Analysis of parametric oscillatory instability in Fabry-Perot cavity with Gauss and Laguerre-Gauss main mode profile

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Outline

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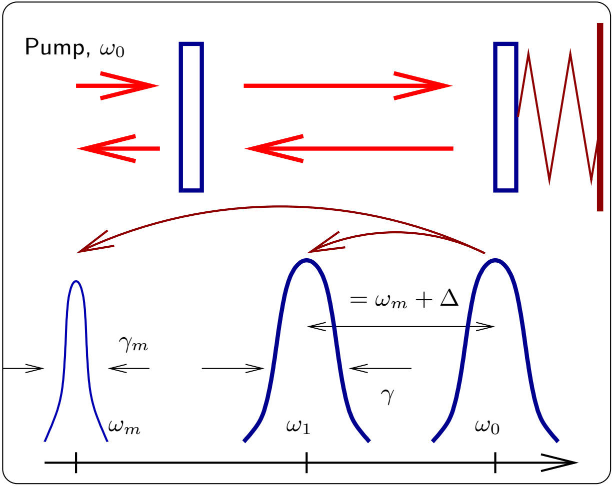
Effect of Parametric Oscillatory Instability in FP Cavity



Condition of Parametric Instability in FP Cavity:

Qualitative consideration:

- Detuning $\Delta = \omega_0 - \omega_1 - \omega_m$ is small: $\Delta \ll \gamma$.

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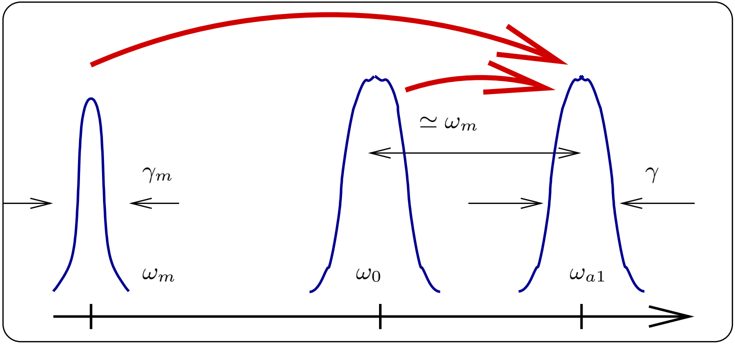
- The existence of anti-Stokes mode can depress Parametric instability.

[1] V. B. Braginsky, S. E. Strigin, and S. P. Vyatchanin, *Physics Letters* **A287**, 331 (2001); gr-qc/0107079.

Anti-Stokes Mode

Existence of the anti-Stokes mode with frequency $\omega_{1\,a} = \omega_0 + \omega_m$ will substantially dump the effect of parametric instability[2]. However, the probability that suitable anti-Stokes mode exists is relatively small[3].



[2] E. D'Ambrosio and W. Kells, *Physics Letter* **A299**, 326 (2002). LIGO-T020008-00-D

[3] V. B. Braginsky, S. E. Strigin and S. P. Vyatchanin, *Physics Letters* **A305**, 111 (2002).

Outline

Mesa beams, conical modes and Laguerre-Gauss modes

In order to lower the thermal noise in gravitational wave antennas

it was proposed to change the mode shape of the laser beam inside the interferometer:

- Mesa beams [4]
- Conical modes [5]
- High order Laguerre-Gauss modes, in particular, LG_{33} mode [6]

The aim of consideration

We estimate the possibility of parametric instabilities in Fabry-Perot cavity of Advanced VIRGO and LIGO interferometers both for Gauss TEM_{00} and Laguerre-Gauss LG_{33} modes as a carriers.

