Investigating Quasi Monte Carlo in Geant4 / Gate simulations

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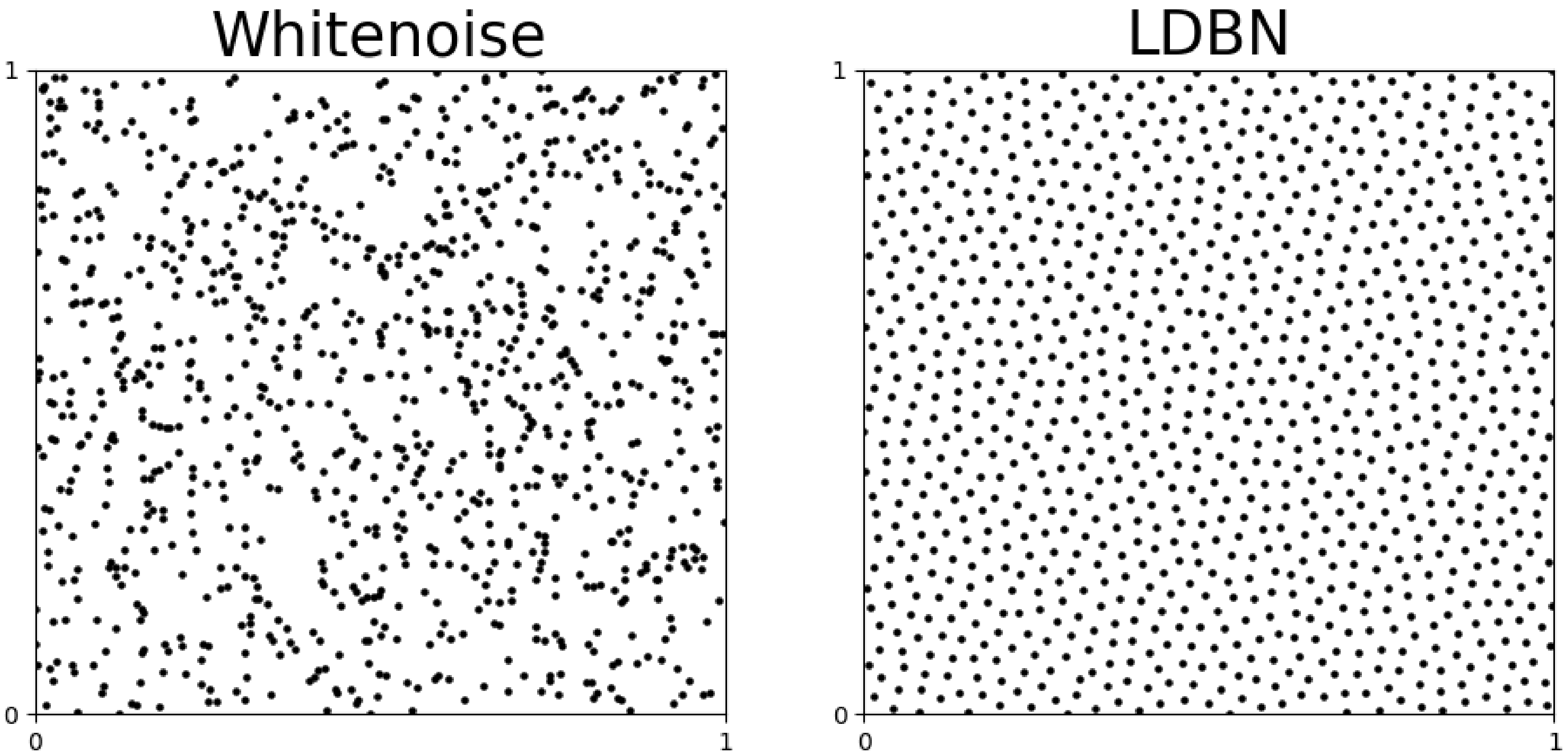
25/10/2022

