Quantum entanglement effects in Geant4 for PET applications

D. P. Watts (1), **J. Bordes** (1), J. R. Brown (1), A. Cherlin (2), R. Newton (1), J. Allison (3, 4), M. Bashkanov (1), N. A. Zachariou (1)

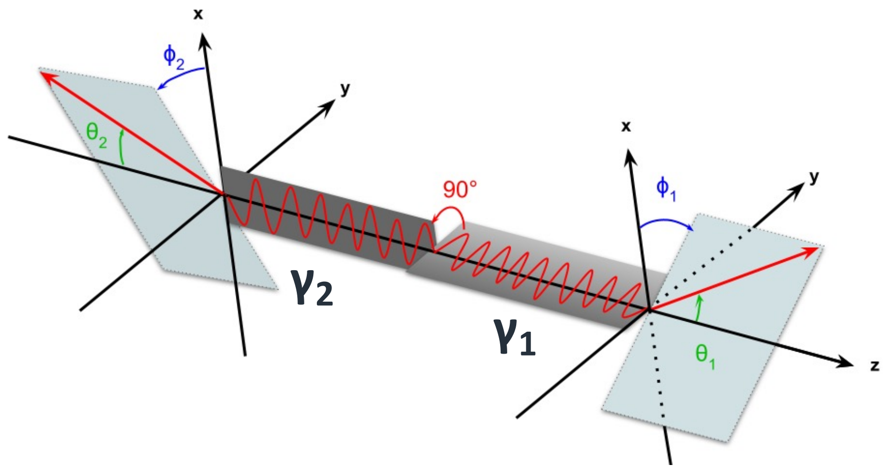
1. University of York
2. Kromek Group
3. Geant4 Associates International Ltd.
4. University of Manchester

Entangled double Compton scattering

- In medical PET, annihilations produce two γ in an entangled polarization state.
- Double Compton scattering cross section for the entangled state:

$$\boxed{\frac{d^2\sigma}{d\Omega_1 d\Omega_2} = \frac{r_0^{\ 4}}{16}\left(K_a(\theta_1\ \theta_2) - K_b(\theta_1\ \theta_2)\ \mathbf{\cos(2\Delta\phi)}\right)}$$

e.g. Pryce and Ward 1947 Nature 160 : 435



- $\mathbf{\cos(2\Delta\phi)}$ modulation: amplitude increased by the entanglement
- Enhancement factor: $R = P(\Delta\phi = \pm 90°)/P(\Delta\phi = 0°)$
- Peaks at 2.85 for $\theta_1 = \theta_2 = 81.7°$

Quantum entangled Geant4 (QE-Geant4)

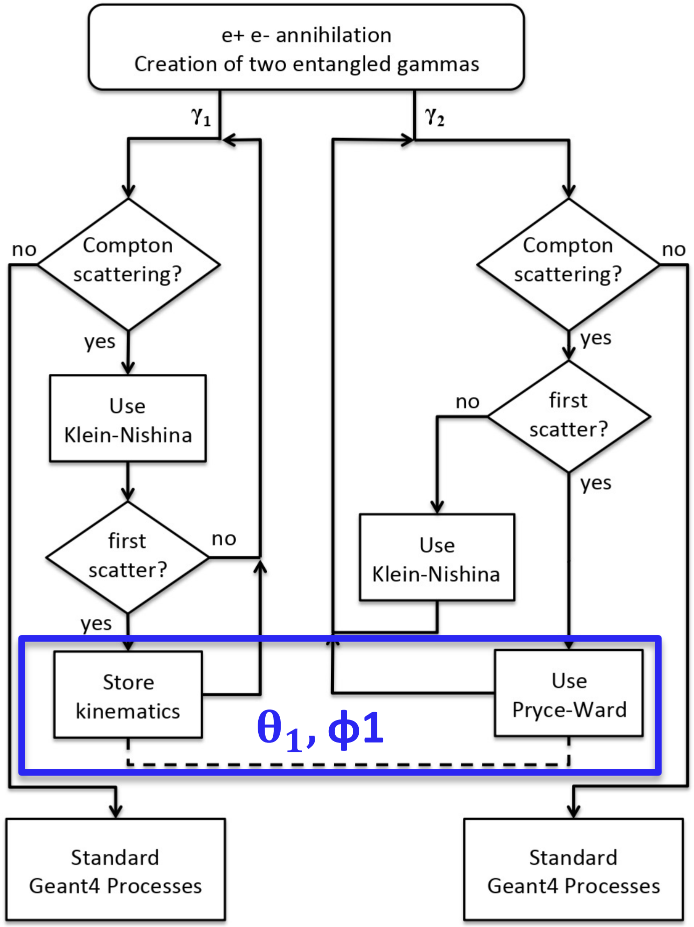
- Quantum entanglement effects in double Compton scattering of annihilation γ added to the Livermore physics models