[4] E. D'Ambrosio, R. O'Shaughnessy, S. E. Strigin, K. Thorne and S. P. Vyatchanin, http://arXiv.org: gr-qc/0409075;
R. O'Shaughnessy, S. E. Strigin and S. P. Vyatchanin, http://arXiv.org: gr-qc/0409050;
M. Bondarescu and K.S. Thorne, Physical Review D74, 082003, 2006

[5] M. Bondarescu, O. Kogan, and Y. Chen, Physical Review D78, 082002, 2008

[6] B. Mours, E. Tournefier, and J.-Y. Vinet, Classical and Quantum Gravity 23, 5777, 2006;
S.Chelkowski, S. Hild, A. Freise, arXiv:0901.4931v1[gr-qs], 2009.

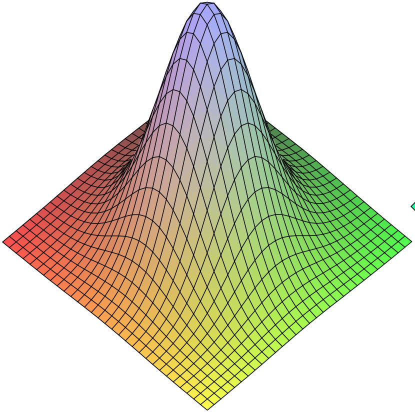
Condition of parametric instability

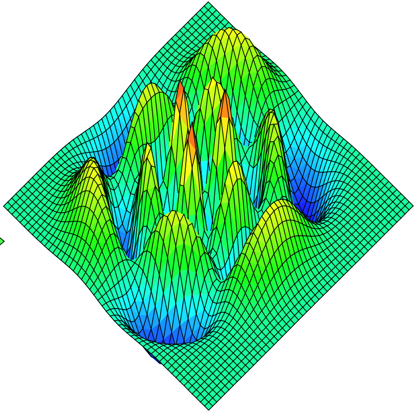
$$\frac{\Lambda_1 W \omega_1}{cLm\omega_m\gamma_m\gamma_1} \times \frac{1}{1+\frac{\Delta^2}{\gamma_1^2}} - \frac{\Lambda_{1a} W \omega_1}{cLm\omega_m\gamma_m\gamma_1} \times \frac{\omega_{1a}\gamma_1}{\omega_1\gamma_{1a}} \times \frac{1}{1+\frac{\Delta_{1a}^2}{\gamma_{1a}^2}} > 1,$$

$$\Lambda_1 = \frac{V\left(\int A_0(r_\perp)A_1(r_\perp)u_z dr_\perp\right)^2}{\int |A_0|^2 dr_\perp \int |A_1|^2 dr_\perp \int |\vec{u}|^2 dV},\ \Lambda_{1a} = \frac{V\left(\int A_0(r_\perp)A_{1a}(r_\perp)u_z dr_\perp\right)^2}{\int |A_0|^2 dr_\perp \int |A_{1a}|^2 dr_\perp \int |\vec{u}|^2 dV}$$

W is a power circulating inside the cavity, c is a speed of light,
L is a distance between the FP mirrors, m is a mirror's mass,
ω_1 is frequency of Stokes mode, ω_{1a} — frequency of anti-Stokes mode,
γ_m, γ_1 are the relaxation rates of elastic and Stokes modes correspondingly,
$\Delta = \omega_0 - \omega_1 - \omega_m$, $\Delta_{1a} = \omega_{1a} - \omega_0 - \omega_m$ are detunings, Λ_1 and Λ_{1a} are overlapping factors.

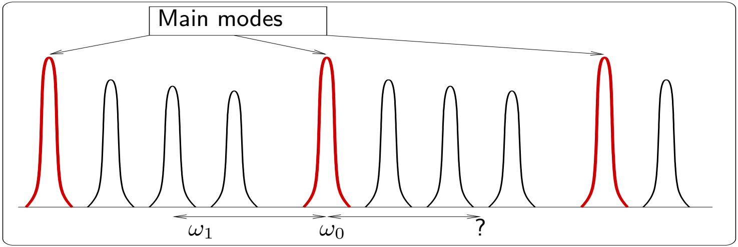
TEM_{00} and LG_{33} modes (amplitude distributions)



