Theoretical foundations of Quantum Information

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Quantum technologies within INFN: status and perspectives

Padova, January 20-21 2020

Outline

- Short presentation of QUit group
- Practical application: quantum tomography algorithm for x-ray medical tomography
- Theoretical tool: the “quantum comb”
- Foundations: “Information” as a paradigm for QT and QFT

QUit contributions to qu-info

QI TOOLS

- quantum tomography of states, transformations and measurements (ancilla-assisted tomography)

QUANTUM ENTANGLEMENT

- entanglement as a tool for improving the precision of q-measurements ⟶ quantum metrology
- first entanglement witness schemes
- entanglement transmission on noisy channels with correlated noise

QUANTUM COMPUTING

- first encodings for quantum error correction
- new quantum algorithms
- "hyper-graph states" used for quantum computation
- quantum memory channels

QUANTUM CRYPTOGRAPHY

- quantum private queries
- quantum privacy amplification in the presence of noise

INFORMATION THEORY

- no info without disturbance
- bosonic channel capacity with noise

NEW DEVICES FOR HIGH SENSITIVITY MEASUREMENTS (QUANTUM METROLOGY)

- quantum radars
- quantum GPS
- quantum frequency measurements

Collaboration with experimental labs: Roma La Sapienza, Northwestern University

OPTIMISATION OF PROTOCOLS

- broadcasting/cloning of states
- phase-estimation for mixed states
- estimation/discrimination of states and transformations
- cloning of phase-states, general states, and transformations
- quantum learning of transformation

OPTIMISATION METHODS

- the "quantum comb": general method for optimisation of for quantum circuit architecture (quantum processing, algorithms, protocols, …)

QUANTUM DEVICES

- quantum RAM
- new uncertainty relations

FOUNDATIONS OF QT, QFT AND IT

- information-theoretic postulates for QT and for Free QFT
- "comb" notion for a new understanding of causality, with impact on:
  - quantum causal inference
  - causal discovery algorithms
  - reconciliation of QT with GR (causal structure itself as dynamical)
  - …

QUit quantum information theory group

ATTRACT

Practical application: quantum tomography algorithm for x-ray medical tomography

Lorenzo Maccone PI, GMD, Nicola Mosco (UniPV)
Giampaolo Stopazzolo (Director Department of Health IT Vicenza, ULSS 8 Berica, Vicenza)

Collaboration with Siemens (Thomas Flohr, Forchheim, Germany)

Quantum homodyne tomography of the radiation state

1-1 connection between density matrix and Wigner

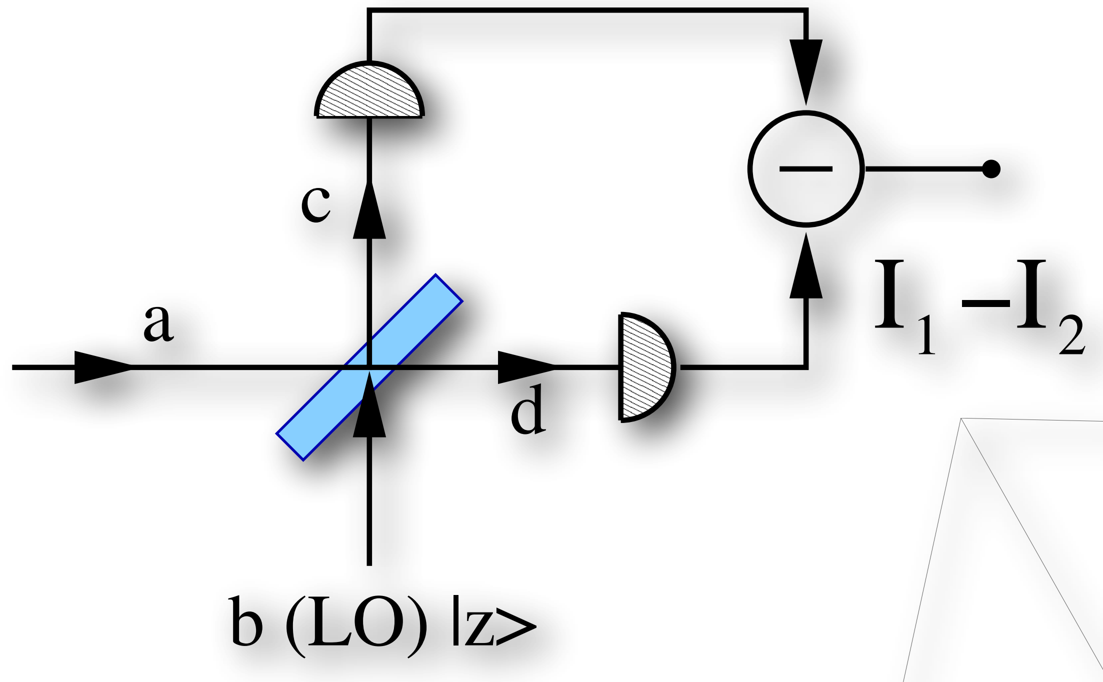
→ Reconstruct Wigner from its projections

$$W(\alpha,\overline{\alpha}) = \frac{2}{\pi}\mathrm{Tr}[\hat{\varrho}D(2\alpha)\exp(i\pi a^\dagger a)]$$

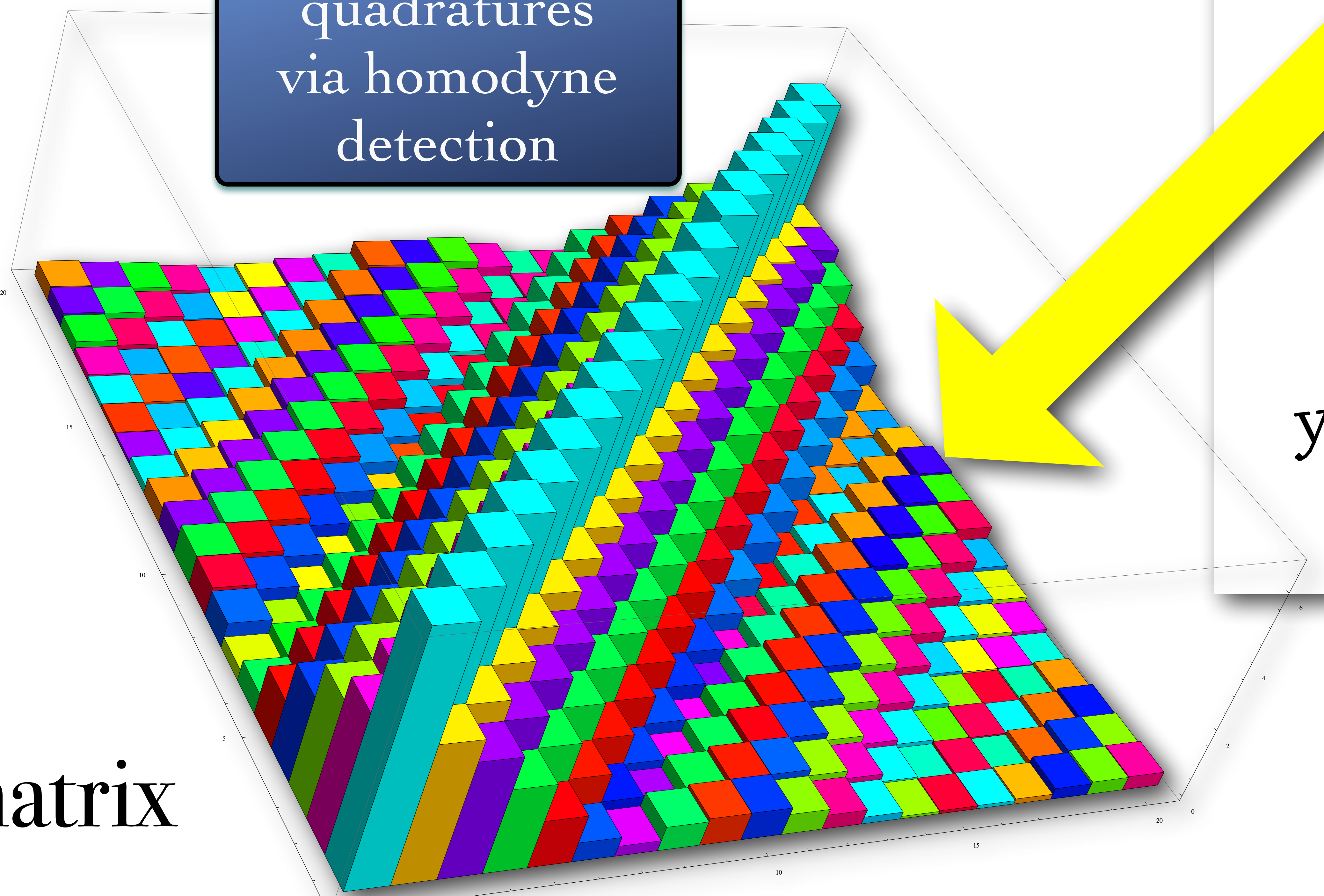
$$\hat{\varrho} = 2\int d^2\alpha W(\alpha,\overline{\alpha})D(2\alpha)(-)^{a^\dagger a}$$

↓ Projections are the pdfs of quadratures

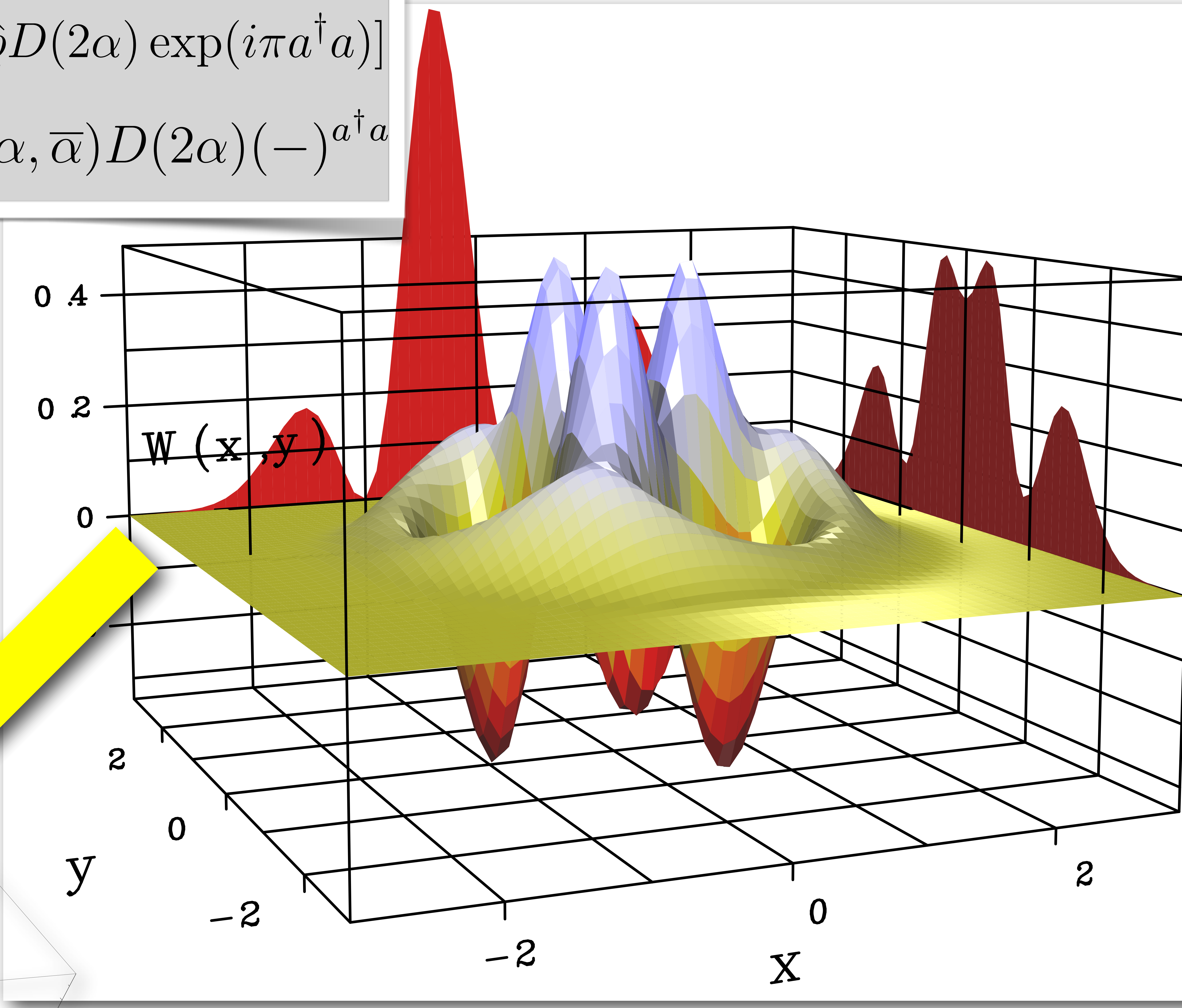
↓ quadratures via homodyne detection



$$\hat{x}_\phi = \frac{1}{2}(a^\dagger e^{i\phi} + a e^{-i\phi})$$



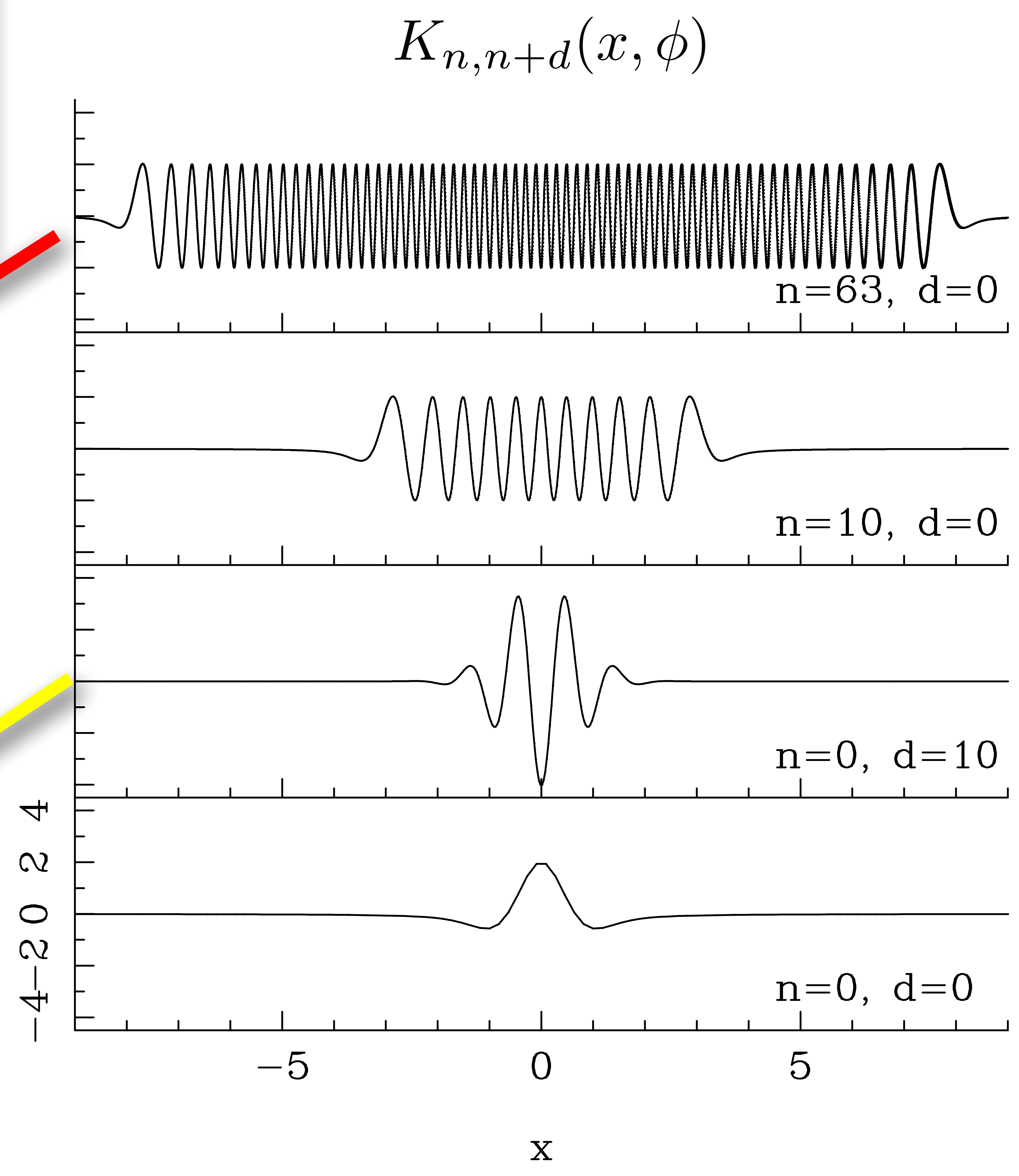
Density matrix

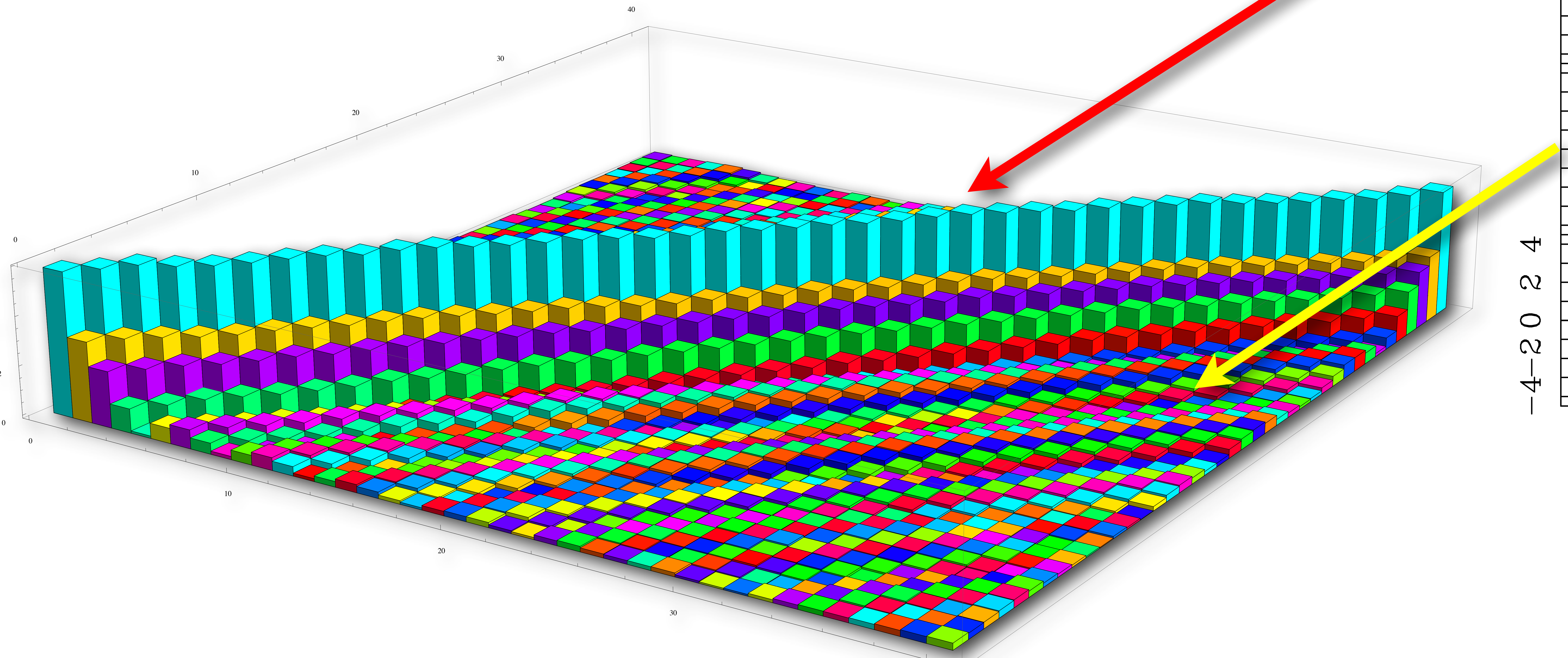


G. M. D'Ariano, U. Leonhardt and H. PRA **52** R1801 (1995)
G. M. D'Ariano, C. Macchiavello and M.G.A. Paris, PRA **50** 4298 (1994)
M. Beck, D. T. Smithey, and M. G. Raymer PRA **48** R890 (1993)
K. Vogel and H. Risken PRA **40** 2847(1989)