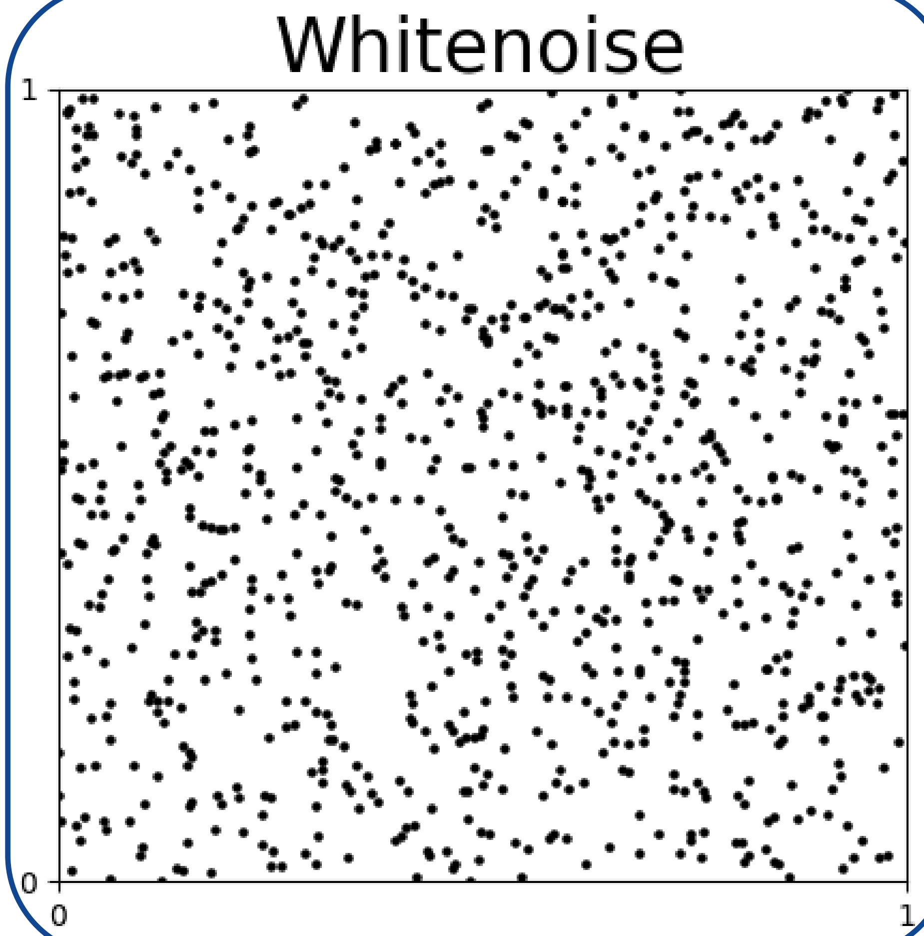
Points Matter

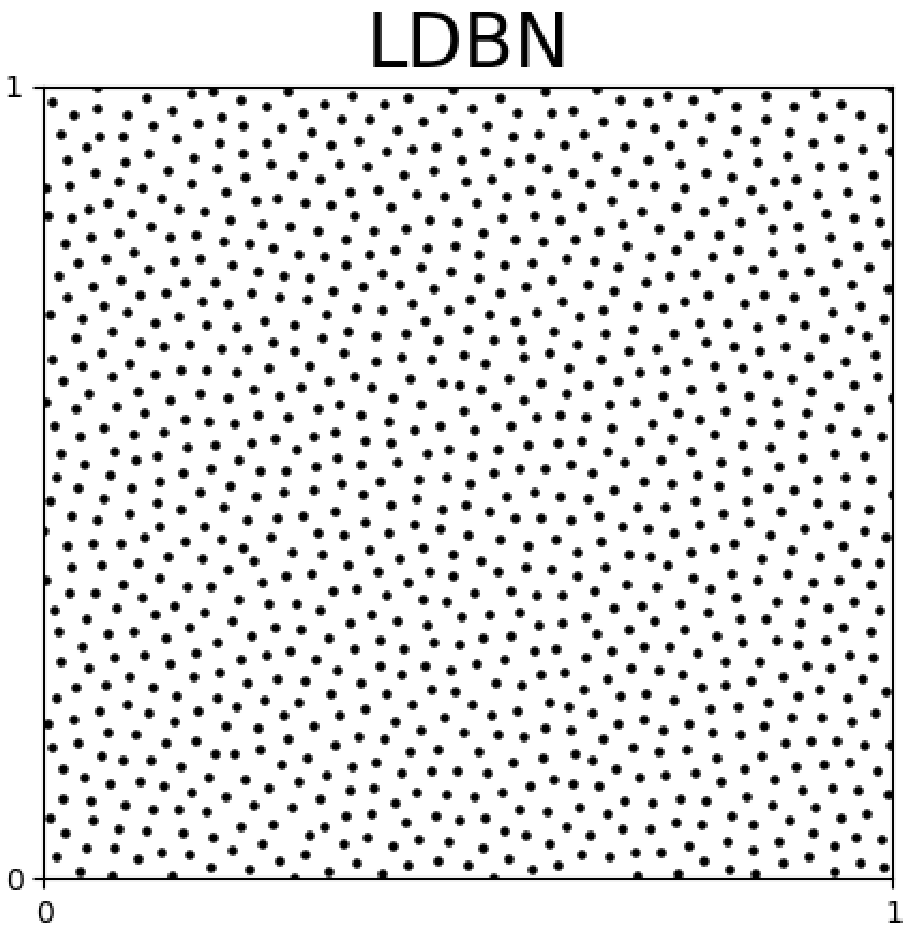


N =1024 points

Points Matter

Pseudo-Random





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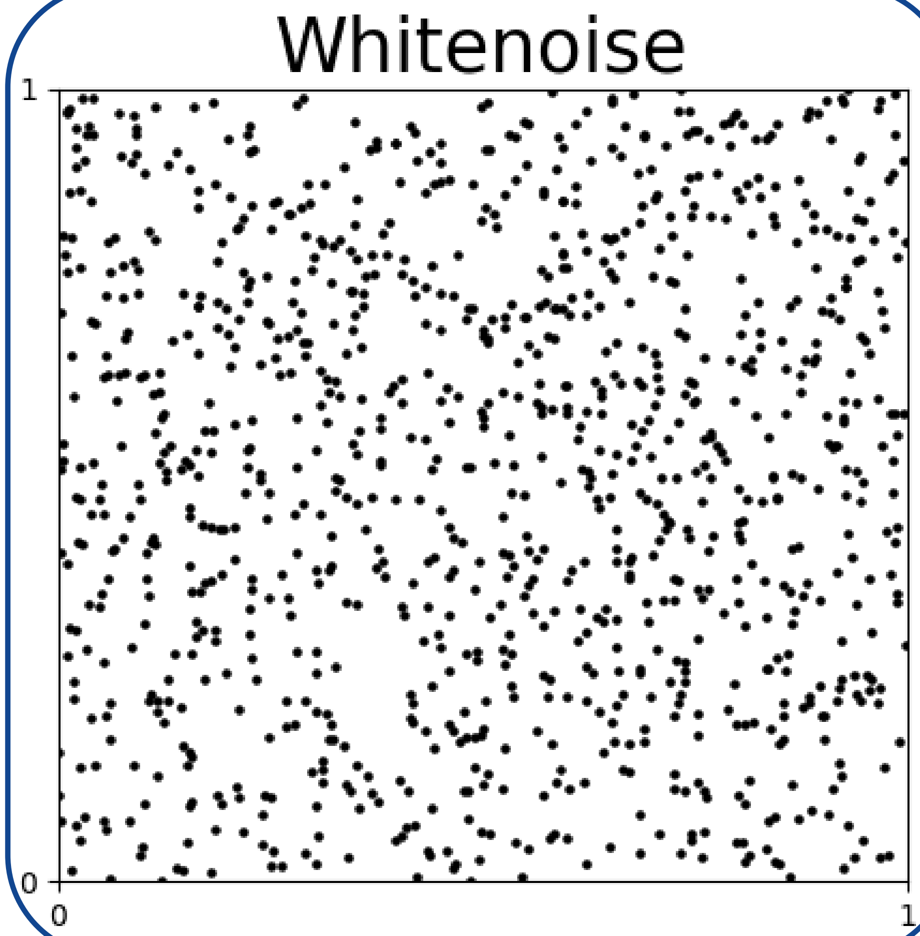
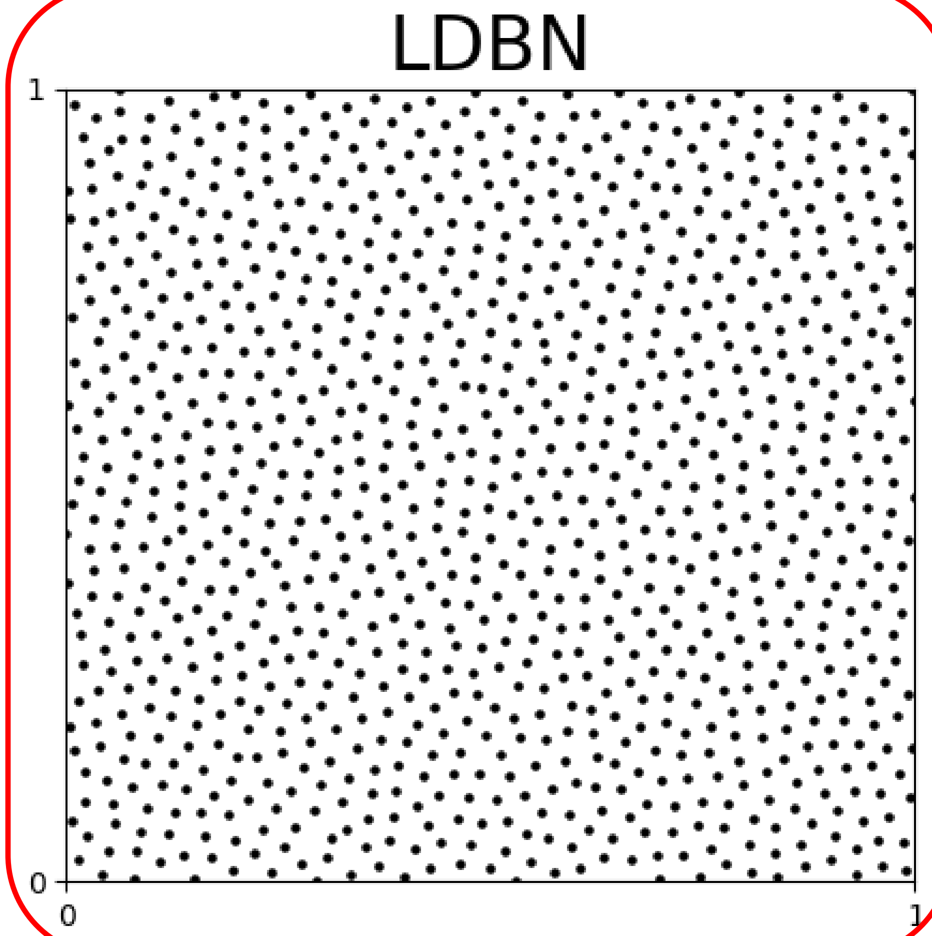