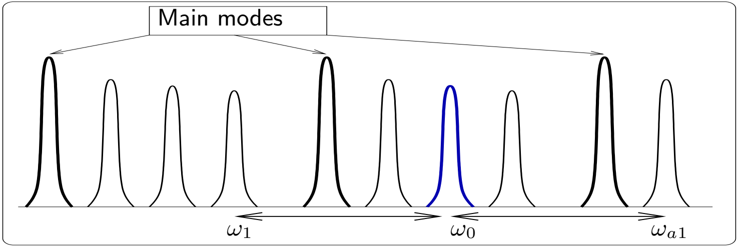


TEM_{00} and LG_{33} modes

TEM_{00}: structure of modes



LG_{33}: structure of modes



Dependence on azimuth angle

TEM_{00}

If the elastic modes have azimuth dependence $\sim e^{im\phi}$ then Stokes modes have to be $\sim e^{im\phi}$. In opposite case overlap factor will be equal to 0.

LG_{33}

If the elastic modes have azimuth dependence $\sim e^{im\phi}$ then Stokes modes have to be $\sim e^{i(m\pm 3)\phi}$. In opposite case overlap factor will be equal to 0.

It is correct

only if cylinder center coincides with laser spot center.
Below we consider this particular case.

Parameters of Advanced LIGO and VIRGO interferometers

In our calculations we used the parameters, listed below

VIRGO	LIGO
$W = 0.76 \times 10^6 W$	$W = 0.83 \times 10^6 W$
$L = 3000m$	$L = 4000\,\mathrm{m}$
$T = 7 \times 10^{-3}$	$T = 14 \times 10^{-3}$
$\lambda = 1064\,\mathrm{nm}$	$\lambda = 1064\,\mathrm{nm}$
$m = 40\,\mathrm{kg}$	$m = 40\,\mathrm{kg}$
$r = 0.17\,\mathrm{m}$	$r = 0.17\,\mathrm{m}$
$H = 0.2\,m$	$H = 0.2\,\mathrm{m}$
$\gamma_m = 6 \times 10^{-3}\,\mathrm{s}^{-1}$	$\gamma_m = 6 \times 10^{-3}\,\mathrm{s}^{-1}$
$\gamma = cT/4L = 175\,\mathrm{s}^{-1}$	$\gamma = cT/4L = 262.5\,\mathrm{s}^{-1}$

W is a power circulating inside the cavity,
L is a distance between the FP mirrors,
m is a mirror's mass, r and H are its radius and thickness,
T are power transmittances, γ is bandwidth of main mode (TEM_{00} or LG_{33}),
γ_m is bandwidth of elastic modes.

Details of calculations

Calculation of elastic modes

The elastic modes were calculated numerically using COMSOL® code on triangle mesh with about 40000 meshing elements.

$$\omega_{\text{elastic}} \leq 2\pi \times 40000\,\mathrm{s}^{-1}$$

(degradation of numerical accuracy of COMSOL® code).

VIRGO

Laser spot radii on mirrors surfaces $w = 6.47$ cm and $w = 3.94$ cm in TEM_{00} and LG_{33} cases correspondingly

LIGO

Laser spot radii on mirrors surfaces $w = 6$ cm and $w = 3.94$ cm in TEM_{00} and LG_{33} cases correspondingly.

Details of calculations

Level of diffractional losses of main mode

Level of diffractional losses in clipping approximation is hold to be $l_{clip} = 1$ ppm (or slightly less for LIGO TEM_{00} case).

Overlap factors

We calculated the overlap factors for all combinations of elastic and optical modes(up to 9^{th} order).
We used azimuth numbers condition in each case.

Losses of Stokes and anti-Stokes modes

We took into account the diffractional losses of Stokes and anti-Stokes modes in clipping approximation.

VIRGO, LG_{33}

One unstable combination

Table: Unstable combination of elastic and Stokes optical modes in FP cavity of VIRGO configuration with LG_{33} as a carrier. In the case of TEM_{00} carrier we did not find any unstable modes in given frequency range due to small overlap factors and large detuning values Δ.