Quantum homodyne tomography of the radiation state

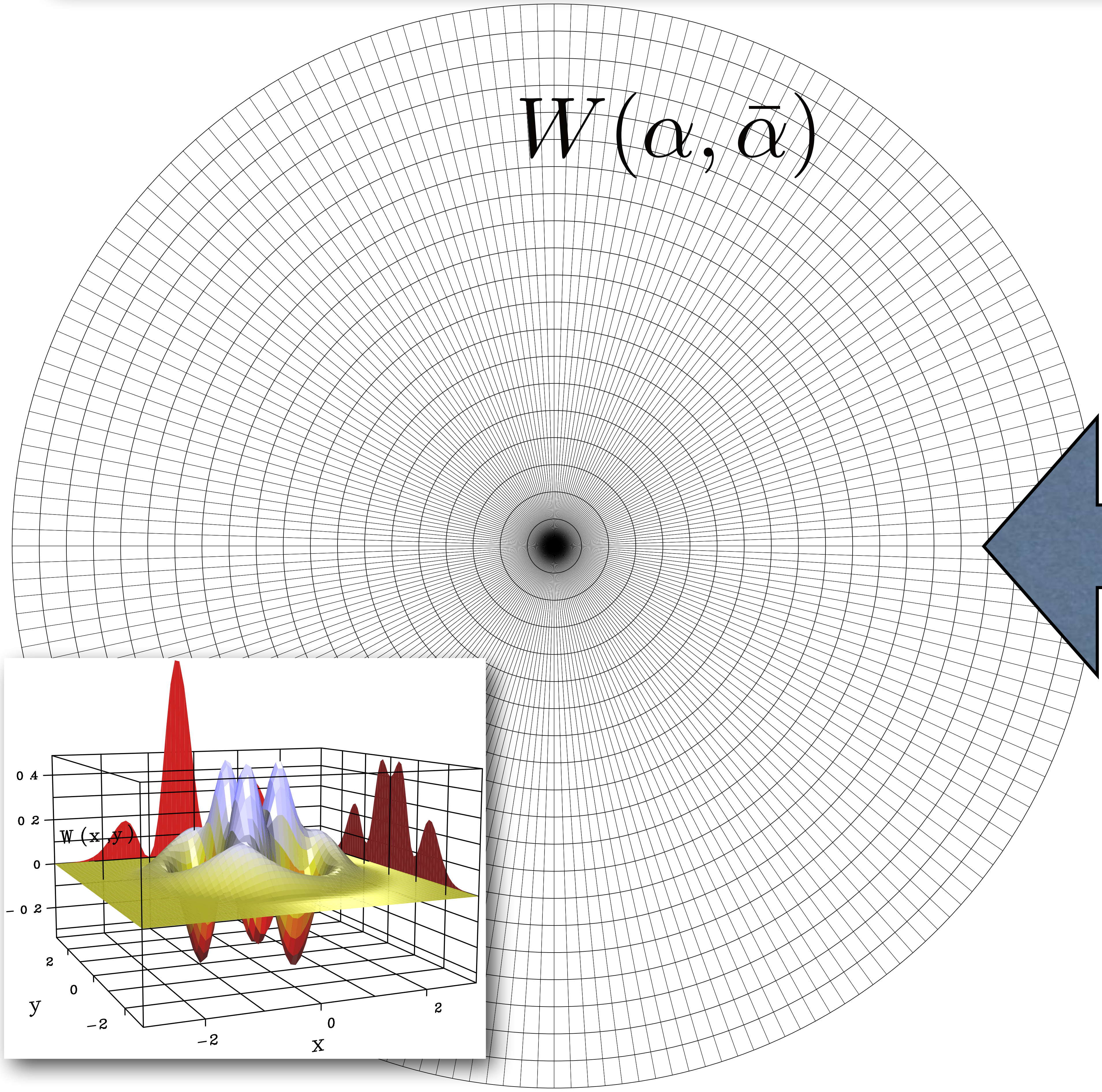
$$\rho_{n,n+d} = \int_0^{\pi} \frac{d\phi}{\pi} \int_{-\infty}^{+\infty} dx\, p(x,\phi)\, K_{n,n+d}(x,\phi)$$

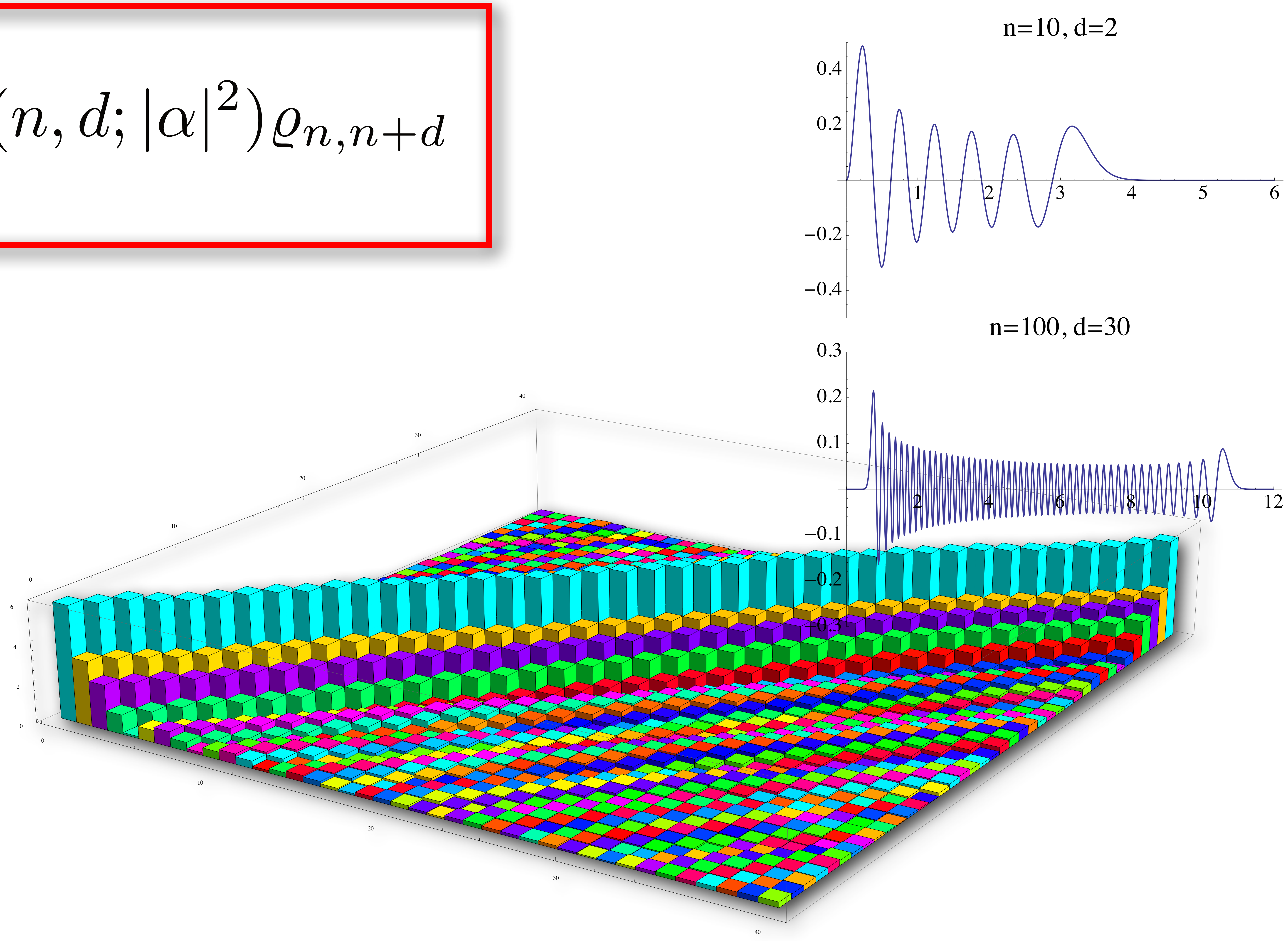




QUANTUM HOMODYNE TOMOGRAPHY OF THE RADIATION STATE

$$W(\alpha,\bar{\alpha}) = \mathrm{Re}\sum_{d=0}^{\infty} e^{id\arg(\alpha)} \sum_{n=0}^{\infty} \Lambda(n,d;|\alpha|^2)\varrho_{n,n+d}$$

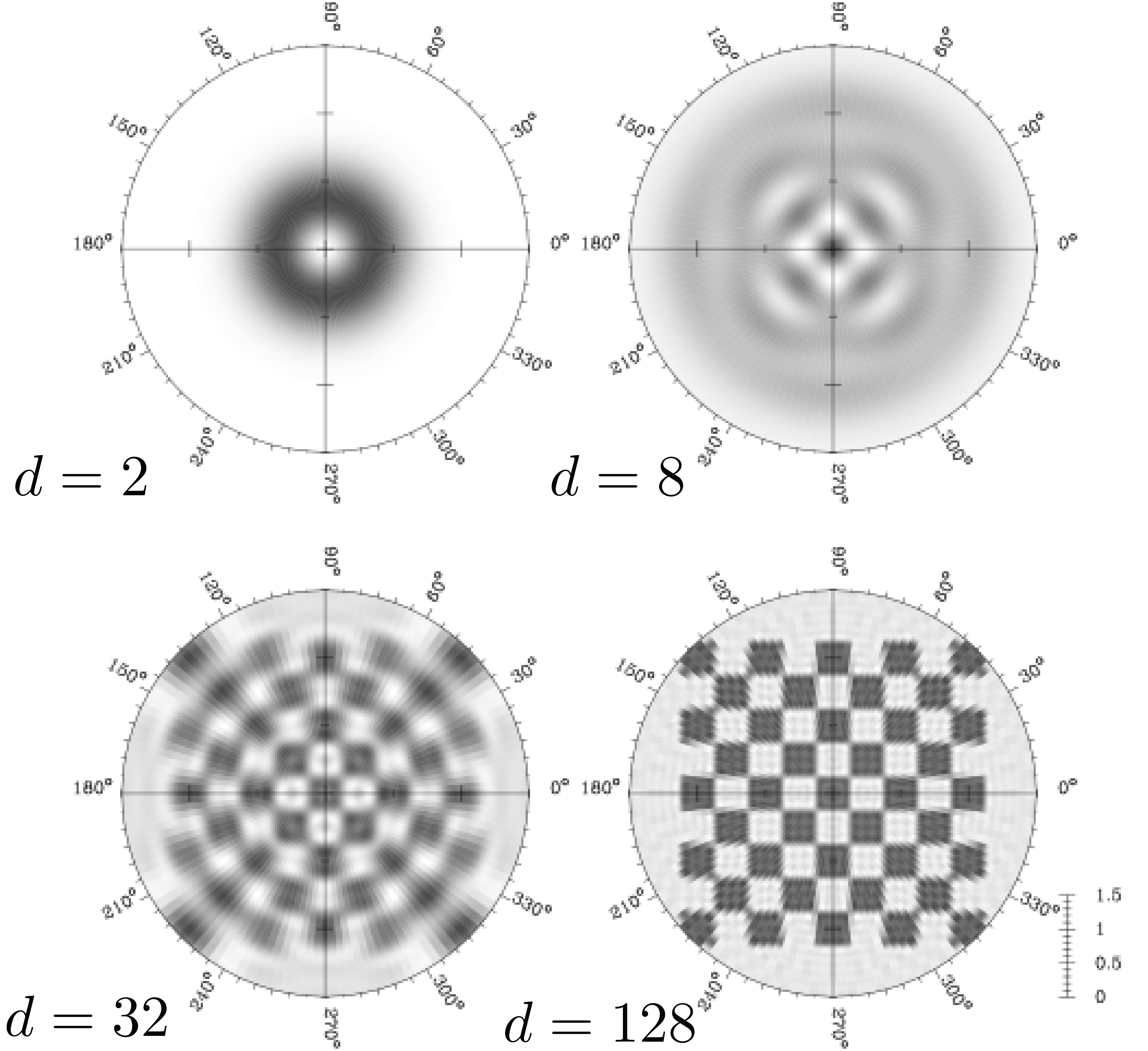




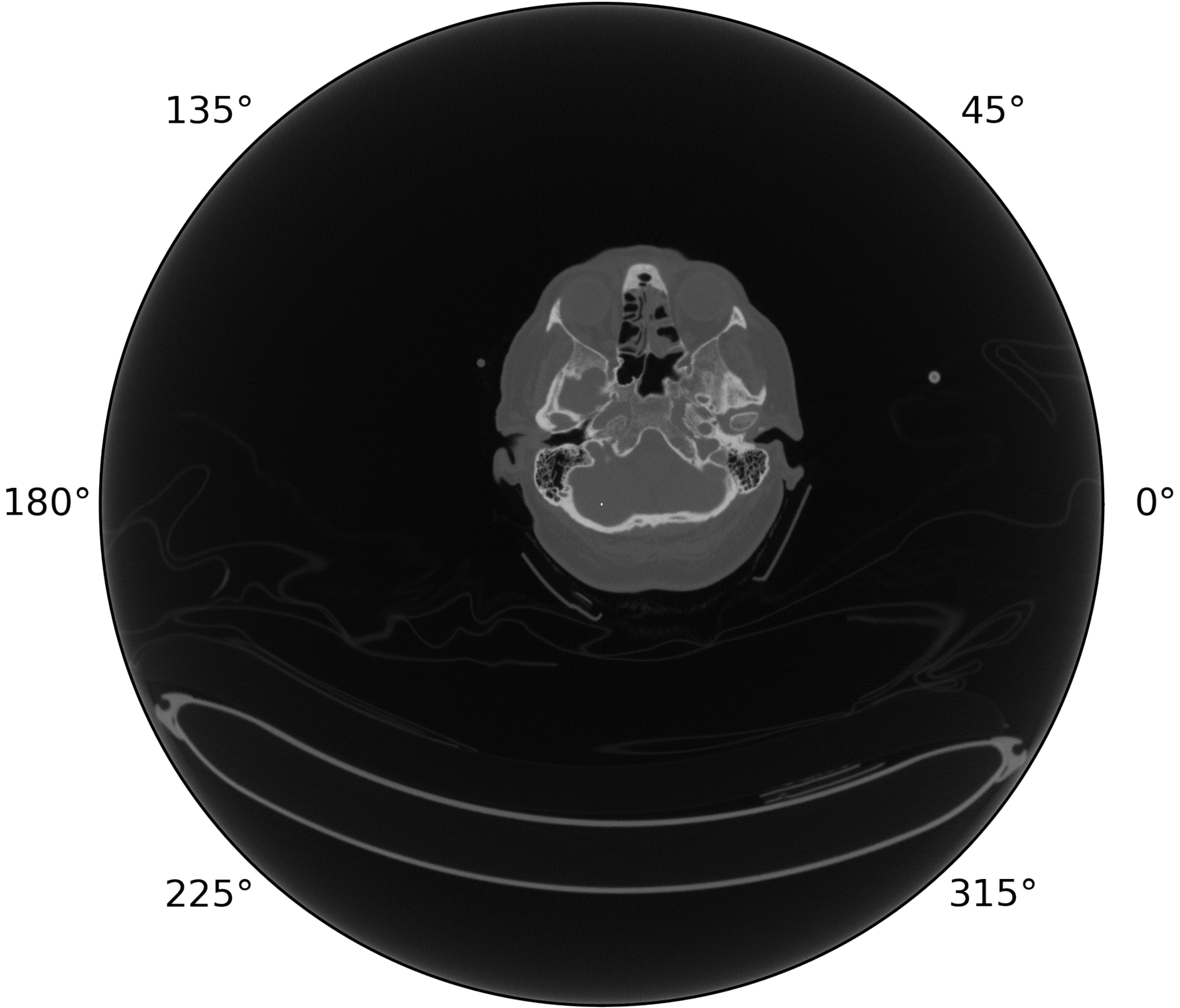
-Structurally very paralleliseable algorithm

