



Transverse polarization of Lambda-hyperons in e^+e^- annihilation processes in a TMD approach

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In collaboration with: Umberto D'Alesio, Francesco Murgia

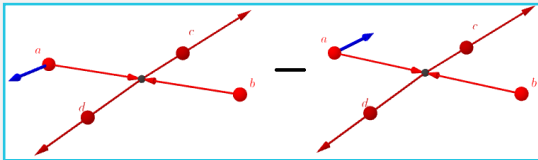
Introduction

Spin and transverse momentum effects expected negligible at high energies.

Single Spin Asymmetry (SSA) at partonic level: $a^\uparrow b \rightarrow cd$

$$a_N = \frac{d\sigma^{a^\uparrow b \rightarrow cd} - d\sigma^{a^\downarrow b \rightarrow cd}}{d\sigma^{a^\uparrow b \rightarrow cd} + d\sigma^{a^\downarrow b \rightarrow cd}}$$

$$a_N \propto \alpha_s \frac{m}{\sqrt{s}} \simeq \alpha_s \frac{m}{p_\perp} \quad [\text{Kane, Pumplin, Repko 1978}]$$



Collinear
pQCD

SSAs at hadronic level:
 $A_N : p^\uparrow p \rightarrow hX$
 $P_T : pp \rightarrow h^\uparrow X$

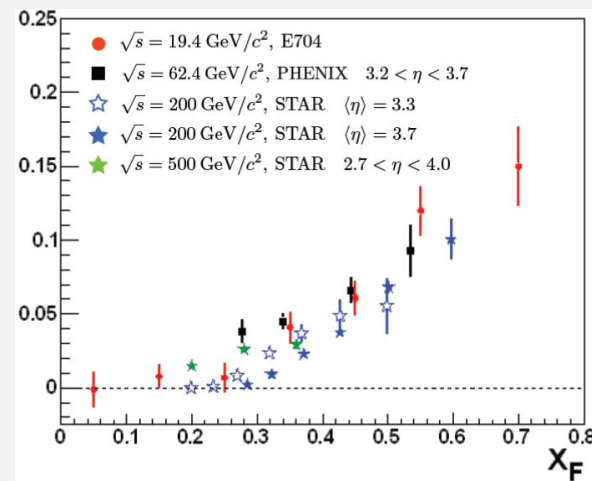
$$A_N/P_T = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} \longrightarrow A_N \leq 1 - 2\%$$

Experimental Data:

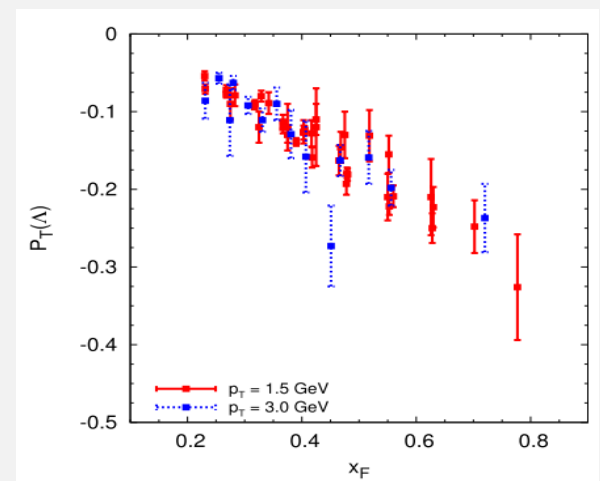
$$A_N \simeq 20\%$$

$$P_T \simeq 30\%$$

$p^\uparrow p \rightarrow \pi X$



$pp \rightarrow \Lambda^\uparrow X$



TMD factorisation

$$\underbrace{d\sigma^{AB \rightarrow CX}}_{\text{Hadronic cross sec.}} \propto \underbrace{d\sigma^{ab \rightarrow cd}}_{\text{Partonic cross sec.}} \otimes \begin{cases} f(x, k_{\perp}) \text{ TMD PDF} \\ D(z, k_{\perp}) \text{ TMD FF} \end{cases}$$

Correlation between intrinsic transverse momenta and spin, leading to a generalisation of PDFs and FFs, with asymmetric azimuthal distributions.

SSAs closely related to our understanding of:

- Strong interaction/confinement
- Inner structure of hadron in terms of quarks and gluons
- Hadronisation processes

New Data in e^+e^- from Belle: 150 data points

Observation of Transverse $\Lambda/\bar{\Lambda}$ Hyperon Polarization in e^+e^- Annihilation at Belle

We report the first observation of the spontaneous polarization of Λ and $\bar{\Lambda}$ hyperons transverse to the production plane in e^+e^- annihilation, which is attributed to the effect arising from a polarizing fragmentation function. For inclusive $\Lambda/\bar{\Lambda}$ production, we also report results with subtracted feed-down contributions from Σ^0 and charm. This measurement uses a dataset of 800.4 fb^{-1} collected by the Belle experiment at or near a center-of-mass energy of 10.58 GeV. We observe a significant polarization that rises with the fractional energy carried by the $\Lambda/\bar{\Lambda}$ hyperon.

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Contents

- TMDs FF with Helicity Formalism
- $e^+e^- \rightarrow h_1^{\uparrow} h_2 X$
- Phenomenology
- Preliminary Fits

Benefits:

- No PDFs - Cleaner process
- Lambda self-analysing
- TMD Factorisation proven

Lambda Polarising FF:
first extraction !