Quasi-Random

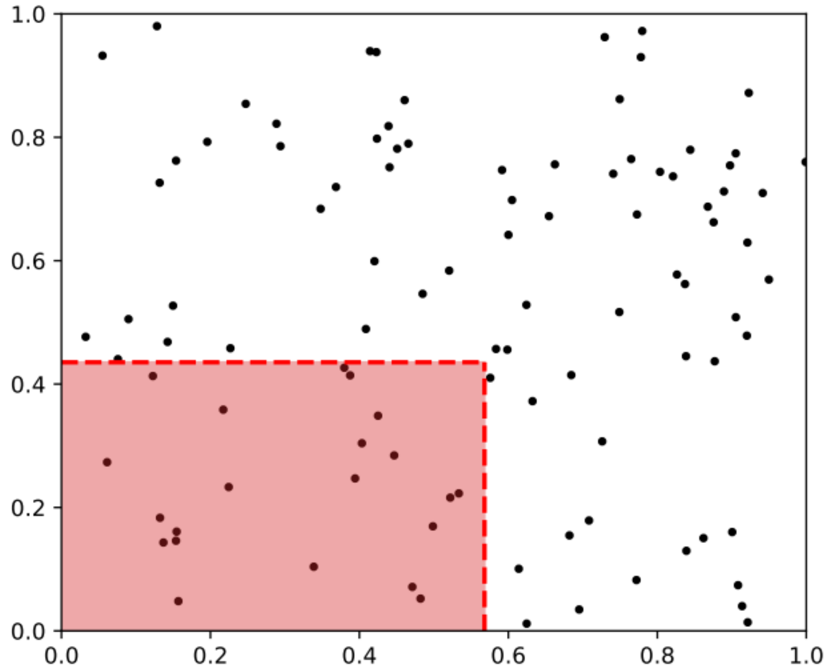
N =1024 points

Low Discrepancy

Koksma-Hlawka inequality :

$$\left| \frac{1}{N} \sum_{i=1}^{N} f(x_i) - \int_{I^d} f(u)\, du \right| \leq V(f) D_N^{\,*}\,(x_1, \ldots, x_N)$$

where $D_N^{\,*}$: “discrepancy”



