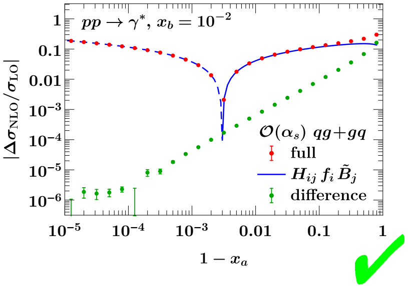
$$= \mathrm{d}z_a \mathrm{d}z_b \Big[\frac{\pi^2}{6}\delta(1-z_a)\delta(1-z_b) - \mathcal{L}_1(1-z_a)\delta(1-z_b) + \mathcal{L}_0(1-z_a)\mathcal{L}_0(1-z_b) - \delta(1-z_a)\mathcal{L}_1(1-z_b) + \delta(1-z_a)\frac{\ln\frac{2z_b}{1+z_b}}{1-z_b} + \delta(1-z_b)\frac{\ln\frac{2z_a}{1+z_a}}{1-z_a} + \frac{1}{(1+z_a)(1+z_b)} \Big]$$

- Several soft threshold factorizations for the rapidity spectrum neglect this term, and conclude $\hat{\sigma}_{ij}(z, y) = [\delta(y) + \delta(1-y)]\,\hat{\sigma}^{\mathrm{soft}}_{ij}(z) + \mathcal{O}(1)$
  [Bolzoni '06; Mukherjee, Vogelsang '06; Becher, Neubert, Xu '07; Bonvini, Forte, Ridolfi '10]

✗ Distributional identity implies this misses leading-power soft terms already at LL.

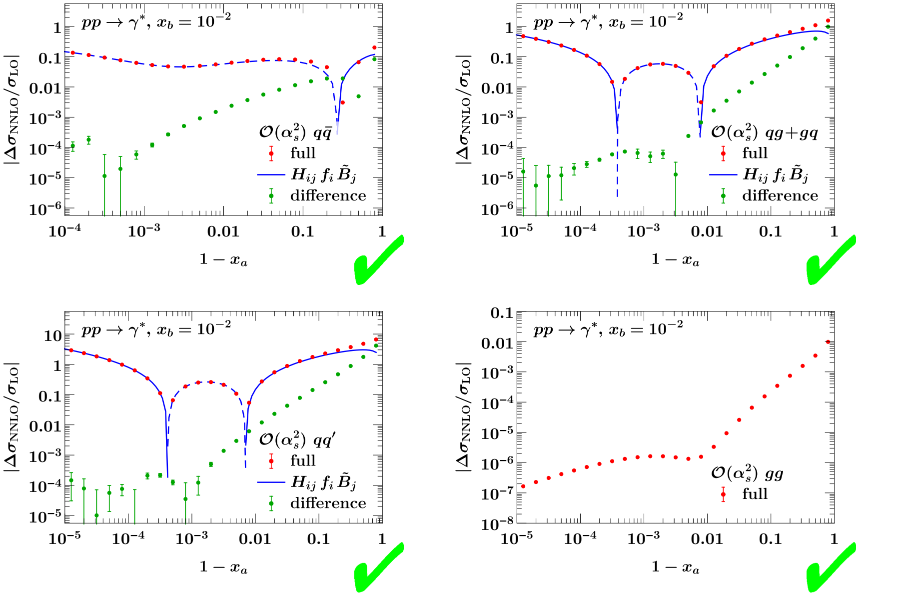
Numerical NLO check.



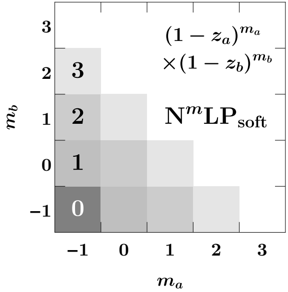


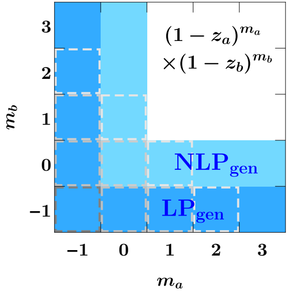
- Use flat PDFs $f_i(x) = \theta(1 - x)$, compare `Vrap 0.9` to $H_{ij} f_i \tilde{B}_j$ as $x_a \to 1$
  - Checks complete singular limit including terms $\sim \delta(1 - z_a)$
  - $\mathcal{L}_n(1 - z_a)$ turns into $\ln^n(1 - x_a)$, $\mathcal{O}(1)$ turns into $\mathcal{O}(1 - x_a)$

Numerical NNLO check.

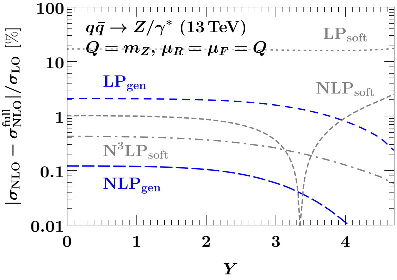


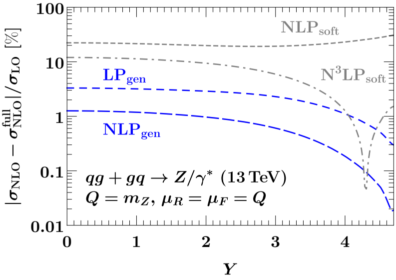
Soft vs. generalized beyond leading power.