-Image analysis in polar coordinates

Truncation of the density matrix

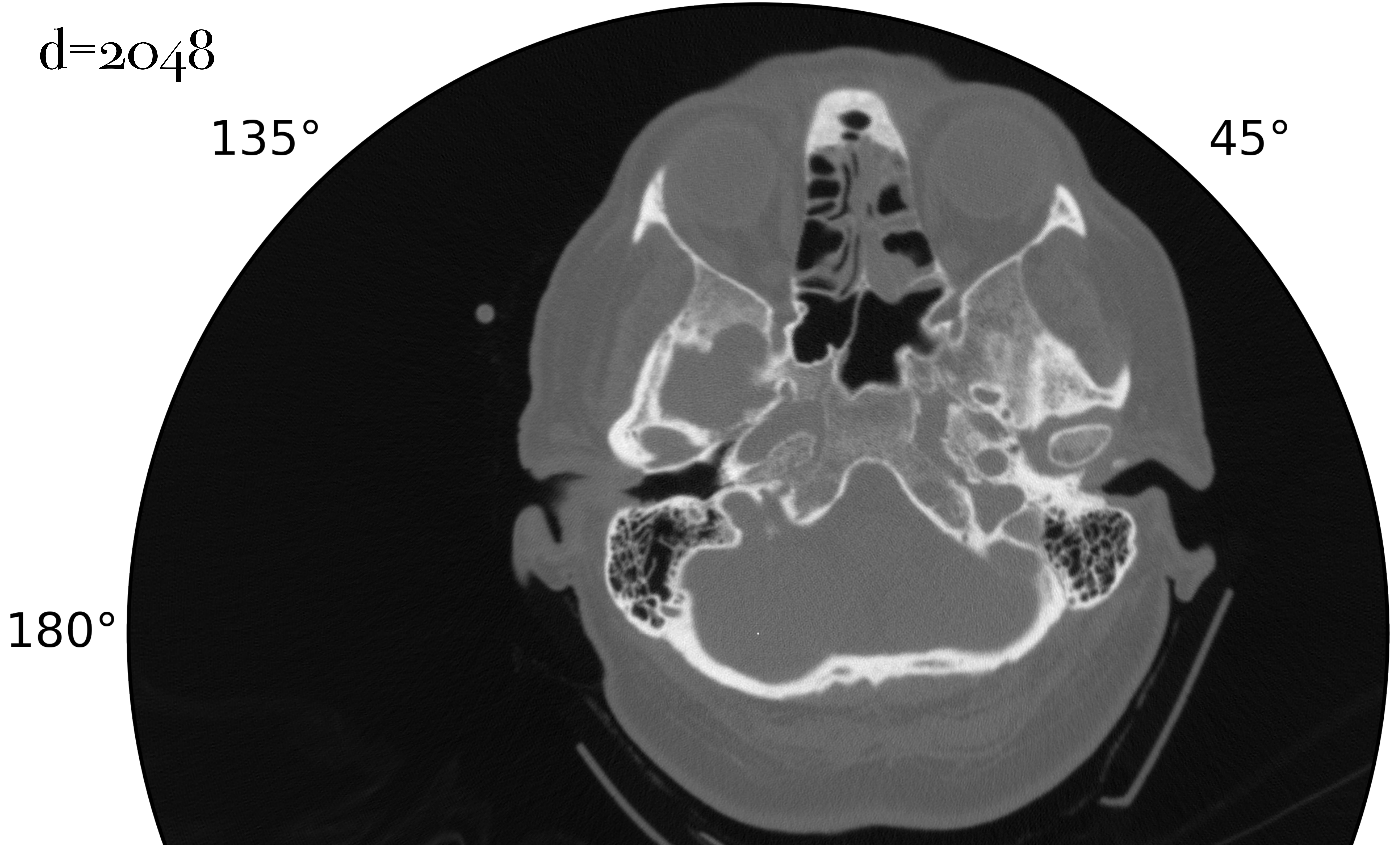


d=2048



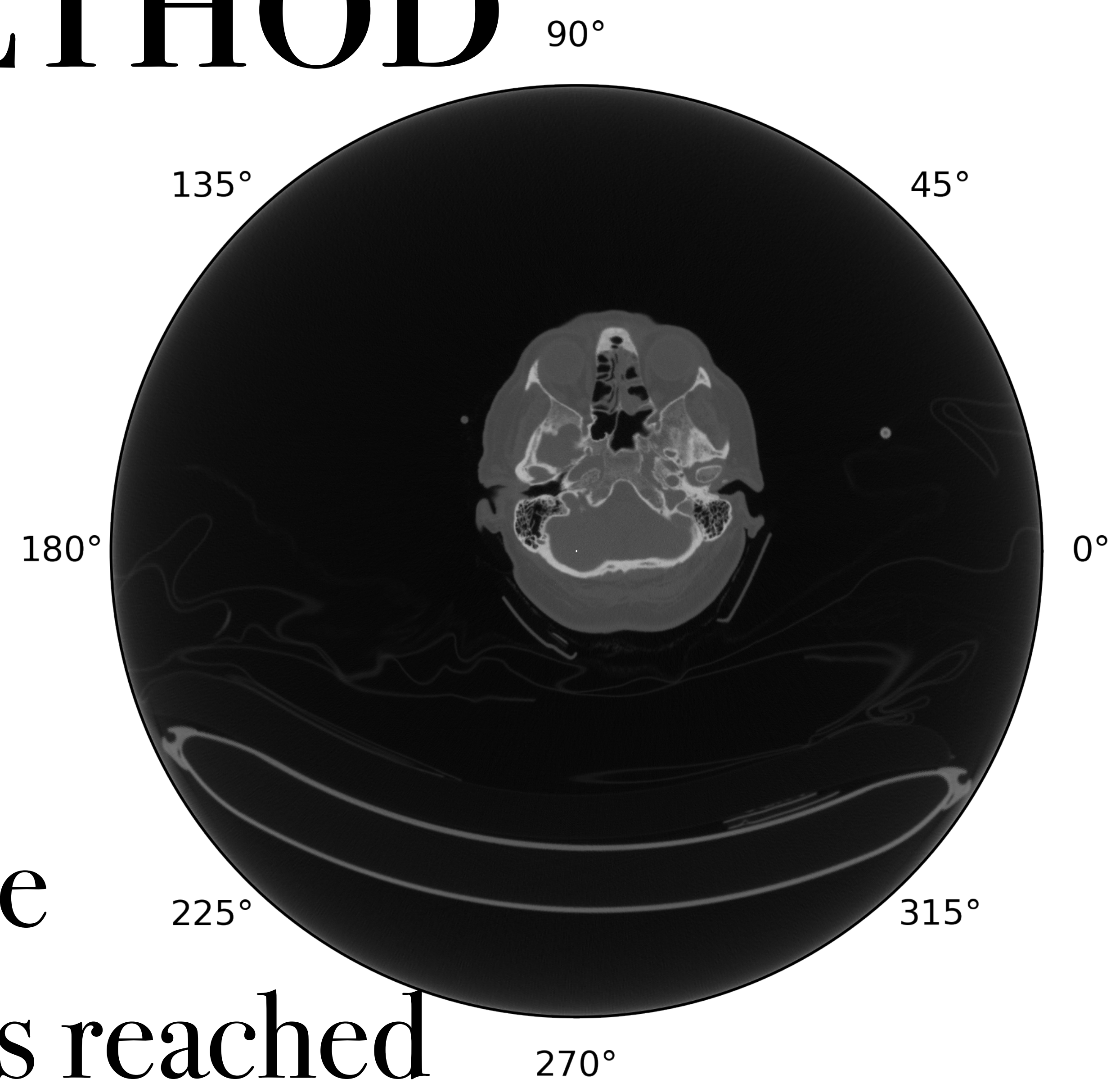
Sinogram from SIEMENS

d=2048



Sinogram from SIEMENS

Possible advantages of the method



-Suitable to photo counting detectors imaging in view of x-ray low-dose
-Acquisition can be stopped on the fly when a sufficient level of detail is reached

-Further iterative/adaptive reconstruction techniques can be used

-Tests needed to check if the method outperforms the inverse Radon for very low radiation doses with single photon detectors

PRIN 2008

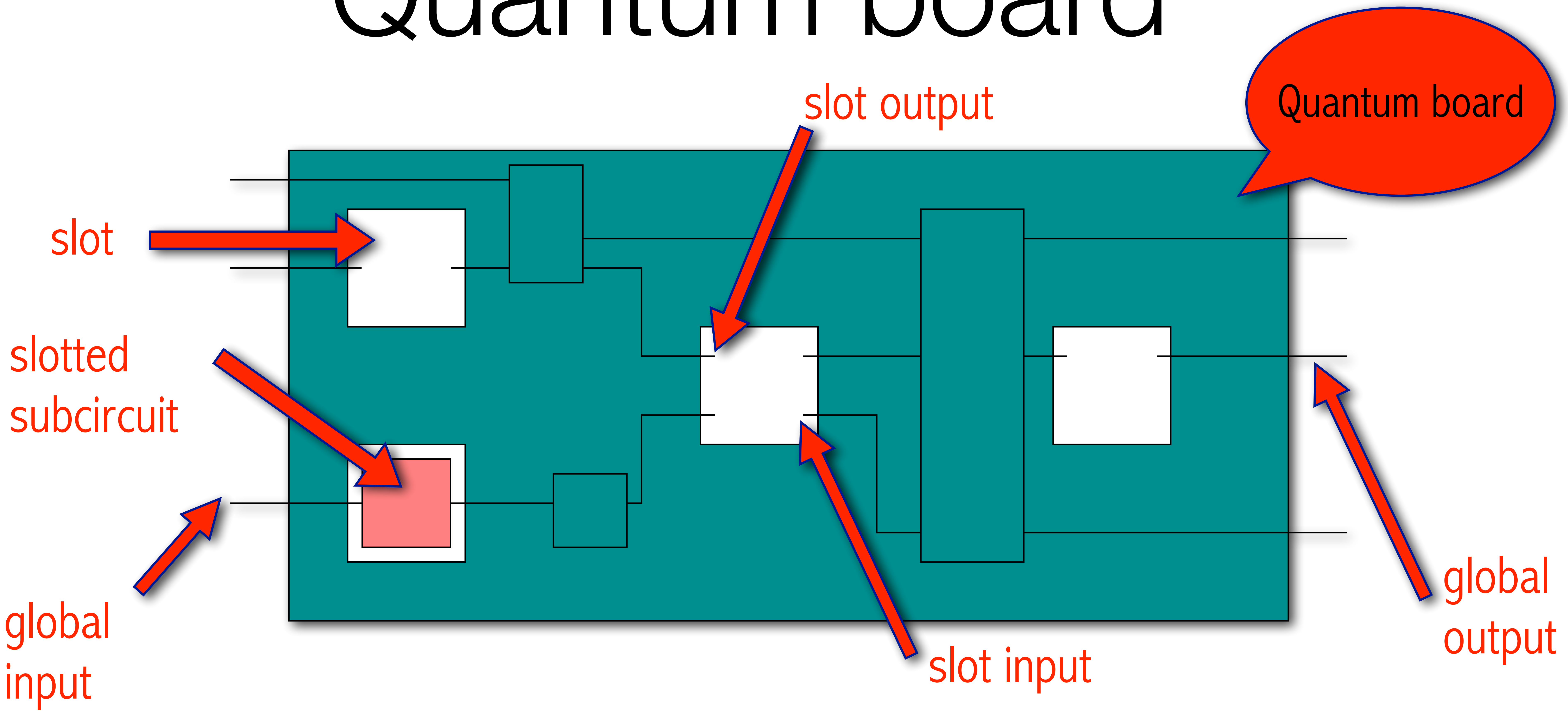
STREPS:
SECOCQ
CORNER
COQUIT

Theoretical tool: the quantum comb

With P. Perinotti and G. Chiribella (HK)

Quantum Circuits Architecture, Phys. Rev. Lett. 101 060401 (2008)

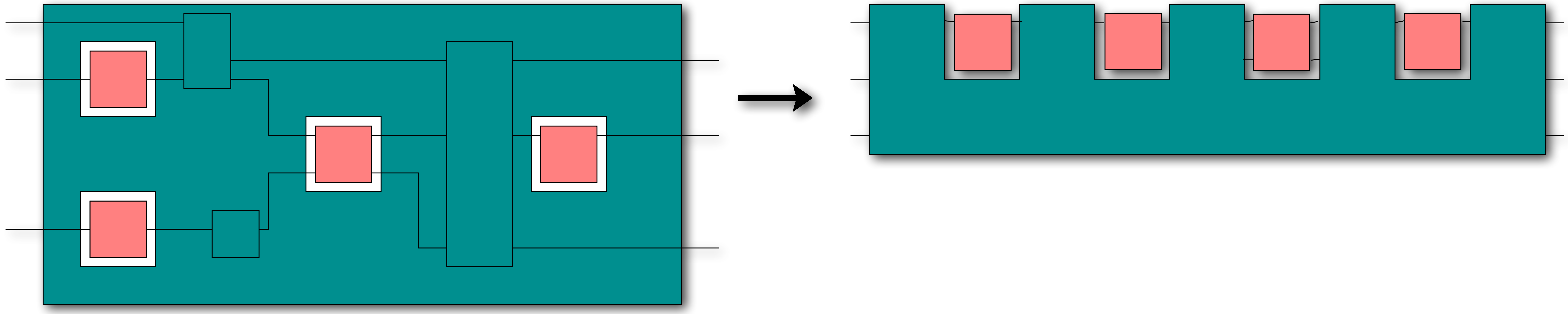
Quantum board

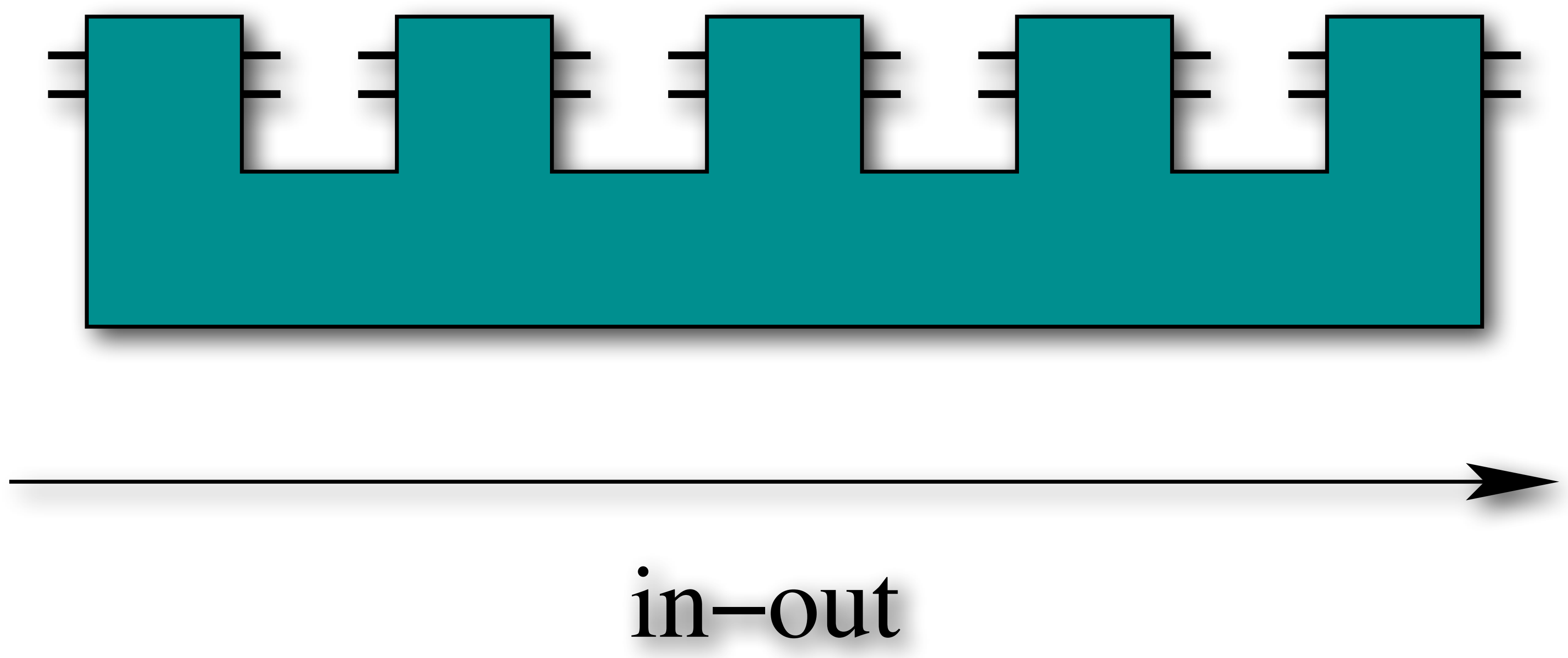


Problem: what is the optimal board for given slots achieving a global input/output task optimally according to a given cost function?

Quantum Combs

The circuits-boards can be reshaped in form of a "comb", with an ordered sequence of slots, each between two successive teeth



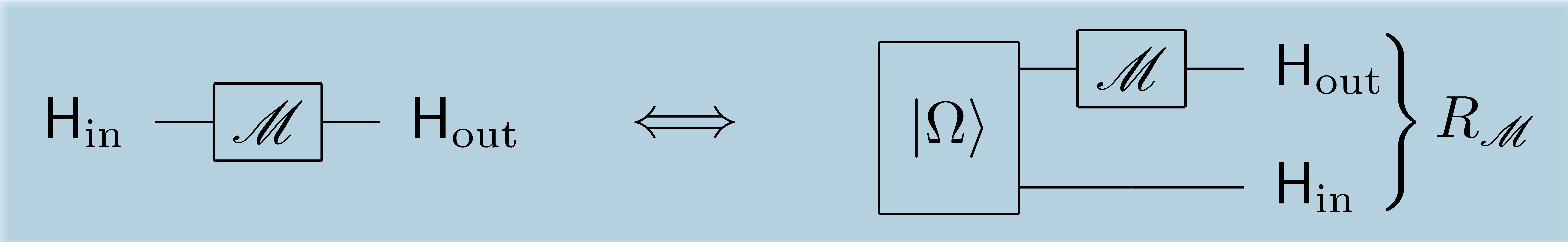


The pins in a quantum comb represent quantum systems, with generally variable dimensions, entering or exiting from the board

Choi representation of q-operation

The input-output quantum operation achieved by any quantum circuit is a CP map, and a suitable representation is provided by the one-to-one correspondence with a positive operator called "Choi-Jamiolkowski operator".

The Choi operator is the output state of the map applied locally to a maximally entangled reference state with suitable normalisation.