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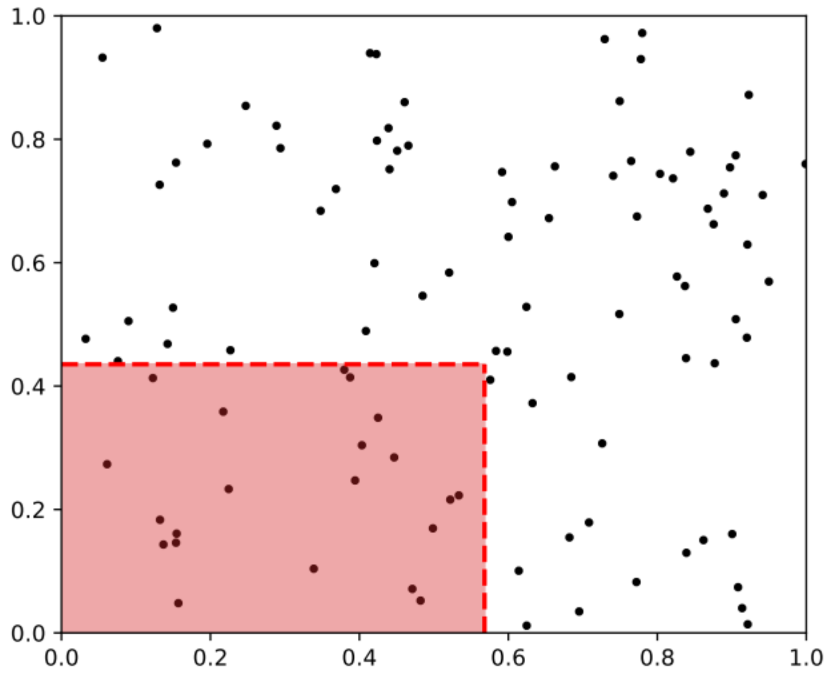

« Whitenoise » (or *Random*) :

$$E[D_N^{\,*} (x_1, \ldots, x_N)] \leq \frac{C_1}{\sqrt{N}}$$

« Low discrepancy » (or *Quasi Random*) if :

$$D_N^{\,*} (x_1, \ldots, x_N) \leq \frac{C_2 (\ln N)^d}{N}$$

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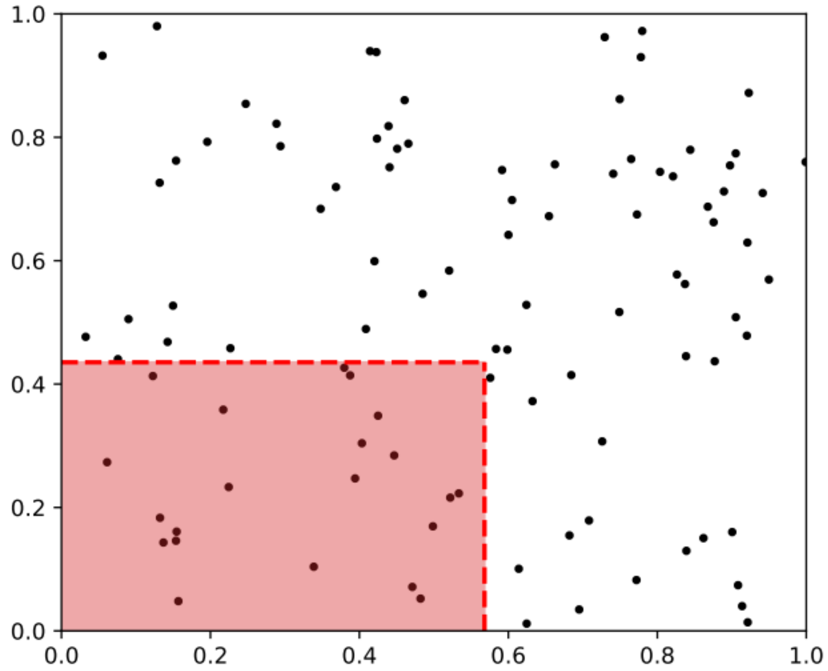

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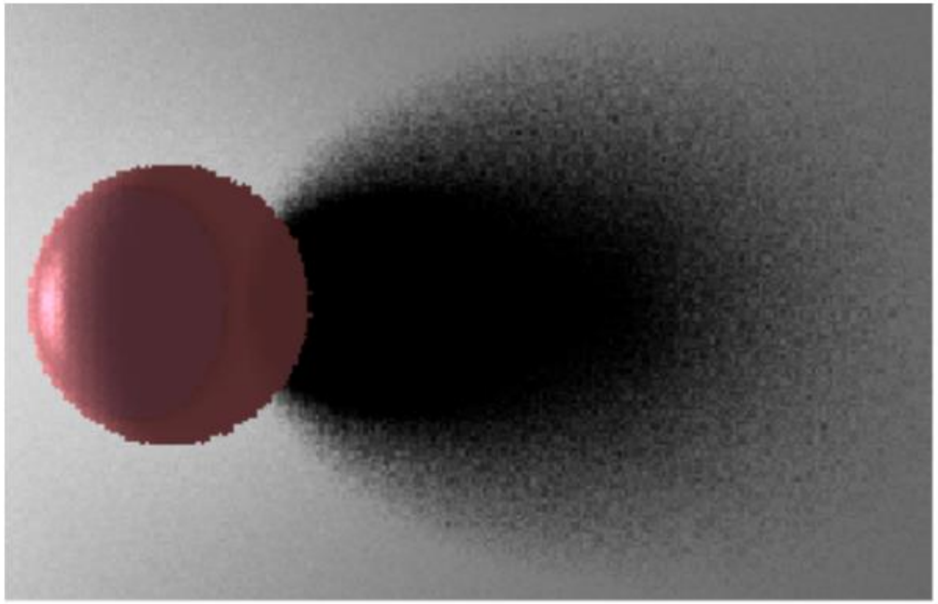
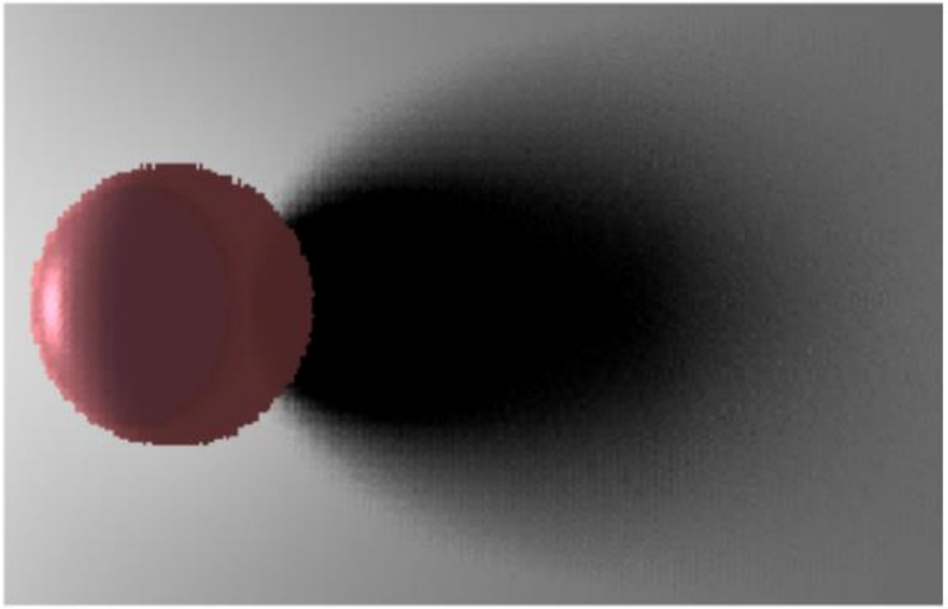
QMC in Rendering (Computer Graphics)