$\omega_m/2\pi, \mathrm{Hz}$	m	Stokes mode	R
38577	1	LG_{32}	2.5

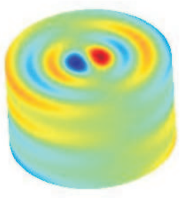


Figure: Displacement vector component distribution u_z of elastic mode with frequency $38577\mathrm{Hz}$ and azimuth index $m = 1$. The numerical calculations of cylinder have been made on triangle mesh with about 40000 meshing elements.

LIGO, LG_{33}

Three unstable combinations

Table: Unstable combinations of elastic and Stokes optical modes in FP cavity of LIGO configuration with LG_{33} as a carrier.

$\omega_m/2\pi, \mathrm{Hz}$	m	Stokes mode	R
36524	6	LG_{03}	1.5
37566	4	LG_{17}	9.3
37566	4	LG_{41}	10.8

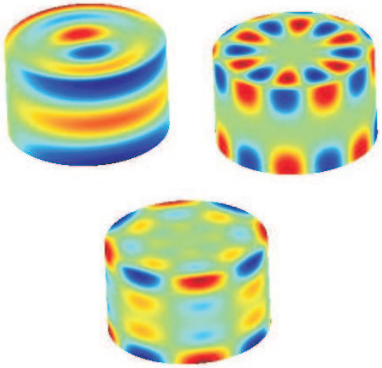


Figure: Displacement vector component distributions u_z of elastic modes with frequencies $23048\mathrm{Hz}$(azimuth index $m = 1$), $36524\mathrm{Hz}$(azimuth index $m = 6$) and $37566\mathrm{Hz}$(azimuth index $m = 4$) correspondingly.

VIRGO and LIGO – TEM_{00}

VIRGO, TEM_{00}

No unstable combinations were found

LIGO, TEM_{00}

Table: One unstable combination of elastic and Stokes optical modes in FP cavity of LIGO configuration with TEM_{00} as a carrier.

$\omega_m/2\pi, \mathrm{Hz}$	m	Stokes mode	R
23048	1	LG_{21}	1.7

Conclusion

Comparison of TEM_{00} and LG_{33} modes as carriers

The number of obtained unstable modes for LG_{33} case is slightly larger than for TEM_{00} case.
Restrictions: account of elastic modes with frequencies up to 40 kHz only.

However,

this difference is not large enough to conclude that LG_{33} case is more dangerous than TEM_{00} case.

Detailed analysis

There is necessity to perform detailed analysis for full scale schemes of Advanced LIGO and VIRGO in LG_{33} case with wide elastic modes frequency range.

Notes

Axis of opticlal and elastic modes

Elastic and the Stokes mode can have different dependence on azimuth angle — we omitted such combinations in our estimations. However, it is important to take into account that only the elastic mode is attached to the mirror axis in contrast to the optical mode which can be shifted from the mirror axis due to non-perfect optical alignment.

Flats and ears

Imperfections from the cylinder shape (flats and suspension ears), should cause splitting of elastic modes with azimuth index $m \geq 1$ into doublets and the difference between doublet frequencies may be large enough or greater as compared with relaxation rate of Stokes mode. It may increase the possibility of parametric instability in Advanced VIRGO and LIGO [7].

Power and signal recycled interferometer

The effect of parametric instability for power recycled interferometer may be larger than for the separate Fabry-Perot cavity because the Stokes mode emitted from the Fabry-Perot cavity throughout its input mirror is not lost irreversible but returns back due to power recycling mirror, therefore, its interaction is prolonged. Detail analysis of parametric instability in full scale Advanced LIGO interferometer for LG_{33} case has to be performed.

[7] S.E. Strigin, D.G. Blair, S. Gras, S.P. Vyatchanin, Phys. Lett. A 372 (2008) 5727; S.E. Strigin, Phys. Lett. A 372 (2008) 6305.

Future plans

It would be good to analyze the effect of parametric oscillatory instability in full scale topology of Einstein telescope scheme (not only in Fabry-Perot cavities) and find all dangerous combinations of elastic and optical Stokes modes.