nature COMMUNICATIONS

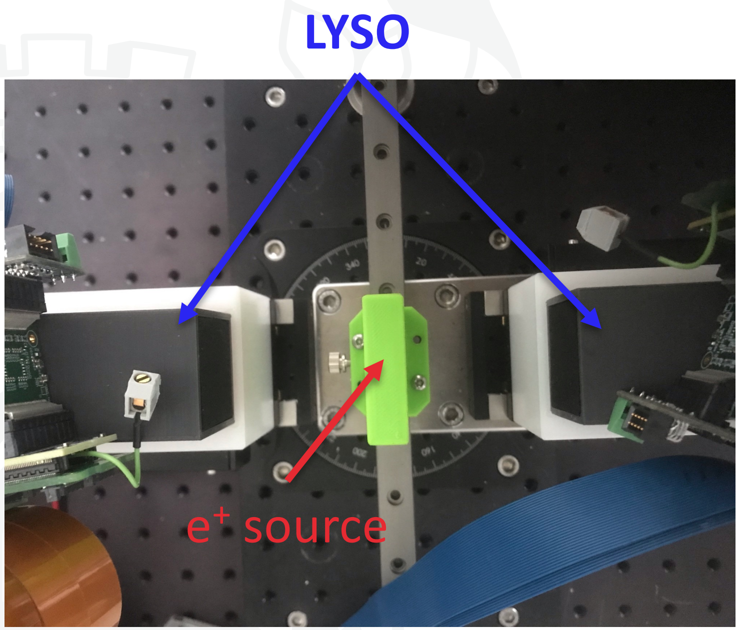
Photon quantum entanglement in the MeV regime and its application in PET imaging

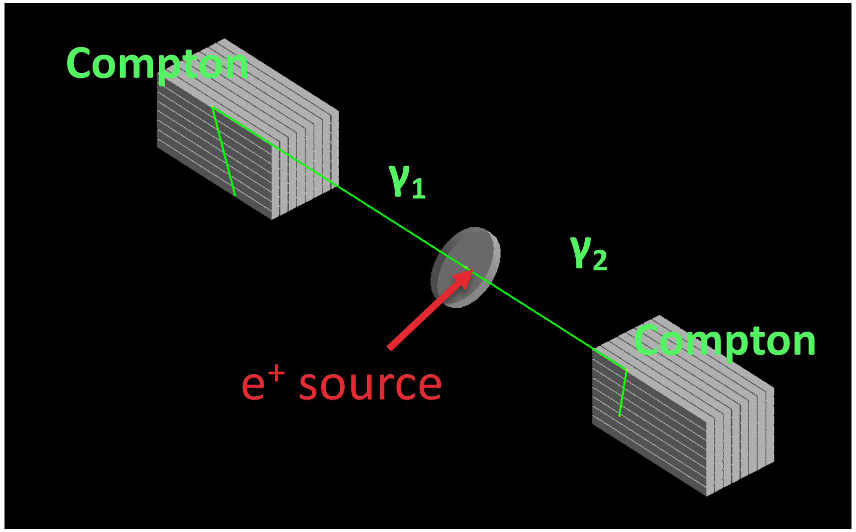
D. P. Watts [1✉], J. Bordes [1], J. R. Brown [1], A. Cherlin [2], R. Newton [1], J. Allison [3,4], M. Bashkanov [1], N. Efthimiou [1,5] & N. A. Zachariou [1]

