# Injection of Round Beams

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# Introduction: Vertical emittance of future low emittance ring light sources

| project | E/GeV | $\varepsilon_x$/pm·rad | $\varepsilon_y$/pm·rad | Emittance ratio | $\varepsilon_y$/pm·rad today |
|---|---|---|---|---|---|
| ESRF-EBS | 6.0 | 130 | 4 | 3% | 4 |
| APS-U | 6.0 | 32 | 32 | 100% | 40 |
| Spring 8-II | 6.0 | 150 | 15 | 10% | ~20 |
| HEPS-IHEP | 6.0 | 35 | 3.5 | 10% | n.a. |
| | | | | | |
| MAXIV | 3.0 | 330 | 8.3 | 2.5% | n.a. |
| Sirius | 3.0 | 250 | ~10 | 4% | n.a. |
| | | | | | |
| SLS-2 | 2.4 | ~120 | 10 | 8% | |
| Elettra 2.0 | 2.0/2.4 | ~200 | ~4 | ~2% | 70-100 |
| ALS-U | 2.0 | ~80 | ~80 | 100% | ~20 |

In future low emittance ring based light sources the vertical emittance in terms of coupling = $\varepsilon_y/\varepsilon_x$ will vary from a few % to 100% (full coupling)

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# Content

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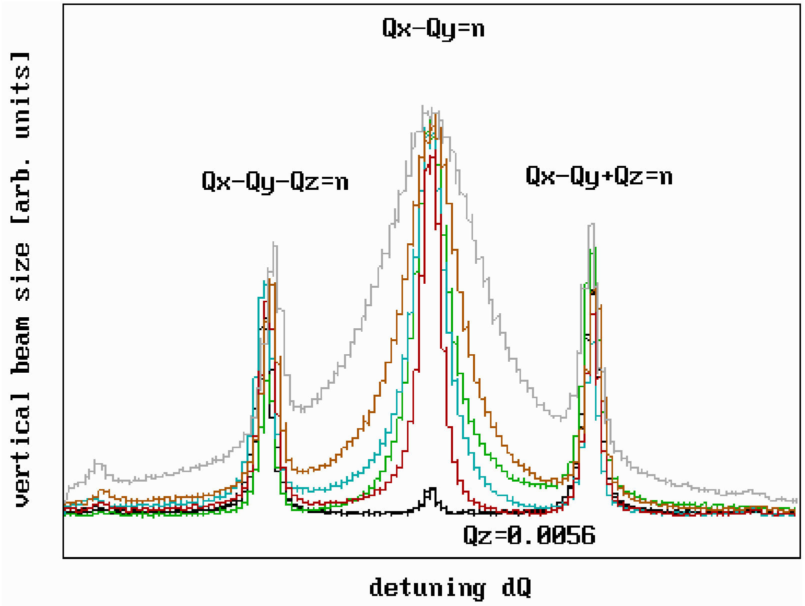


Linear coupling is modeled by mapping the second-order moments of the particle distributions (K. Hirata, *Particle Accelerators,* 1987, Vol. 22, pp. 57-79). Start with simple, one dimensional example, the uncoupled horizontal particle motion. The relevant second-order moments of the beam distribution are <*xx*>, <*xx'*>, and <*x'x'*>. *x'* should be interpreted as the conjugate horizontal momentum, $p_x$. The following single particle, turn-by-turn tracking equations are used in order to construct these moments:

$$x_1 = R_{11}x_0 + R_{12}x'_0$$

$$x'_1 = (R_{21}x_0 + R_{11}x'_0) \cdot l_x + R_x \cdot \sqrt{1 - l_x^2} \cdot \sqrt{\frac{\varepsilon_{x_0}}{\beta_x}}$$

$$l_x = e^{-2T_0/\tau_x} \approx \left(1 - 2\,{}^{T_0}/_{\tau_x}\right)$$

$R_{ij}$- are the elements of the 2x2 one turn matrix, the Twiss parameter $\alpha_x$ is assumed to be zero,($R_{22}$=$R_{11}$), $ß_x$ is the beta function, $T_0$, the revolution time, $\tau_x$, the damping time, and $\varepsilon_{x0}$, the natural emittance. Impact of synchrotron radiation is modeled by a diffusion term which has a Gaussian random distribution and the resulting damping which are both added to the horizontal momentum each turn. $R_x$ is a random number with a mean value of $\langle R_x \rangle = 0$ and a standard deviation, $\langle R_x R_x \rangle = 1$. This set of equations is very common for modelling transverse instabilities by tracking individual electrons forming an ensemble.

Moments of the distribution functions are obtained by integrating over all particles, indicated by <…>, leading to a set of equations for the second-order moments:

$$< xx >_1 = R_{11}^2 < xx >_0 + 2R_{11}R_{12} < xx' >_0 + R_{12}^2 < x'x' >_0$$

$$< xx' >_1 = (R_{11}R_{21} < xx >_0 + (R_{11}R_{11} + R_{12}R_{21}) < xx' >_0 + R_{11}R_{12} < x'x' >_0) \cdot l_x$$

$$< x'x' >_1 = (R_{21}^2 < xx >_0 + 2R_{11}R_{21} < xx' >_0 + R_{11}^2 < x'x' >_0) \cdot l_x^2 + (1 - l_x^2) \cdot \frac{\varepsilon_{x_0}}{\beta_x}$$

<*xx'*>=0 for $\alpha_x = 0$. For a stable, steady-state particle distribution the moments do not change from turn-by-turn and the subscripts of the moments can be ignored:

$$< xx > = R_{11}^2 < xx > + R_{12}^2 < x'x' >$$

$$< x'x' > = (R_{21}^2 < xx > + R_{11}^2 < x'x' >) \cdot l_x^2 + (1 - l_x^2) \cdot \frac{\varepsilon_{x_0}}{\beta_x}$$

From this set of equations the moments <xx> and <x'x'> are determined with the expected result:

$$< xx > = \beta_x \cdot \varepsilon_{x_0}.$$

With linear coupling the 4x4 ring matrix R has non-zero off-diagonal elements. One obtains for the turn-by-turn mapping of the transverse coordinates:

$$x_1 = R_{11}x_0 + R_{12}x'_0 + R_{13}y + R_{14}y'$$

$$x'_1 = (R_{21}x_0 + R_{11}x'_0 + R_{23}y + R_{24}y') \cdot l_x + R_x \cdot \sqrt{1 - l_x^2} \cdot \sqrt{\frac{\varepsilon_{x_0}}{\beta_x}}$$

$$y_1 = R_{31}x_0 + R_{32}x'_0 + R_{33}y_0 + R_{34}y'_0$$

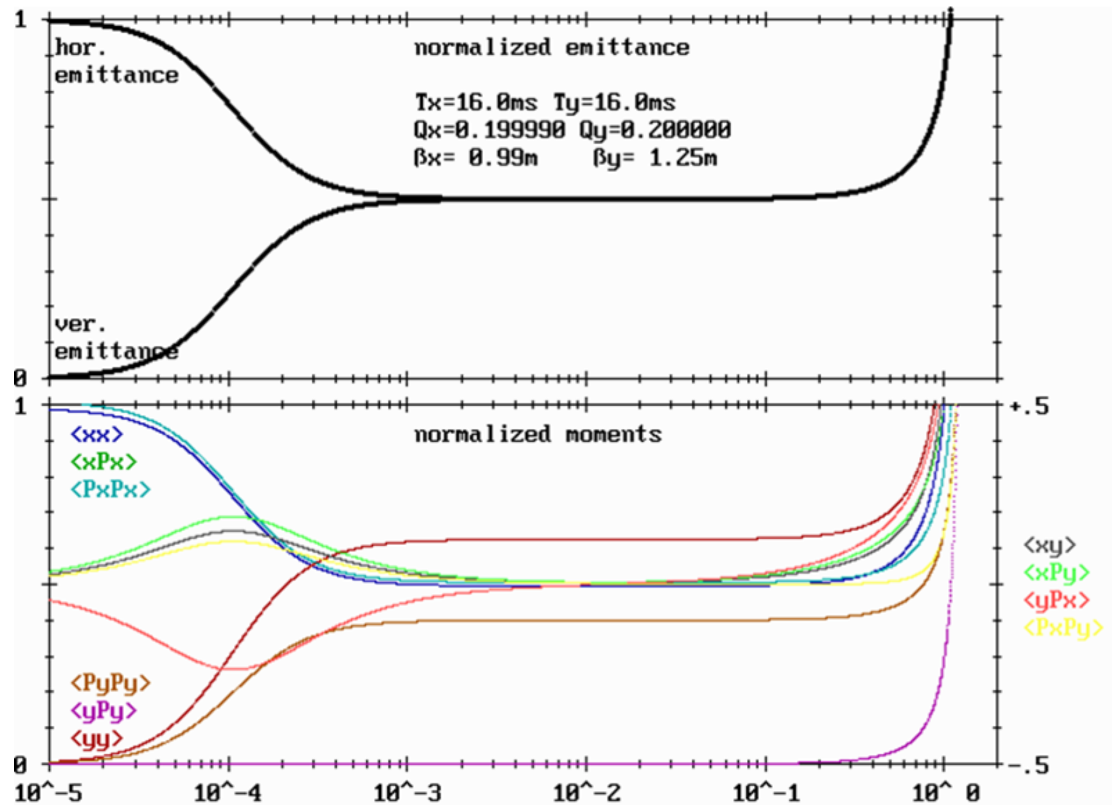
$$y'_1 = (R_{41}x_0 + R_{42}x'_0 + R_{43}y + R_{33}y') \cdot l_y + R_y \cdot \sqrt{1 - l_y^2} \cdot \sqrt{\frac{\varepsilon_{y_0}}{\beta_y}}$$

with the vertical Twiss parameters $\beta_y$ and with $\alpha_y$=0. The other parameters have the same meaning as in the horizontal plane. Following a similar route as in the introduction of the simple one dimensional, horizontal, case we can construct a set of 10 different second-order moments which all dependent on the 9 other moments. Taking into account the damping and the diffusion effects, the 10 second-order moments can be calculated from this set of equations by inverting a 10x10 matrix. Some of these moments are used to estimate the emittance, as usually given by:

$\varepsilon_x = \sqrt{< xx >< x'x' > - < xx' >^2}$ and similarly for the vertical emittance.

