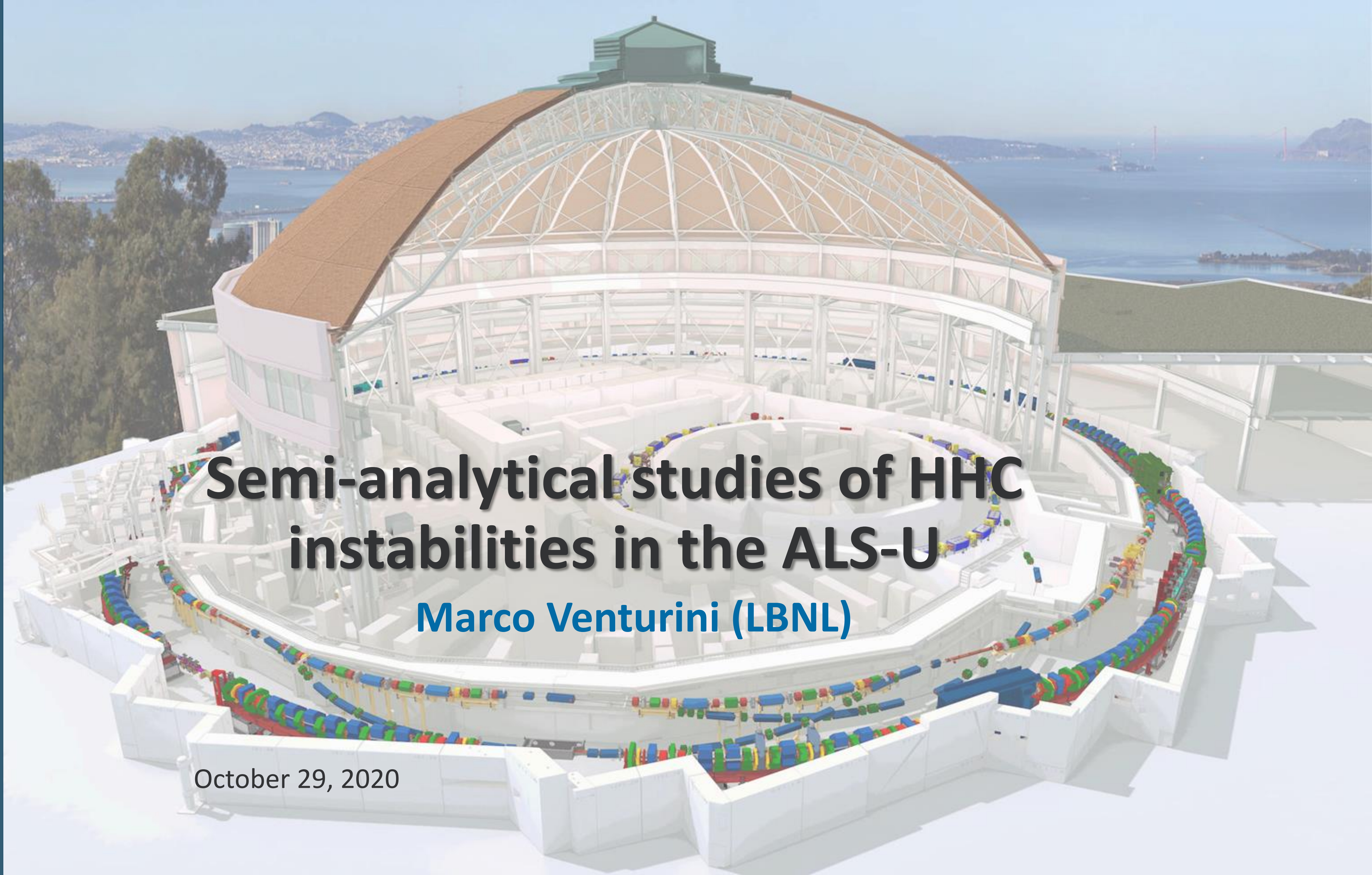

Semi-analytical studies of HHC instabilities in the ALS-U

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U.S. DEPARTMENT OF ENERGY
Office of Science
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ADVANCED LIGHT SOURCE

LER8 Workshop, Frascati, Octber 26-30, 2020.

Q: Why higher-harmonic (aka "Landau") cavities in low-emittance storage rings?

- **A**: **Primarily for bunch lengthening → smaller IBS effects, longer Touschek lifetime**

- **Q**: **How about the "Landau" aspect of HHCs, to help against instabilities?**
- **A**: **It's complicated …**
  - HHCs can benefit or hurt the beam dynamics (or be neutral) depending on the particular type of instability and specific (HHC design, beam) parameters.
    - *A full discussion is left for another time …*
  - HHCs can introduce instabilities on their own through the fundamental mode (this talk; see also Alexis Gamelin's talk at this workshop) or by adding more HOMs to those of the main rf cav.

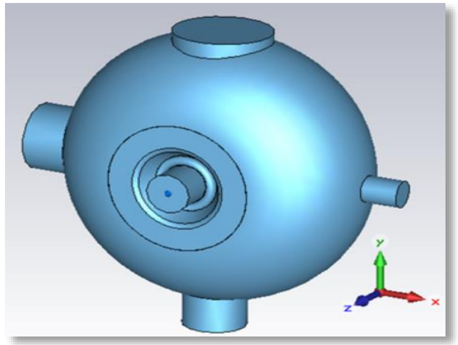
Outline

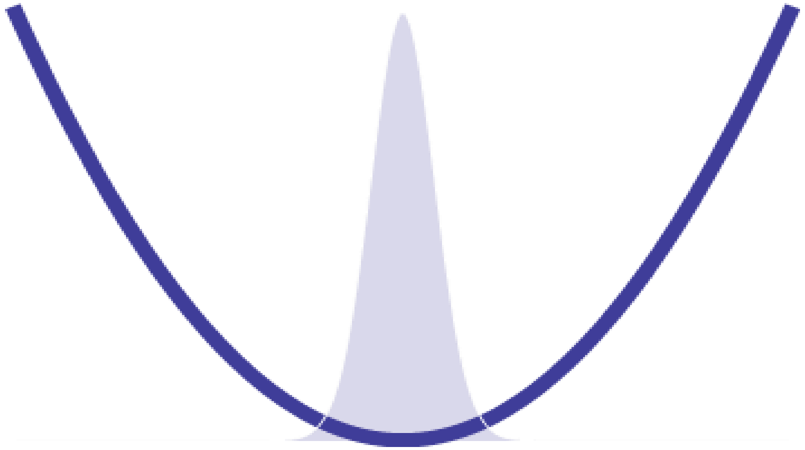
- Introduction
- Finding the beam equilibrium
  - Uniform beam fill
  - Arbitrary beam fill
- An interesting formula: beam-power dissipated in the HHC
- A useful formula for the dipole, $\ell = 1$ multi-bunch longitudinal instability
- The dipole, $\ell = 0$ (Robinson) instability revisited
- Status of the ALS-U 3HC design

Note & disclaimer: Focus of this talk is on normal-conducting, passive HHC. Not all results are necessarily applicable to SC HHCs.