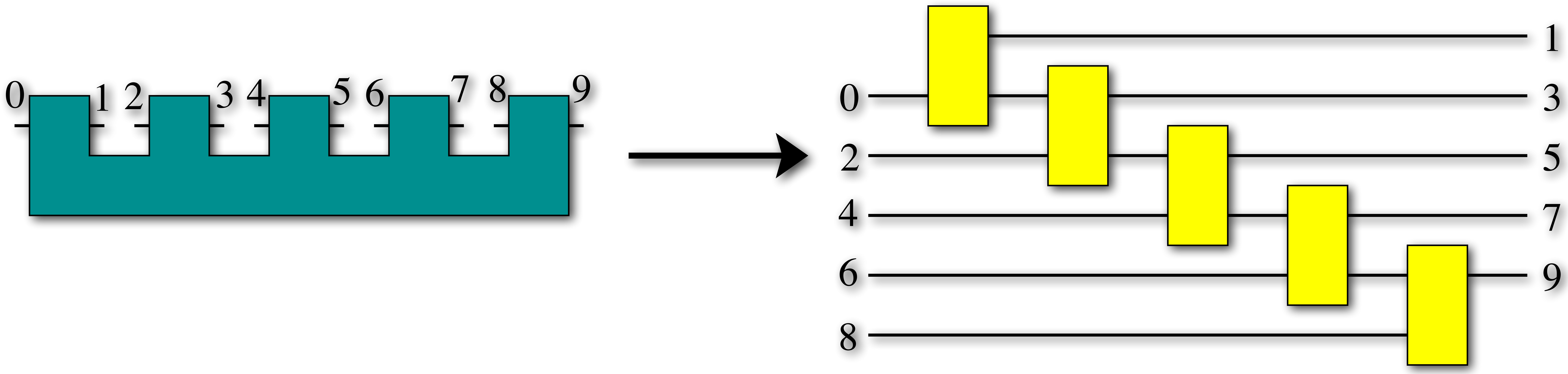
$$|\Omega\rangle \in \mathsf{H}_{\mathrm{in}} \otimes \mathsf{H}_{\mathrm{in}}$$

$$R_{\mathscr{M}} \in \mathsf{B}(\mathsf{H}_{\mathrm{in}} \otimes \mathsf{H}_{\mathrm{out}})$$

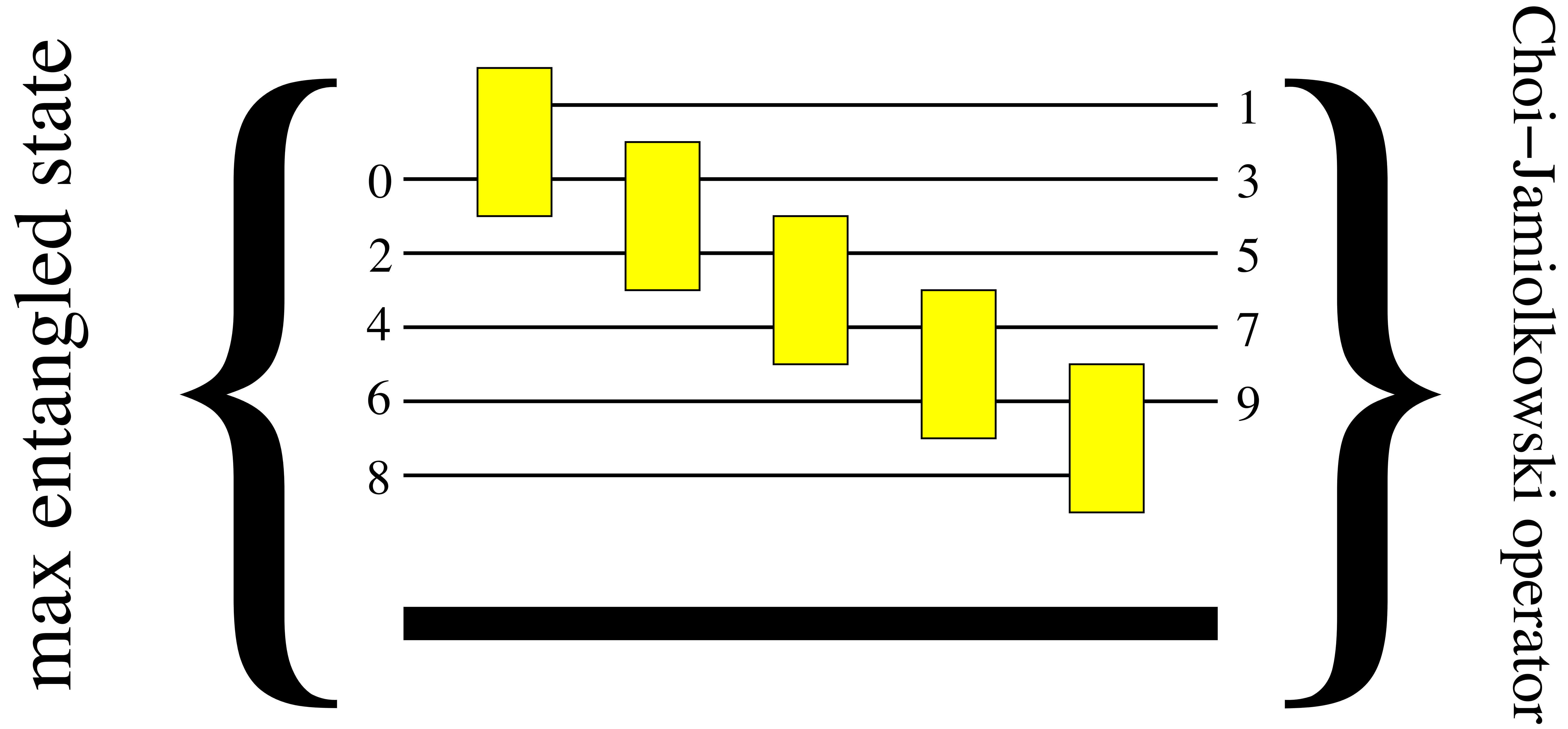
$$\mathrm{Tr}_{\mathrm{out}}[R_{\mathscr{M}}] = I_{\mathrm{in}}$$

Causal networks

To a comb we associate the Choi operator of the quantum operation of the causal network equivalent to the comb, with all inputs on the left and all outputs on the right



Choi representation



Causality constraints: (N+1 inputs/outputs)

$$\mathrm{Tr}_{2n+1}\left[R^{(n)}\right] = I_{2n} \otimes R^{(n-1)}, \quad n = 0, 1, N,$$

$$R^{(N)} \equiv R, \; R^{(-1)} = 1$$

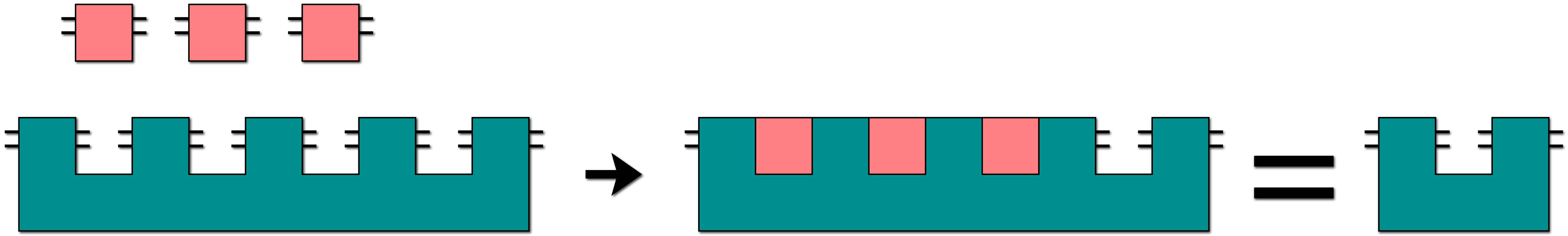
Supermaps

A quantum comb performs a transformation that is a generalization of the quantum operation: the so called "supermap"

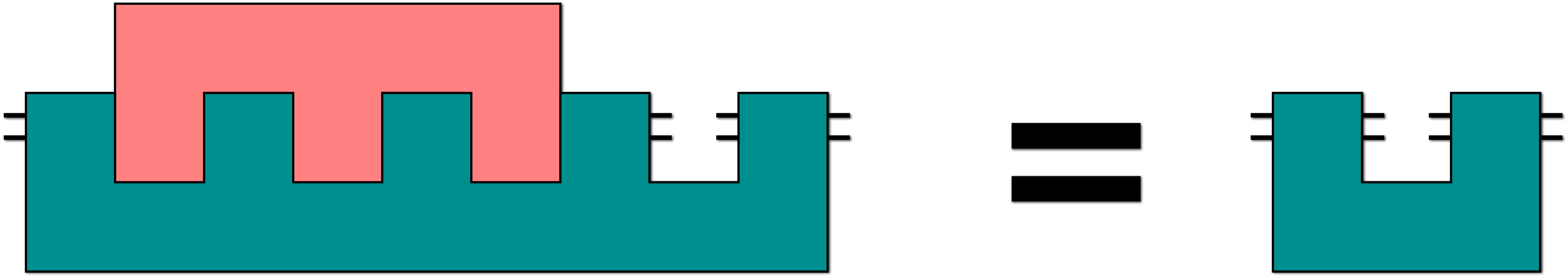
A supermap sends a series of N channels to one channel. Mathematically it is represented by a completely positive N-linear map which sends N Choi operators to one Choi operator, and with his own Choi operator satisfying the causality constraints.

Supermaps

More generally, quantum combs map series of channels into combs

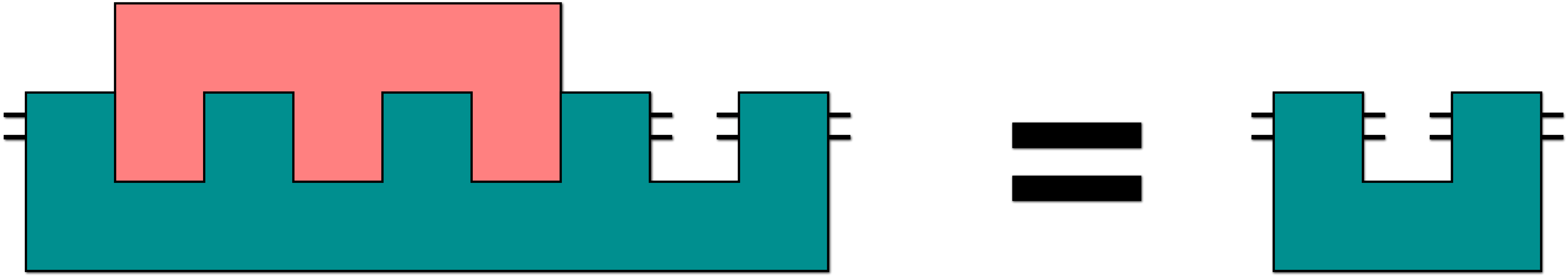


or even more generally combs into combs

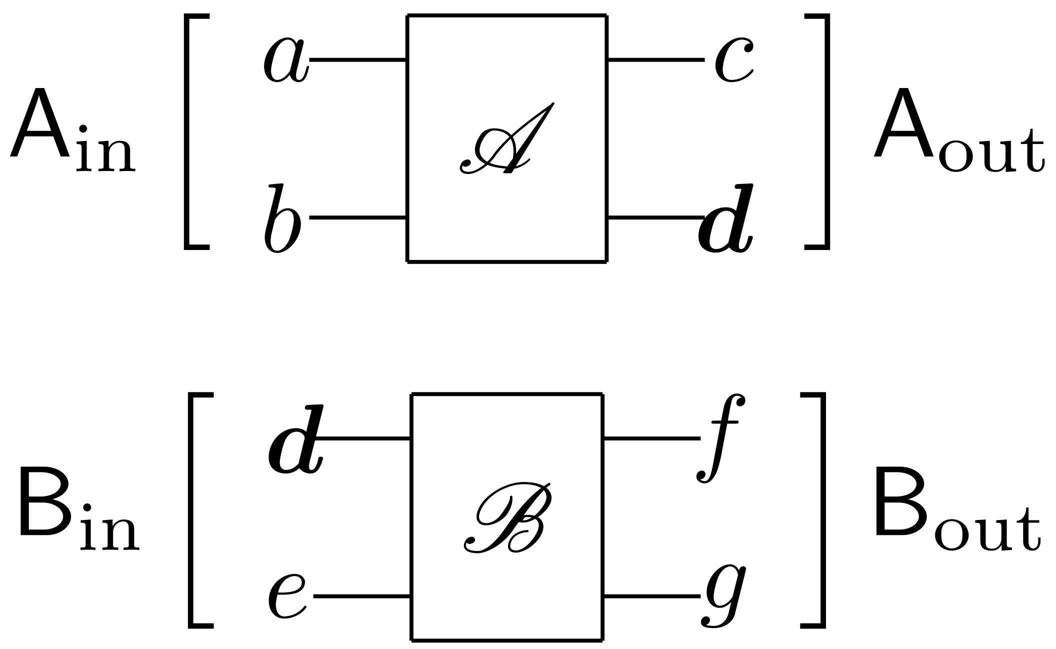


Supermaps

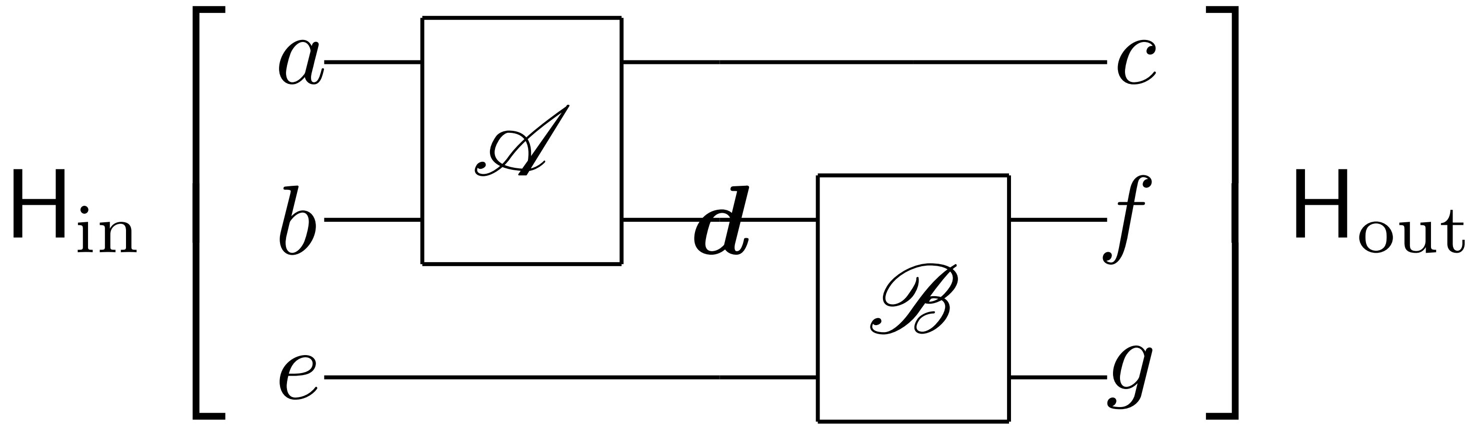
The notion of supermap is the last level of generalization, i.e. "super-supermaps" (mapping supermaps to supermaps) are still supermaps = quantum combs.



Link product



$\Longrightarrow$



Choi-operator calculus

$$A \in \mathsf{B}(\mathsf{A}_{\mathrm{out}} \otimes \mathsf{A}_{\mathrm{in}}) = \mathsf{B}(\mathsf{H}_a \otimes \mathsf{H}_b \otimes \mathsf{H}_c \otimes \mathsf{H}_{\boldsymbol{d}}), \qquad \mathsf{J} \equiv \mathsf{H}_{\boldsymbol{d}}$$

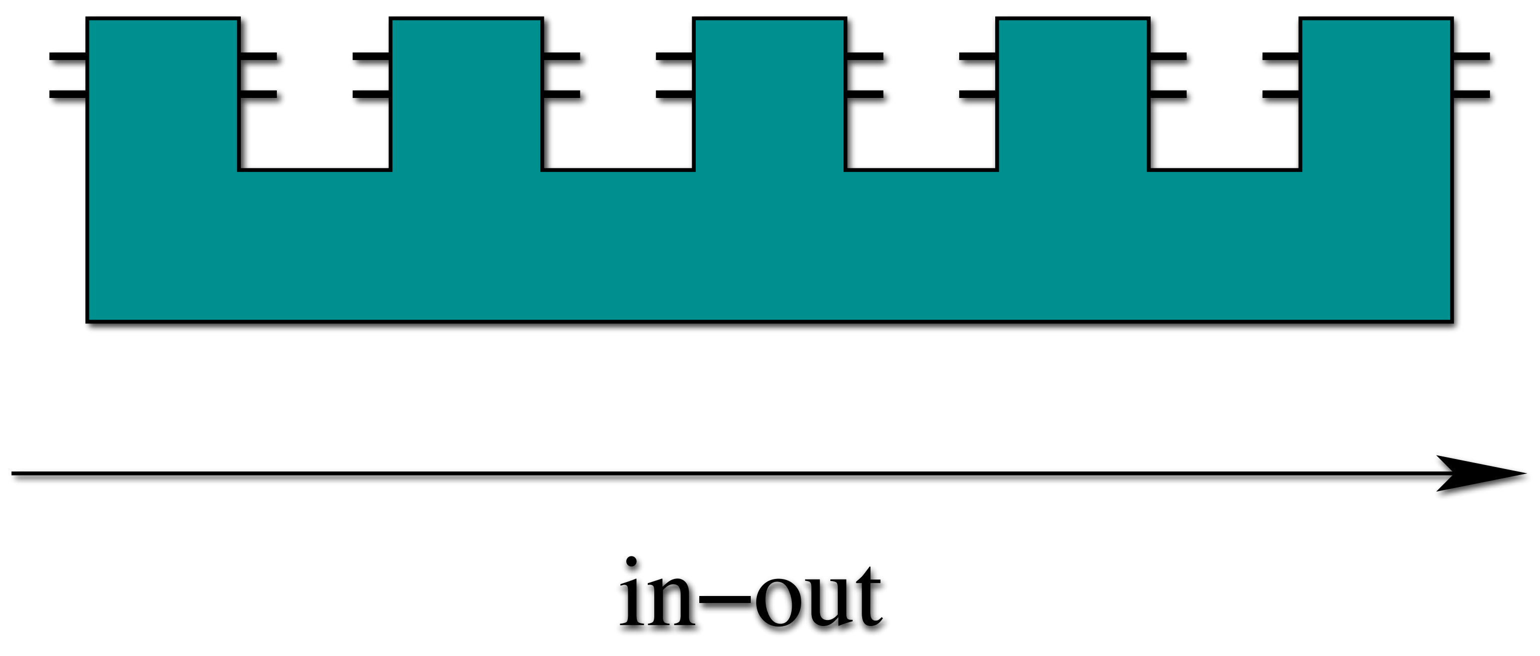
$$B \in \mathsf{B}(\mathsf{B}_{\mathrm{out}} \otimes \mathsf{B}_{\mathrm{in}}) = \mathsf{B}(\mathsf{H}_{\boldsymbol{d}} \otimes \mathsf{H}_e \otimes \mathsf{H}_f \otimes \mathsf{H}_g)$$

$$A * B = \mathrm{Tr}_{\mathsf{J}}[A^{\theta_{\mathsf{J}}} B] \in \mathsf{B}(\mathsf{H}_{\mathrm{out}} \otimes \mathsf{H}_{\mathrm{in}})$$