Helicity Formalism: TMD Fragmentation Functions for quarks

From Quark Polarisation to Hadron Polarisation

$$\underbrace{\rho_{\lambda_h, \lambda'_h}^{h, S_h}}_{\text{Hadron h.m.}} \hat{D}_{h/q, s_q}(z, \mathbf{k}_{\perp h}) = \sum_{\lambda_q, \lambda'_q} \underbrace{\rho_{\lambda_q, \lambda'_q}^{q, s_q}}_{\text{Parton h.m.}} \hat{D}_{\lambda_q, \lambda'_q}^{\lambda_h, \lambda'_h}(z, \mathbf{k}_{\perp h})$$

$$P_j^h \hat{D}_{h/q, s_q} = \hat{D}_{S_j/q, s_q}^h - \hat{D}_{-S_j/q, s_q}^h = \Delta \hat{D}_{S_j/s_q}^h.$$

Helicity density matrix

$$\rho_{\lambda_i, \lambda'_i}^{i, s_i} = \frac{1}{2} \begin{pmatrix} 1 + P_z^i & P_x^i - iP_y^i \\ P_x^i + iP_y^i & 1 - P_z^i \end{pmatrix}$$

8 independent TMD Fragmentation Functions

		Hadron		
	Pol. States	U	L	T
Quark	U	$\hat{D}_{h/q}$		$\Delta \hat{D}_{S_Y/q}^h$
	L		$\Delta \hat{D}_{S_Z/s_L}^{h/q}$	$\Delta \hat{D}_{S_X/s_L}^{h/q}$
	T	$\Delta^N D_{h/q^\uparrow}$	$\Delta \hat{D}_{S_Z/s_T}^{h/q}$	$\Delta \hat{D}_{S_X/s_T}^{h/q} / \Delta^- \hat{D}_{S_Y/s_T}^{h/q}$

Helicity Formalism: TMD Fragmentation Functions for quarks

From Quark Polarisation to Hadron Polarisation

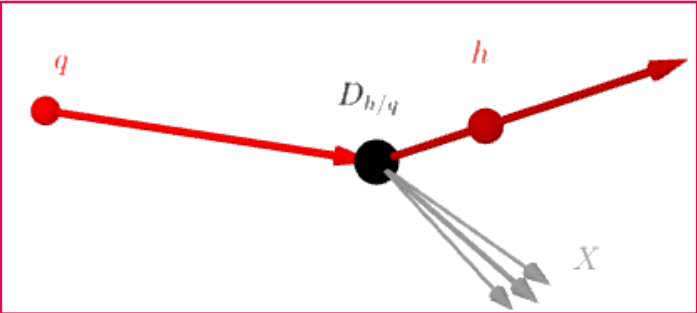
$$\underbrace{\rho_{\lambda_h, \lambda'_h}^{h, S_h}}_{\text{Hadron h.m.}} \hat{D}_{h/q, s_q}(z, \mathbf{k}_{\perp h}) = \sum_{\lambda_q, \lambda'_q} \underbrace{\rho_{\lambda_q, \lambda'_q}^{q, s_q}}_{\text{Parton h.m.}} \hat{D}_{\lambda_q, \lambda'_q}^{\lambda_h, \lambda'_h}(z, \mathbf{k}_{\perp h})$$

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Unpolarized FF



8 independent TMD Fragmentation Functions

		Hadron		
	Pol. States	U	L	T
Quark	U	$\hat{D}_{h/q}$		$\Delta \hat{D}_{S_Y/q}^h$
	L		$\Delta \hat{D}_{S_Z/s_L}^{h/q}$	$\Delta \hat{D}_{S_X/s_L}^{h/q}$
	T	$\Delta^N D_{h/q^\uparrow}$	$\Delta \hat{D}_{S_Z/s_T}^{h/q}$	$\Delta \hat{D}_{S_X/s_T}^{h/q} / \Delta^{-} \hat{D}_{S_Y/s_T}^{h/q}$

Helicity Formalism: TMD Fragmentation Functions for quarks

From Quark Polarisation to Hadron Polarisation

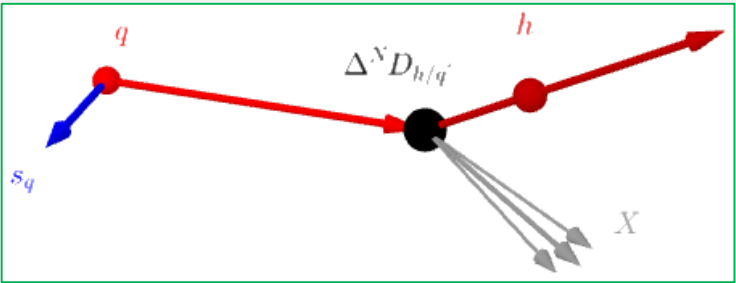
$$\underbrace{\rho_{\lambda_h, \lambda'_h}^{h, S_h}}_{\text{Hadron h.m.}} \hat{D}_{h/q, s_q}(z, \mathbf{k}_{\perp h}) = \sum_{\lambda_q, \lambda'_q} \underbrace{\rho_{\lambda_q, \lambda'_q}^{q, s_q}}_{\text{Parton h.m.}} \hat{D}_{\lambda_q, \lambda'_q}^{\lambda_h, \lambda'_h}(z, \mathbf{k}_{\perp h})$$

$$P_j^h \hat{D}_{h/q, s_q} = \hat{D}_{S_j/q, s_q}^h - \hat{D}_{-S_j/q, s_q}^h = \Delta \hat{D}_{S_j/s_q}^h.$$

Helicity density matrix

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Collins FF



8 independent TMD Fragmentation Functions

		Hadron		
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Quark	U	$\hat{D}_{h/q}$		$\Delta \hat{D}_{S_Y/q}^h$
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Helicity Formalism: TMD Fragmentation Functions for quarks

From Quark Polarisation to Hadron Polarisation

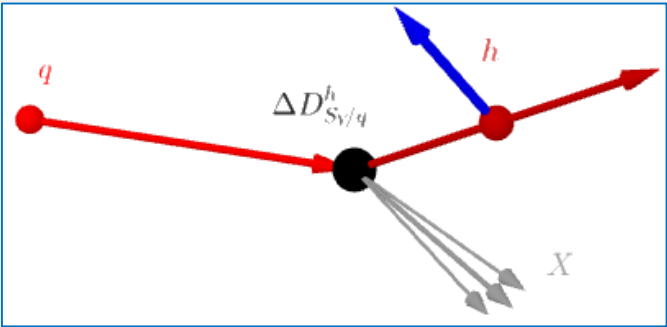
$$\underbrace{\rho_{\lambda_h, \lambda'_h}^{h, S_h}}_{\text{Hadron h.m.}} \hat{D}_{h/q, s_q}(z, \mathbf{k}_{\perp h}) = \sum_{\lambda_q, \lambda'_q} \underbrace{\rho_{\lambda_q, \lambda'_q}^{q, s_q}}_{\text{Parton h.m.}} \hat{D}_{\lambda_q, \lambda'_q}^{\lambda_h, \lambda'_h}(z, \mathbf{k}_{\perp h})$$

$$P_j^h \hat{D}_{h/q, s_q} = \hat{D}_{S_j/q, s_q}^h - \hat{D}_{-S_j/q, s_q}^h = \Delta \hat{D}_{S_j/s_q}^h.$$