HHCs lengthen the bunches by flattening the RF potential well (zeroing the rf voltage slope)

Main RF cavity only

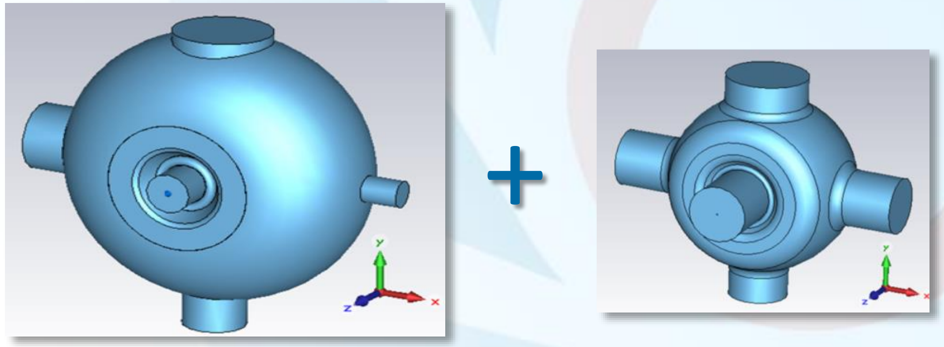




Quadratic RF potential well

Locally linear RF voltage

Main + Higher-Harmonic Cavity



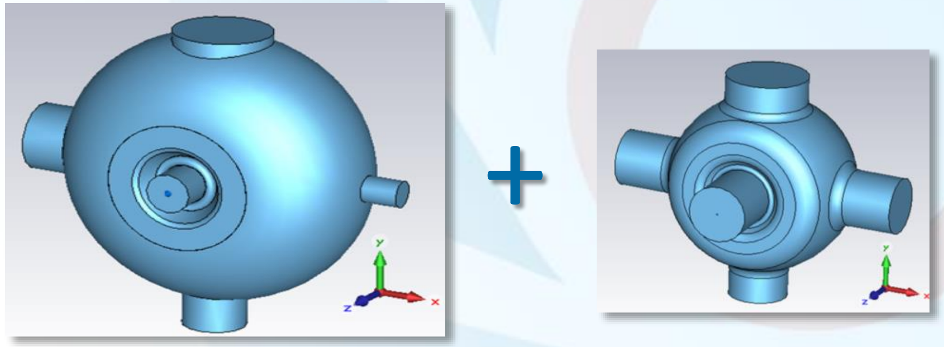
Flat RF potential well

Vanishing RF voltage slope

The beam HHC interaction is well described by a narrow-band resonator impedance model

$$Z(\omega) = \frac{R_s}{1 + iQ\left(\frac{\omega_r}{\omega} - \frac{\omega}{\omega_r}\right)} \equiv R_s \cos\psi_\omega e^{-i\psi_\omega},$$

Main + Higher-Harmonic Cavity



- The tuning angle ψ measures proximity to resonance ($\psi = 0$ ➔ on resonance)

Cavity higher-harmonic no.

HHC resonance frequency

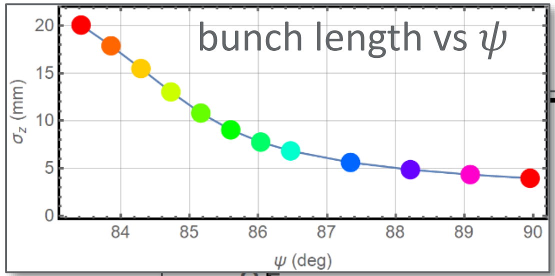
RF generator frequency

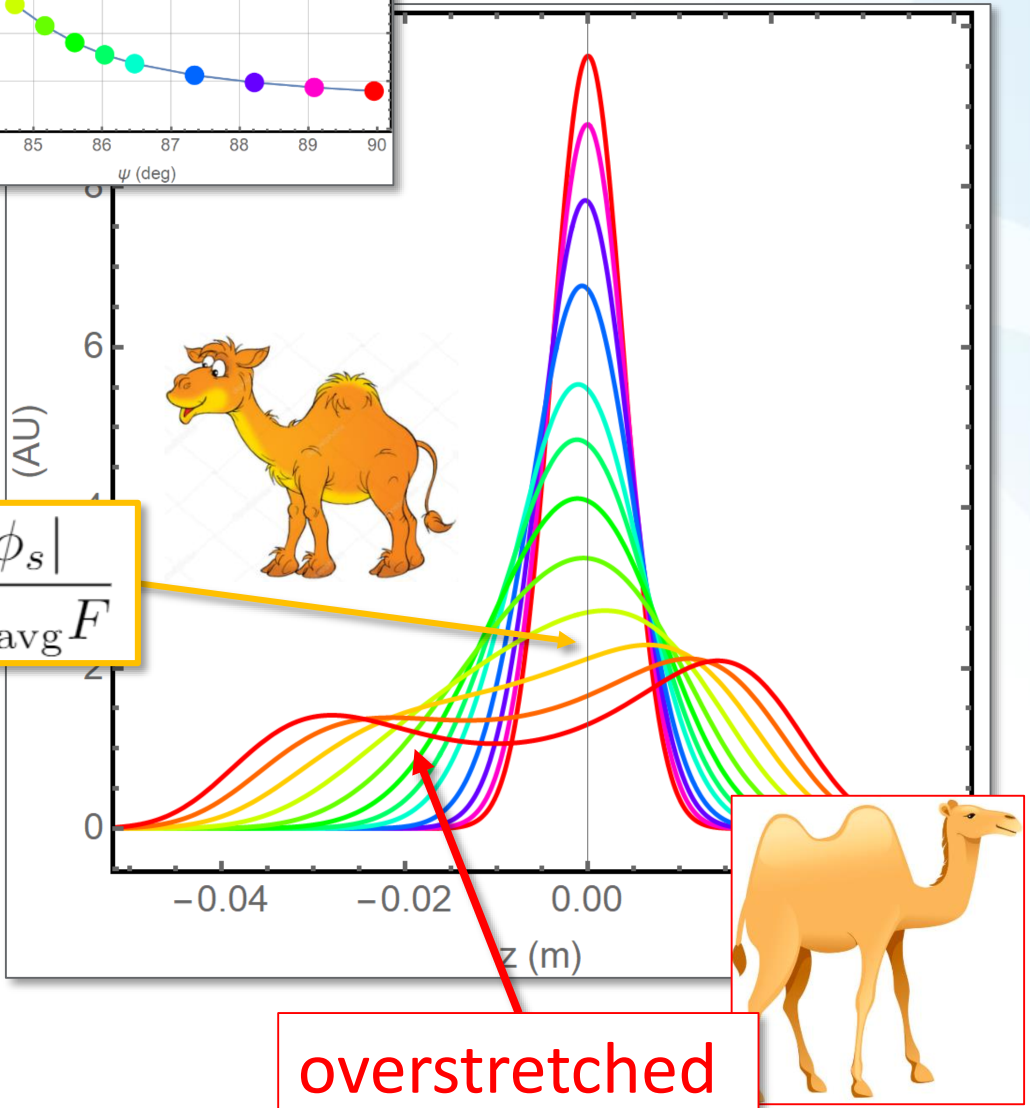
$$\tan\psi \simeq 2Q\,\frac{\omega_r - 3\omega_{\mathrm{rf}}}{\omega_r}$$