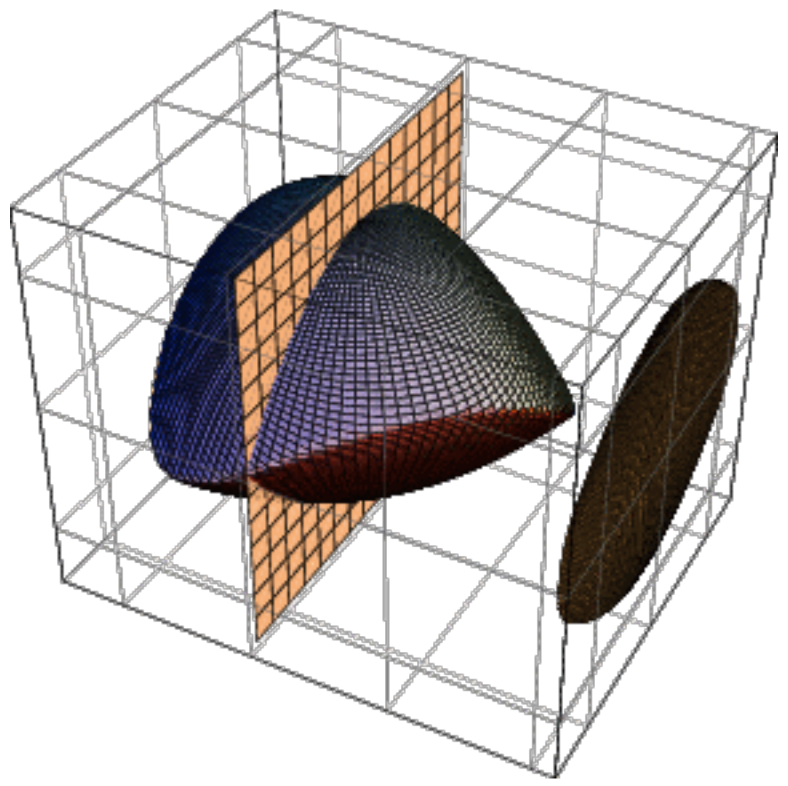
$$AB := (A \otimes I_{e,f,g})(I_{a,b,c} \otimes B)$$

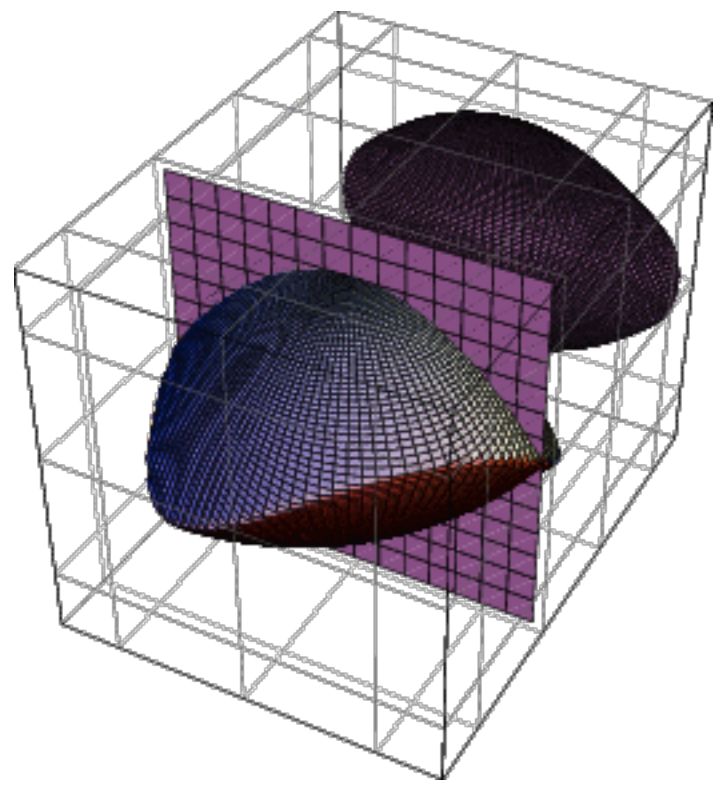
The link-product is commutative!

Circuits Architecture Optimization



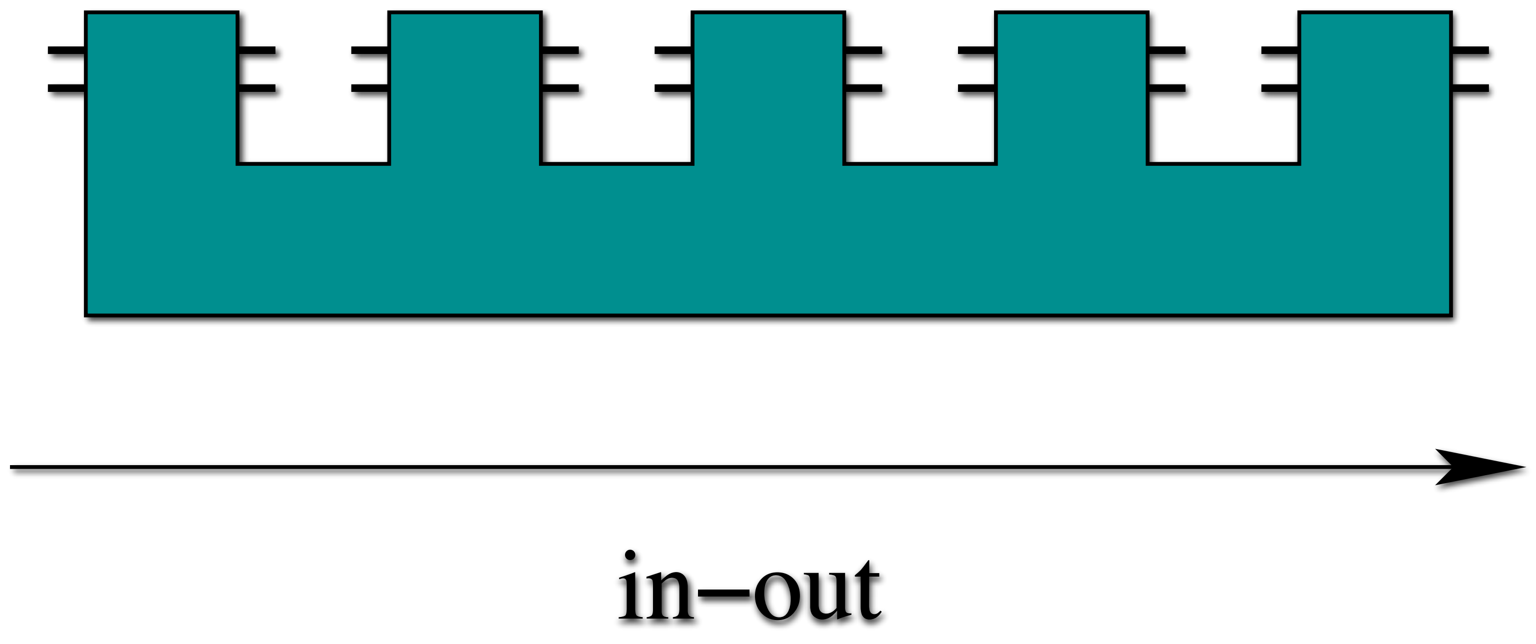
- The Choi operators of a fixed input-output comb structure make a convex set

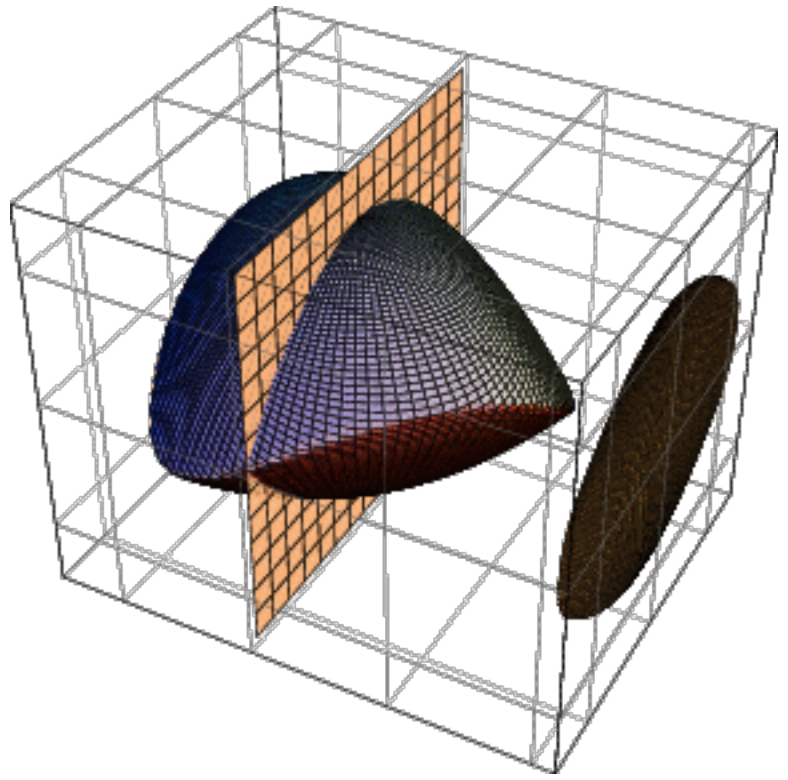




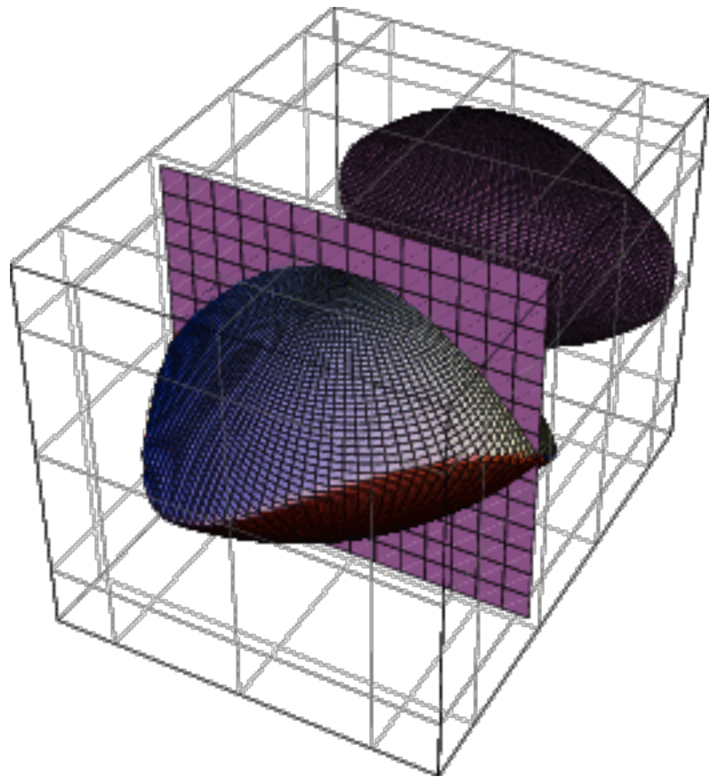
G. Chiribella, GMD, P. Perinotti, Phys. Rev. Lett. **101** 180501 (2008)

Circuits Architecture Optimization



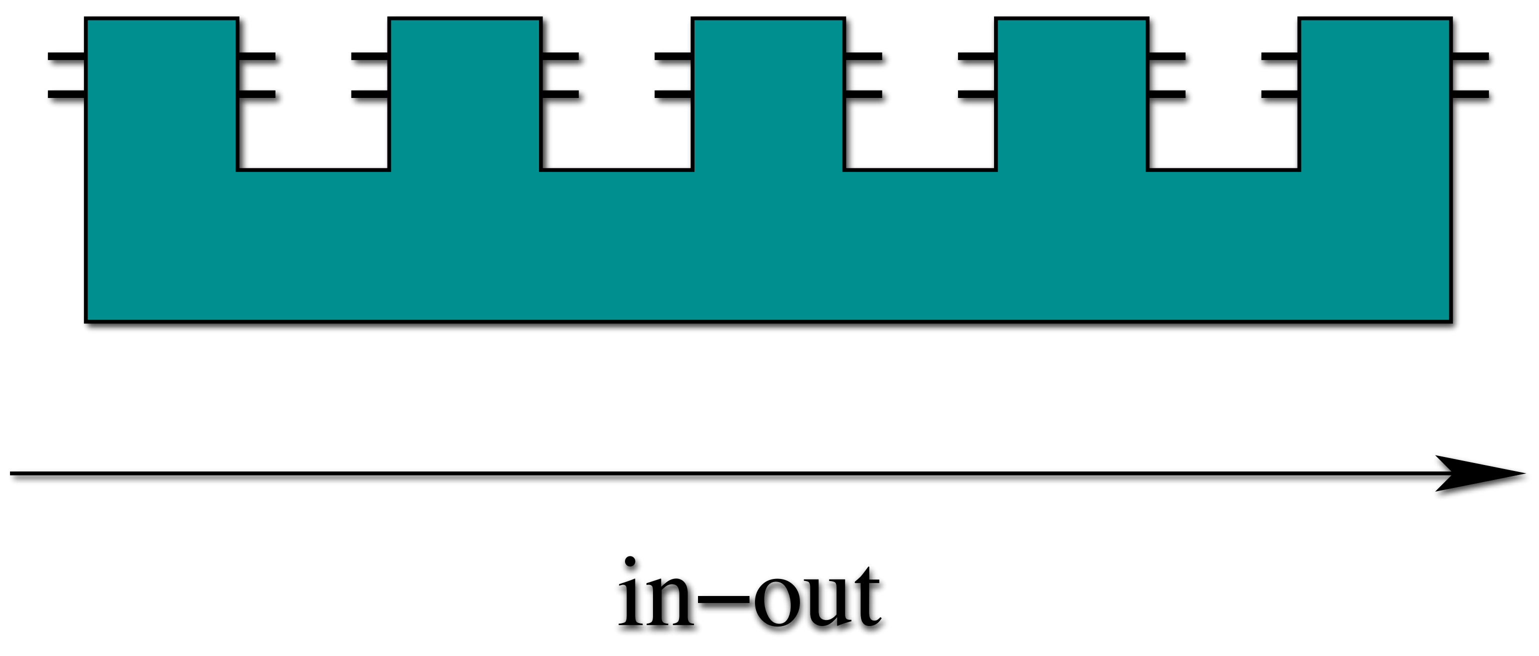


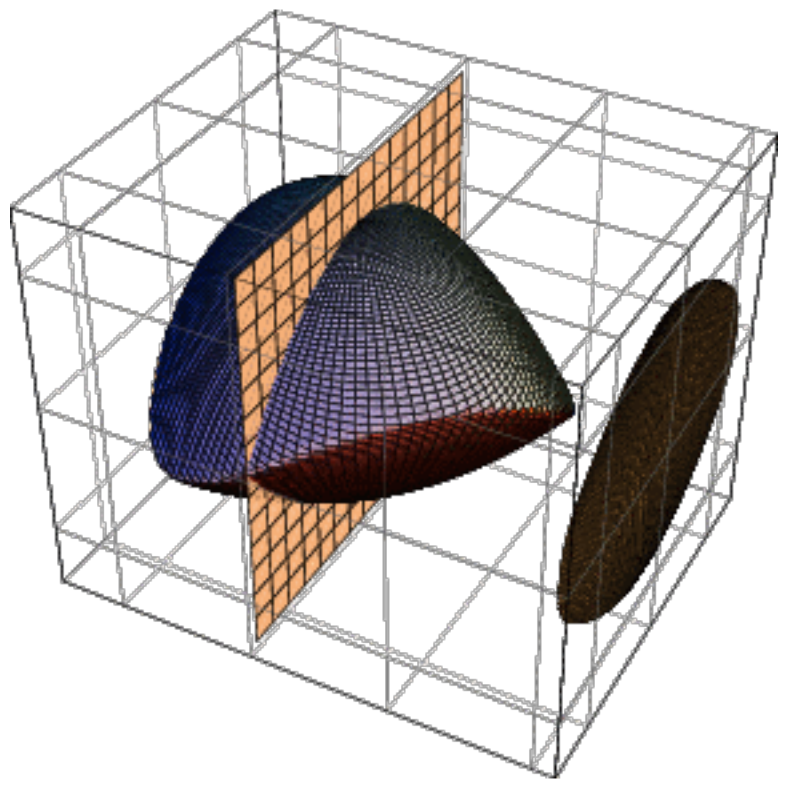
- The Choi operators of a fixed input-output comb structure make a convex set
- Causality constraints correspond to a hyperplane section of the convex
- The border of the section is the section of the border, and extremals of the section belong to the original border



G. Chiribella, GMD, P. Perinotti, Phys. Rev. Lett. **101** 180501 (2008)

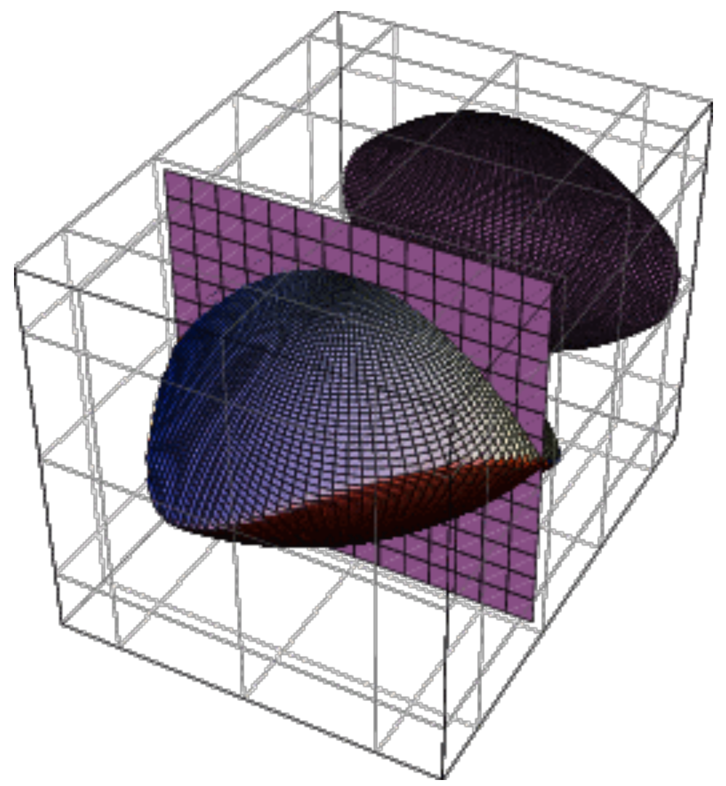
Circuits Architecture Optimization





- The Choi operators of a fixed input-output comb structure make a convex set
- Causality constraints correspond to a hyperplane section of the convex
- The border of the section is the section of the border, and extremals of the section belong to the original border
- Group-covariance gives another linear constraint:

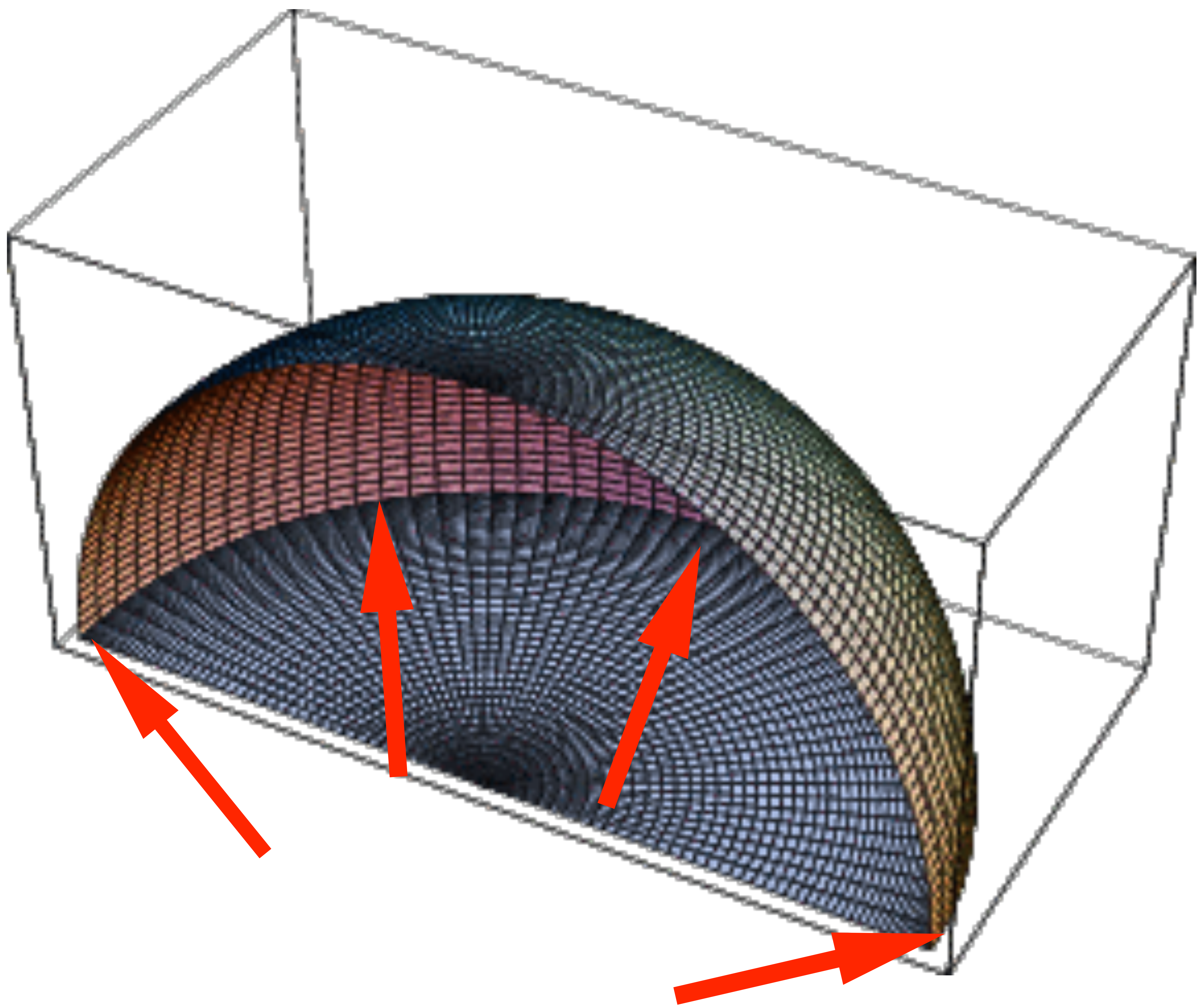
$$[R, V_g] = 0 \implies R = \bigoplus_j R_j \otimes \mathbb{1}_{m_j}$$



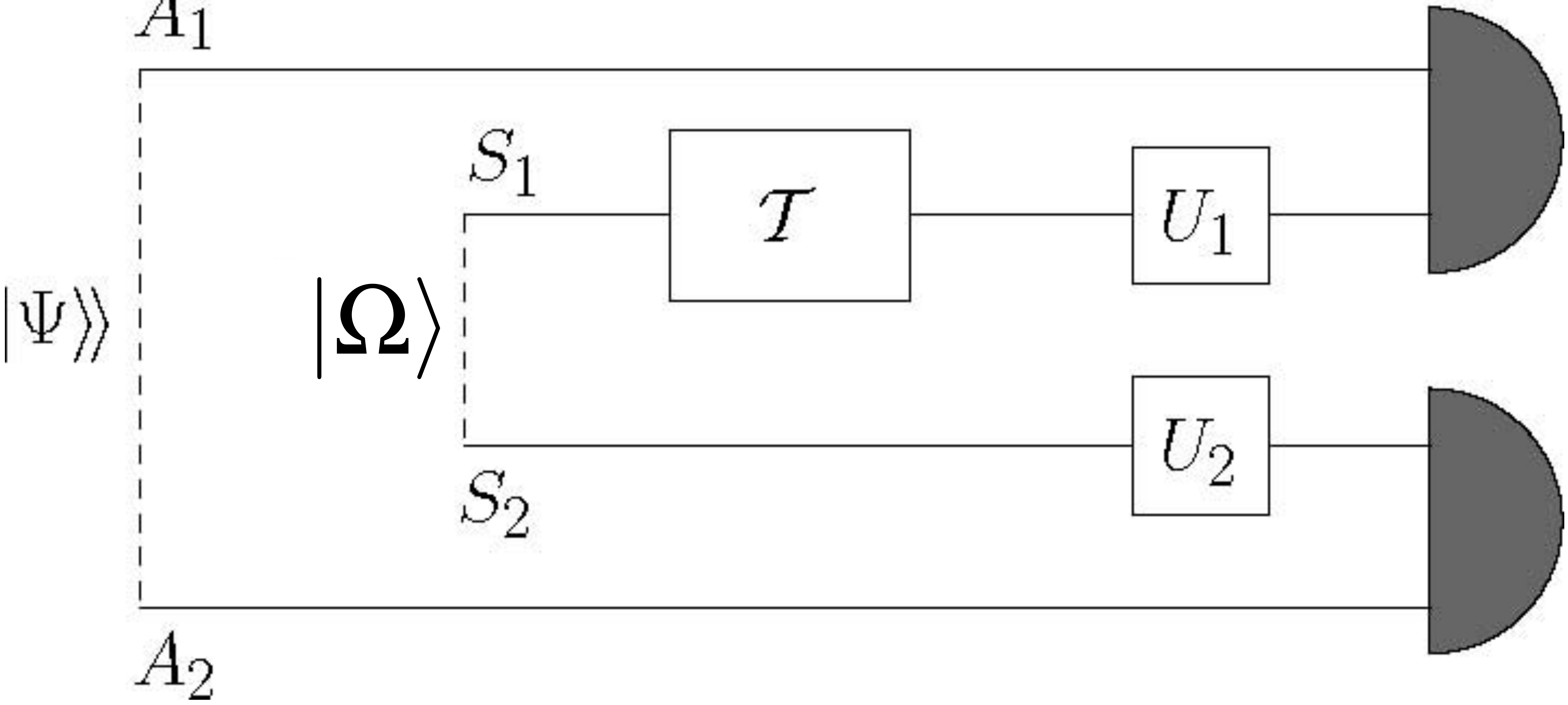
G. Chiribella, GMD, P. Perinotti, Phys. Rev. Lett. **101** 180501 (2008)

Circuits Architecture Optimization

- Generally the cost-function for optimisation is a concave function over the convex set of Choi.
- The optimal combs are thus achieved upon minimising the cost-function over the set of extremal points.



Optimal quantum tomography of transformations

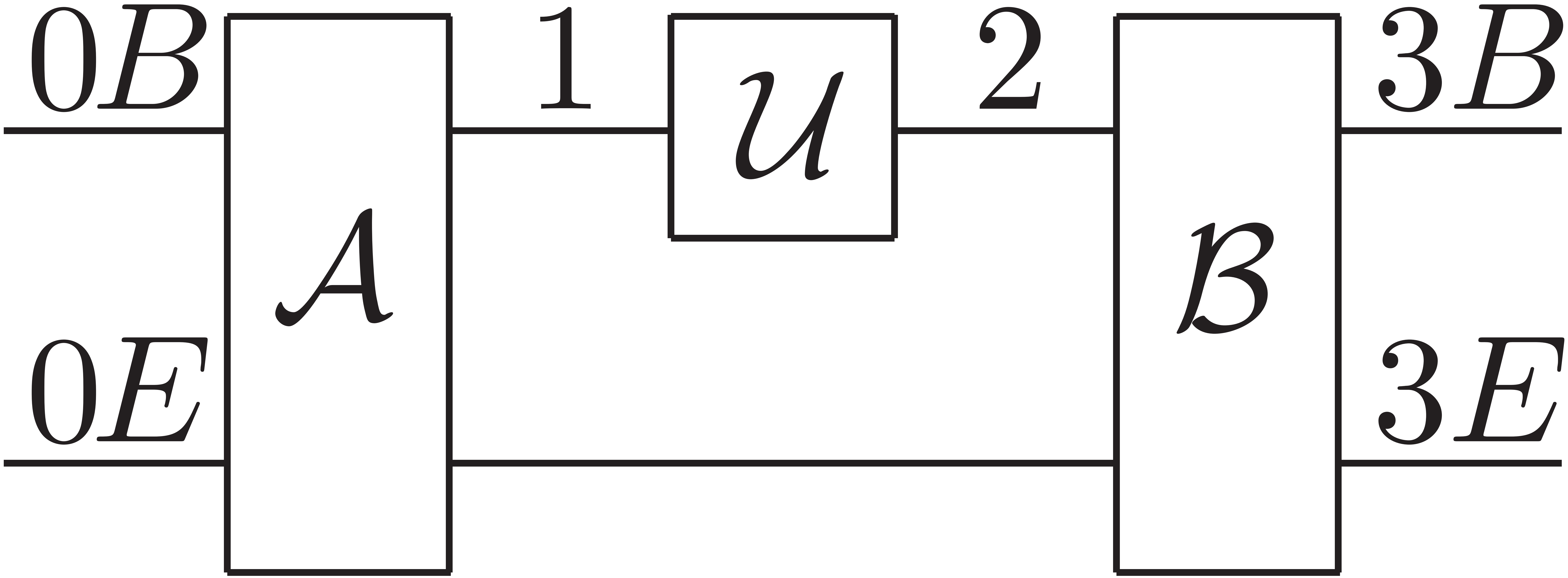


A. Bisio, G. Chiribella, GMD, S. Facchini, P. Perinotti, Phys. Rev. Lett. **102** 010404 (2009)

IEEE J.Sel.Top. Quant. Electr.**15** 1646 (2009)

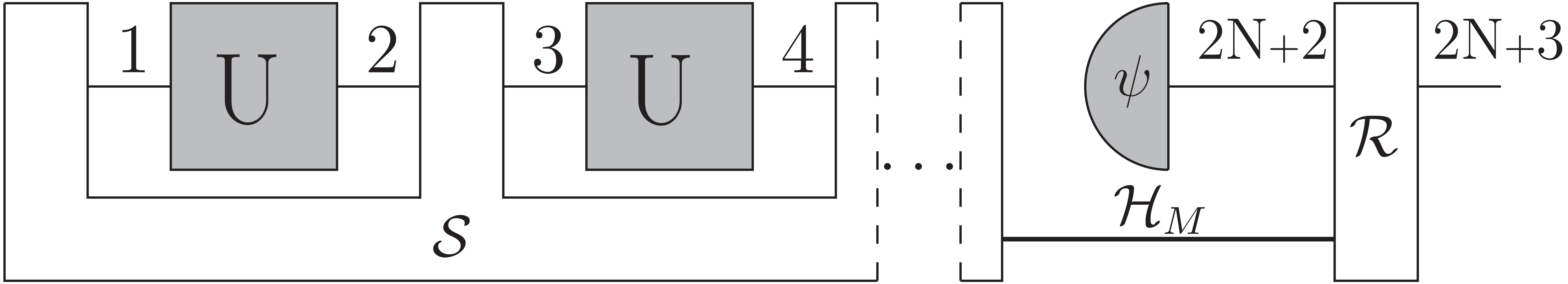
Optimal cloning of unitaries

1—> 2 cloning of unitaries



G. Chiribella, GMD, P. Perinotti, Phys. Rev. Lett. **101** 180504 (2008)

Optimal learning of unitaries



A. Bisio, G. Chiribella, GMD, S. Facchini, P. Perinotti, Phys. Rev. A **81** 032324 (2010)

Foundations:

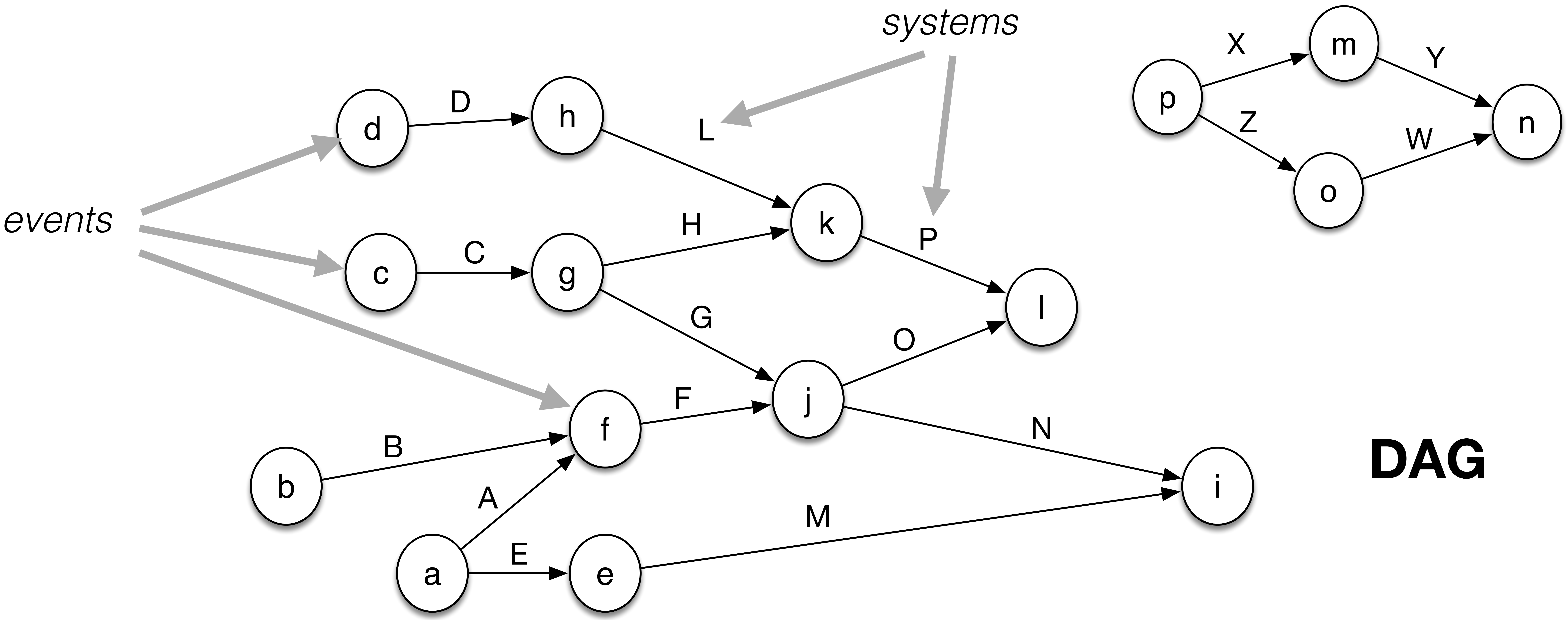
“information” as a paradigm for QT and QFT

With P. Perinotti, A. Bisio, A. Tosini, M. Erba, N. Mosco, …

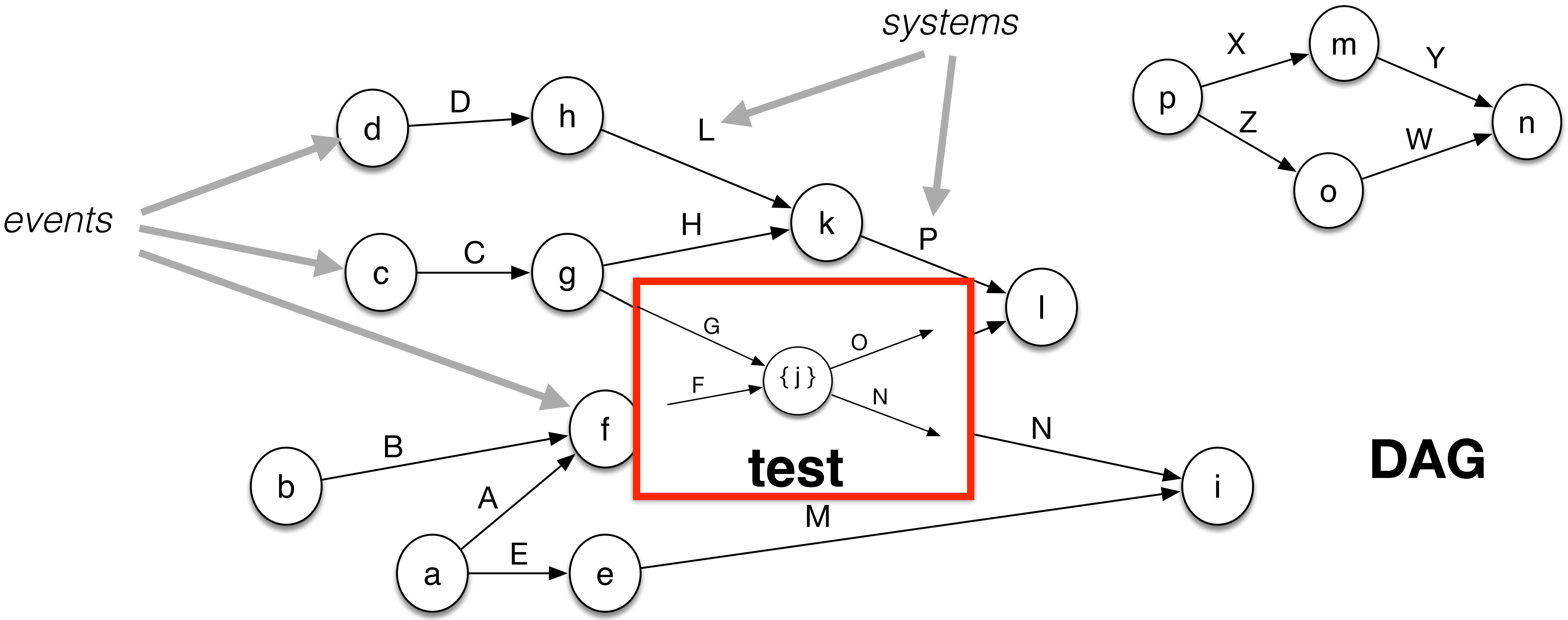
Quantum Theory: the “grammar” of Physics