Helicity density matrix

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Polarising FF



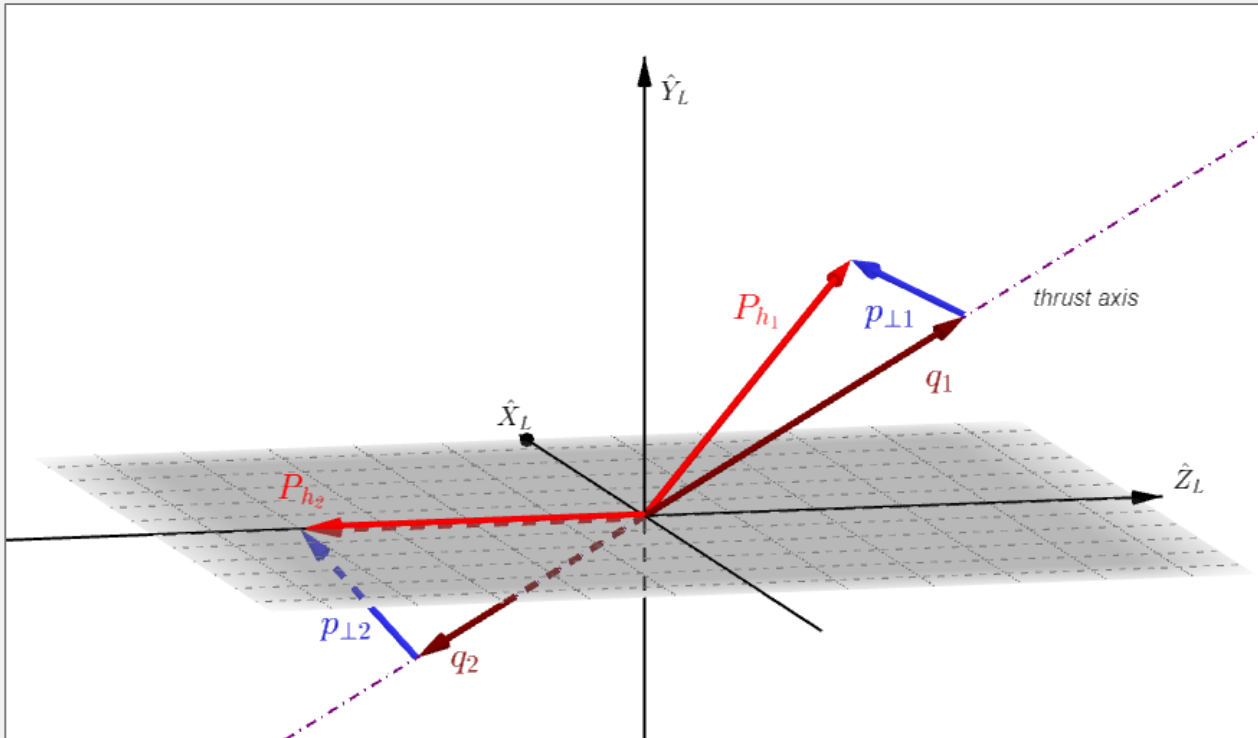
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8 independent TMD Fragmentation Functions

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Quark	U	$\hat{D}_{h/q}$		$\Delta \hat{D}_{S_Y/q}^h$
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$$e^+e^- \rightarrow h_1^\dagger h_2 X$$

$$\begin{aligned}
 & \overbrace{\rho_{\lambda_{h_1}, \lambda'_{h_1}}^{h_1, S_{h_1}} \rho_{\lambda_{h_2}, \lambda'_{h_2}}^{h_2, S_{h_2}}}^{\text{Helicity matrices}} \frac{d\sigma^{e^+e^- \rightarrow h_1 h_2 X}}{d\cos\theta dz_1 d^2\mathbf{p}_{\perp h_1} dz_2 d^2\mathbf{p}_{\perp h_2}} \\
 &= \sum_{q_c} \sum_{\{\lambda\}} \frac{3}{32\pi s} \frac{1}{4} \underbrace{M_{\lambda_c \lambda_d, \lambda_a \lambda_b} M_{\lambda'_c \lambda'_d, \lambda_a \lambda_b}^*}_{\text{Scattering Amplitudes}} \underbrace{\hat{D}_{\lambda_c, \lambda'_c}^{\lambda_{h_1}, \lambda'_{h_1}}(z_1, \mathbf{p}_{\perp h_1}) \hat{O}_{\lambda_d, \lambda'_d}^{\lambda_{h_2}, \lambda'_{h_2}}(z_2, \mathbf{p}_{\perp h_2})}_{\text{TMD Fragmentation Functions}}
 \end{aligned}$$



Polarisation along the hadron helicity axes:

$$\hat{X}_{h_1} = \hat{Y}_{h_1} \times \hat{Z}_{h_1}$$

$$\hat{Y}_{h_1} = \frac{\hat{q}_1 \times P_{h1}}{|\hat{q}_1 \times P_{h1}|}$$

$$\hat{Z}_{h_1} = \frac{P_{h1}}{|P_{h1}|}$$

Polarisation vector

$$\mathcal{P}^{h_1} = P_x^{h_1} \hat{X}_1 + P_y^{h_1} \hat{Y}_1 + P_z^{h_1} \hat{Z}_1$$

Advantages:

- hadron 2 allows to weight the FF of hadron 1 from (anti)quark 1 ;