Whitenoise (MC)



Sobol (QMC)

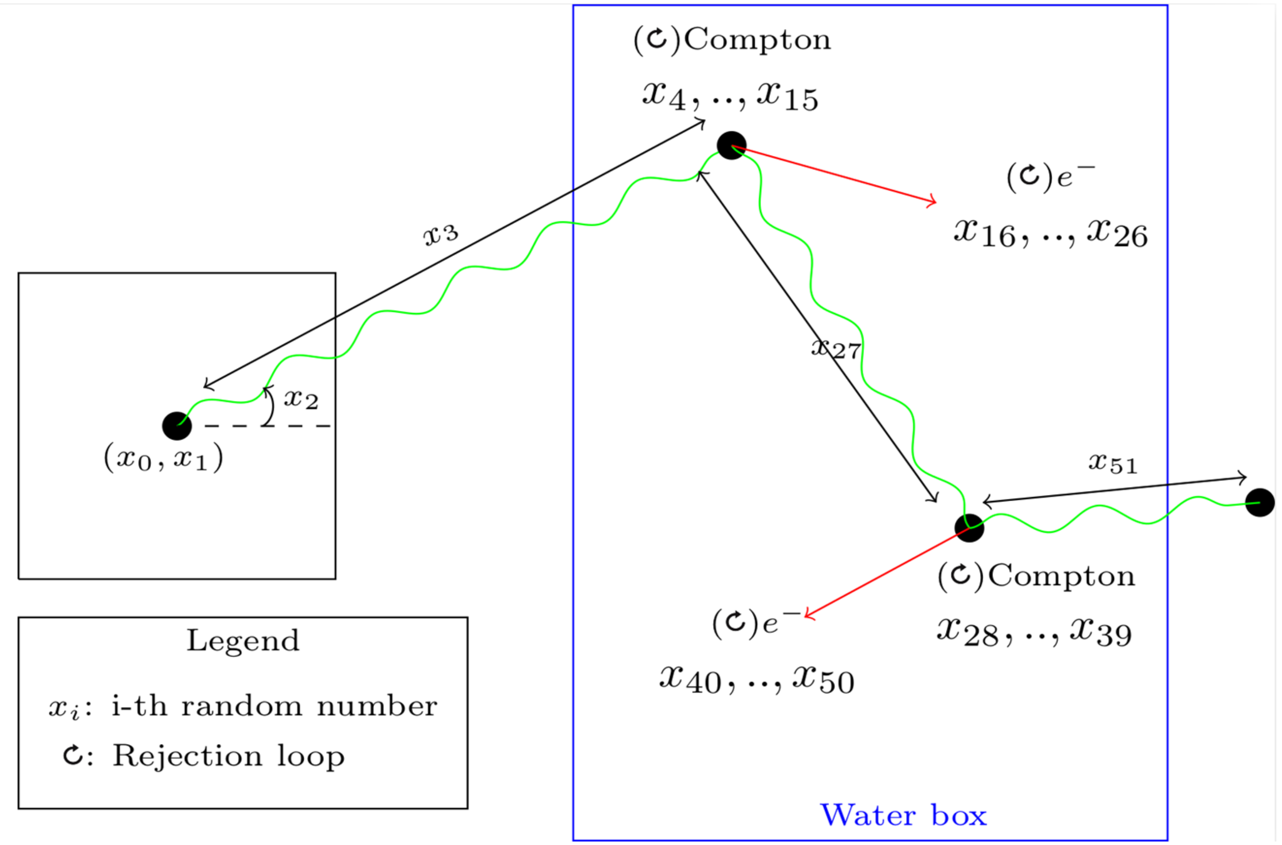


36 rays per pixel

QMC in Geant4

k^{th} point of Sobol' sequence in dimension d : $(x_0^k, x_1^k, \ldots, x_{d-1}^k)$