Quantum Theory is an OPT

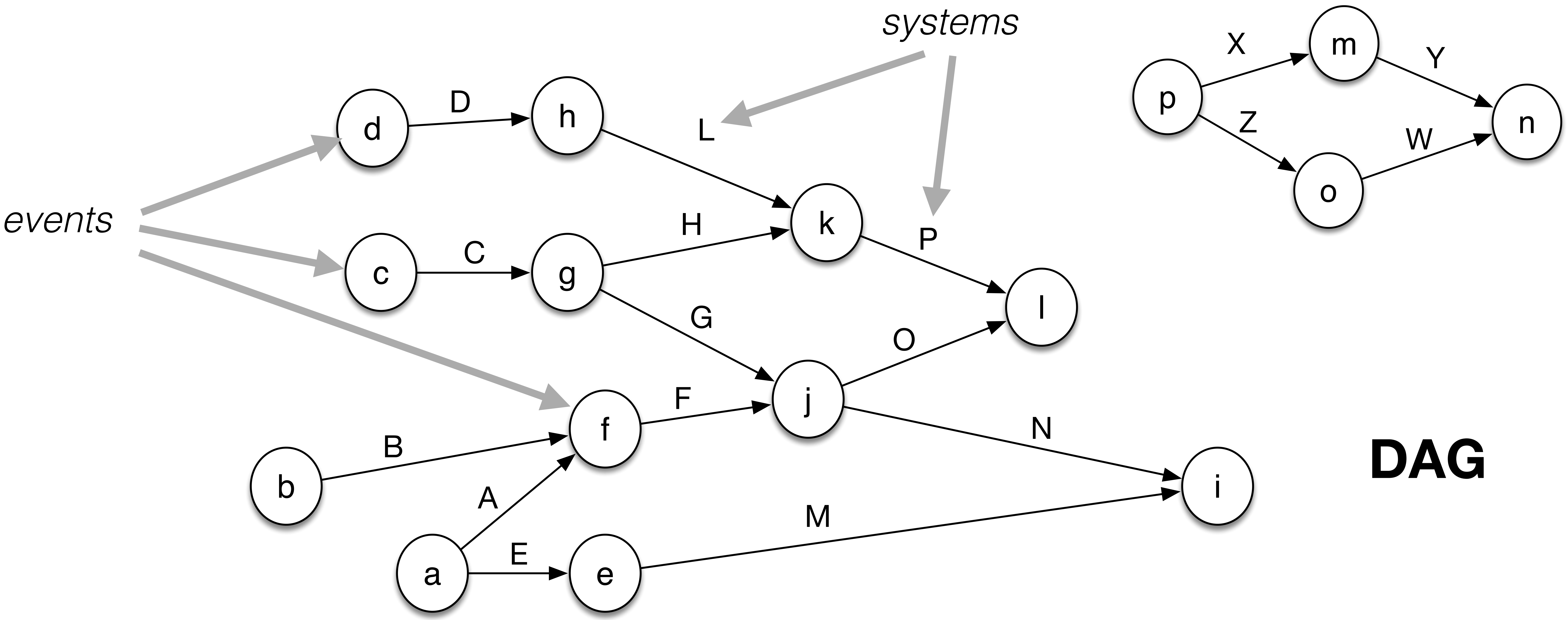
Operational probabilistic theory (OPT)



Operational probabilistic theory (OPT)

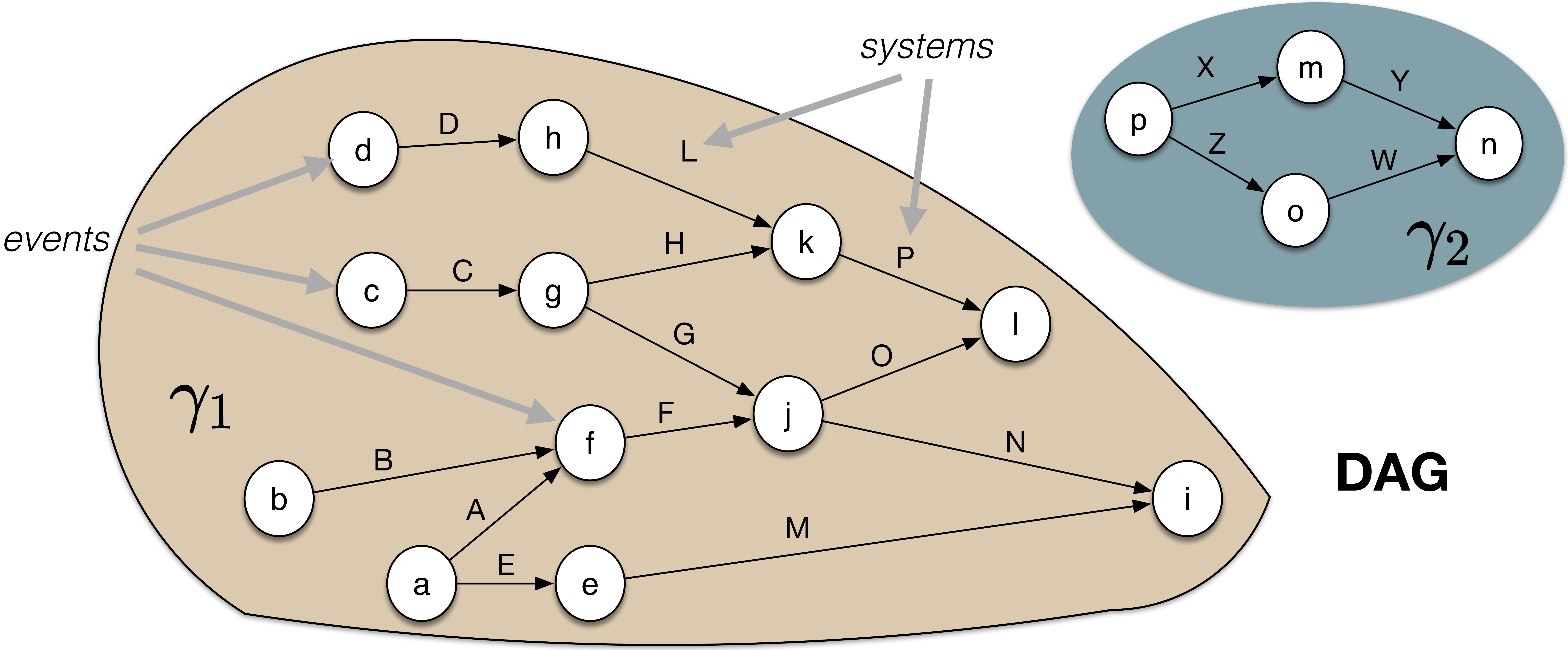


Operational probabilistic theory (OPT)



NOTICE: marginals depend on the marginalised part of the graph!

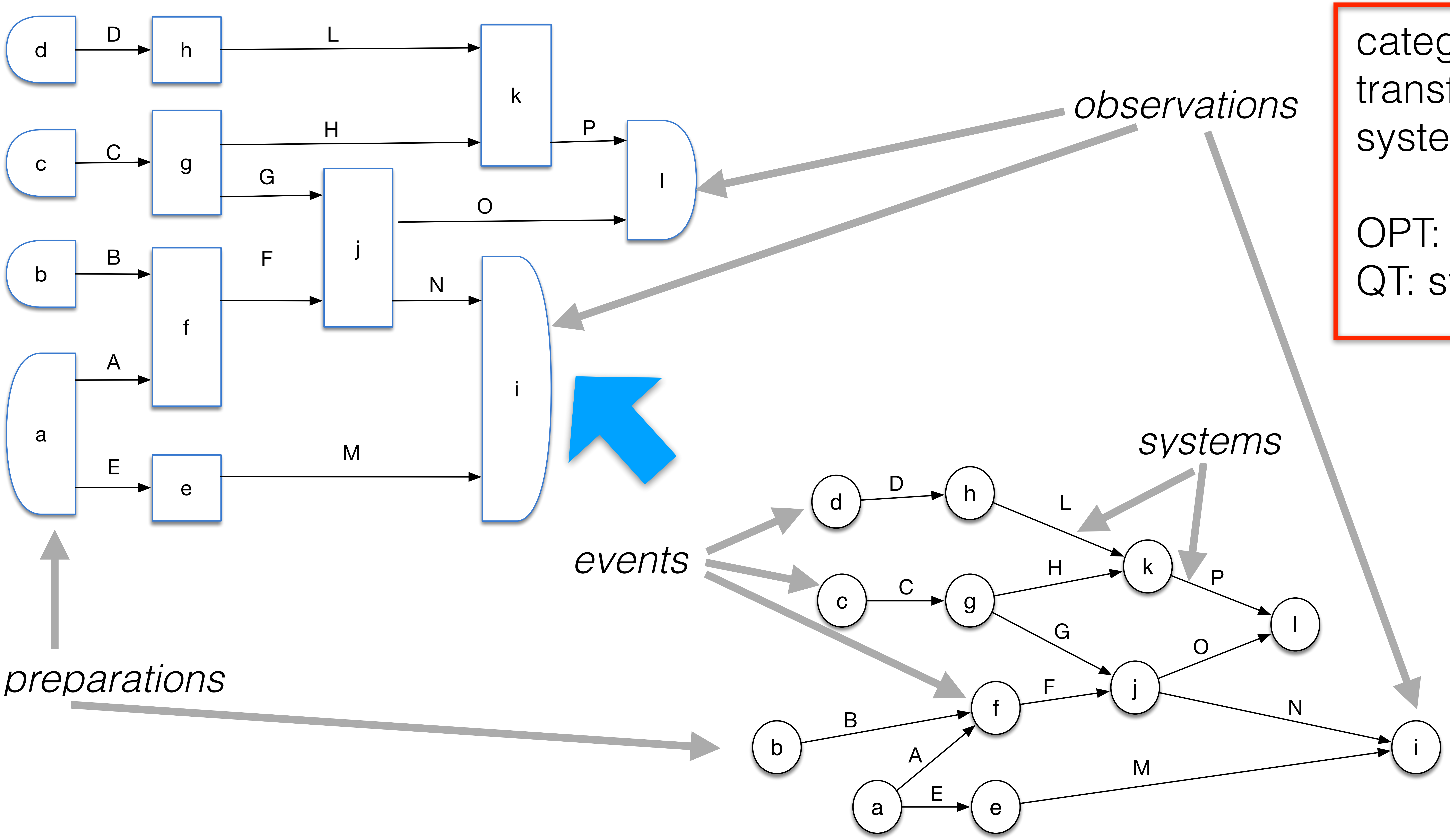
Operational probabilistic theory (OPT)



$$p(abc,\ldots,o|\gamma_1 \cup \gamma_2) = p(abc,\ldots,l|\gamma_1)p(n,\ldots,p|\gamma_2)$$

NOTICE: marginals depend on the marginalised part of the graph!

An OPT is an Information Theory



category theory:
transformations ➡ morphisms
systems ➡ objects

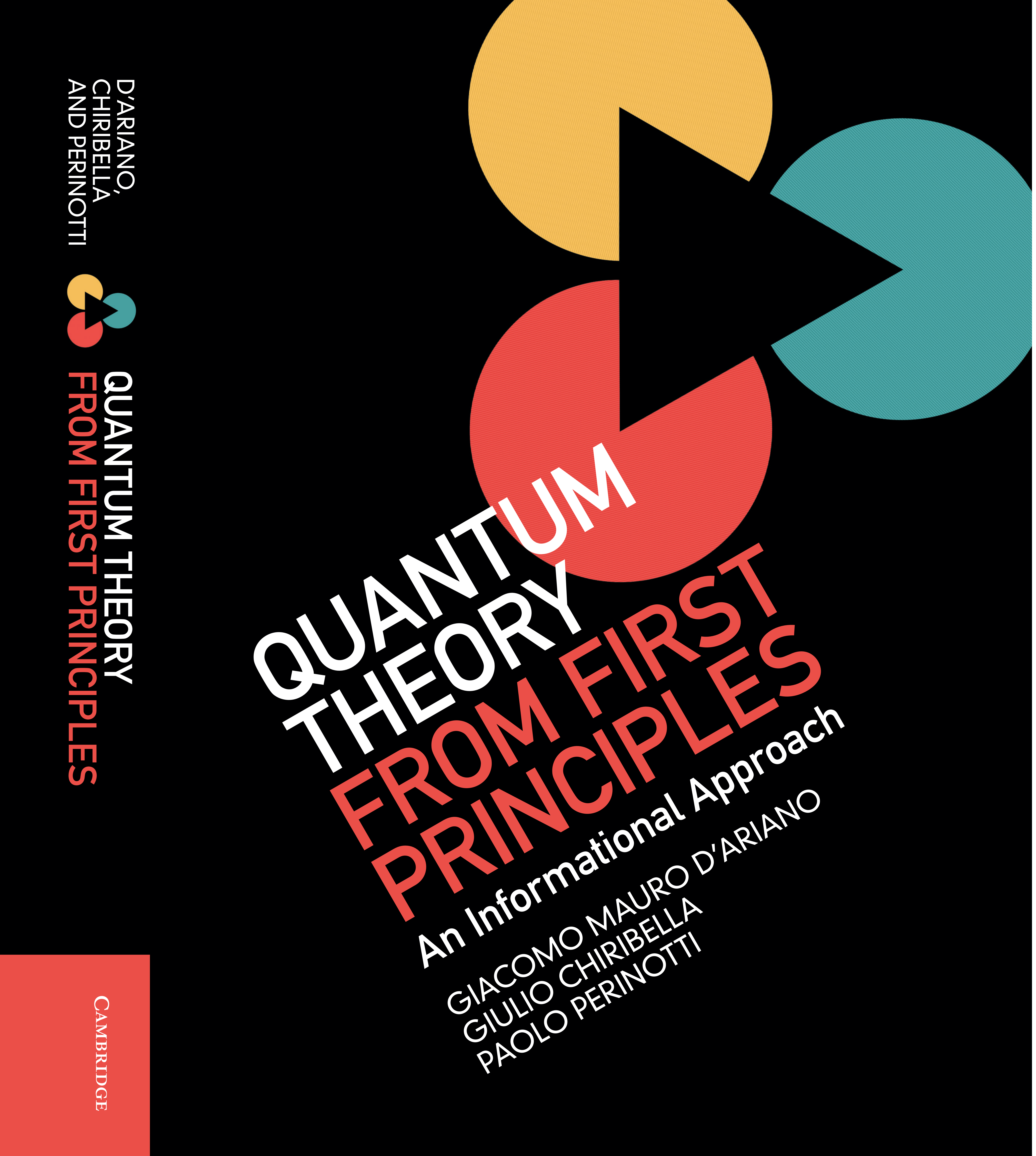
OPT: strict monoidal braided category
QT: symmetrical OPT

finite DAG

OPT=information theory

Quantum Theory as OPT		
system	A	$\mathscr{H}_{\mathrm{A}}$
system composition	AB	$\mathscr{H}_{\mathrm{AB}} = \mathscr{H}_{\mathrm{A}} \otimes \mathscr{H}_{\mathrm{B}}$
transformation	$\mathscr{T} \in \mathrm{Transf}(\mathrm{A} \to \mathrm{B})$	$\mathscr{T} \in \mathrm{CP}_{\leq}(\mathrm{T}(\mathscr{H}_{\mathrm{A}}) \to \mathrm{T}(\mathscr{H}_{\mathrm{B}}))$

Theorems		
trivial system system	I	$\mathscr{H}_{\mathrm{I}} = \mathbb{C}$
deterministic transformation	$\mathscr{T} \in \mathrm{Transf}_1(\mathrm{A} \to \mathrm{B})$	$\mathscr{T} \in \mathrm{CP}_{=}(\mathrm{T}(\mathscr{H}_{\mathrm{A}}) \to \mathrm{T}(\mathscr{H}_{\mathrm{B}}))$
states	$\rho \in \mathrm{St}(\mathrm{A}) \equiv \mathrm{Transf}(\mathrm{I} \to \mathrm{A})$	$\rho \in \mathrm{T}^{+}_{\leq 1}(\mathscr{H}_{\mathrm{A}})$
	$\rho \in \mathrm{St}_1(\mathrm{A}) \equiv \mathrm{Transf}_1(\mathrm{I} \to \mathrm{A})$	$\rho \in \mathrm{T}^{+}_{=1}(\mathscr{H}_{\mathrm{A}})$
	$\rho \in \mathrm{St}(\mathrm{I}) \equiv \mathrm{Transf}(\mathrm{I} \to \mathrm{I})$	$\rho \in [0,1]$
	$\rho \in \mathrm{St}_1(\mathrm{I}) \equiv \mathrm{Transf}(\mathrm{I} \to \mathrm{I})$	$\rho = 1$
effects	$\varepsilon \in \mathrm{Eff}(\mathrm{A}) \equiv \mathrm{Transf}(\mathrm{A} \to \mathrm{I})$	$\varepsilon(\cdot) = \mathrm{Tr}_{\mathrm{A}}[\cdot E],\ 0 \leq E \leq I_A$
	$\varepsilon \in \mathrm{Eff}_1(\mathrm{A}) \equiv \mathrm{Transf}_1(\mathrm{A} \to \mathrm{I})$	$\varepsilon = \mathrm{Tr}_{\mathrm{A}}$



Principles for Quantum Theory

P1. Causality

P2. Local discriminability

P3. Purification

P4. Atomicity of composition

P5. Perfect distinguishability

P6. Lossless Compressibility

G. Chiribella, G. M. D'Ariano, P. Perinotti, *Probabilistic Theories with Purification* Phys. Rev. A **81** 062348 (2010)

G. Chiribella, G. M. D'Ariano, P. Perinotti, *Informational derivation of Quantum Theory* Phys. Rev A **84** 012311 (2011)

Other OPTs

	Caus.	Perf. disc.	Loc. discr.	n-loc. discr.	At. par. comp.	At. seq. comp.	Compr.	∃ Purification	∃! Purification	NIWD
QT	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
CT	✓	✓	✓	✓	✓	✓	✓	✗	✗	✗
QBIT	✓	✓	✓	✓	✓	✓	✗	✓	✓	✓
FQT	✓	✓	✗	✓	✓	✓	✗	✓	✓	✓
RQT	✓	✓	✗	✓	✓	✓	✓	✓	✓	✓
NSQT	?	?	✗	✗	?	?	?	?	?	?
PR	✓	?	✓	✓	✓	?	✗	✗	✗	✓
DPR	✓	?	✓	✓	✓	?	✗	✗	✗	✓
HPR	✓	?	✓	✓	✓	✓	✓	✓	✓	✓
FOCT	✗	?	✓	✓	✓	?	?	✗	✗	?
FOQT	✗	?	?	✓	?	?	?	?	?	?
NLCT	✓	✓	✗	✓	✗	?	✓	✗	✗	✗
NLQT	?	?	?	✓	?	?	?	?	?	?