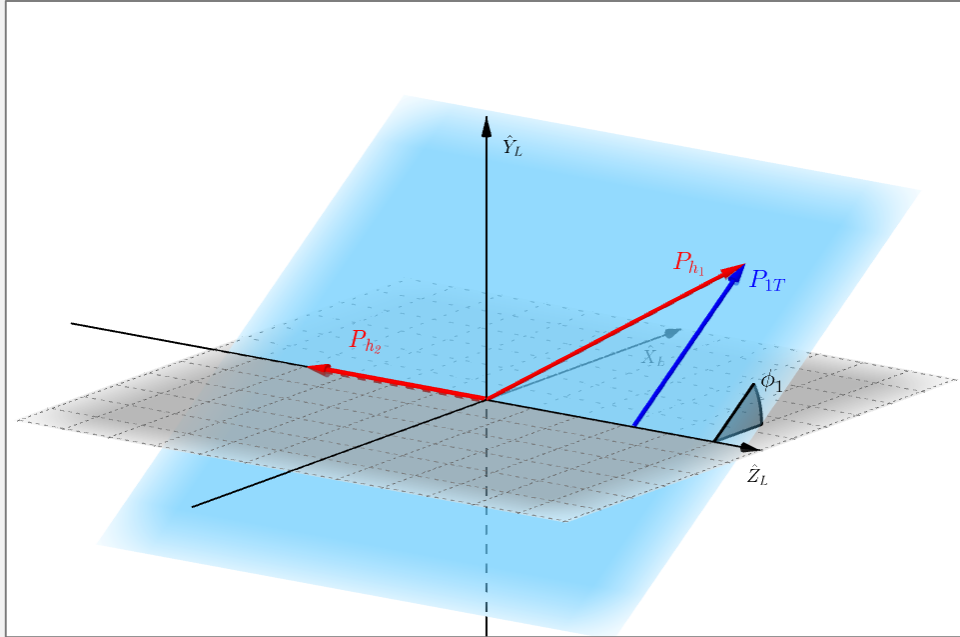
Example:

- h1 : $\Lambda(u,d,s)$ – h2: $\pi^+(u,d\bar{b})$;

Enhancement of Λ from d quark (at moderate z)

Polarisation: $e^+e^- \rightarrow h_1^\uparrow h_2 X$

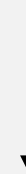
Experimental configuration:



- Polarisation measured perpendicularly to the plane of the 2 hadrons
- Matching between theory and experiment
- Data depends only on Energy Fraction z

The polarisation is measured along:

$$\hat{n} = -\hat{P}_{h_2} \times \hat{P}_{h_1}$$



The polarisation projection along $\hat{n}$:

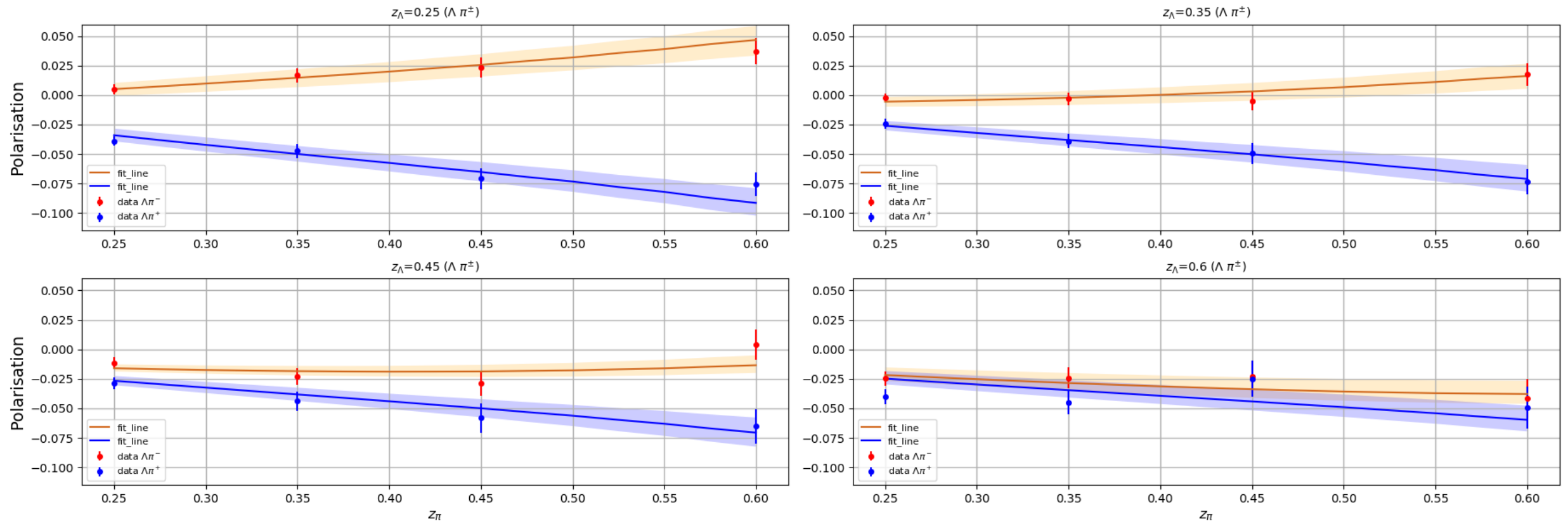
$$\mathcal{P}^{h_1} \cdot \hat{n} = P_x^{h_1} \cos \tilde{\phi} + P_y^{h_1} \sin \tilde{\phi}$$

This projection depends *deeply* on the kinematical variables

$$\mathcal{P}^{h_1} \cdot \hat{n} :: \frac{\sum_q \Delta D_{S_Y/q}^h(z) D_{h_2/\bar{q}}(z_2)}{\sum_q D_{h_1/q}(z_1) D_{h_2/\bar{q}}(z_2)} \times \frac{\sqrt{2e\pi}}{2M_p} \frac{\langle p_\perp^2 \rangle_p^2}{\langle p_\perp^2 \rangle_1} \frac{z_2}{\sqrt{z_1^2 \langle p_\perp^2 \rangle_2 + z_2^2 \langle p_\perp^2 \rangle_p}}$$