Each dimension $i < d$ → one *physical* dimension of the simulation



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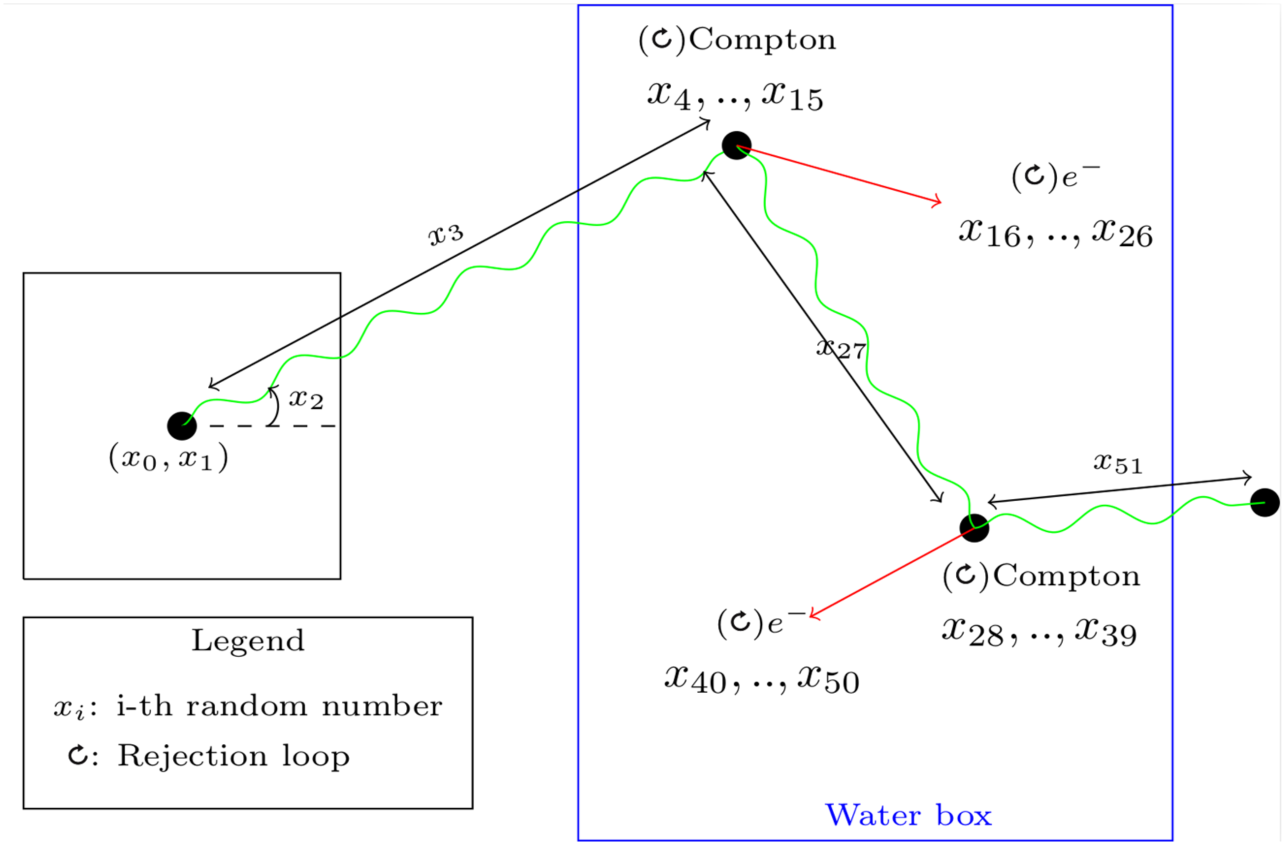
HepRandomEngine

…

double flat(*std::source_location*)

…



"Calling context" = dimension

Experience 1

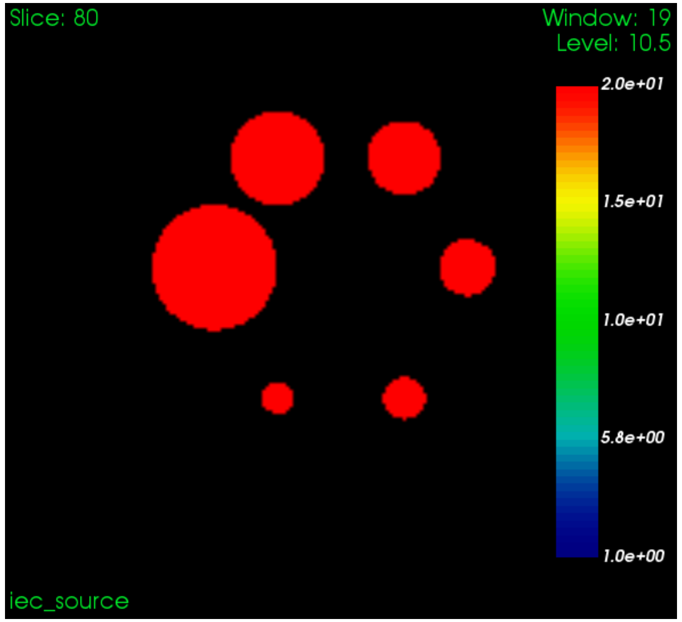
Toy SPECT experiment.

Source :

- 6 spheres
- gamma particles
- 140 keV
- source-to-background ratio : 20:1

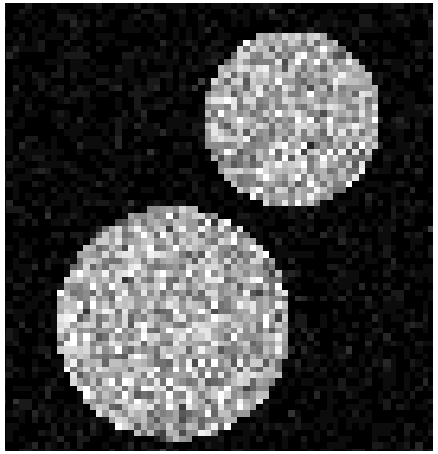
Medium : **vacuum**

Sampling : **5 QMC dimensions** (initial position / direction)