- Innovative extensions enabling communication between the individual annihilation γ tracking processes
- A complete loss of entanglement is assumed after a double Compton scattering – discussed in a later slide



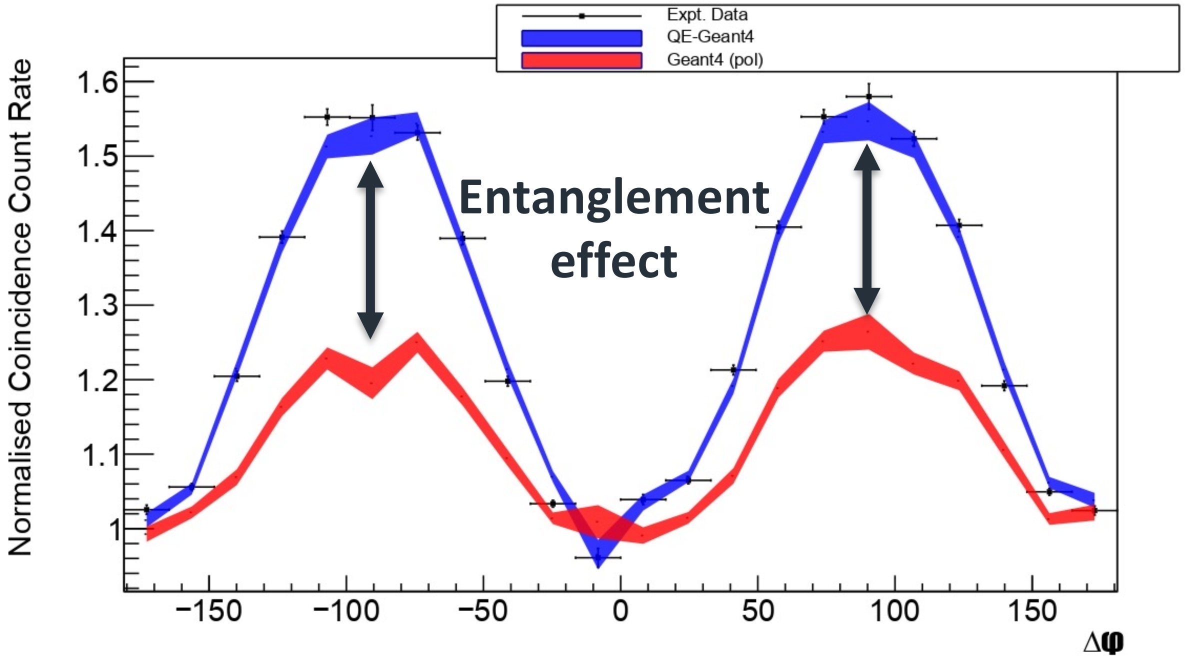
Validation – No scatter before detection