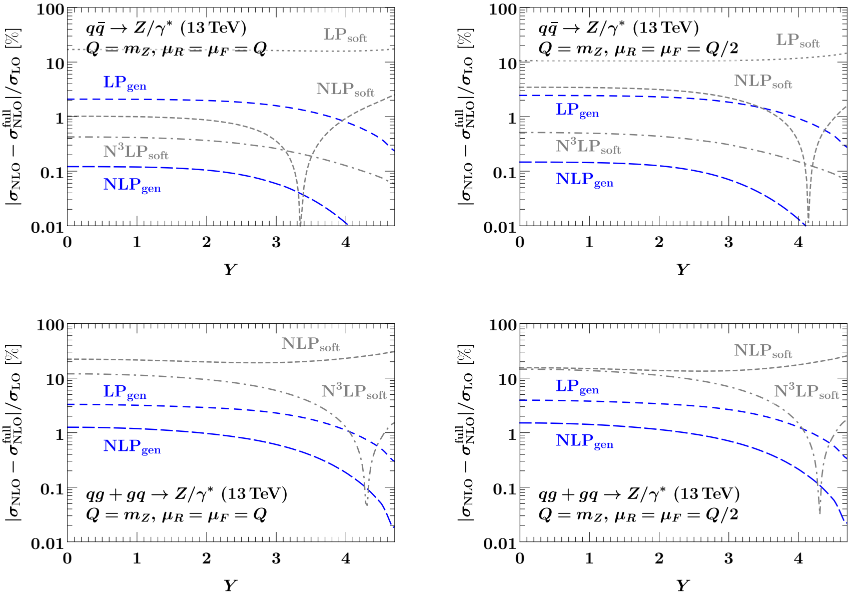


- Generalized threshold expansion converges much faster for all Y
- Perfect convergence at large Y by construction
- For qg channel, $\mathrm{LP}_{\mathrm{gen}}$ already better than $\mathrm{N}^3\mathrm{LP}_{\mathrm{soft}}$

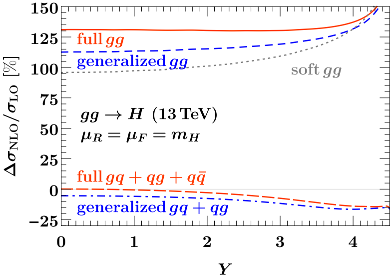


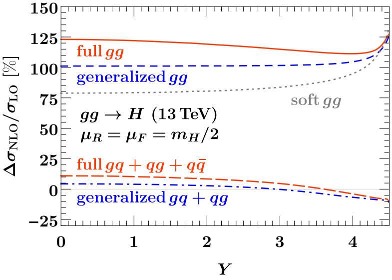


More on NLO $pp \to Z$ beyond leading power.

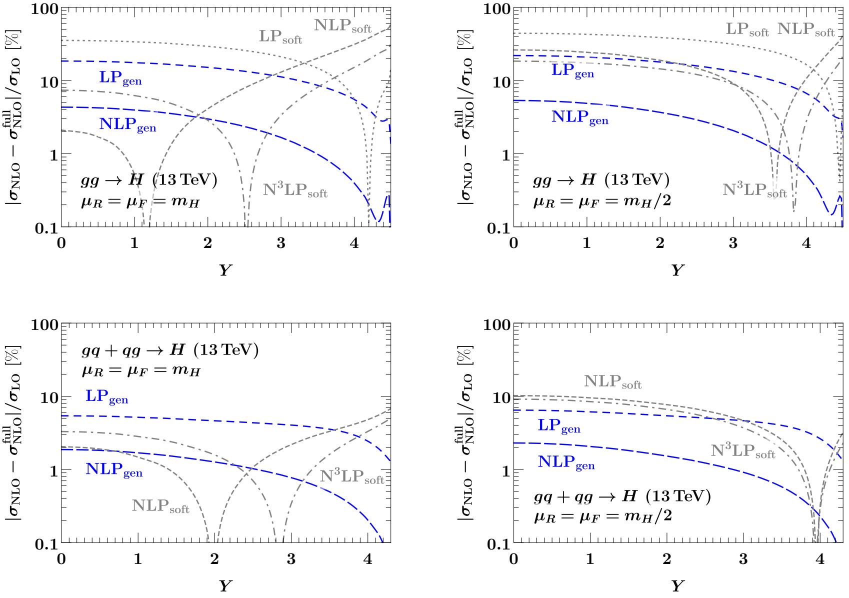


NLO $gg \to H$ approximants.





NLO $gg \to H$ at subleading power.



Translating $\mathcal{L}_0(1-z)[\mathcal{L}_0(y)+\mathcal{L}_0(1-y)]$.