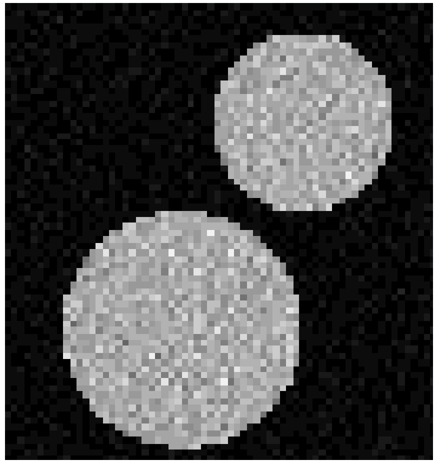


Experience 1

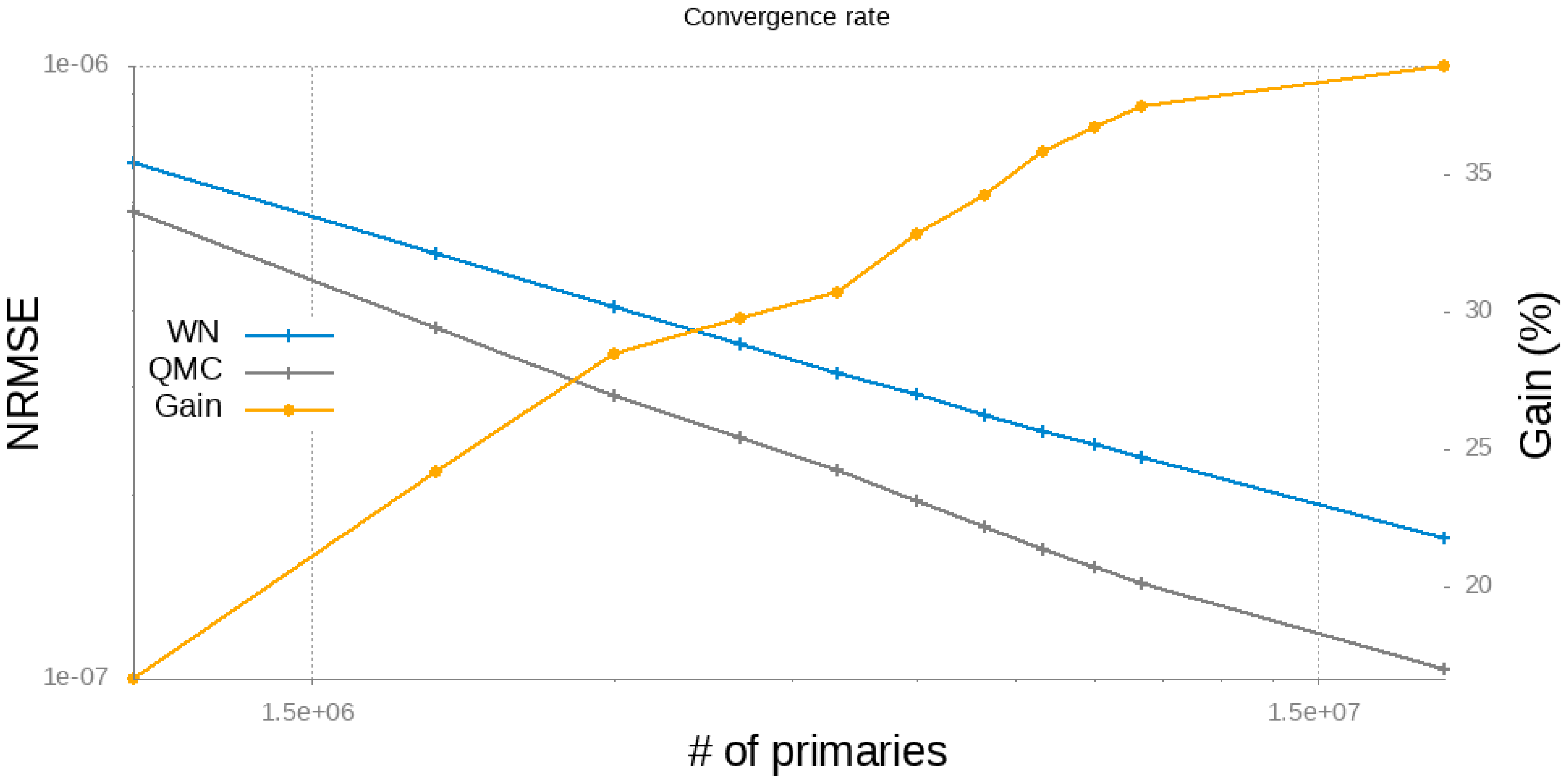
MC



QMC



N = 2.10^6



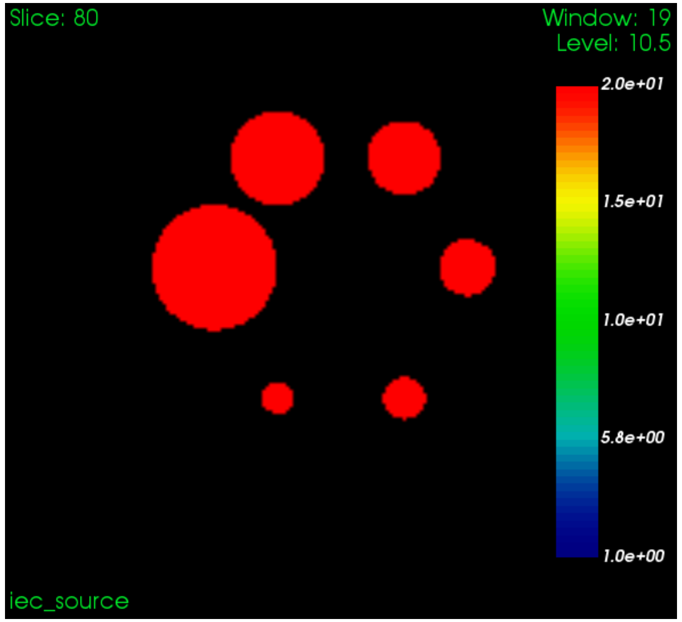
Experience 2

Source :