QT: Quantum theory

CT: Classical theory

QBIT: Qubit theory

FQT: Fermionic quantum theory

RQT: Real quantum theory

NSQT: Number superselected quantum theory

PR: PR-boxes theory

DPR: Dual PR-boxes theory

HPR: Hybrid PR-boxes theory

FOCT: First order classical theory

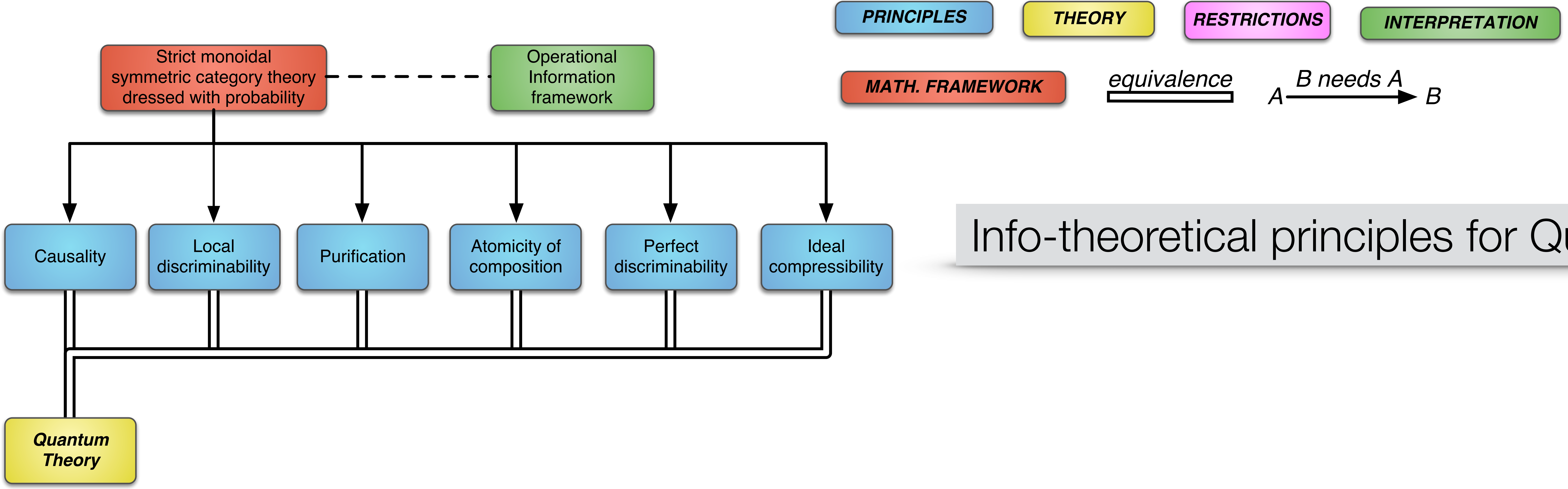
FOQT: First order quantum theory

NLCT: Non-local classical theory

NLQT: Non-local quantum theory

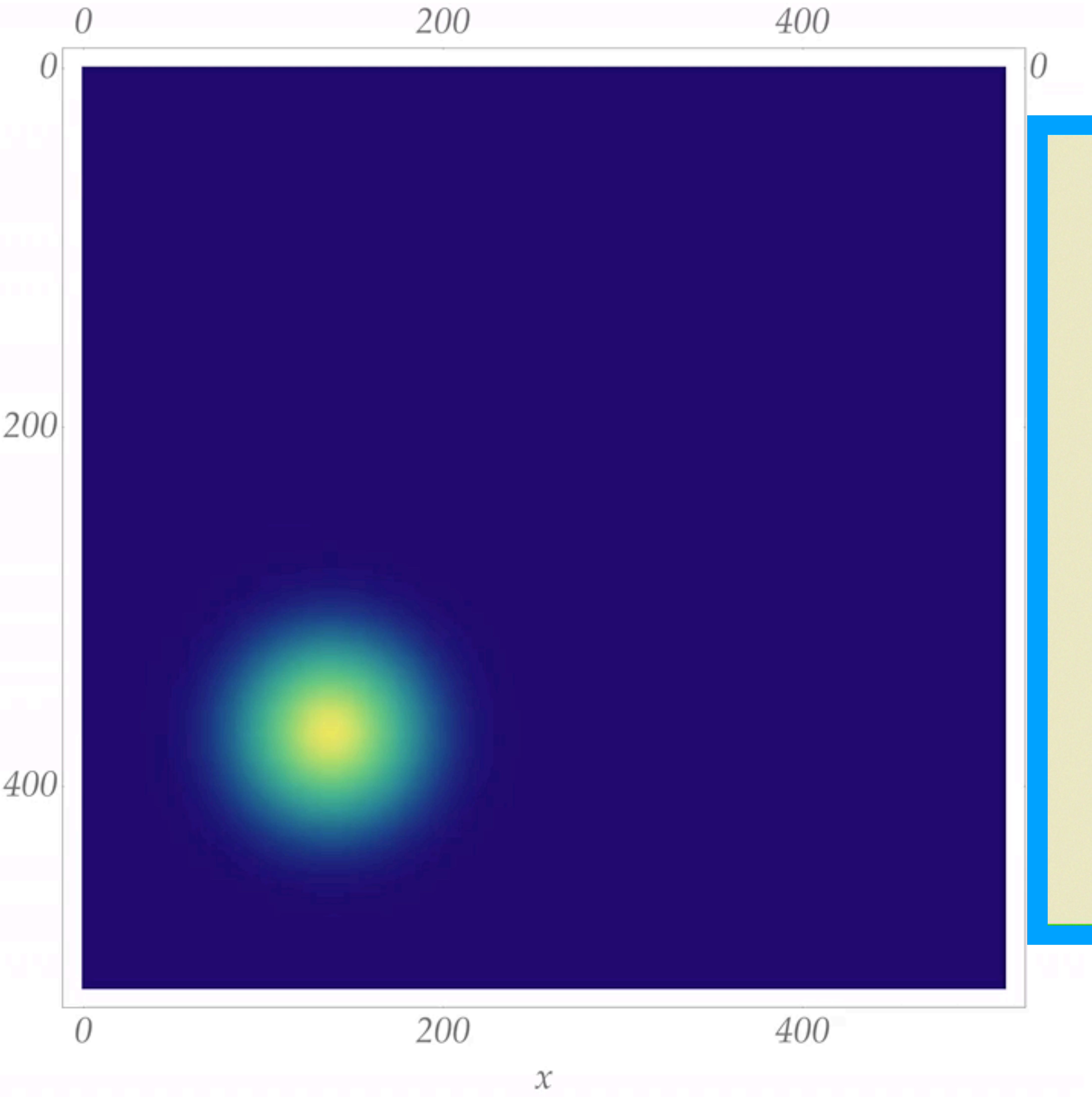
“HOW TO GET THE “MECHANICS?”

QUANTUM FIELD THEORY: an ultra-short account

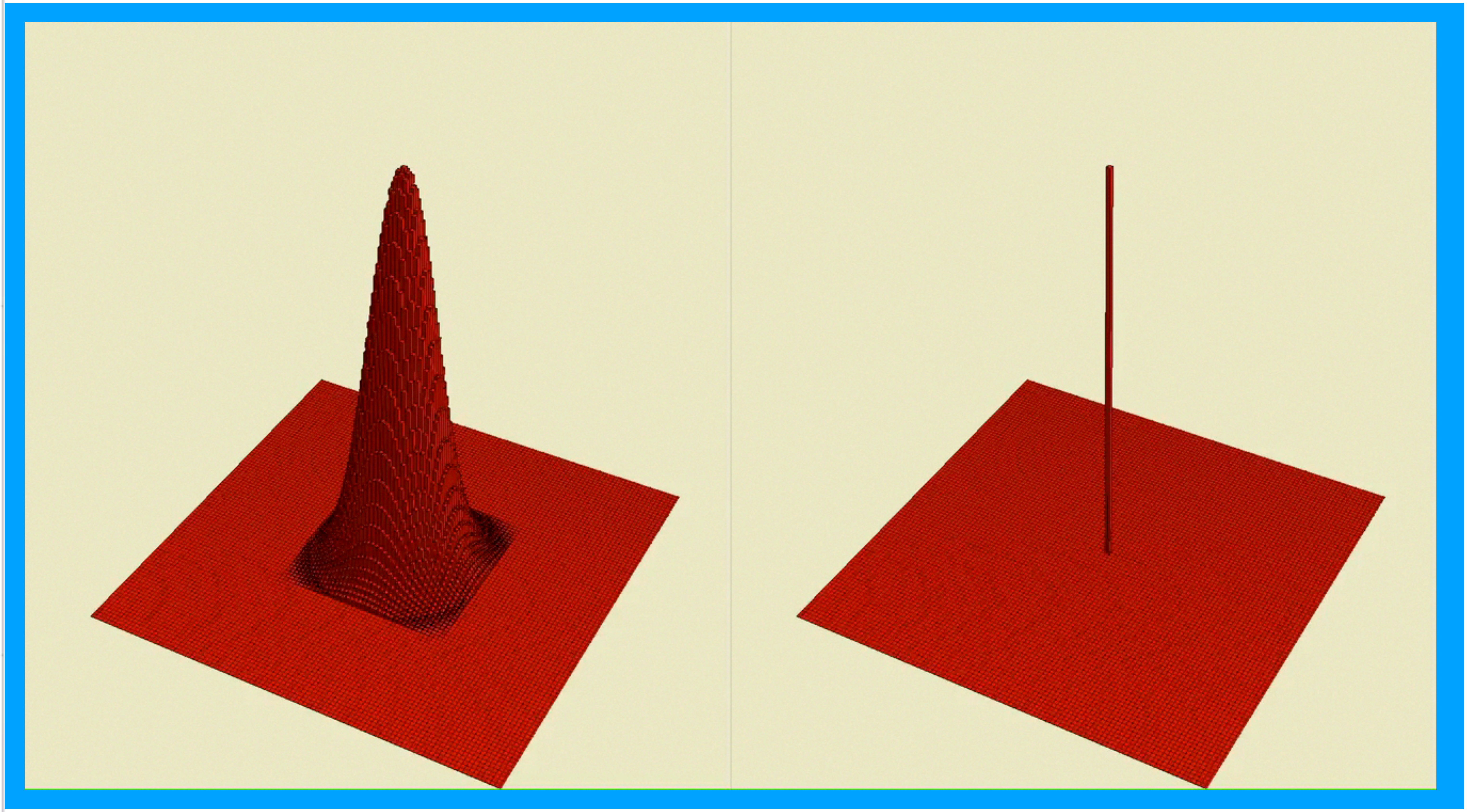


Info-theoretical principles for Quantum Field Theory

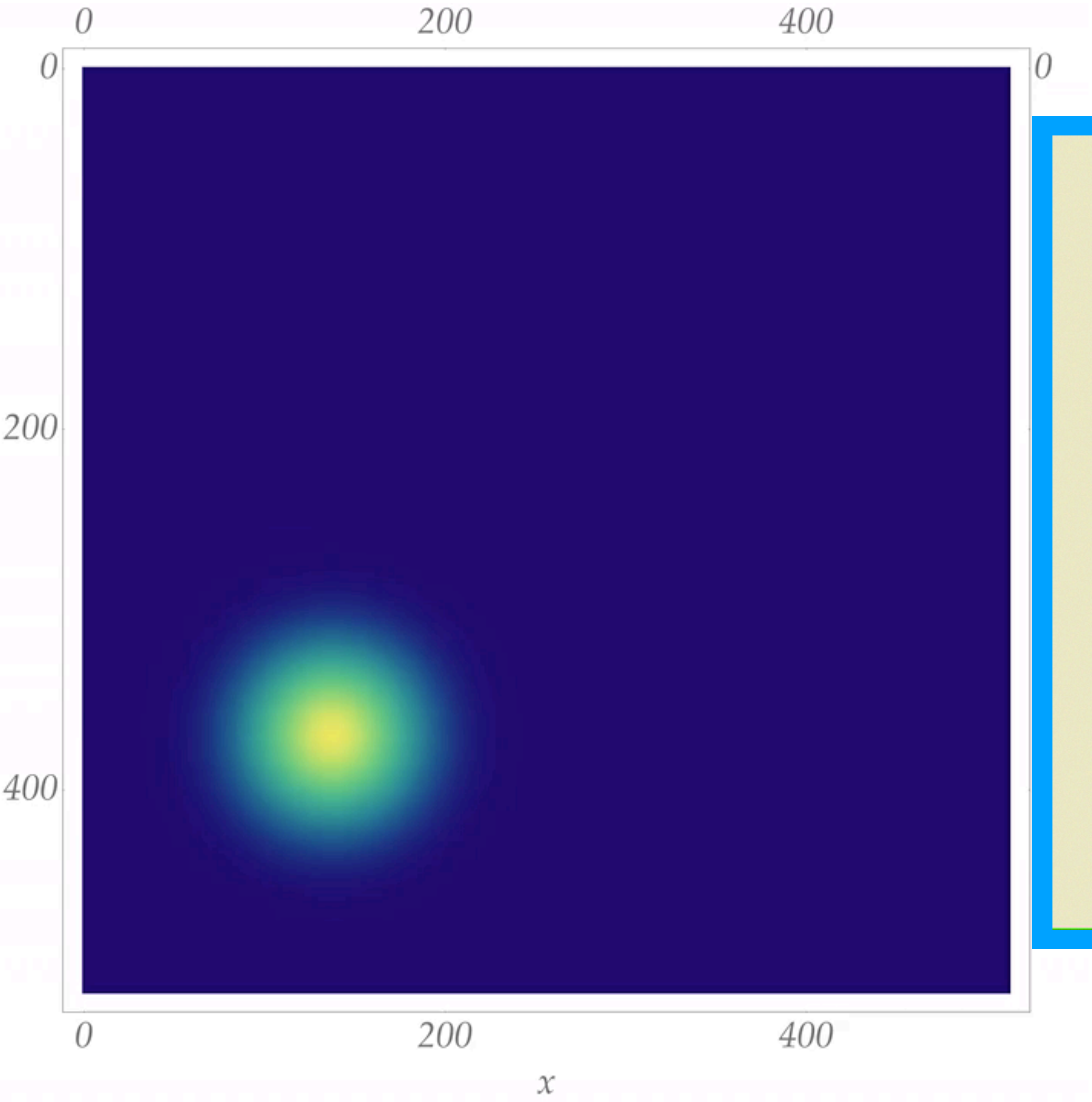
PRINCIPLES
THEORY
RESTRICTIONS
INTERPRETATION
MATH. FRAMEWORK
equivalence
B needs A
A
B
Info-theoretical principles for Quantum Field Theory
Strict monoidal symmetric category theory dressed with probability
Operational Information framework
Causality
Local discriminability
Purification
Atomicity of composition
Perfect discriminability
Ideal compressibility
Quantum Theory
Locality
Homogeneity
Isotropy
Unitarity
Quantum Cellular Automata on a Cayley graph of G
Linearity
Quantum Walk on Cayley graph of G
Cayley graph quasi-isometrically embeddable in Euclidean space
G virtually Abelian
Quantum Walk on Cayley graph of Abelian G
Relativity Principle without space-time
m>0: deformed De Sitter
Particle as Poincaré invariant
m=0: deformed Lorentz
GR heuristics
Relativistic limit
Free Quantum Field Theory
LTM Standards
$m \in S^1$ discrete proper time

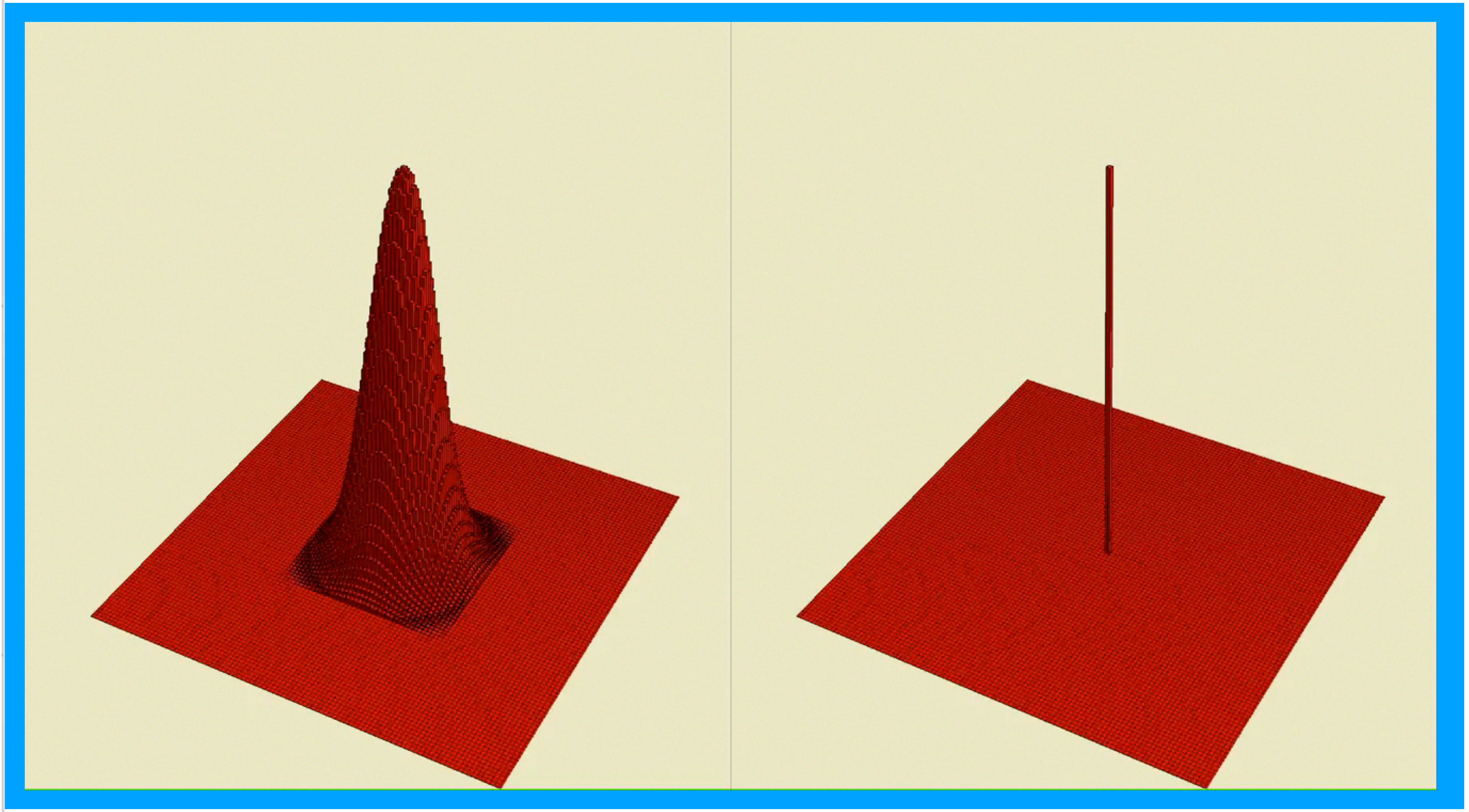


Info-theoretical principles for Quantum Field Theory



Info-theoretical principles for Quantum Field Theory





Thank you for your attention