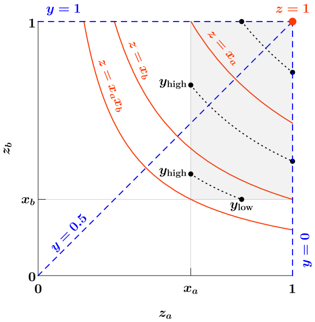
To derive this, with $\mathcal{L}_n(x) \equiv \left[\frac{\ln^n x}{x}\right]_+$:

$$\mathrm{d}z\,\mathrm{d}y\;\; \mathcal{L}_0(1-z)[\mathcal{L}_0(y)+\mathcal{L}_0(1-y)]$$

$$= \mathrm{d}z_a\mathrm{d}z_b \Big[\frac{\pi^2}{6}\delta(1-z_a)\delta(1-z_b) - \mathcal{L}_1(1-z_a)\delta(1-z_b) + \mathcal{L}_0(1-z_a)\mathcal{L}_0(1-z_b)$$

$$- \delta(1-z_a)\mathcal{L}_1(1-z_b) + \delta(1-z_a)\frac{\ln\frac{2z_b}{1+z_b}}{1-z_b} + \delta(1-z_b)\frac{\ln\frac{2z_a}{1+z_a}}{1-z_a} + \frac{1}{(1+z_a)(1+z_b)}\Big]$$

1. Make an ansatz that agrees with the LHS in the bulk, i.e., at $z_a, z_b < 1$.
   - Boundary condition at $z_a = 1$ is a function of z_b & vice versa.
   - Overall boundary condition at $z_a = z_b = 1$ is also unknown.
2. Fix all boundary conditions by integrating LHS and the ansatz over the domain $x_a \leq z_a \leq 1$, $x_b \leq z_b \leq 1$.



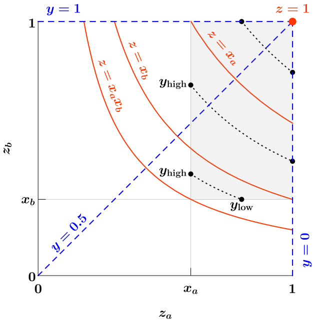
Soft limit: Why y matters.

- y and $1 - y$ multiply the leading dependence on $1 - z$ in the PDF arguments:

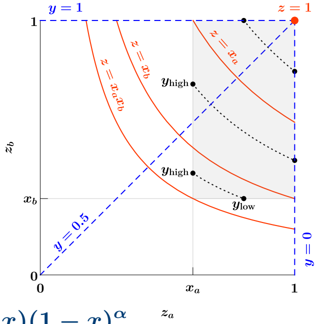
$$f_i\left(\frac{x_a}{z_a(z,y)}\right) = f_i\left(x_a\left\{1 + y(1-z) + \mathcal{O}[(1-z)^2]\right\}\right)$$
$$f_j\left(\frac{x_b}{z_b(z,y)}\right) = f_j\left(x_b\left\{1 + (1-y)(1-z) + \mathcal{O}[(1-z)^2]\right\}\right)$$

- Fixing y to 0 and 1 (or $1/2$) amounts to neglecting (or multiplying with an arbitrary factor) the leading dependence on $1 - z$ in the inclusive luminosity:

$$ff\left(\frac{\tau}{z}\right) = ff\left(\tau\left\{1 + (1-z) + \mathcal{O}[(1-z)^2]\right\}\right)$$



Soft limit: Why y matters.



- Assume hadronic threshold $1 - x_{a,b} \sim 1 - z \ll 1$,

$$a(z,y) \equiv \frac{1}{2}\mathcal{L}_0(1-z)\big[\delta(y) + \delta(1-y)\big]$$

$$b(z,y) \equiv \mathcal{L}_0(1-z)\,\delta\Big(y - \frac{1}{2}\Big)$$

- If the y dependence were subleading as $z \to 1$, we would expect equal contributions from a and b to the hadronic rapidity spectrum up to power corrections.

- However, for a simple power-law PDF $f(x) \equiv \theta(1-x)(1-x)^\alpha$,

$$\int \mathrm{d}z\,\mathrm{d}y\, a(z,y)\, f\Big[\frac{x_a}{z_a(z,y)}\Big] f\Big[\frac{x_b}{z_b(z,y)}\Big]$$

$$= f(x_a) f(x_b) \Big[\frac{1}{2}\ln(1-x_a) + \frac{1}{2}\ln(1-x_b) - H_\alpha + \mathcal{O}(1-x_a, 1-x_b)\Big],$$

$$\neq \int \mathrm{d}z\,\mathrm{d}y\, b(z,y)\, f\Big[\frac{x_a}{z_a(z,y)}\Big] f\Big[\frac{x_b}{z_b(z,y)}\Big]$$

$$= f(x_a) f(x_b) \Big[\ln(1 - \max\{x_a, x_b\}) - H_\alpha + \mathcal{O}(1-x_a, 1-x_b)\Big]$$

- ▶ Results differ as soon as $x_a \neq x_b$ ($Y \neq 0$).