- 6 spheres
- gamma particles
- 140 keV
- source-to-background ratio : 20:1

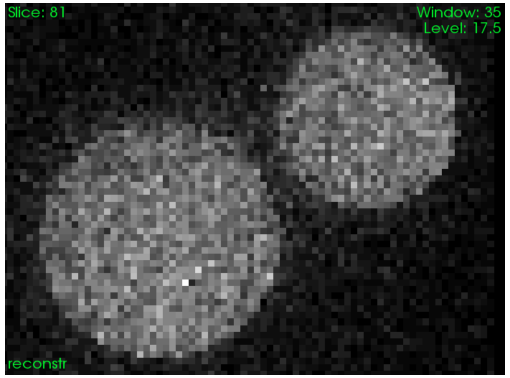
Medium : **Water** but Limited Physics List (only Compton)

Sampling : **23 QMC dimensions** (primary dimension + 1 Compton max)

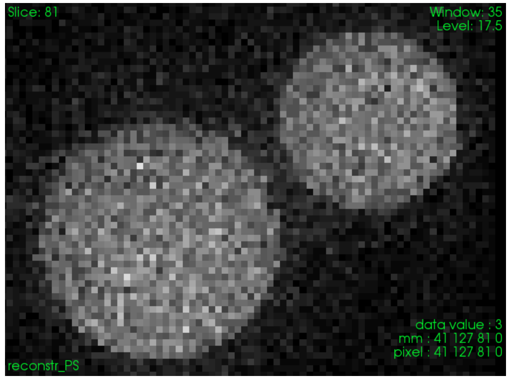


Experience 2

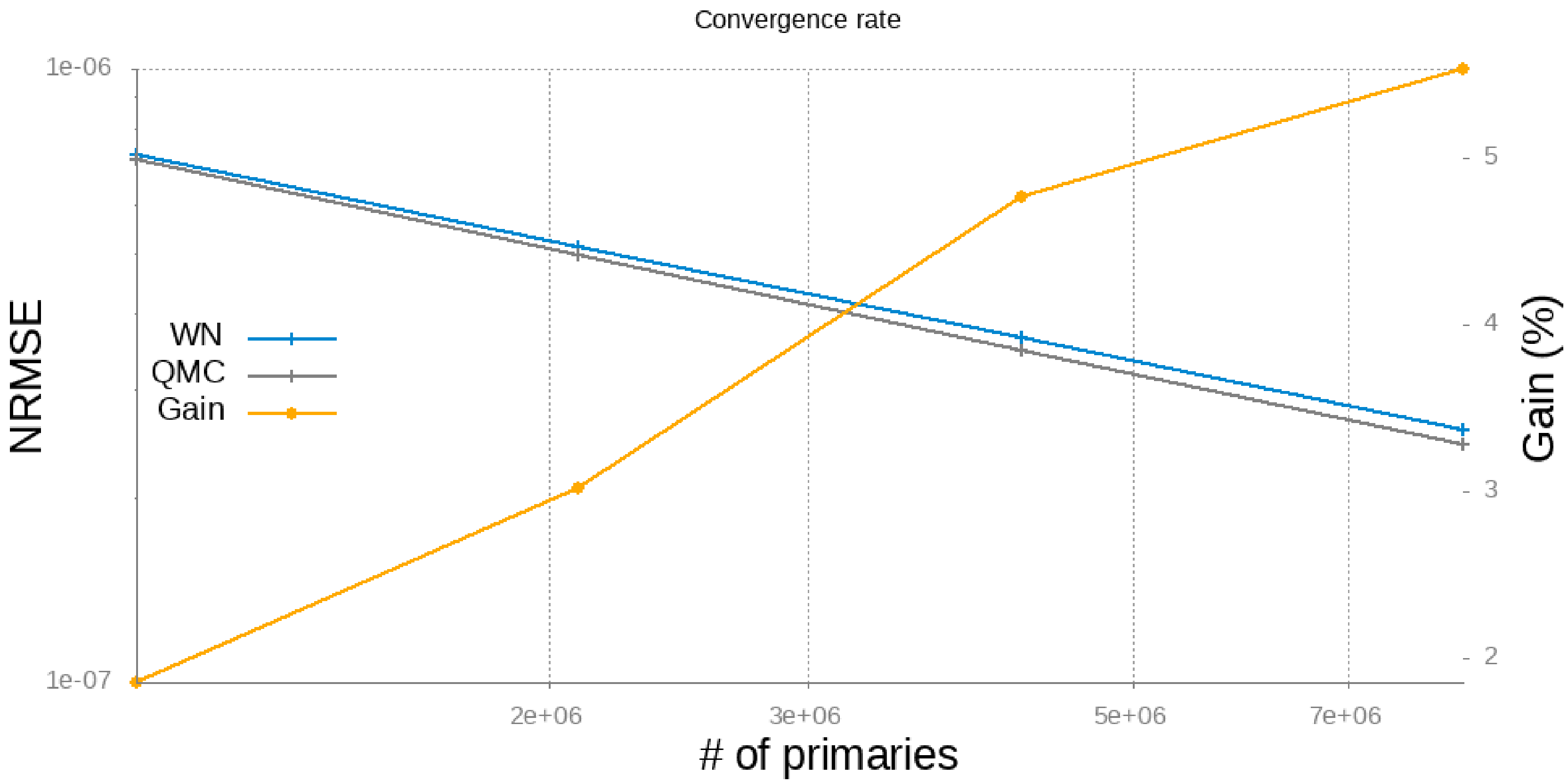
MC



QMC



$N \approx 4.10^6$



Conclusion

Potential Advantages

- QMC as G4RandomEngine
- Variance Reduction Technique
- Generalisable

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Limitations

- Dependance to dimensions
- Prototype implementation (still slower)

Thanks