Quality factor

In NC HHCs, ψ is the control parameter

One-hump, two-hump, and maximally-flat bunches





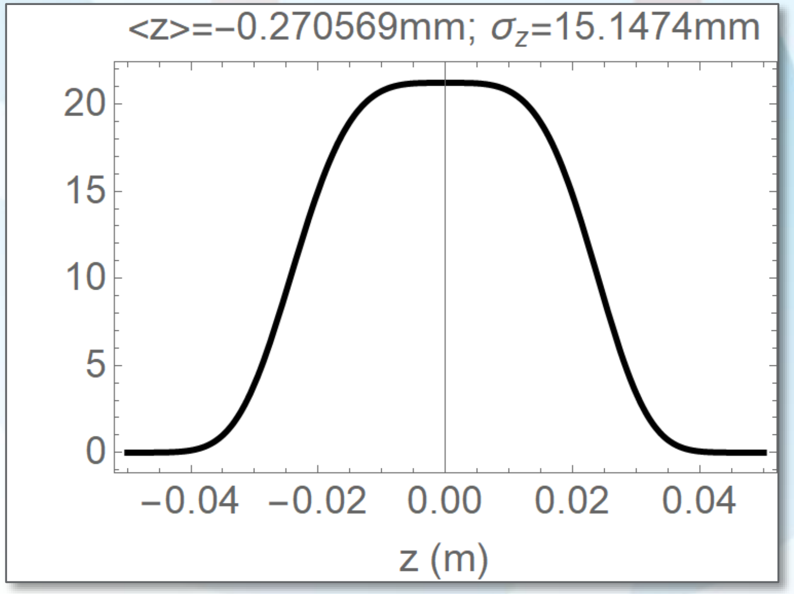
tuning angle for the bunch profile to develop two humps (vanishing rf voltage slope, $V'_{\rm rf} = 0$):

$$\sin 2\psi_{\rm crit} \simeq \frac{V_{\rm rf}|\cos\phi_s|}{n_{\rm harm} R_s I_{\rm avg} F}$$

Typical form factor at transition: $F \sim 0.9$

overstretched

For $\psi = \psi_{\rm crit}$ and for a given current there's a special value of R_s such that $V''_{\rm rf} = 0$ as well ➔ quartic rf potential ➔ flat profile (often referred as "optimal")



- Largest lifetime is, in fact, for two-hump ("overstretched") bunches.
  - But overstretching comes with its own perils … (see later slides)
- Analysis based on uniform-fill

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A Haissinski-like equation governs the form of the beam equilibrium

- Thermal equilibrium: $f \sim e^{-H/\beta}$:

$$\rho_0(z) = \frac{e^{-u(z;F,\Phi)}}{\int e^{-u(z';F,\Phi)}dz'}.$$

$$\tilde{\rho}_0(\omega_3) = Fe^{i\Phi}$$

Fourier transform of bunch profile at higher-harmonic of RF generator: $\omega_3 \equiv 3\omega_{\mathrm{rf}}$.

The importance of the phase Φ *(Tavares et al., PRAB, 2014)*

rf potential $u(z;F,\Phi) = -\int^z [eV_{\mathrm{rf},0}(z';\rho_0) - U_0]dz'/(\alpha c\sigma_\delta^2 E_0 T_0)$

Total RF voltage includes main cavity + HHC

$$V_{\mathrm{rf},0}(z;\rho_0) = V_{10}\sin(k_1 z + \phi_1) - 2I_{\mathrm{avg}}R_s F\cos\psi\cos(k_3 z + \psi - \Phi).$$

The three unknowns

Two eq.'s for F and Φ:

$$F\cos\Phi = \frac{\int \cos(k_3 z)e^{-u(z;F,\Phi)}dz}{\int e^{-u(z;F,\Phi)}dz},$$

$$F\sin\Phi = \frac{\int \sin(k_3 z)e^{-u(z;F,\Phi)}dz}{\int e^{-u(z;F,\Phi)}dz}.$$

+ eq. for synchronous phase (energy balance)

$$V_{10}\sin\phi_1 = U_0 + 2I_{\mathrm{avg}}R_s F\cos\psi\cos(\psi - \Phi).$$

$k_1 = \omega_{\mathrm{rf}}/c, \;\; k_3 = 3\omega_{\mathrm{rf}}/c$

An interesting expression for the dissipated beam power for "optimum" HHC settings

- Remarkably, for optimum HHC parameters the expression for the beam-power dissipated to HHCs exhibits no explicit dependence on R_s and ψ
  - General expression for arbitrary R_s, ψ: $\boldsymbol{P_{HHC} = 2I_{avg}^2 R_s F^2 \cos^2\psi}$

$$\boxed{P_{\mathrm{HHC}} = \frac{F}{n^2 - 1} P_{\mathrm{rad}}}$$

For 3rd HC:

$$\boxed{P_{\mathrm{3HC}} \simeq 0.1 \times P_{\mathrm{rad}}}$$

$n =$ higher-harmonic number

Form factor: $F \simeq 1 - (3\omega_{\mathrm{rf}}\sigma_t)^2/2 \simeq 0.9$

Disclaimer: formula is not applicable to SC HHCs, which tend to operate far from "optimum"

Recent progress on beam equilibrium for non-uniform fill

- When gaps are present in the beam fill, the form of bunch equilibrium varies bunch to bunch
- The problem of finding the equilibrium is numerically more complicated but its formulation is formally similar to that of the uniform beam fill

3HC voltage

Uniform beam fill:

$$V(\tau) = -2I_{\text{avg}} F R_s \cos\psi \cos(\omega_3\tau + \psi - \Phi),$$

Arbitrary beam fill of n_b bunches:

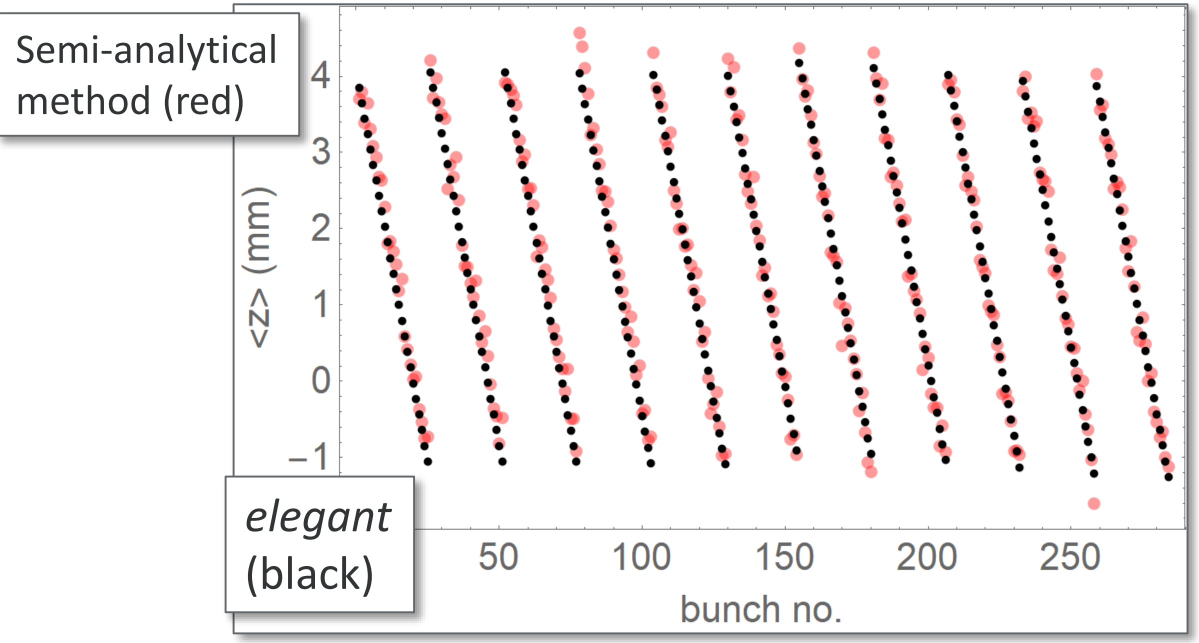
$$V_n(\tau) = -2I_{\text{avg}} \sum_{n'=0}^{n_b-1} \frac{N_{n'}}{N_{\text{tot}}} F_{n'} |Z_{n,n'}| \cos(\omega_3\tau + \psi_{n,n'} - \Phi_{n'})$$

$$\tilde{\rho}_n(\omega_3) = F_n e^{i\Phi_n}$$

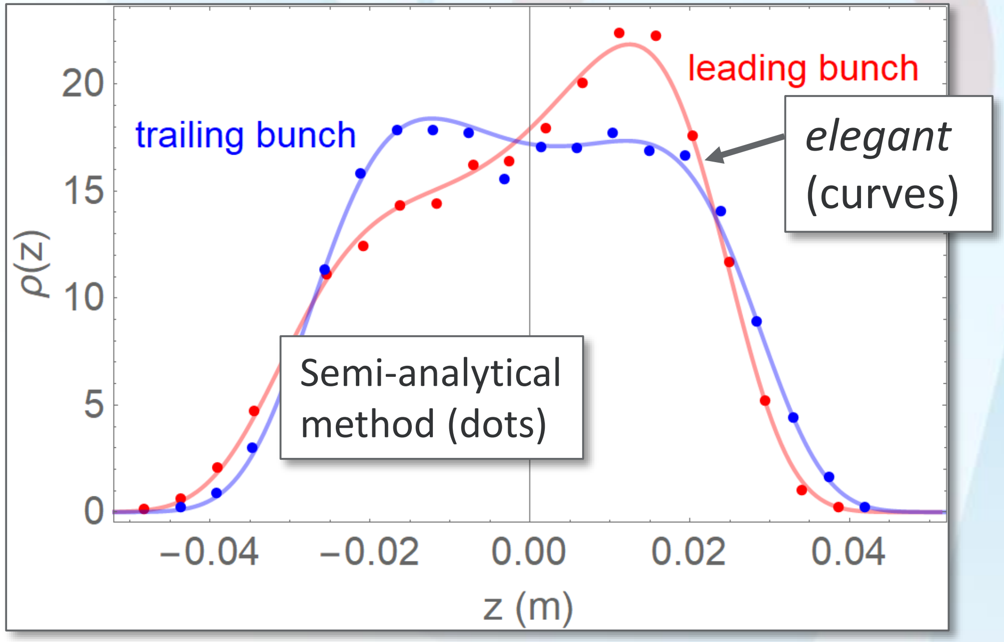
- $2n_b$ equations for $2n_b$ unknowns (form-factor parameters F_n and Φ_n for the n-bunch).
  - + 1 equation for main rf cav. phase
- Newton-method (with derivatives calculated from symbolic expressions) is effective and robust (R. *Warnock and M.V., PRAB **23**, 064403, 2020):*
  - "stiffer" cases (typically those that involve extreme lengthening) can be handled by ramping up the beam average current adiabatically
  - Even for extreme lengthening, in all cases tried so far the algorithm has never failed to converge

Semi-analytical method reproduces transient beam-loading simulations results fairly well.

Bunch centroids slew along 284-bunch, 11-train beam



Profiles of first and last bunch of first train

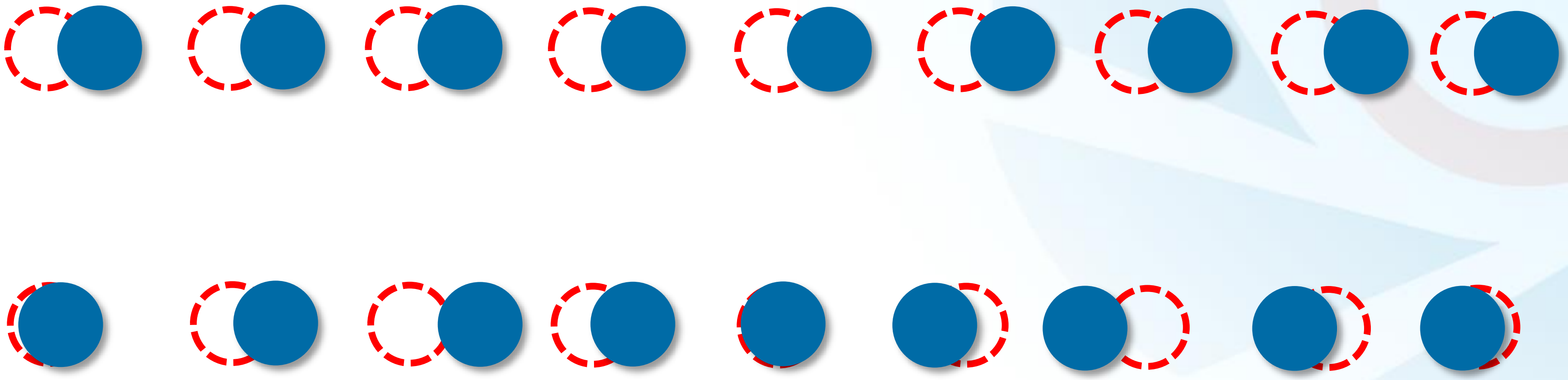


- ALS-U beam-fill example: 284 bunches, 11 trains; 10ns gaps (4 empty rf buckets)
  - Macro-particle simulations by *elegant*
- Compared to ALS fill (with one long train and relatively long gap) transient effects from 3HC beam loading, i.e. bunch to bunch variations, are relatively smaller
  - In ALS transient effects prevent the attainment of the bunch lengthening that would be possible with uniform fill