Typical result for these moments as a function of the coupling constant, $|\kappa|$. In electron storage rings the damping has to be overcome by the coupling in order to create substantial emittance variations. For very strong coupling the sum-resonance dominates and both emittances blow up and particles will be lost. In the region in between the beam is fully coupled and the emittance in both planes is equal, $\varepsilon_x=\varepsilon_y=\varepsilon_{x,y}$, with the value $\varepsilon_x + {}^{\tau_x}/_{\tau_y} \cdot \varepsilon_y = \varepsilon_0$.

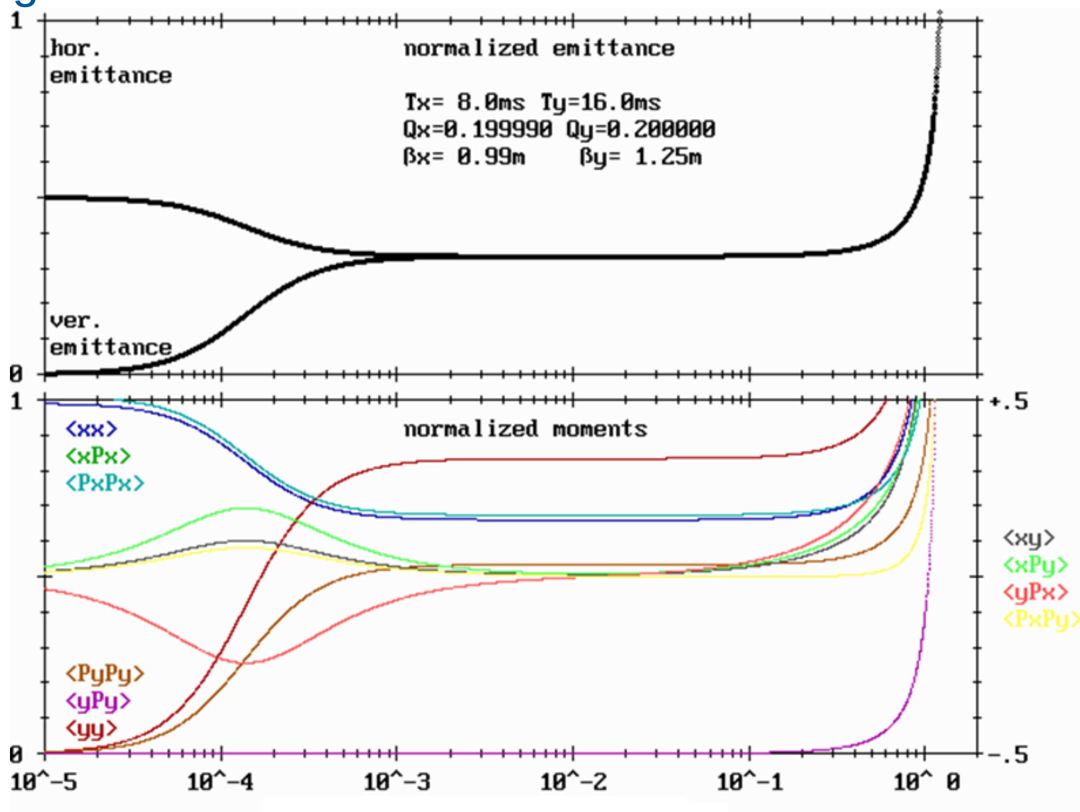
Horizontal and vertical emittance (top) and all 10 moments (bottom) as a function of coupling strength. The horizontal $\beta_x$ was chosen slightly different from 1.0 so that the moments <xx> and <PxPx> differ from each other. Tunes are close but not on the resonance.




$$skew\ Q = \frac{L}{B\rho}\frac{\partial B_x}{\partial x}\sqrt{\beta_x\beta_y}$$

Note the behavior of the second-order moments for increasing coupling strength at the bottom of this and the previous Figure: In the region where the transverse emittances are approaching emittance sharing the mixed coordinate moments, <xy>, <xPy>, <yPx> and <PxPy> are non-zero. In an intermediate range of coupling with emittance sharing these second-order moments are near zero. For very strong coupling all moments increase to very large values and in reality particles will be lost due to the excitation of the fatal sum-resonance, $Q_x+Q_y$=integer.

Like the previous Figure, however, the horizontal damping is twice as strong as the vertical. This reduces the horizontal emittance to 50% of the value with $\tau_x=\tau_y$. With full coupling horizontal and vertical emittance are equal and the horizontal emittance is reduced by 33.3%.




$$skew\ Q = \frac{L}{B\rho}\frac{\partial B_x}{\partial x}\sqrt{\beta_x\beta_y}$$

## Rate equation for the transverse emittance with coupling

The steady-state solution for the coupling can be found with the moment mapping approach. Identical results are obtained with a rate equation approach. S.Y. Lee gives the heuristic rate equations for the horizontal and vertical emittance, $\varepsilon_{x,y}$, under linear coupling (p. 464, exercise 7):

$$\boxed{\begin{aligned}\dot{\varepsilon}_x &= -\alpha_x(\varepsilon_x - \varepsilon_{x_0}) - \alpha_c(\varepsilon_x - \varepsilon_y)\\ \dot{\varepsilon}_y &= -\alpha_y(\varepsilon_y - \varepsilon_{y_0}) + \alpha_c(\varepsilon_x - \varepsilon_y).\end{aligned}}$$

These two differential equations are coupled through the coupling rate, $\alpha_c$, and the coupling is proportional to the difference between $\varepsilon_x$ and $\varepsilon_y$. Without coupling, $\alpha_c=0$, the emittances would approach their equilibrium values, $\varepsilon_{x0}$ and $\varepsilon_{y0}$, with the decay rates $\alpha_{x,y}$ given by $\alpha_x=1/\tau_x$ and $\alpha_y=1/\tau_y$. If everything is settled, $t\rightarrow\infty$, the time derivatives must be zero and the resulting emittances are:

$$\varepsilon_x(t\to\infty) = \frac{\alpha_x(\alpha_c+\alpha_y)}{\alpha_c\left(\alpha_x+\alpha_y\right)+\alpha_x\alpha_y}\varepsilon_{x_0} + \frac{\alpha_c\alpha_y}{\alpha_c(\alpha_x+\alpha_y)+\alpha_x\alpha_y}\varepsilon_{y_0}$$

$$\varepsilon_y(t\to\infty) = \frac{\alpha_x\alpha_c}{\alpha_c\left(\alpha_x+\alpha_y\right)+\alpha_x\alpha_y}\varepsilon_{x_0} + \frac{\alpha_y(\alpha_c+\alpha_x)}{\alpha_c(\alpha_x+\alpha_y)+\alpha_x\alpha_y}\varepsilon_{y_0}$$

If we assume that the initial emittances are given by $\varepsilon_{x0}=\varepsilon_0$, $\varepsilon_{y0}=0$, and if the coupling rate is much larger then the transverse decay rates we obtain:

$$\varepsilon_x(t \to \infty) = \frac{\alpha_x \alpha_c}{\alpha_c(\alpha_x + \alpha_y)} \varepsilon_0 = \frac{\alpha_x}{(\alpha_x + \alpha_y)} \varepsilon_0$$

$$\varepsilon_y(t \to \infty) = \frac{\alpha_x \alpha_c}{\alpha_c(\alpha_x + \alpha_y)} \varepsilon_0 = \frac{\alpha_x}{(\alpha_x + \alpha_y)} \varepsilon_0$$

And the emittances are equal in both planes.

Up to moderate coupling values and without significant excitation of the sum resonance the coupling rate, $\alpha_c$, can be found by the comparison of the results with the mapping approach:

$$\alpha_c = |C|^2 \frac{\sqrt{l_x l_y} \cdot (1/\tau_x + 1/\tau_y)}{\left(T_0/\tau_x + T_0/\tau_y\right)^2 + (2\pi \cdot \Delta Q)^2},$$

where C is the slightly modified coupling constant:

$$C = \frac{1}{2} \oint ds \frac{1}{B\rho} \frac{\partial B_x}{\partial x}(s) \cdot \sqrt{\beta_x(s)\beta_y(s)} \cdot e^{i(\mu_x(s) - \mu_y(s))}.$$

$T_0$ is the revolution time and $\Delta Q = Q_x - Q_y$.