Fit: Lambda-pion

Fit Lambda-hadron $\chi^2/\text{dof} = 1.3$



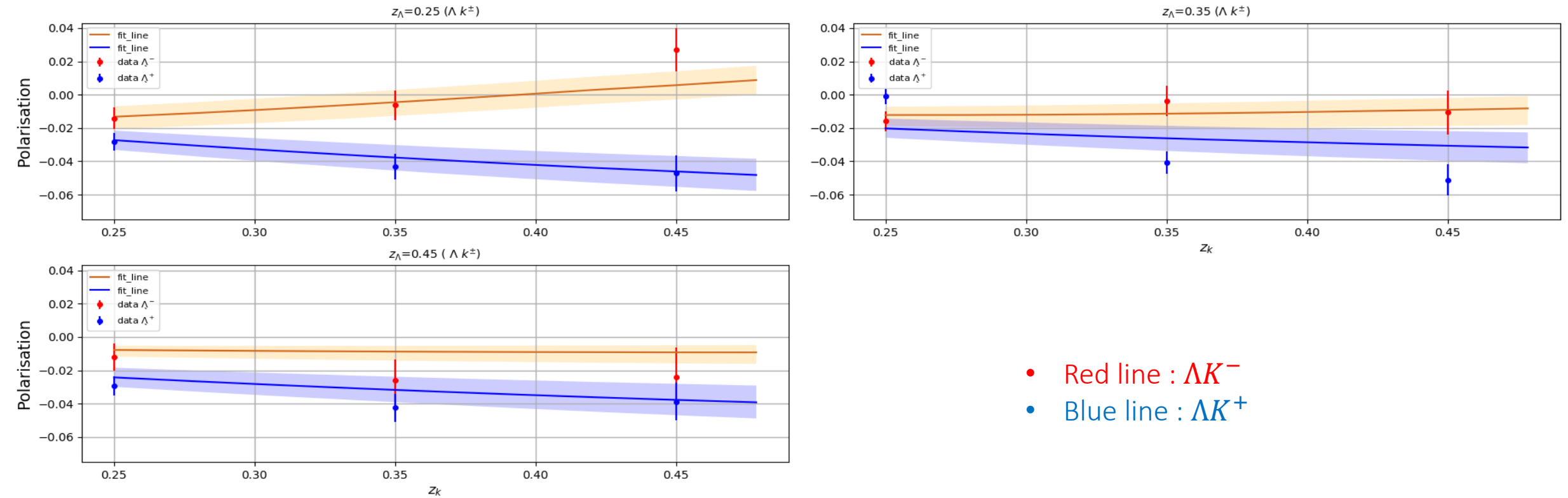
- Red line : $\Lambda\pi^-$
- Blue line : $\Lambda\pi^+$

z_Λ bins : (0.25,0.35,0.45,0.6)

Preliminary

Fit: Lambda-kaon

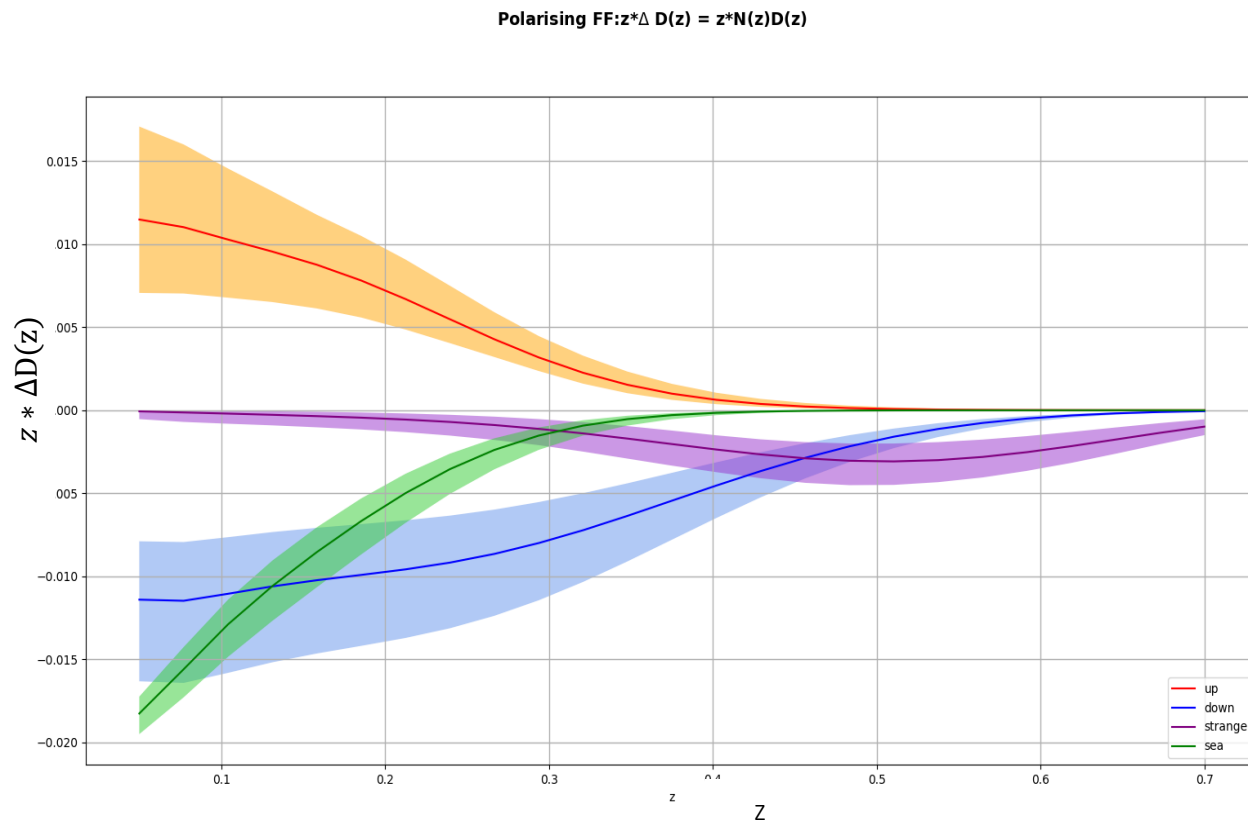
Fit Lambda-hadron $\chi^2/dof = 1.3$



z_Λ bins : (0.25,0.35,0.45)

Preliminary

Polarising FFs



- Up
- Down
- Strange
- Sea

Preliminary

up vs. down:

- different normalisation and shape (improve the fit)
- Relative opposite sign
- Comparable size at intermediate z
- Faster decrease of up w.r.t down at large z (cancellation vs. role of electric charges)

strange

- dominant at large z
- small at low z

sea

- Large at small z

Conclusions

- Complete helicity formalism within a TMD approach for $e^+e^- \rightarrow h_1(\text{jet})X$ and $e^+e^- \rightarrow h_1^\uparrow h_2 X$
- First extraction of Lambda Polarising FF from Belle data : D'Alesio, Murgia, Zaccheddu