Pictorial view of the dipole $\ell = 0$ and $\ell = 1$ coupled-bunch longitudinal modes

$\ell = 0$

$\ell = 1$



Phase of oscillation varies as $e^{2\pi i n \ell / n_b}$ along beam

HHCs and multi-bunch longitudinal beam instabilities

- **Conventional linear-Vlasov methods apply to uniform beam fill (identical bunches)**
  - Robust algorithms for finding equilibrium for non-uniform fill beams open the door to theories applicable to the more general case
  - Technical problems (perturbation theory in double-well potential) + large no. of degrees of freedom
- **Existing theory (e.g. Bosch, et al, PRAB 2001) is for multi-bunch instabilities with coupled-bunch mode $\ell = 0$ and dipole, quadrupole, … dipole/quadrupole azimuthal modes (Robinson instability)**
  - Purely HHC quartic rf potential
- **MV PRAB 2018: analysis of $\ell = 0$ and $\ell = 1$ (dipole) coupled-bunch modes based on exact numerical solution of unperturbed motion in arbitrary HHC rf potential**
  - Results are very close to those we get by assuming a quartic rf potential
  - Clarified that HHC contribution to the Robinson (dipole) instability is never Landau damped
- **In the next few slides:**
  1. Analyze $\ell = 1$ and present an approximate expression for the critical HHC R_s/Q at the onset of instability
  2. Comment on the expression for the HHC Robinson instability growth rate

From the standard analytic tool box: dispersion equation, effective impedance, Keil-Schnell diagram, and the "onion"

Scaled effective impedance:

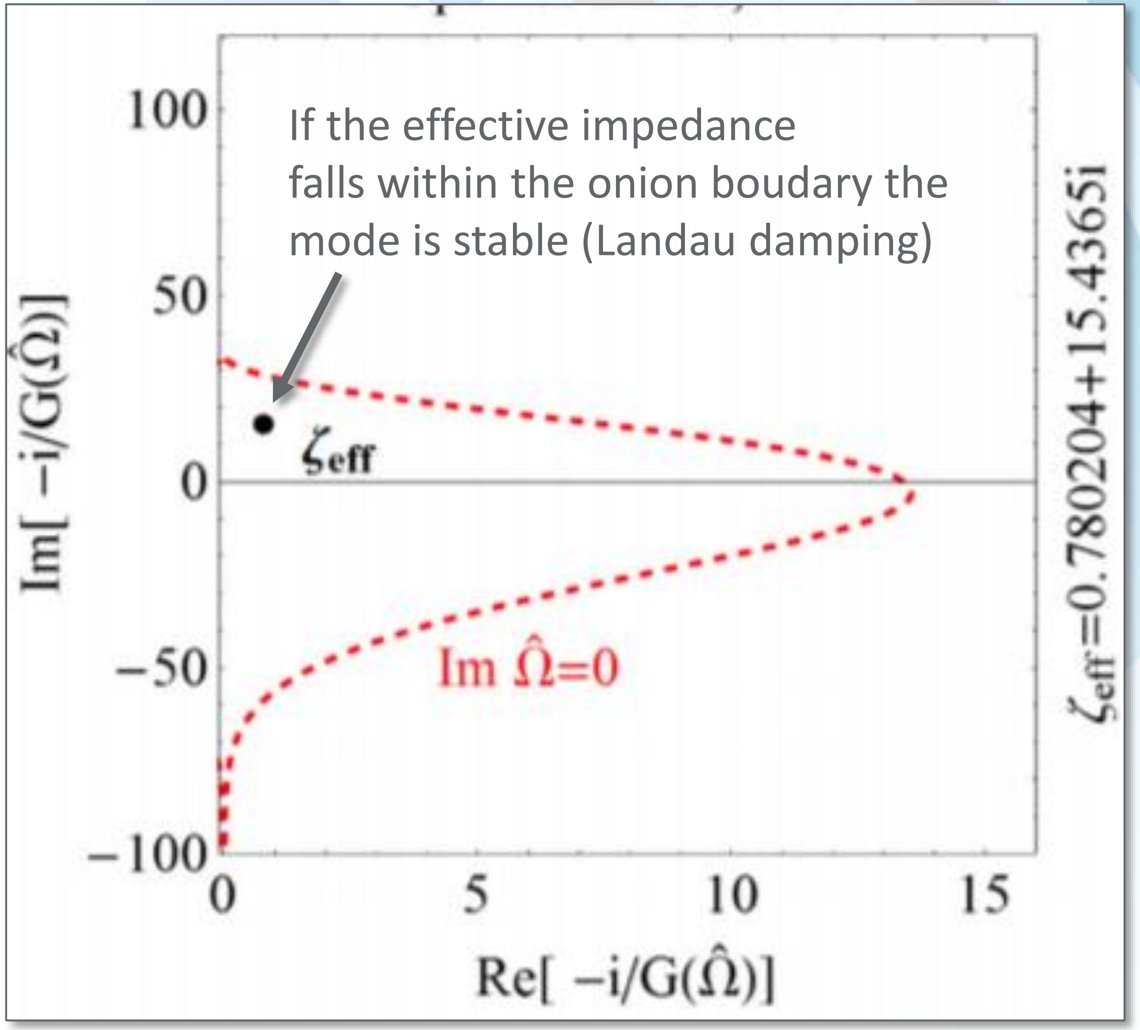
$$\zeta_{\text{eff}}(\hat{\Omega}) = \frac{64\pi}{9\Gamma^3_{1/4}} \frac{eI_{\text{avg}}\alpha\omega_1}{E_0T_0\langle\omega_s\rangle^2(\sigma_z k_1)^2} Z_{\text{eff}}\left(\frac{\langle\omega_s\rangle\hat{\Omega}}{0.816}\right)$$

Dispersion equation:

$$\boxed{\frac{-i}{G(\hat{\Omega})} = \zeta_{\text{eff}}(\hat{\Omega})}$$

$$G(\hat{\Omega}) = \int_0^\infty dx \frac{x^4 J_1^2(5.16\times k_1\sigma_z x)e^{-x^4}}{\hat{\Omega}^2 - x^2}$$

Particle motion in quartic rf potential; $\ell = 1$ couple-bunch mode



The beam-cavity interaction driving the instability is captured by the effective impedance

$$Z_{\mathrm{eff},\ell}(\Omega) \simeq 3Z(\omega_{3,\ell} + \Omega) - 3Z(\omega_{-3,\ell} + \Omega)$$