Under the assumption of small coupling the final emittances are given by:

$$\frac{\varepsilon_y}{\varepsilon_0} = \frac{|C|^2\sqrt{l_x l_y}\cdot(1+\tau_y/\tau_x)}{|C|^2\sqrt{l_x l_y}\cdot\frac{(\tau_x+\tau_y)^2}{\tau_x\cdot\tau_x}+\left(T_0/\tau_x+T_0/\tau_y\right)^2+(2\pi\cdot\Delta Q)^2}$$ , and:

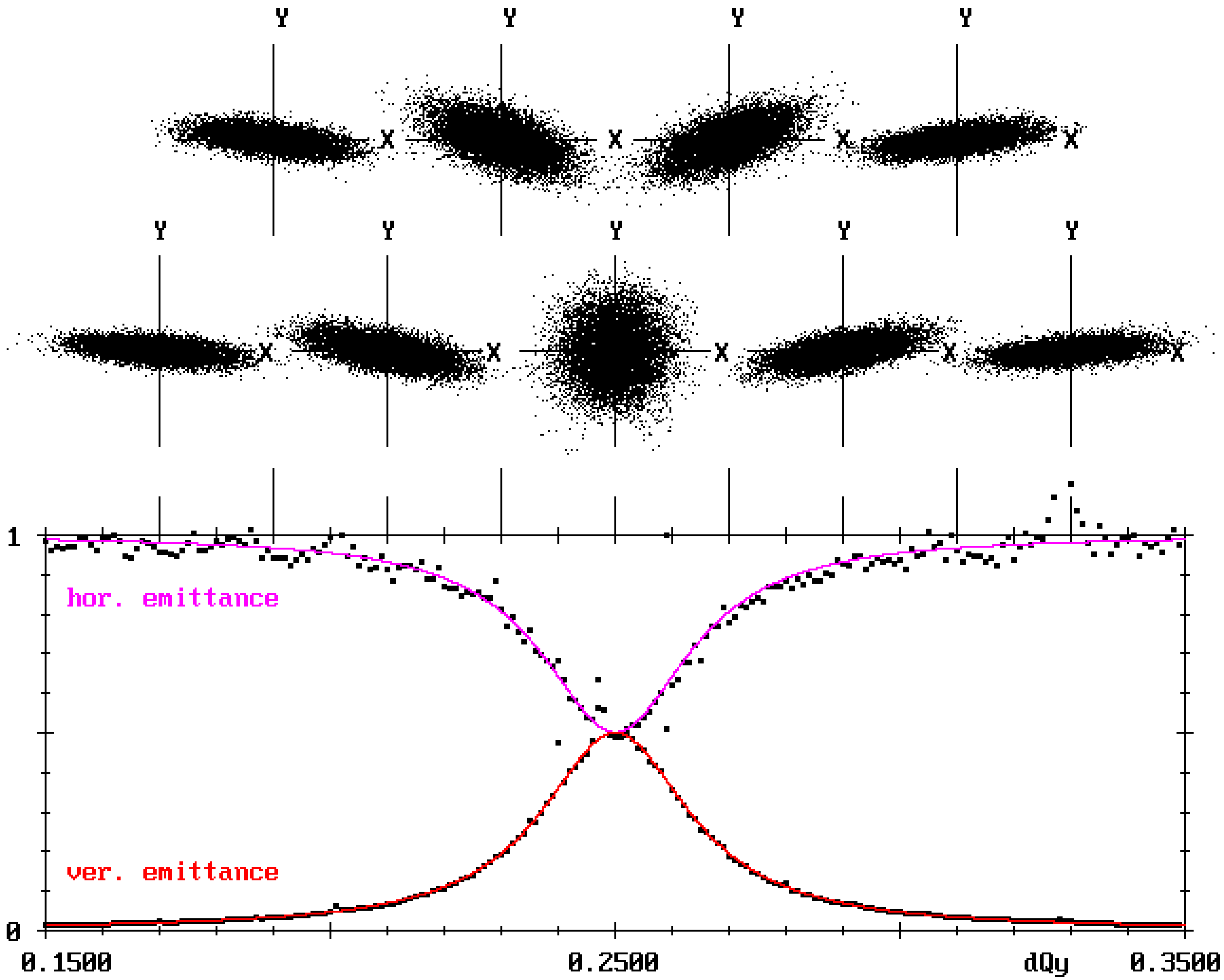
$$\frac{\varepsilon_x}{\varepsilon_0} = \frac{|C|^2\sqrt{l_x l_y}\cdot(1+\tau_y/\tau_x)+\left(T_0/\tau_x+T_0/\tau_y\right)^2+(2\pi\cdot\Delta Q)^2}{|C|^2\sqrt{l_x l_y}\cdot\frac{(\tau_x+\tau_y)^2}{\tau_x\cdot\tau_x}+\left(T_0/\tau_x+T_0/\tau_y\right)^2+(2\pi\cdot\Delta Q)^2}.$$

The result for the full width at half maximum, FWHM, of the vertical resonance curve is:

$$\Delta Q_{FWHM} = \frac{1}{\pi}\sqrt{|C|^2\sqrt{l_x l_y}\cdot\frac{(\tau_x+\tau_y)^2}{\tau_x\cdot\tau_x}+\left(T_0/\tau_x+T_0/\tau_y\right)^2}.$$

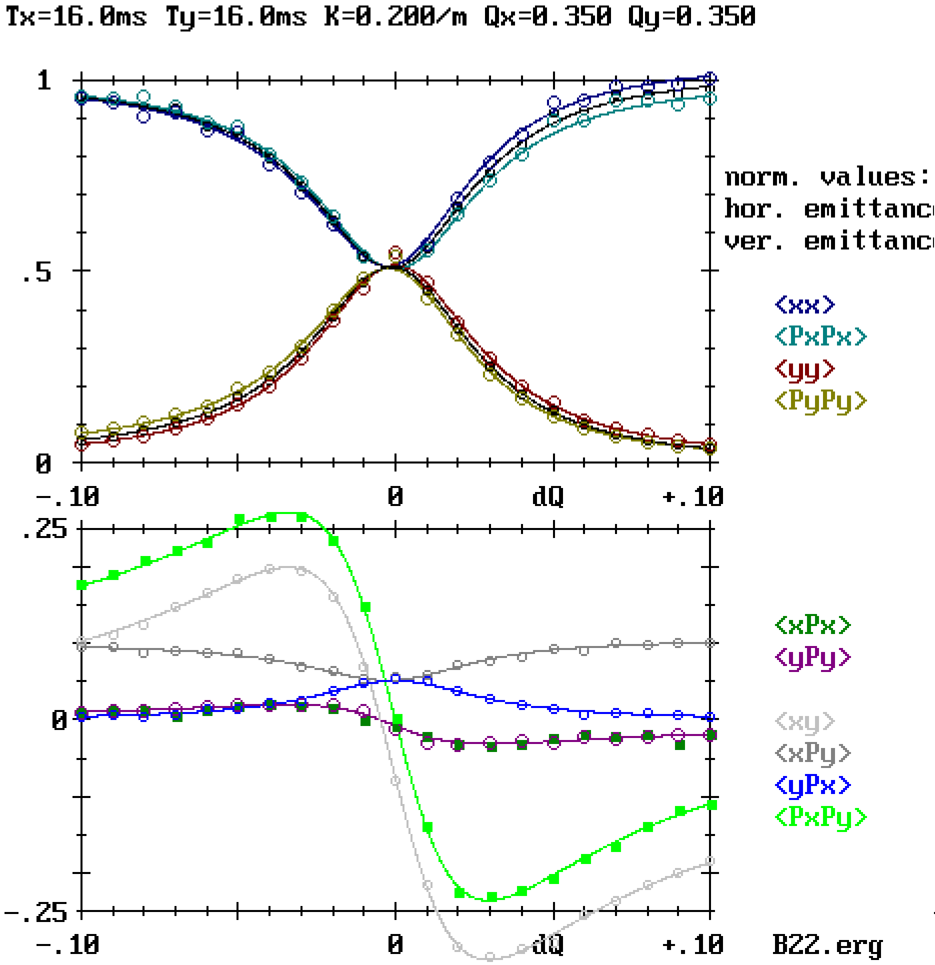
The above three formulas assume that the initial vertical emittance is zero and that the invariant emittance is $\varepsilon_0$.There is a small contribution to the line width from the finite lifetime of the transverse single particle motion.