Outlook

- Different Gaussian width for the unpolarised and/or the polarising FF:
z dependence, flavour dependence.
- Functional form of the polarising FF



Grazie

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$$\begin{aligned}
& \frac{d\sigma^{e^+e^- \rightarrow h_1 h_2 X}}{d \cos \theta dz_1 d^2 p_{\perp 1} dz_2 d^2 p_{\perp 2}} \\
&= \frac{6e^4 e_q^2}{64\pi \hat{s}} \left\{ D_{h_1/q}(z_1, p_{\perp 1}) D_{h_2/\bar{q}}(z_2, p_{\perp 2}) (1 + \cos^2 \theta) \right. \\
& \quad \left. + \frac{1}{4} \sin^2 \theta \Delta^N D_{h_1/q^\uparrow}(z_1, p_{\perp 1}) \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2, p_{\perp 2}) \cos(2\varphi_2 + \phi_1^{h_1}) \right\}
\end{aligned}$$

$$\begin{aligned}
& P_Y^{h_1} \frac{d\sigma^{e^+e^- \rightarrow h_1 h_2 X}}{d \cos \theta dz_1 d^2 p_{\perp 1} dz_2 d^2 p_{\perp 2}} \\
&= \frac{6e^4 e_q^2}{64\pi \hat{s}} \left\{ \Delta D_{S_Y/q}^{h_1}(z_1, p_{\perp 1}) D_{h_2/\bar{q}}(z_2, p_{\perp 2}) (1 + \cos^2 \theta) \right. \\
& \quad \left. + \frac{1}{2} \sin^2 \theta \Delta^- D_{S_Y/s_T}^{h_1}(z_1, p_{\perp 1}) \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2, p_{\perp 2}) \cos(2\varphi_2 + \phi_1^{h_1}) \right\}
\end{aligned}$$

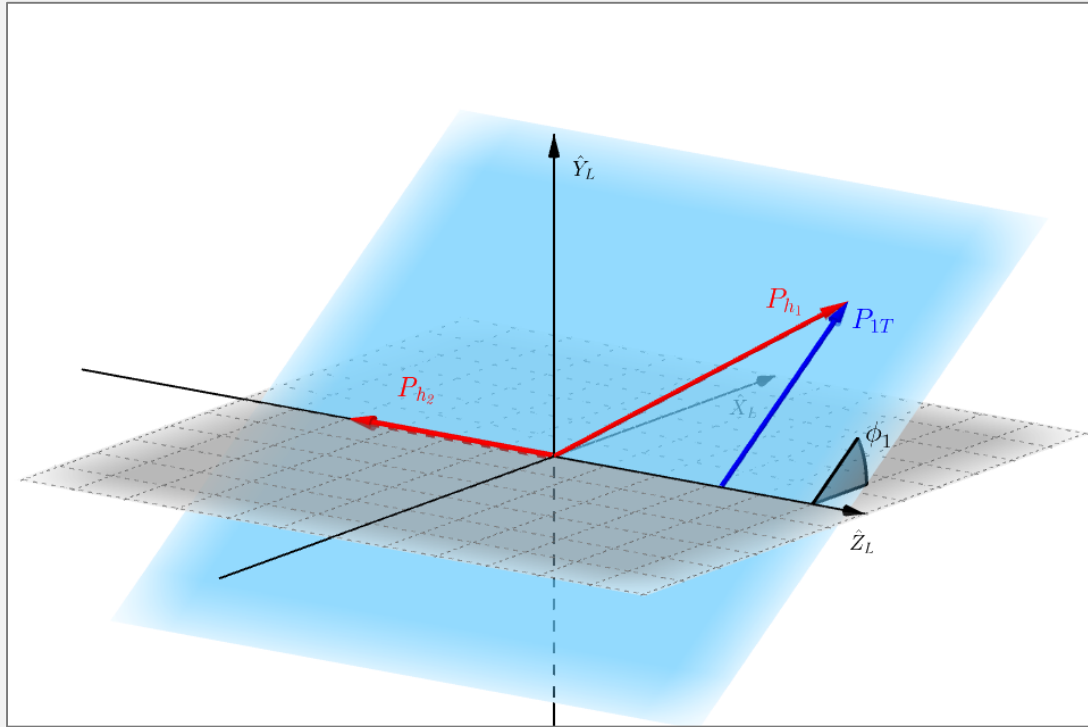
$$\begin{aligned}
& P_X^{h_1} \frac{d\sigma^{e^+e^- \rightarrow h_1 h_2 X}}{d \cos \theta dz_1 d^2 p_{\perp 1} dz_2 d^2 p_{\perp 2}} \\
&= \frac{3e^4 e_q^2}{64\pi \hat{s}} \Delta D_{S_X/s_T}^{h_1}(z_1, p_{\perp 1}) \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2, p_{\perp 2}) \sin^2 \theta \sin(2\varphi_2 + \phi_1^{h_1})
\end{aligned}$$

$$\begin{aligned}
& P_Z^{h_1} \frac{d\sigma^{e^+e^- \rightarrow h_1 h_2 X}}{d \cos \theta dz_1 d^2 p_{\perp 1} dz_2 d^2 p_{\perp 2}} \\
&= \frac{3e^4 e_q^2}{64\pi \hat{s}} \Delta D_{S_Z/s_T}^{h_1}(z_1, p_{\perp 1}) \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2, p_{\perp 2}) \sin^2 \theta \sin(2\varphi_2 + \phi_1^{h_1})
\end{aligned}$$