HHC fundamental-mode impedance is sampled at these beam harmonics:

$$\omega_{\pm 3,\ell} = \pm 3h\omega_0 + \ell\omega_0$$

- We are interested in coupled-bunch mode $\ell = 1$
- If $\ell \neq 0$, we can neglect the coherent mode-frequency $\Omega \ll \omega_0$ in the expression for Z_{eff}
- The effective impedance depends on the HHC tuning
  - Choose tuning angle ψ for vanishing RF voltage slope (at transition from single to double-hump profile):

$$\sin 2\psi_{\mathrm{crit}} \simeq \frac{V_{\mathrm{rf}}|\cos\phi_s|}{n_{\mathrm{harm}} R_s I_{\mathrm{avg}} F}$$

A simplified but reasonably accurate expression for the $\ell = 1$ effective impedance of the HHC fundamental mode

- Exploit $Q \gg 1$ to workout a limiting expression for the effective impedance valid for $Q \to \infty$ and finite R_s/Q:

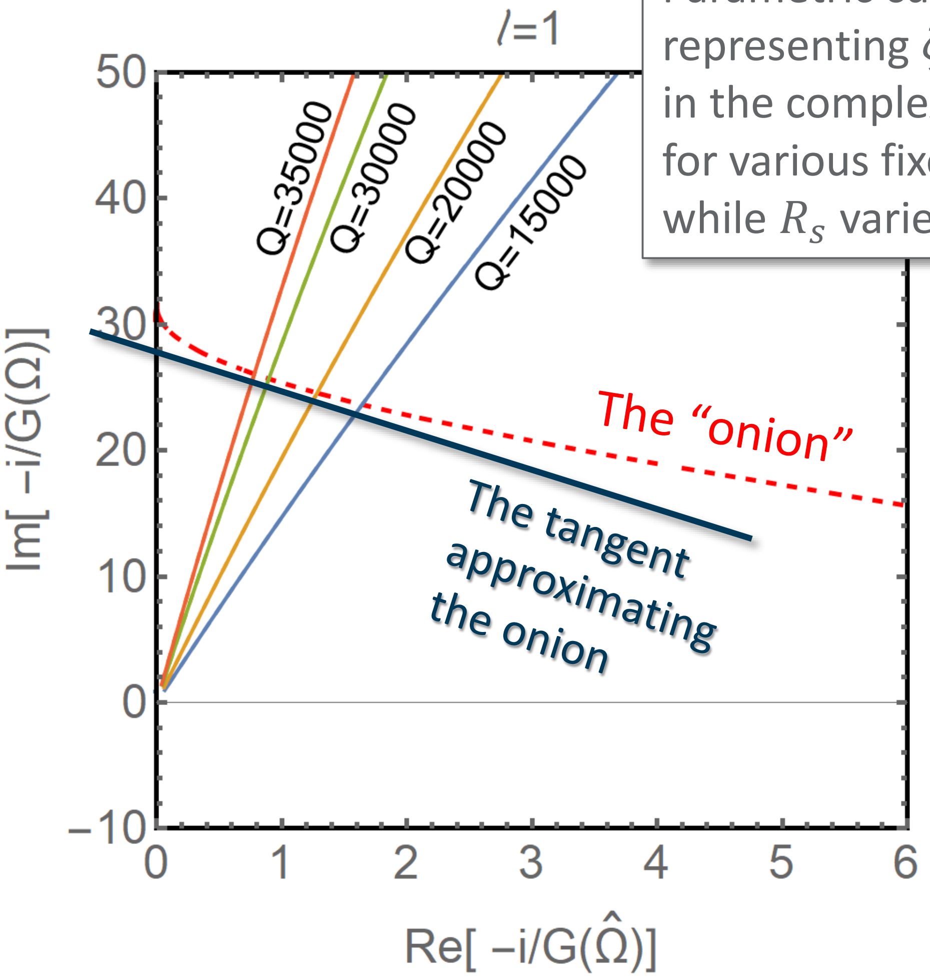
$$\boxed{Z_{\mathrm{eff},\ell=1} \simeq \left(\frac{R_s}{Q}\right)^2 \frac{I_{\mathrm{avg}} F h^2 n^4}{V_{\mathrm{rf}} |\cos\phi_1|} \left[\frac{hn}{Q} + i\right]}$$

$n =$ higher-harmonic number

- The above expression is valid if R/Q is not too large; (not very restrictive in practice: R/Q less than a few 100s Ω)

$$\eta \equiv \frac{R_s}{Q} \ll \frac{V_{\mathrm{rf}}^2 \cos^2\phi_1}{I_{\mathrm{avg}}^2 F^2 h^2 n^4}$$

Solve dispersion equation by approximating the boundary of Keil-Schnell's onion with local tangent



Parametric curves representing ζ_{eff} in the complex plane for various fixed Q while R_s varies

- Exploit the observation that in the range of interest for Q (normal-conducting cavities) the curves representing Z_{eff} intersect the onion boundary in a relatively narrow region where we can approximate the boundary by the tangent:
  - $$y \simeq -2.8x + \frac{1.31}{(\sigma_z k_{\text{rf}})^{5/3}}$$
  - This is a fairly accurate approximation for $\sigma_z k_{\text{rf}}$ in the range between 0.1 and 0.2 but it is still OK over a somewhat larger range. For ALS-U, $\sigma_z k_{\text{rf}} \simeq 0.16$

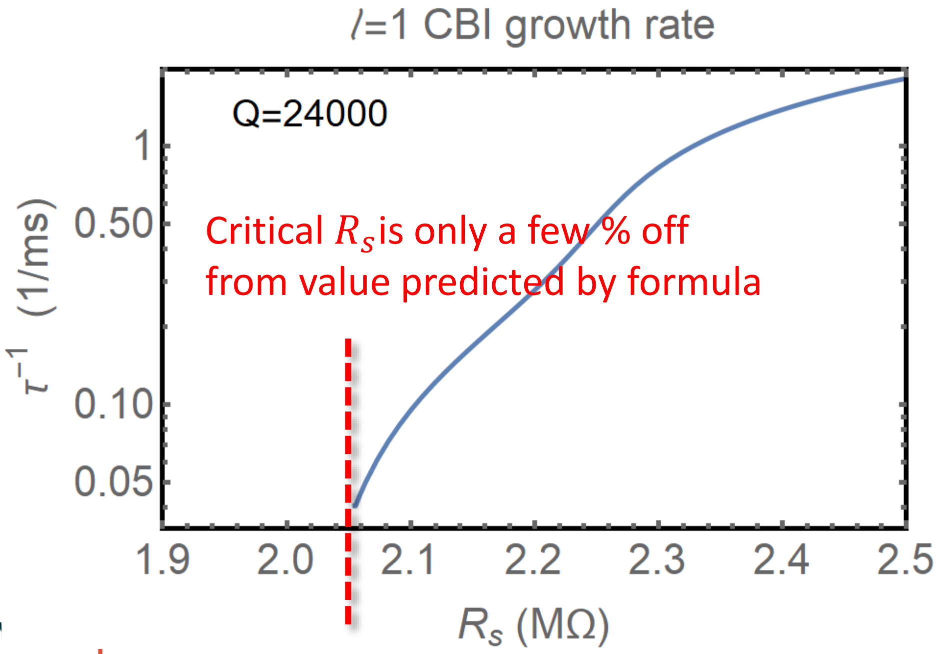
- The critical R_s for instability is found by solving a system of linear algebraic equations (intersection of the tangent to the "onion" and Z_{eff})