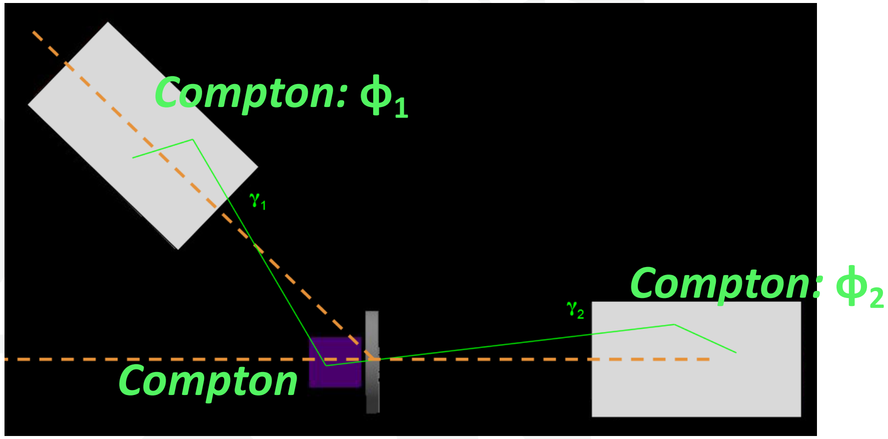
- ^{22}Na at 2.4 MBq
- Back-to-back high density LYSO detectors (7.1 g/cm^3) separated by 140 mm
- 8 x 8 arrays of 3 x 3 x 50 mm^3 crystals
- Geant4 and experimental data analysed with the same code and cuts

Validation – No scatter before detection



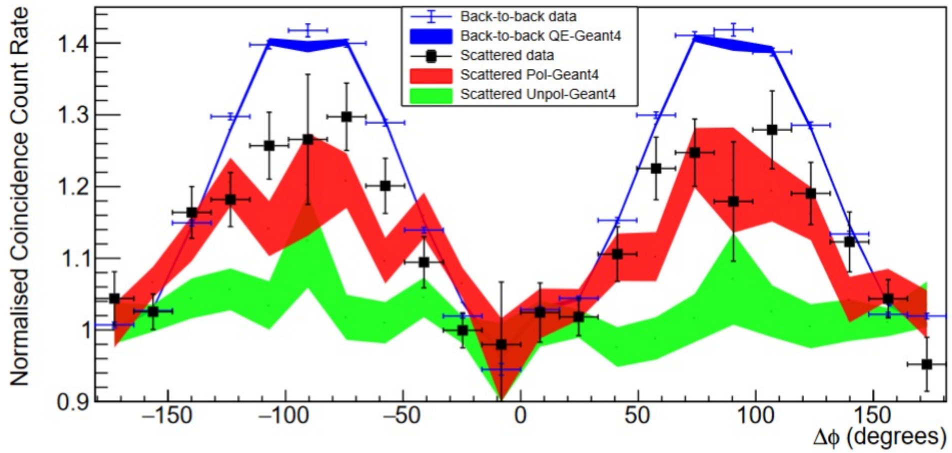
- LYSO directly images the cos(2Δϕ) distribution, $R = 1.56$
- The experimental data:
  - agree with QE-Geant4 ($R = 1.54$)
  - clearly disagree with unentangled Livermore polarised Geant4 ($R = 1.25$)

Testing the entanglement loss assumption



- Addition of an active LYSO scatterer and rotation of one detector by 45°
- Comparing $\Delta\phi = \phi_1 - \phi_2$ with the back-to-back setup and non-entangled Geant4

- After a $\theta \sim 45°$ scattering, the $\Delta\phi$ correlation:
  - agrees with unentangled Geant4
  - is lower than the back-to-back (entangled) case
- Dependence on θ ? Data for smaller θ are being analysed