The polarisation is measured along:

$$\hat{n} = -\hat{P}_{h_2} \times \hat{P}_{h_1}$$

Exp. Datas depend only on energy fraction z_1, z_2

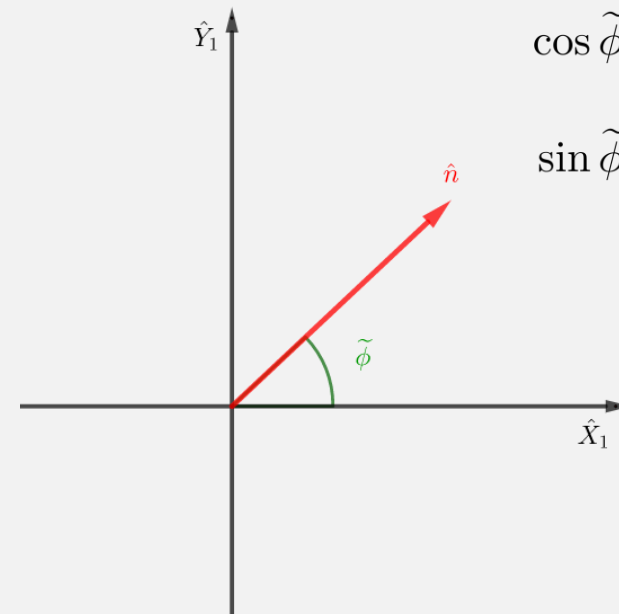


$$\begin{cases} P_1 = (P_{1T} \cos \phi_1, P_{1T} \sin \phi_1, P_{1L}) \\ p_{\perp 1}^2 \simeq P_{1T}^2 + \left(\frac{z_1}{z_2}\right)^2 p_{\perp 2}^2 - 2 \frac{z_1}{z_2} P_{1T} p_{\perp 2} \cos(\phi_1 - \varphi_2) \end{cases}$$

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The polarisation projection along $\hat{n}$:

$$\mathcal{P}^{h_1} \cdot \hat{n} = P_x^{h_1} \cos \tilde{\phi} + P_y^{h_1} \sin \tilde{\phi}$$



$$\begin{aligned} \cos \tilde{\phi} &\simeq \frac{z_{h_1}}{z_{h_2}} \frac{p_{\perp 2}}{p_{\perp 1}} \sin(\phi_1 - \varphi_2) \\ \sin \tilde{\phi} &\simeq \frac{P_{1T}}{p_{\perp 1}} - \frac{z_{h_1}}{z_{h_2}} \frac{p_{\perp 2}}{p_{\perp 1}} \cos(\phi_1 - \varphi_2) \end{aligned}$$

Phenomenology

From datas we can extract different information, particularly:

- $\Lambda(jet)X$: Lambda polarising width $\langle p_{\perp}^2 \rangle_p$
- $\Lambda\pi X$: Polarising FF (u,d)
- $\Lambda k X$: Polarising FF (u,s)

Fitted parameters:

Flav.	$\mathcal{N}_q^p$	α_q	β_q	$\langle p_{\perp}^2 \rangle_p$
u	$\mathcal{N}_u^p$	α_s	β_{sea}	$\langle p_{\perp}^2 \rangle_p$
d	$\mathcal{N}_d^p$			
s	$\mathcal{N}_s^p$			
sea	$\mathcal{N}_{sea}^p$			

Polarising parametrization:

$$\Delta D_{S_Y/q}^h(z) = \mathcal{N}_q^p(z) D_{h/q}(z)$$

$$\mathcal{N}_q^p(z) = \mathcal{N}_q^p z^{\alpha_q} (1-z)^{\beta_q} \frac{(\alpha_q + \beta_q)^{(\alpha_q + \beta_q)}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}}$$

Since we did not have a set of FF that separates the Λ and $\bar{\Lambda}$:

$$D_q^{\Lambda} = D_q^{\Lambda^0} + D_q^{\bar{\Lambda}^0} \longrightarrow D_q^{\Lambda} = D_q^{\Lambda^0} + D_{\bar{q}}^{\Lambda^0}$$

$$D_{\bar{q}}^{\Lambda^0} = (1-z)^{\alpha} D_q^{\Lambda^0}$$

$$D_q^{\Lambda^0} = \frac{1}{1 + (1-z)^{\alpha}} D_q^{\Lambda} \quad \alpha = 1, 2$$

$$D_{\bar{q}}^{\Lambda^0} = \frac{(1-z)^{\alpha}}{1 + (1-z)^{\alpha}} D_q^{\Lambda}$$

FF properties

$$\begin{aligned}
 \hat{D}_{h/q}(z, \mathbf{k}_{\perp, h}) &= D_{h/q} = (D_{++}^{++} + D_{--}^{++}) \\
 \bar{\hat{D}}_{h/q, s_T}(z, \mathbf{k}_{\perp, h}) &= \hat{D}_{h/q} + \frac{1}{2} \Delta \hat{D}_{h/q, s_T} \\
 \Delta \hat{D}_{h/q, s_T}(z, \mathbf{k}_{\perp, h}) &= \Delta^N D_{h/q}^{\dagger} \sin(\phi_{s_q} - \phi_h) = 4Im D_{+-}^{++} \sin(\phi_{s_q} - \phi_h) \quad [Collins] \\
 \Delta \hat{D}_{S_Z/s_L}^{h/q}(z, \mathbf{k}_{\perp, h}) &= \Delta D_{S_Z/s_L}^{h/q} = (D_{++}^{++} - D_{--}^{++}) \\
 \Delta \hat{D}_{S_Z/s_T}^{h/q}(z, \mathbf{k}_{\perp, h}) &= \Delta D_{S_Z/s_T}^{h/q} \cos(\phi_{s_q} - \phi_h) = 2Re D_{+-}^{++} \cos(\phi_{s_q} - \phi_h) \\
 \Delta \hat{D}_{S_X/s_L}^{h/q}(z, \mathbf{k}_{\perp, h}) &= \Delta D_{S_X/s_L}^{h/q} = 2Re D_{++}^{+-} \\
 \Delta \hat{D}_{S_X/s_T}^{h/q}(z, \mathbf{k}_{\perp, h}) &= \Delta D_{S_X/s_T}^{h/q} \cos(\phi_{s_q} - \phi_h) = (D_{+-}^{+-} + D_{-+}^{+-}) \cos(\phi_{s_q} - \phi_h) \\
 \Delta \hat{D}_{S_Y/q}^h(z, \mathbf{k}_{\perp, h}) &= \Delta D_{S_Y/q}^h = -2Im D_{++}^{+-} \quad [Polarizing] \\
 \Delta \hat{D}_{S_Y/s_T}^{h/q}(z, \mathbf{k}_{\perp, h}) &= \Delta \hat{D}_{S_Y/c}^{h/q} + \Delta^- \hat{D}_{S_Y/s_T}^{h/q} \\
 \Delta^- \hat{D}_{S_Y/s_T}^{h/q}(z, \mathbf{k}_{\perp, h}) &= \Delta^- D_{S_Y/s_T}^{h/q} \sin(\phi_{s_q} - \phi_h) = (D_{+-}^{+-} - D_{-+}^{+-}) \sin(\phi_{s_q} - \phi_h)
 \end{aligned}$$