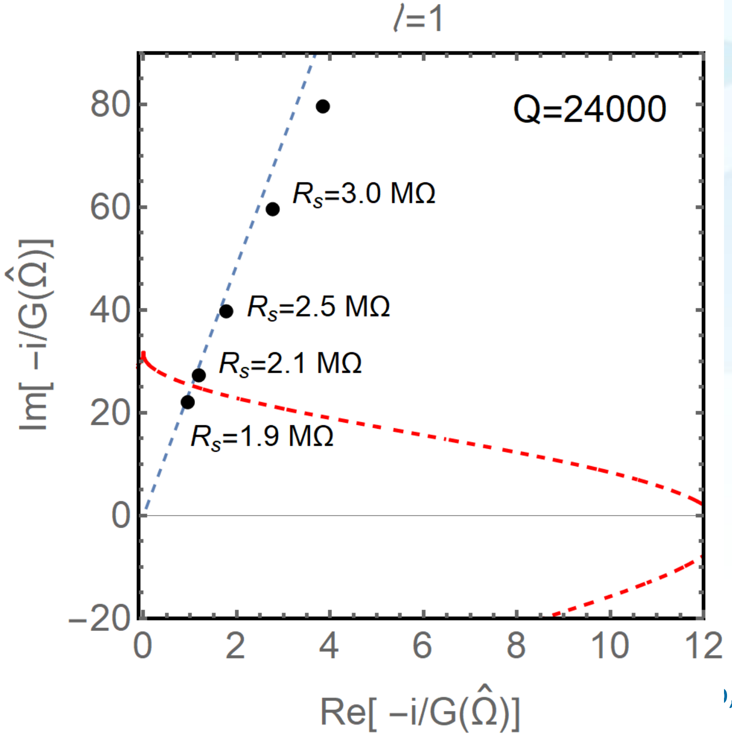
End result: critical R/Q for the $\ell = 1$ CBI instability

$$\left(\frac{R_s}{Q}\right)_{\rm crit} \simeq 3.35 \times \frac{\sigma_\delta}{n^2 (k_{\rm rf}\sigma_z)^{5/3}} \left[\frac{E_0 \alpha_c V_{\rm rf} |\cos\phi_1|}{e I_{\rm avg}^2 F h (1 + 2.8 \times nh/Q)}\right]^{1/2}$$

- RHS is typically a very weak function of Q: $nh/Q = 3 \times 328/24000 \simeq 0.04 \ll 1$; i.e. to a very good approx. the instability depends exclusively on the ratio R_s/Q *
- Disclaimer: The formula is expected to be accurate for parameters in the neighborhood of the ALS-U HHC
  - For more general parameters the formula should be verified against numerical solutions of the exact dispersion equation and/or simulations

Exact numerical solution of dispersion equation





Note: beam loading from the main cavity can also contribute; For ALS-U cavity parameter this is negligible

HHC contribution to the dipole Robinson instability revisited

$$\frac{1}{\tau_{\text{Robinson}}} \simeq \left[2 \times \frac{6\pi^3}{\Gamma^4\left(\frac{1}{4}\right)}\left(1 - \frac{15\Gamma^4\left(\frac{1}{4}\right)}{32\pi^2}\sigma_z k_{\text{rf}}\right)\frac{eI_{avg}\alpha_c}{E_0 T_0}\right] \times R_s Q \cos^2\psi \sin 2\psi$$

Erratum: Eq. (45) in M.V. PRAB 21, 114404 (2018) and related eq.'s have an error in the numerical coefficient (off by few % for parameters of interest).

- Quartic rf potential.
- Robinson from HHC always falls outside the "onion" – no Landau damping
  - Both main rf cav and HHC contribute significantly (main cav contributes damping …)

- In situations where ψ is closer to 90^o than 0 (e.g. for ALS-U current 3HC design $\psi \sim 80^o$), and for vanishing rf voltage slope we have:

$$\frac{1}{\tau_{\text{Robinson}}} \propto R_s Q \cos^2\psi \sin 2\psi \propto R_s Q \left(\frac{V_{\text{rf}}|\cos\phi_1|}{nR_s\, I_{avg\ F}}\right)^3 \propto \frac{1}{Q}\left(\frac{Q}{R_s}\right)^2$$

Formula is not very accurate but the basic scaling is right

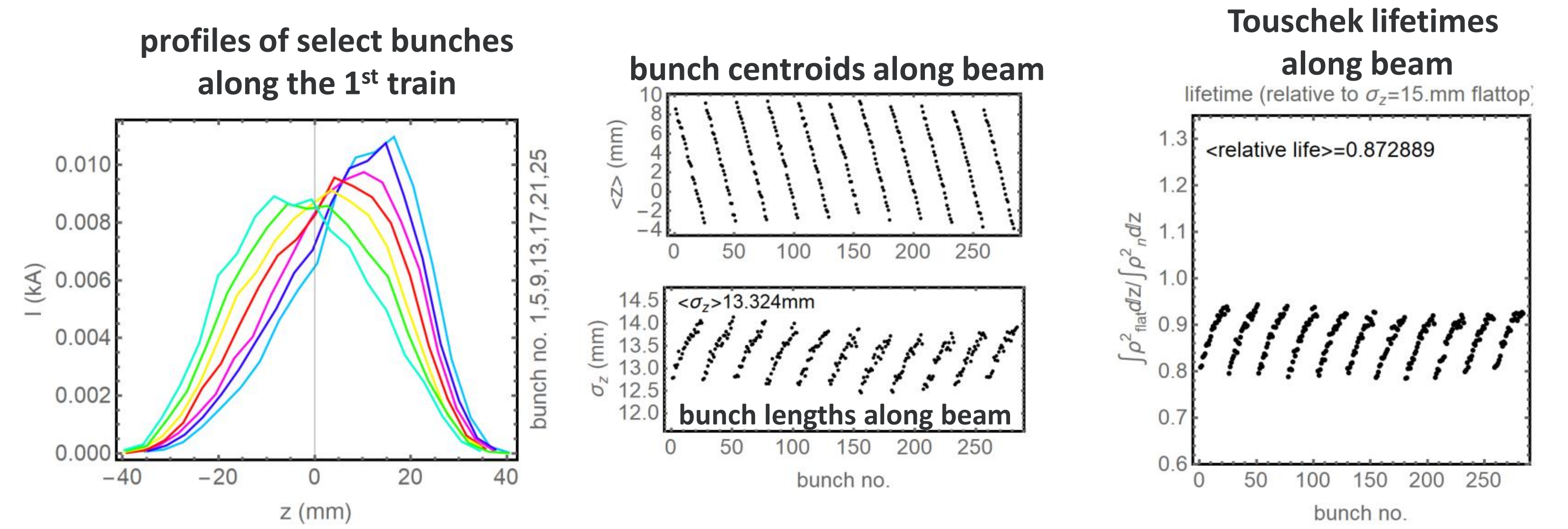
- Somewhat surprisingly, for fixed Q, the instability growth rate scales inversely with R/Q
- Reason is, everything else being equal, for larger R_s the required HHC tuning is achieved farther from resonance, where the instability becomes weaker