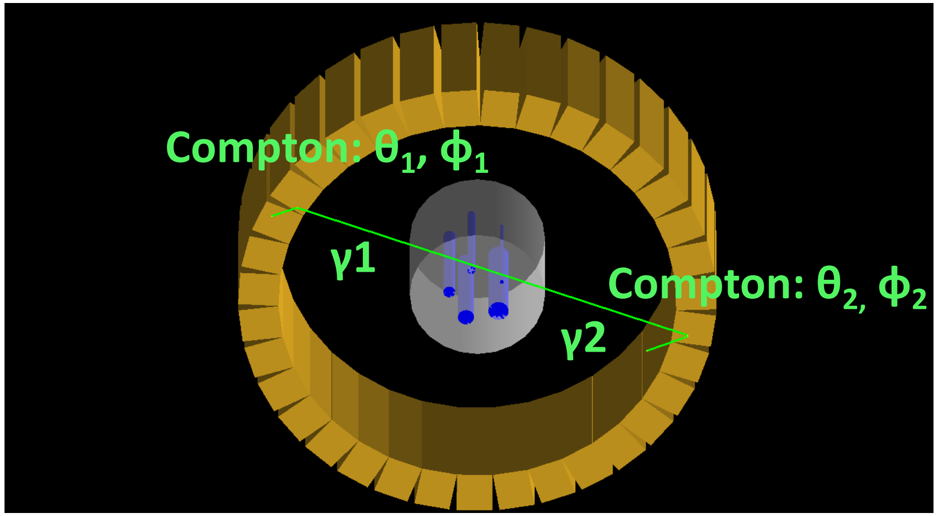
Benefits of QE-PET

- To illustrate the interest of the quantum entanglement in PET, we produced images from a QE-Geant4 simulation of a preclinical system

- Focus on double Compton scattering coincidences (see illustration)