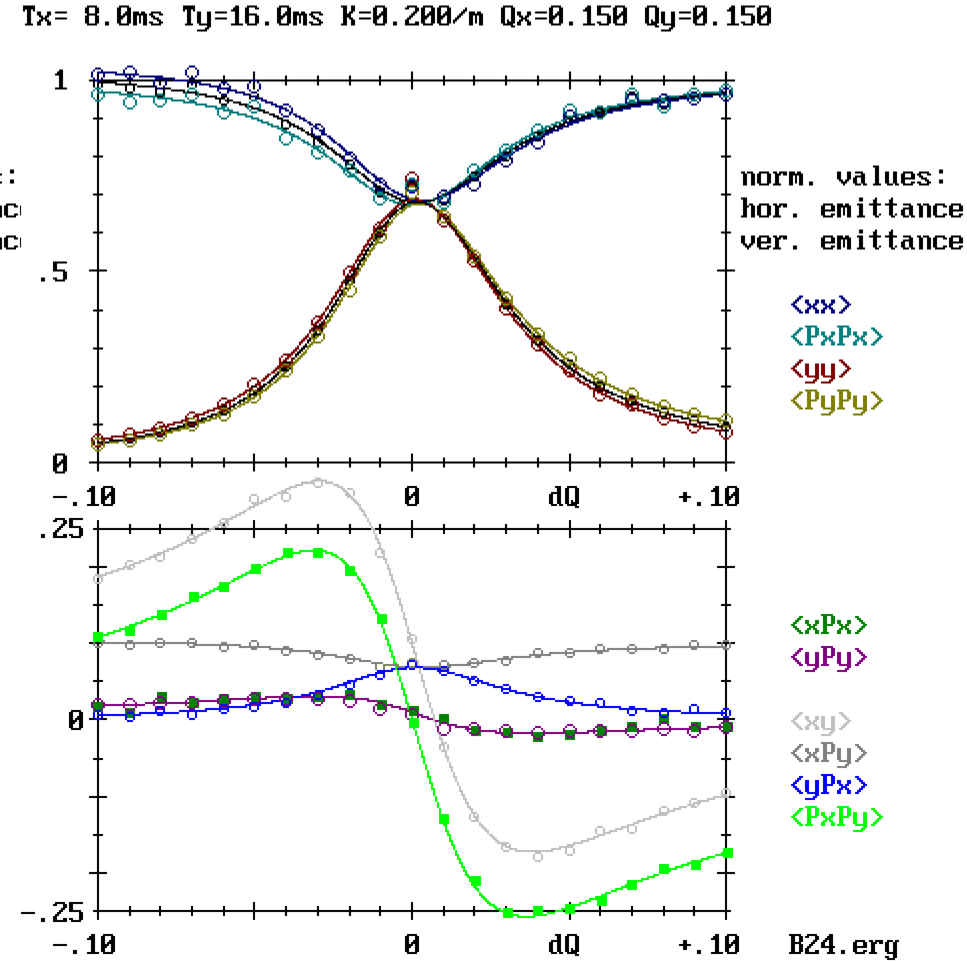
Note, that rate equation and moment mapping do not describe the correct development of the emittances for shorter times. Usually this is not really a problem since the skew gradient errors in a storage ring are static and we observe only the equilibrium situation anyway. However, it is important for the fast process of emittance exchange.




Particle distributions in x,y-space as a function of detuning – even for coupling at the %-level the beam is noticeable tilted

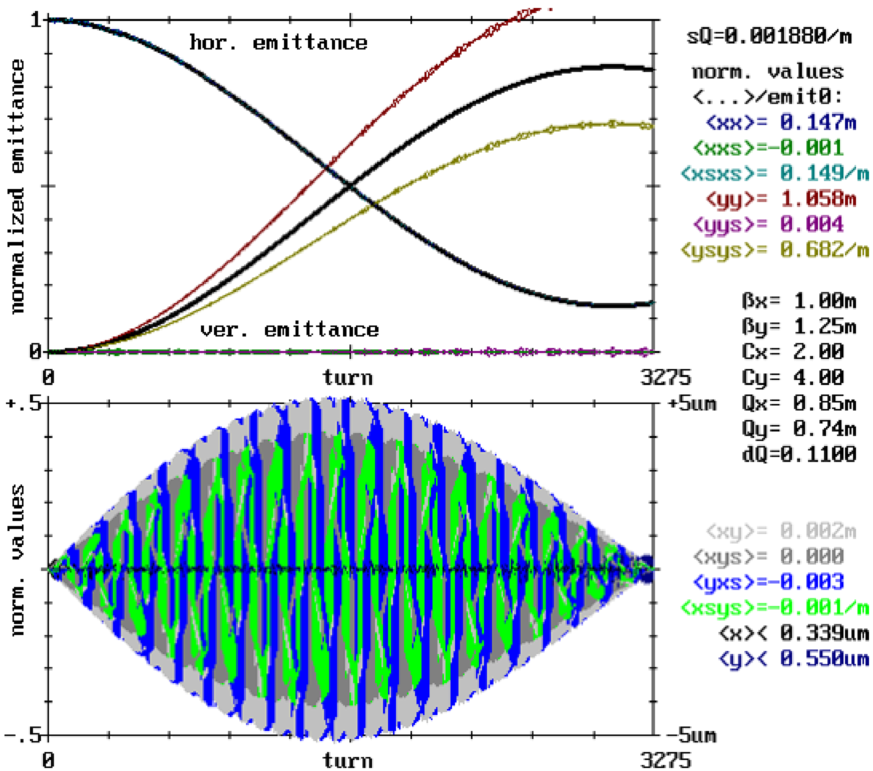
# I. Linear Coupling Theory

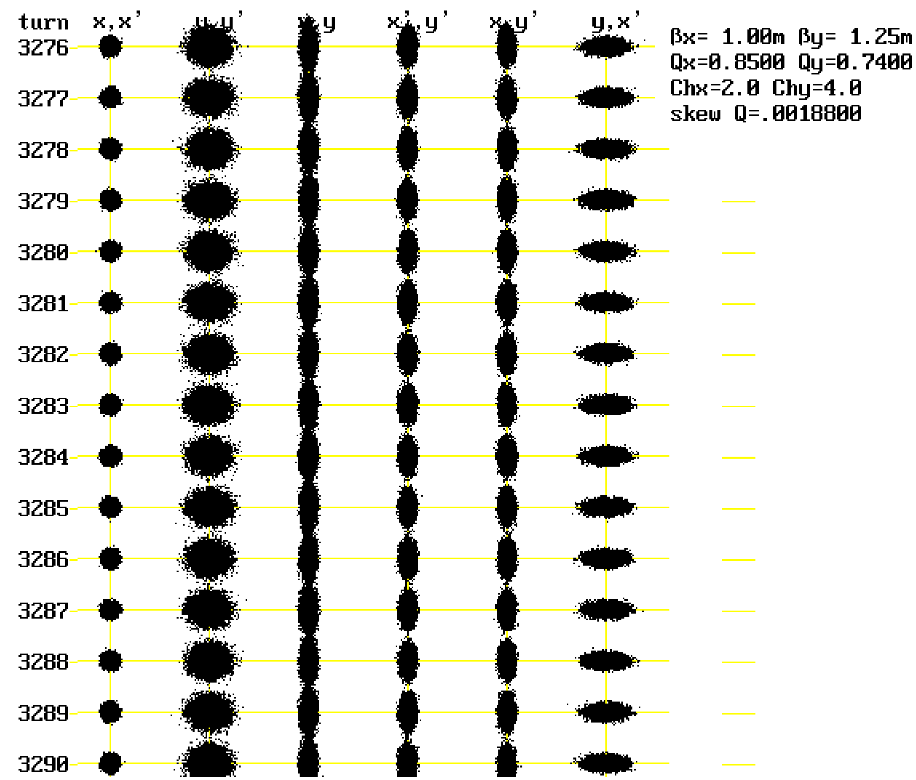



The correlations of particle coordinates near the coupling resonance and changes of these correlations could impact the performance of beamlines.
Results of tracking studies (shown as dots) are in excellent agreement with the analytical solution.