The ALS-U 3HC status: beam-dynamic studies and cavity design

- **Analytical models are useful but no substitute for simulation work …**
  - Extensive numerical studies being done with *elegant*
- **In the range of HHC design-parameters of interest for the ALS-U, we've found that overstretching is invariably associated with an instability**
  - We do not believe that existing theories are adequate at predicting this
  - Instability typically saturates into ~steady-state centroid/bunch-length/energy spread oscillations (next slide)
- **Early on we identified a promising 3HC design with relatively high $R_s/Q \simeq 80\Omega$**
  - Single cavity, manageable power dissipation
  - Avg. bunch length of stable equilibrium somewhat below the design target $\sigma_z = 15mm$
  - Unfortunately it doesn't seem we can control the "overstretching" instability with a LFB

Beam equilibrium from *elegant* simulations

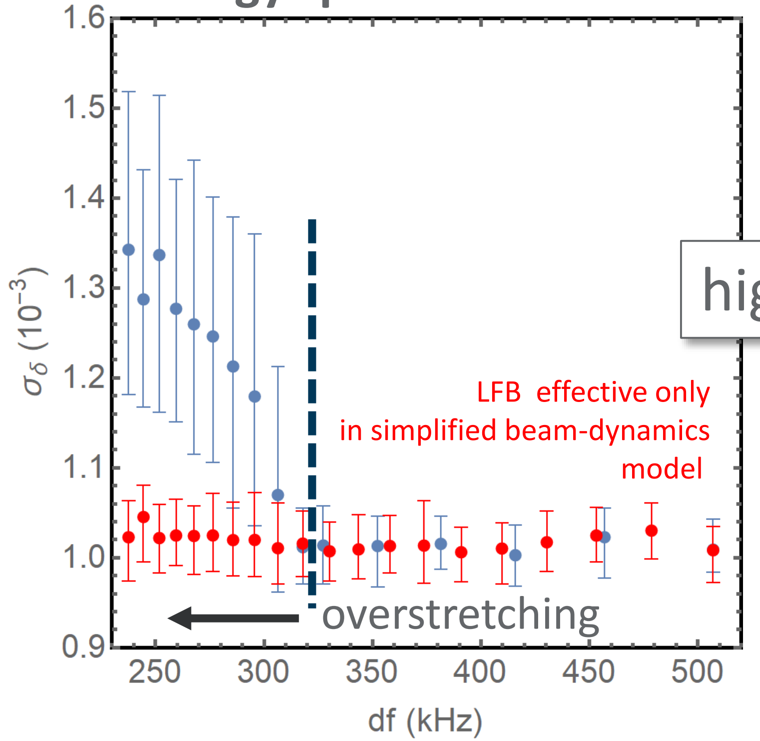
(beam loading from main rf cavity included)

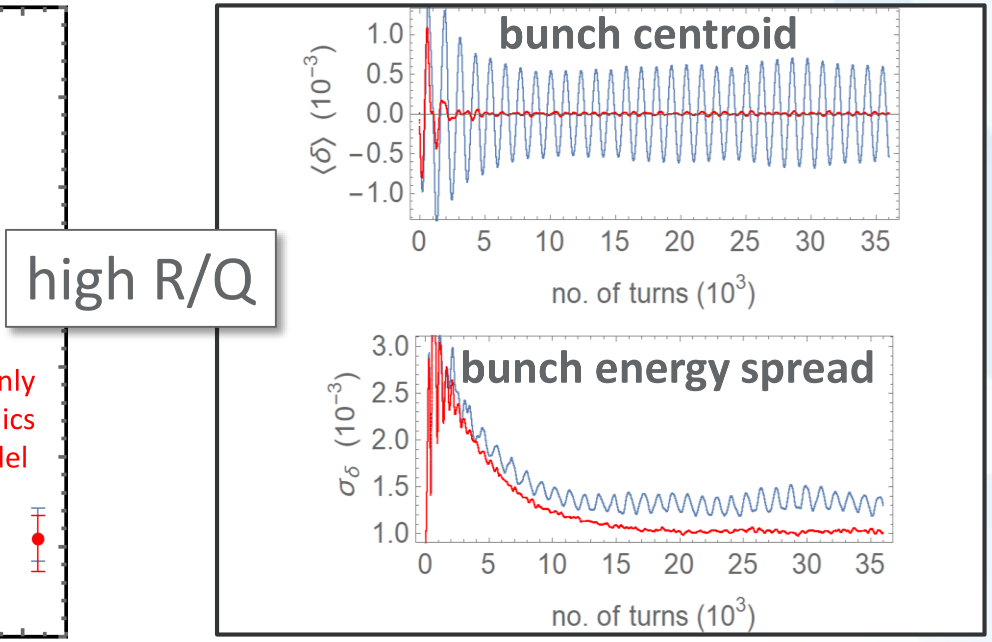


Natural bunch length $\sigma_{z0} = 3.9mm$

The "overstretching" instability and a low-R/Q 3HC design option

Bunch energy spread vs. 3HC detuning





Steady-state oscillations settle in when overstretching

Only in simulations where we force all bunches to be identical an LFB becomes effective, suggesting that $\ell = 1$ plays an important role.

Instability is likely a coupled combination of $\ell = 0{,}1$ and azimuthal dipole, quad, and possibly higher-order modes

- **Feasibility of a low $R_s/Q \simeq 40\Omega$ 3HC design now under study**
  - Would require 2 cavities
  - The "overstretching instability" is still present but now an LFB is found effective at controlling the instability
    - Conventional LFB modelled after the system installed in present ALS
  - Potential for 25-30% additional (average) Touschek lifetime improvement
  - Lower R/Q also helps by reducing transient beam-loading and bunch-to-bunch variations

Summary

- Robust method for finding equilibrium of arbitrary beam-fill
- Critical R/Q for $\ell = 1$ multi-bunch instability
- Progress report on ongoing ALS-U 3HC studies