- Flag the true (entangled) and random (unentangled) coincidences

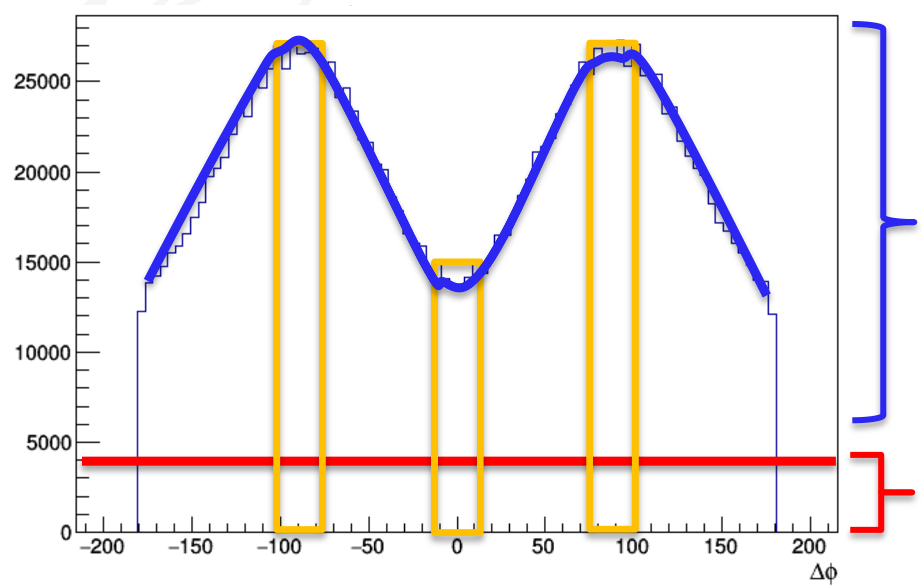
- Filtered back projection



Geant4 simulation: NEMA-NU4 (PMMA, hot rods), 4-rings scanner

Benefits of QE-PET – scatter coincidences

- $|\Delta\Phi|$ distribution from the preclinical PET simulation



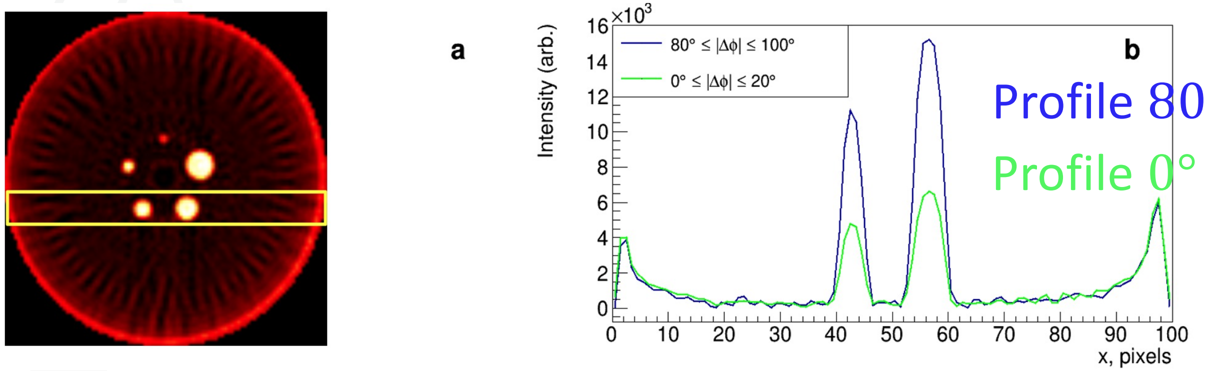
$\cos(2\Delta\phi)$ modulation from the (entangled) true coincidences

Baseline from the (unentangled) random coincidences

- Applying a window around:
  - $|\Delta\Phi| = 90°$ yields the highest true fraction
  - $|\Delta\Phi| = 0°$ yields the lowest true fraction

Benefits of QE-PET – random coincidences

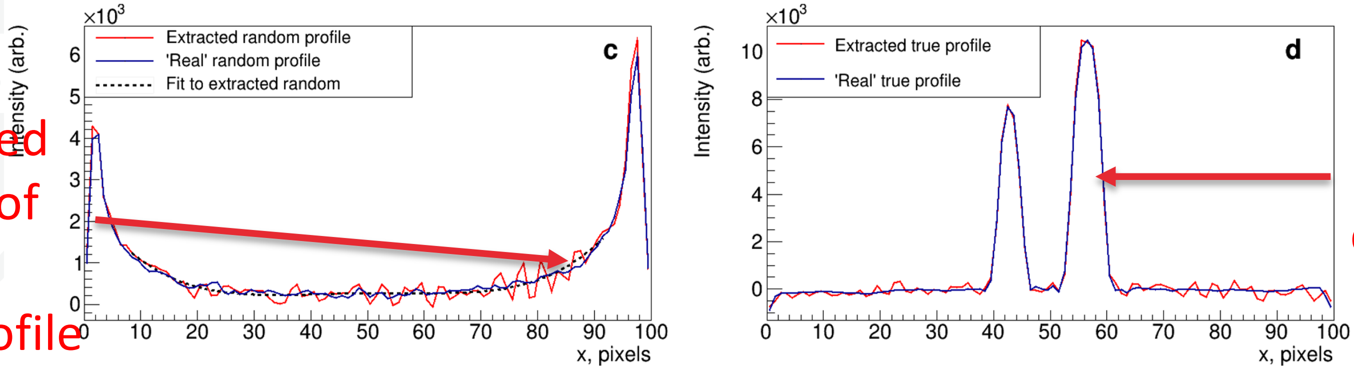
- Profiles for the highest/lowest true fraction |ΔΦ| windows



Profile $80° \leq |\Delta\Phi| \leq 100°$

Profile $0° \leq |\Delta\Phi| \leq 20°$

- Random/true coincidence profiles extracted using these |ΔΦ| profiles and weighting factors from QE-Geant4



Spatially resolved determination of random coincidence profile

Isolating true coincidence profile

Conclusion

- Entanglement effects in double Compton scattering of annihilation gammas were modelled with Geant4

- Improves the realism of the PET simulations. Ongoing work to check the impact on the scanner hit position in case of double Compton scattering

- If we observe that the entanglement is not completely lost at low scattering angles, we will upgrade QE-Geant4 to reproduce the effect of the "residual" entanglement

- Proof-of-principle with a preclinical acquisition - spatially resolved extraction of both the shape and magnitude of the random background

Verification and impact of QE-Geant4

- QE-Geant4 reproduces accurately the entangled theory

- Lowest Δϕ correlation with unentangled Geant4 and theory ($R = 1.63$)