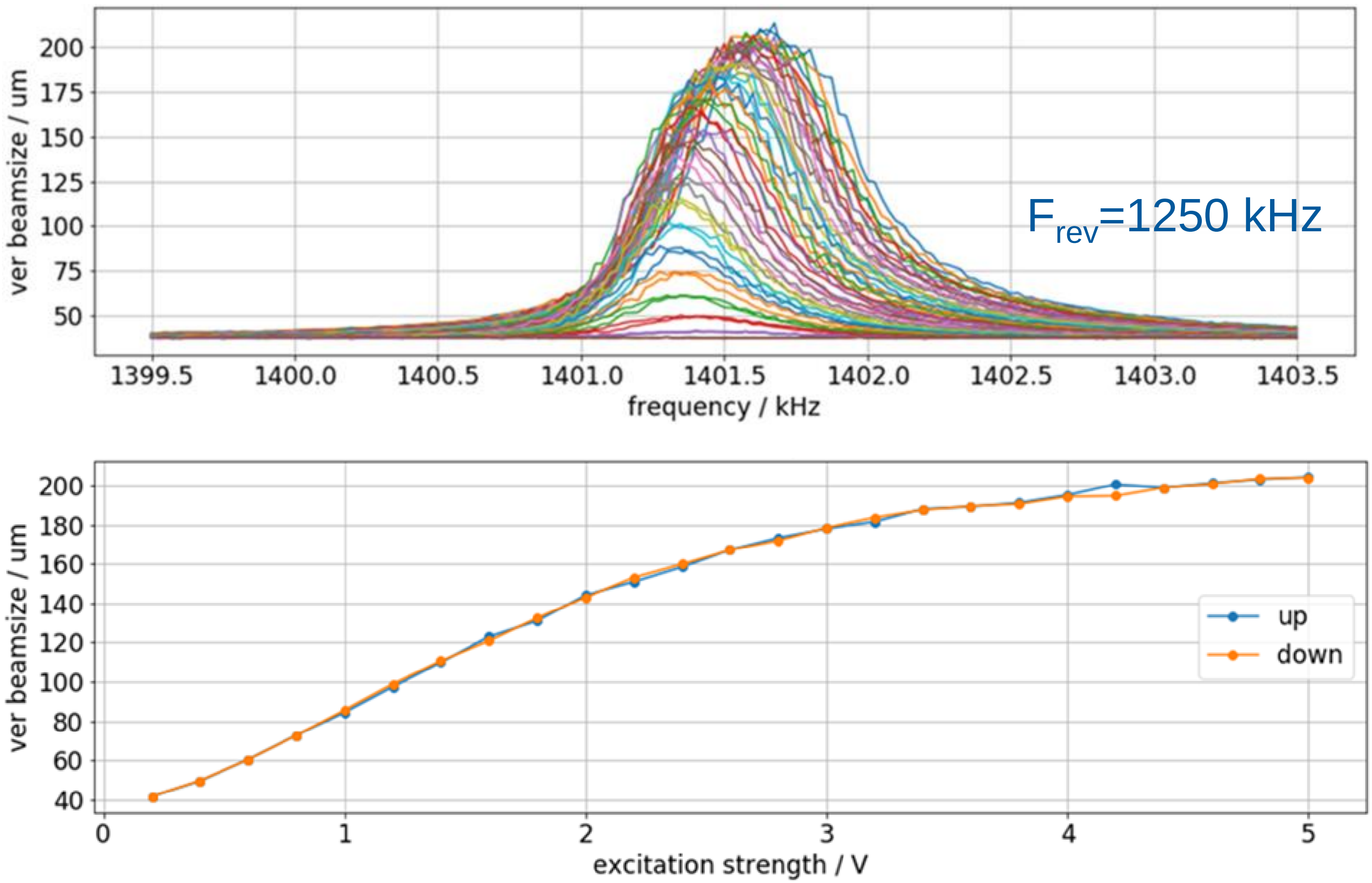
- Note, that rate equation and moment mapping do not describe the correct development of the emittances for shorter times (time $<<\tau_{x,y}$) which is important for the fast process of emittance exchange.
- AC-excitation as an additional techniques for emittance exchange (“Transverse emittance exchange for improved injection efficiency“, P. Kuske and F. Kramer, IPAC2016, WEOAA01). AC-excitation of a stripline ensemble creating a sufficiently strong skew quadrupole field can be switched on very fast and transient effects are observable (actual conditions at BESSY II):



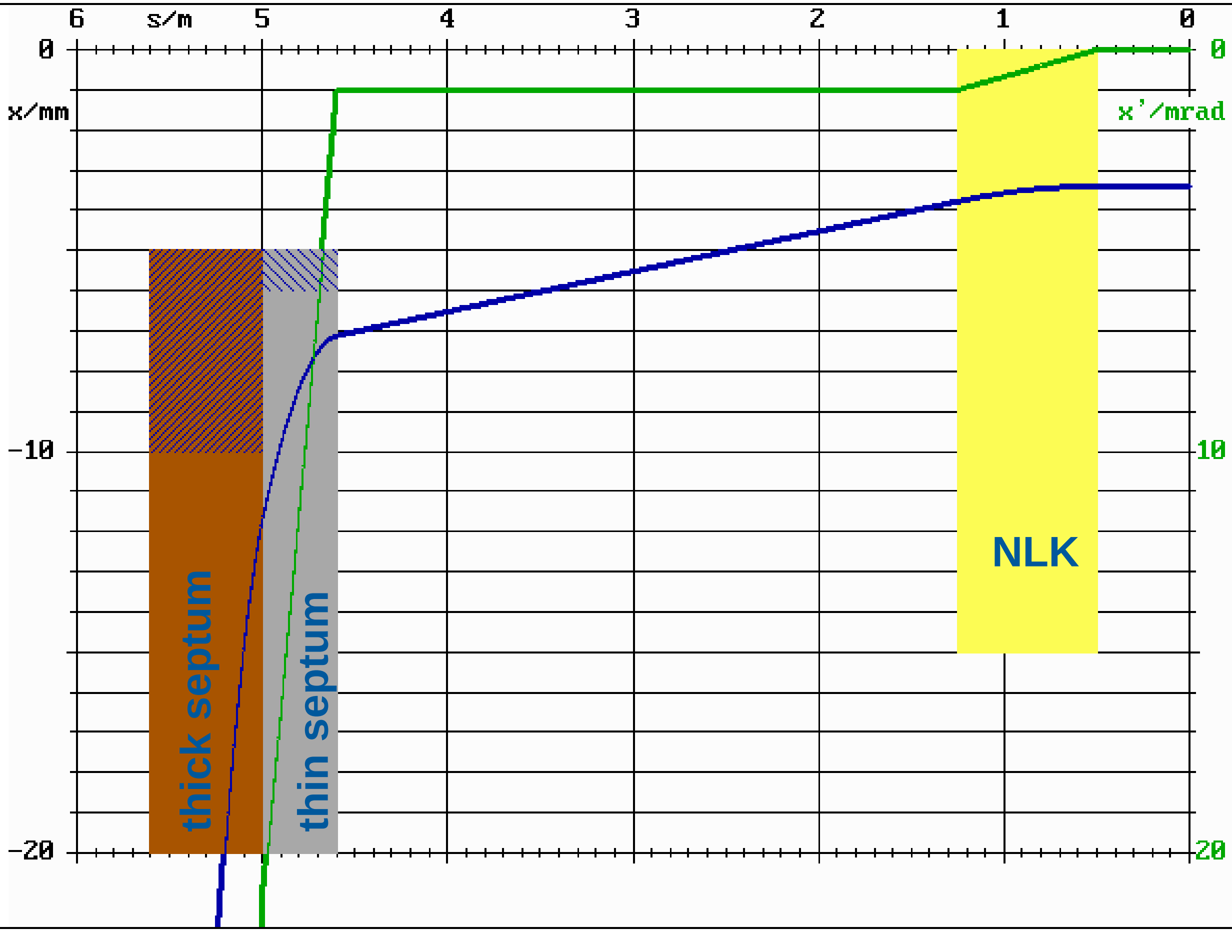
# I.1 Emittance Exchange with AC-Skew Quadrupole

➢ Result of experiments performed at BESSY II by M. Ries and his team:




➢ Results: a) full coupling reached, b) unexpected and unexplained shift of the resonance, c) the theoretically expected emittance inversion could not be observed due to insufficient diagnostics, ms-rise times could be detected, d) 100% injection efficiency with full excitation.

## II. Injection of Round Beams (Soleil Upgrade)



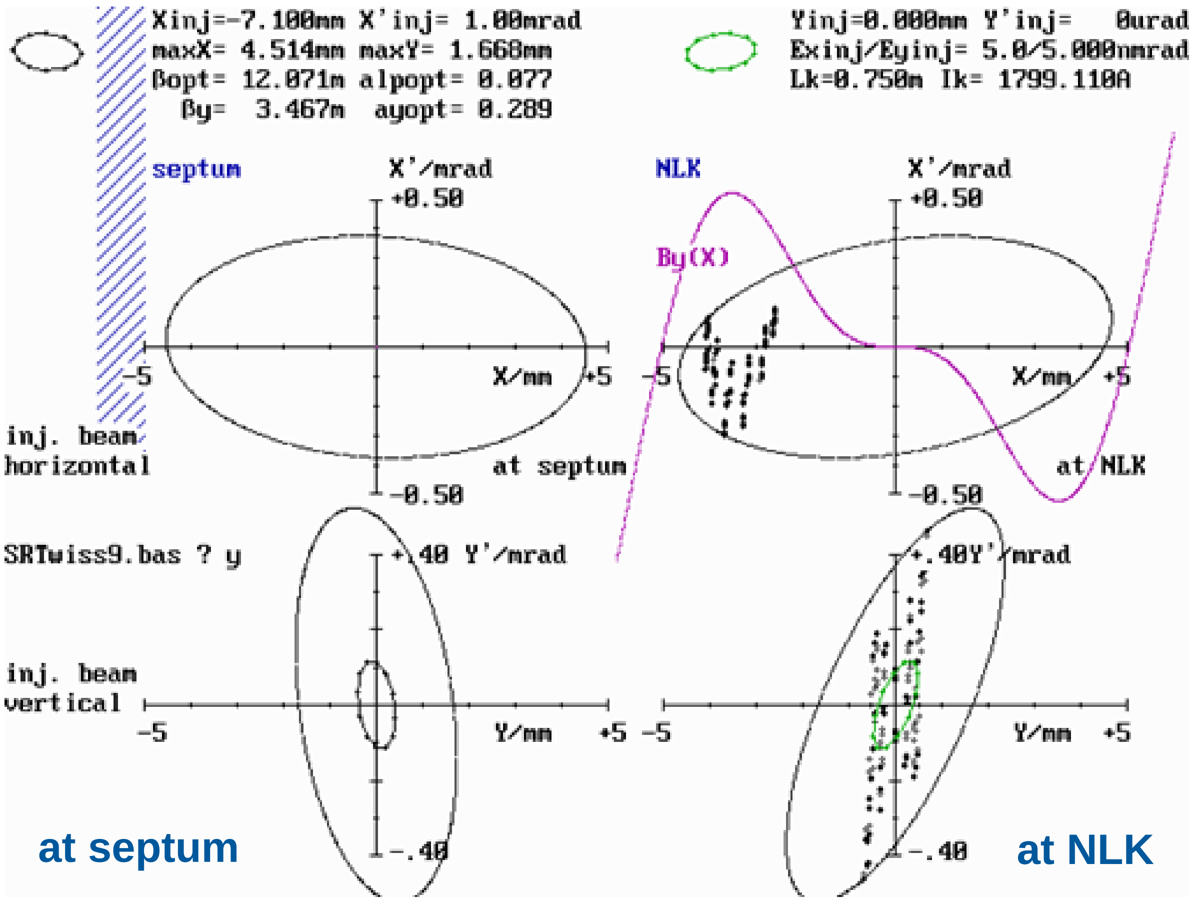

Proposed injection scheme for horizontal off-axis injection of the Soleil upgrade. The combination of a thick and a thin septum inject the beam under an angle of 1 mrad and a newly developed non-linear injection kicker with a field peak at 3.5 mm takes out this angle so that the center of the injected beam has a distance of 3.4 mm to the stored beam.