$$\begin{aligned}
 D_{++}^{++} &= D_{--}^{--} \\
 D_{--}^{++} &= D_{++}^{--} \\
 D_{+-}^{++} &= -D_{-+}^{--} \\
 D_{-+}^{++} &= -D_{+-}^{--} \\
 D_{++}^{+-} &= -D_{--}^{-+} \\
 D_{--}^{+-} &= -D_{++}^{-+} \\
 D_{+-}^{+-} &= D_{-+}^{-+} \\
 D_{-+}^{+-} &= D_{+-}^{-+} \\
 D_{+-}^{++} &= (D_{-+}^{++})^* \\
 D_{-+}^{+-} &= (D_{++}^{-+})^*
 \end{aligned}$$

$$P_T \cdot \hat{n} = P_X^{h_1} \cos \tilde{\phi} + P_Y^{h_1} \sin \tilde{\phi}$$

Polarisation: $e^+e^- \rightarrow h_1^\dagger h_2 X$

Experimental data depend only on energy fractions, z :

We introduce for the FF the following parametrisations:

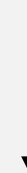
$$\Delta D_{S_Y/q}^h(z, p_\perp) = \underbrace{\Delta D_{S_Y/q}^h(z)}_{\text{z dependence}} \underbrace{\sqrt{2}e \frac{p_\perp}{M_p} \frac{e^{-p_\perp^2 / \langle p_\perp^2 \rangle_p}}{\pi \langle p_\perp^2 \rangle_h}}_{\text{Gaussian dependence on } p_\perp}$$

$$D_{h/q}(z, p_\perp) = \underbrace{D_{h/q}(z)}_{\text{z dependence}} \frac{e^{-p_\perp^2 / \langle p_\perp^2 \rangle_h}}{\pi \langle p_\perp^2 \rangle_h} \quad \frac{1}{\langle p_\perp^2 \rangle_p} = \frac{1}{M_p^2} + \frac{1}{\langle p_\perp^2 \rangle_h}$$

Data depends only on Energy Fraction z

The polarisation is measured along:

$$\hat{n} = -\hat{P}_{h_2} \times \hat{P}_{h_1}$$



The polarisation projection along $\hat{n}$:

$$\mathcal{P}^{h_1} \cdot \hat{n} = P_x^{h_1} \cos \tilde{\phi} + P_y^{h_1} \sin \tilde{\phi}$$

$$P^{h_1} \cdot \hat{n} :: \frac{\sum_q \Delta D_{S_Y/q}^h(z) D_{h_2/\bar{q}}(z_2)}{\sum_q D_{h_1/q}(z_1) D_{h_2/\bar{q}}(z_2)}$$

$$\times \frac{\sqrt{2}e\pi}{2M_p} \frac{\langle p_\perp^2 \rangle_p^2}{\langle p_\perp^2 \rangle_1} \frac{z_2}{\sqrt{z_1^2 \langle p_\perp^2 \rangle_2 + z_2^2 \langle p_\perp^2 \rangle_p}}$$

Phenomenology and Fit

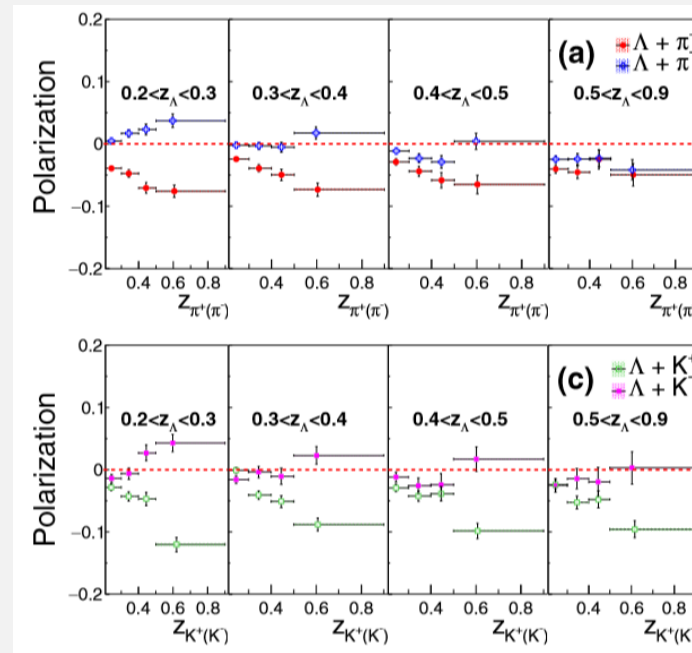
Polarising parametrization:

$$\Delta D_{S_Y/q}^h(z) = \mathcal{N}_q^p(z) \overbrace{D_{h/q}(z)}^{\text{Unpolarized FF}}$$

$$\mathcal{N}_q^p(z) = \mathcal{N}_q^p z^{\alpha_q} (1-z)^{\beta_q} \frac{(\alpha_q + \beta_q)^{(\alpha_q + \beta_q)}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}}$$

- Normalisation factor: $\mathcal{N}_q^p$, $|\mathcal{N}_q^p| \leq 1$
- Shape for high and low z : α_q β_q

Preliminary



Data selection to Fit:

- Lambda-hadron
- 100 data points
- $\Lambda(u,d,s)$ (+ h)
- h = $\pi(u,d)$
- h = $K(u,s)$

Fitted Parameters Value

Nu	0.51 ± 0.09
Nd	-0.45 ± 0.09
Ns	-0.30 ± 0.10
Nsea	-1.00 ± 0.25
α_s	1.6 ± 0.3
$\beta_{\{sea\}}$	6.1 ± 0.6
β_u	3.0 ± 0.3
$p_{\perp}$	0.11 ± 0.03

Minimisation method:
MINUIT

8 parameters

$\chi^2/dof = 1.3$