Key component is the MIK=NLK developed by Patrick Alexandre at Soleil

# II. Injection of Round Beams (Soleil Upgrade)
## Twiss parameters of the Injected Beam Matched to the SR Values

horizontal phase space

vertical phase space



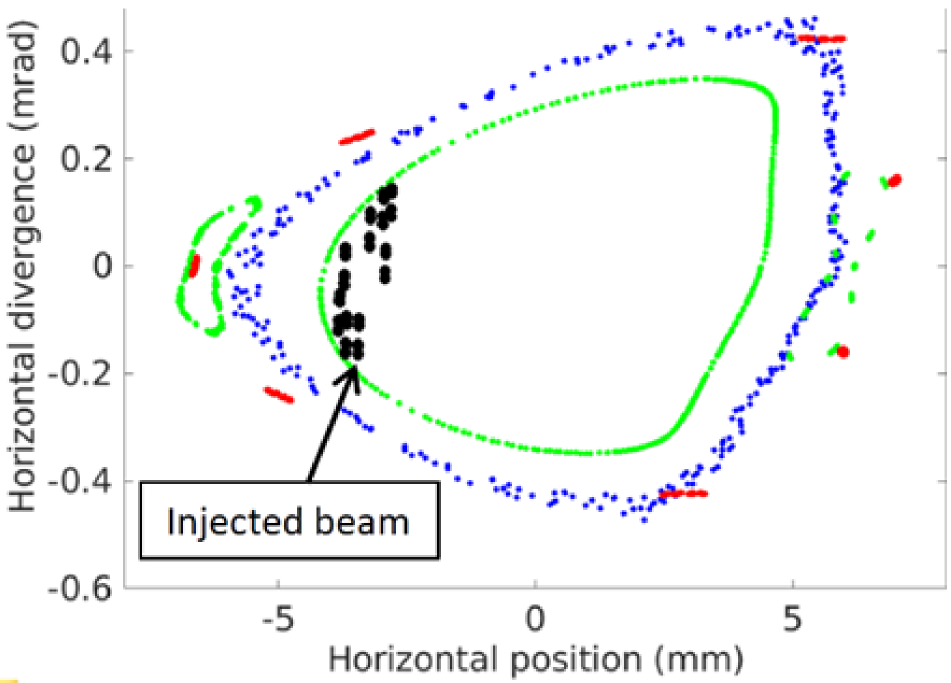

vertical field of the NLK in magenta

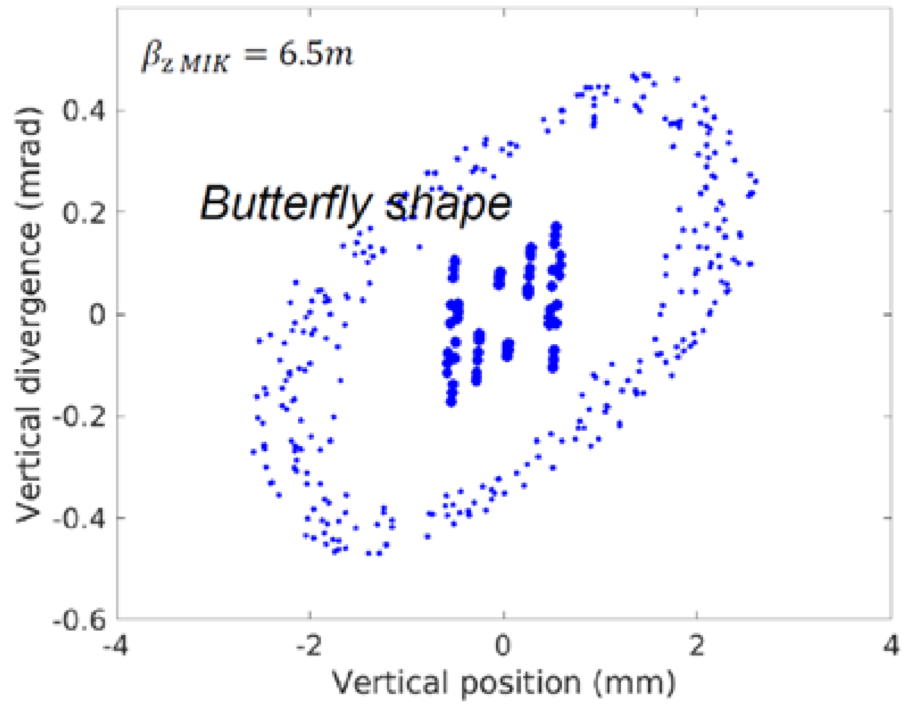
Each dot represents 144 additional particles which have identical coordinates in the plane shown. These additional particles represent the distribution in the orthogonal transverse and longitudinal plane (12 particles each).

In green particle distribution at the entrance of the Non-linear kicker

Simulated particles are shown as ellipses before and with a banana or butterfly shape after the interaction with the non-linear field of the kicker. The 12 points are split up into many more points due to the non-linear coupling fields of the kicker. The momentum distribution plays a minor role. In all dimensions a width of the distributions of $3{\cdot}\sigma$ is chosen. (Marie-Agnes Tordeux, Soleil)

## Determination of Injection Efficiency



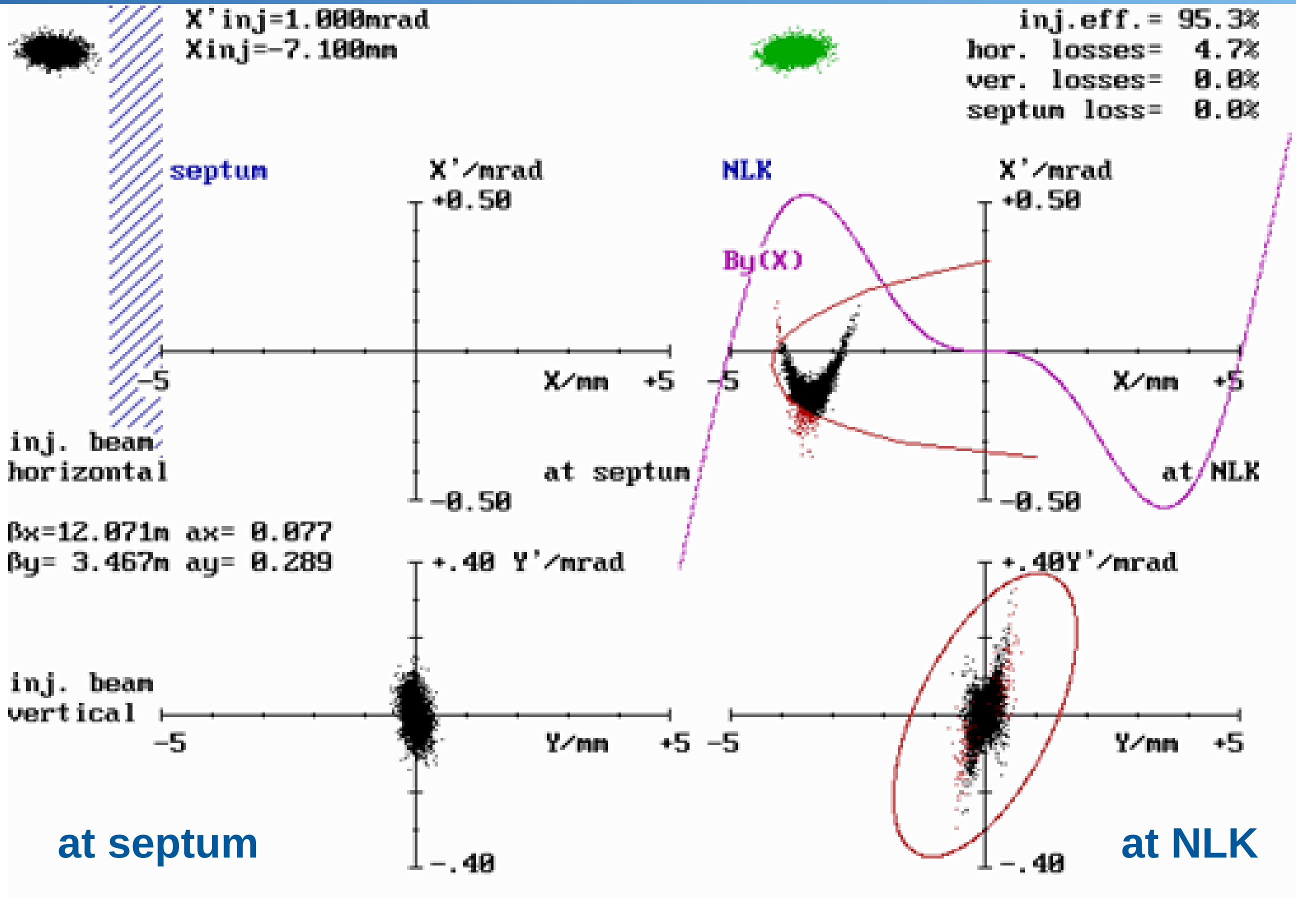




Horizontal (left) and vertical (right) acceptance at the exit of the NLK. In blue without errors and in green with certain higher multipole errors in the permanent quadrupole magnets. In the following simulations the green curve is taken for the horizontal acceptance. In the vertical plane the inner part of the distribution of dots is used and a linear vertical aperture of 1.5 mm is assumed. (Figures from M-A. Tordeux, Soleil)

Bunch length of injected beam has to be short or at best, as short as in the storage ring.

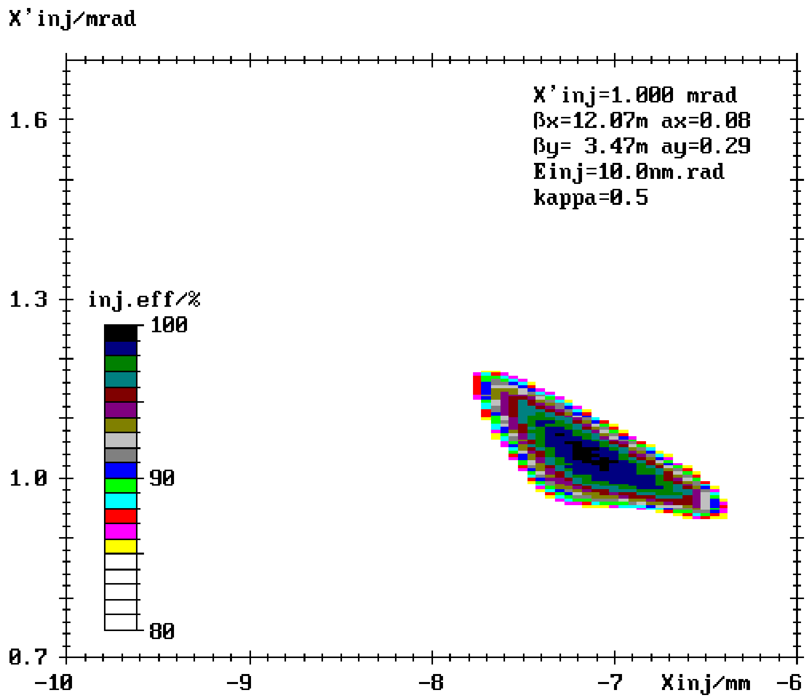
horizontal phase space

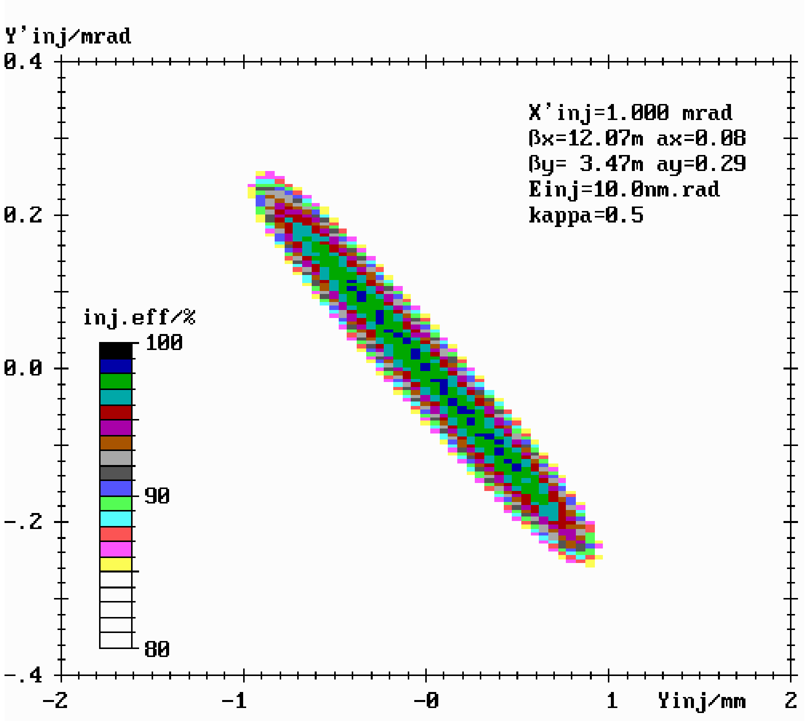
vertical phase space.



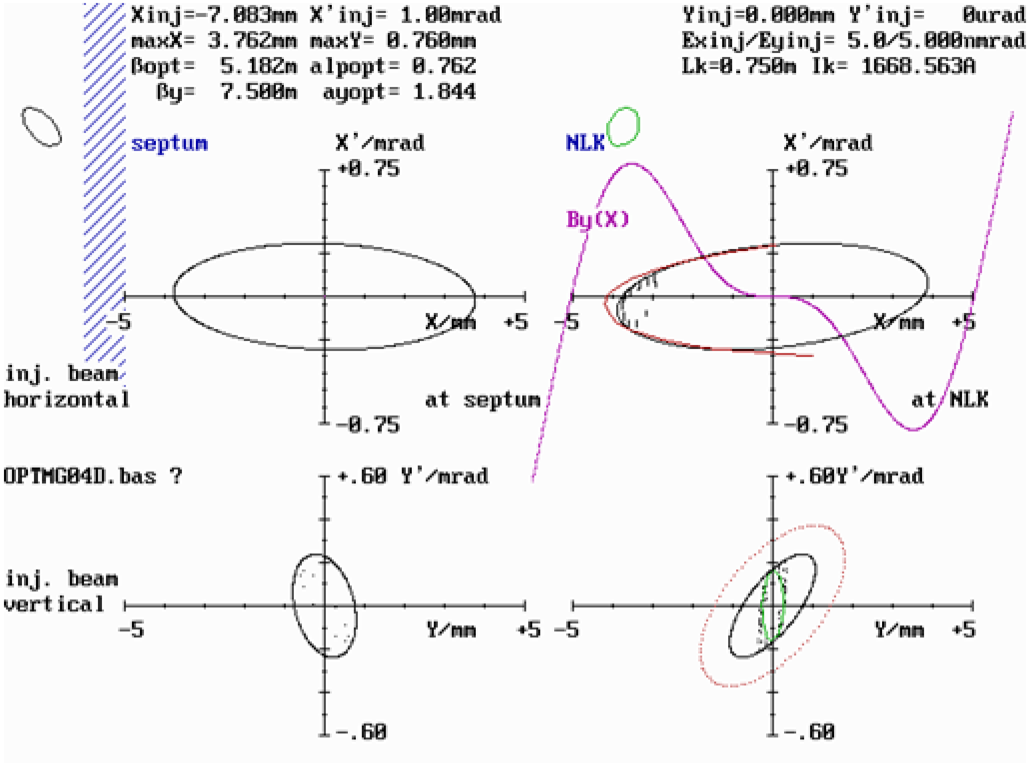

Distributions of 5000 particles with Twiss parameter matched to the SR and the dynamic acceptance of previous slide, shown in red. The injection efficiency is determined based on the surviving particles inside the acceptance of the ring. These particles are shown in black, lost particle in red. Losses can occur at the septum blade, in the vertical or the horizontal plane. The horizontal beam dynamics is dominating the injection efficiency.

# II. Injection of Round Beams – Acceptance (Soleil Upgrade)

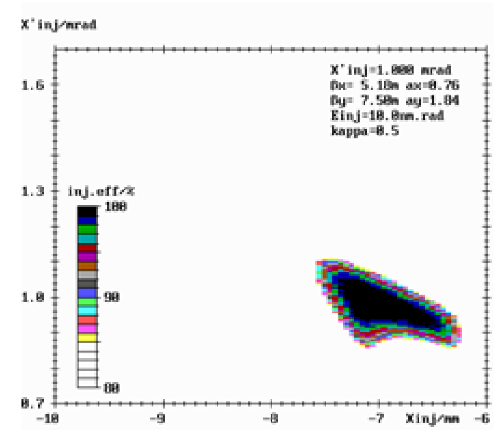


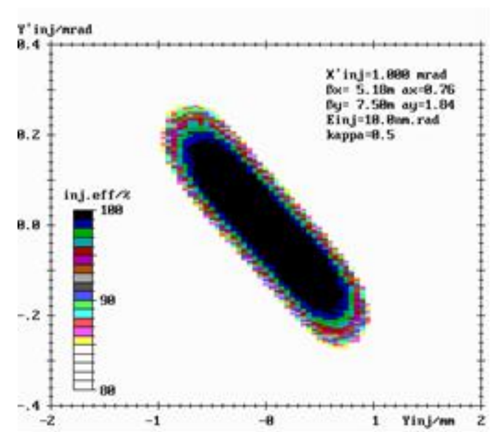




Impact of horizontal (left) and vertical (right) variations of the injection parameters on the injection efficiency – a round beam is injected with the Twiss parameters of the injected beam matched to the values in the injection straight. Matching the Twiss parameters to the values in the ring gives very little safety margin for efficient injection.



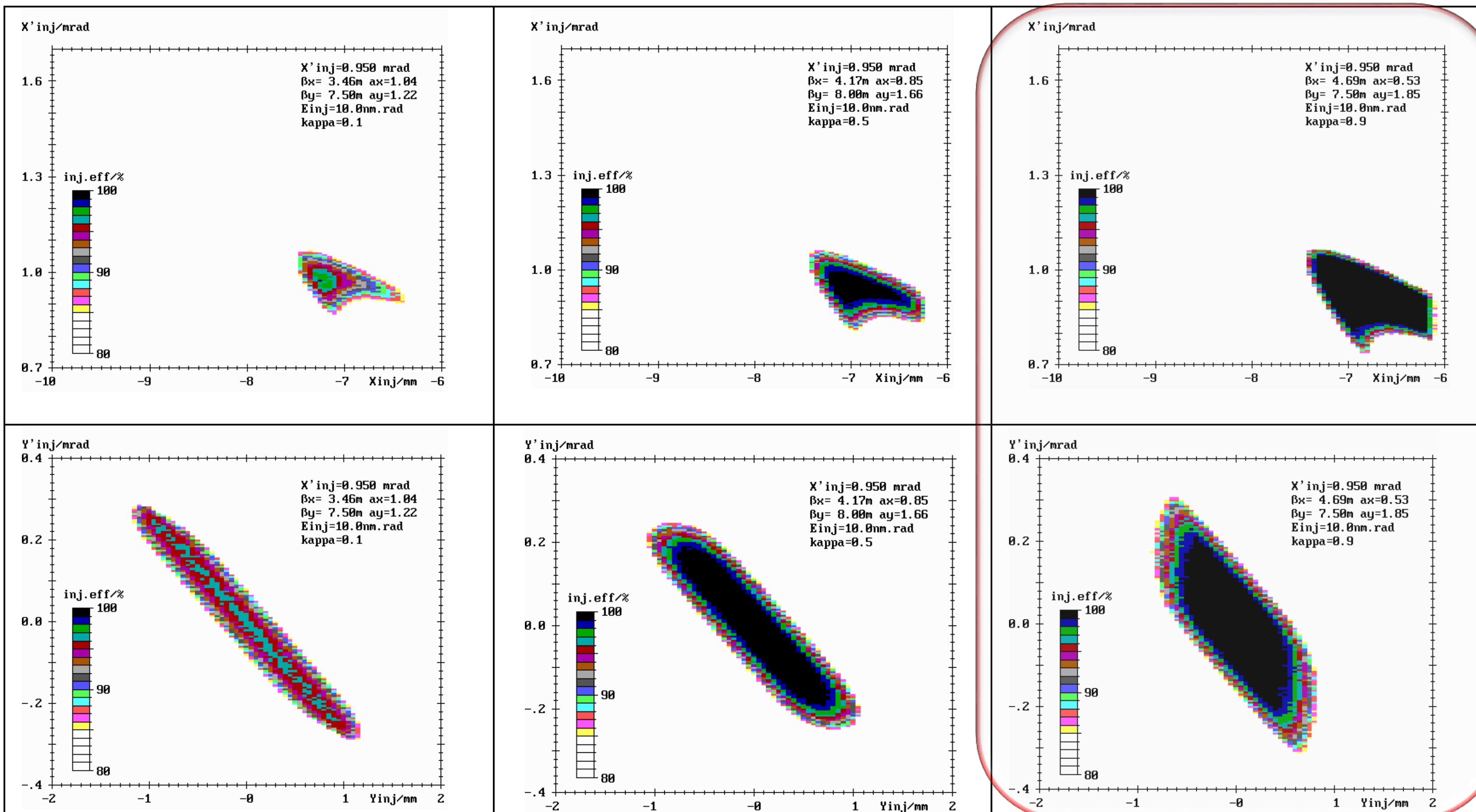

Twiss parameters of the injected beam optimized for small footprints in horizontal and vertical phase space. The encircling ellipses are drawn assuming pure linear motion




Injecting a round beam with optimized Twiss parameters – horizontal (top) as well as vertical acceptance (bottom) are increased. The safety margin for efficient injection is increased.

# II. Injection of Flat, Round, and Beams After Emittance Exchange



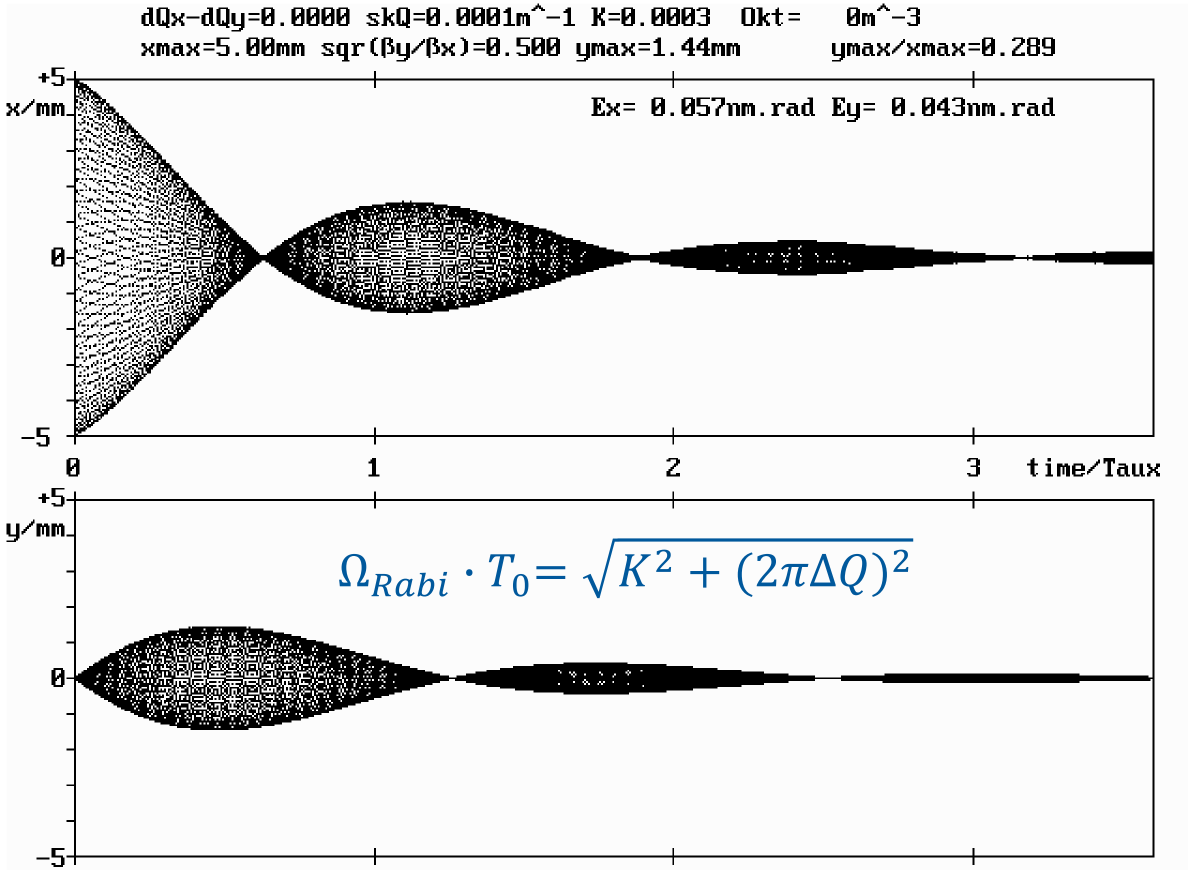

Horizontal (top) and vertical (bottom) injection windows for flat, round thin upright injected beams from left to right. Surprisingly, the reduction of the horizontal emittance, which goes hand in hand with an increase of the vertical emittance of the injected beam, does increase the vertical window. Horizontal losses dominate the injection efficiency and due to the non-linear field in the kicker the smaller horizontal beam size out weights the increased vertical beam size of the injected beam.

- Clear advantage of reduced horizontal emittance at the expense of the vertical

## III. Injecting Into Round Beams

- Round beams are created by operating on or near the linear coupling resonance
$$\Delta Q_x - \Delta Q_y = \text{integer}$$
- The particular resonance has to excited by either one skew quadrupole magnet or a suitable family of skew quadrupole magnets.
- The transverse coupling can cause losses of particles injected off-axis. With too strong coupling the initial momentum in the injection plane can be coupled to the orthogonal plane where the aperture might be limited. For example, with an off-axis injection in the horizontal plane and vertical apertures restricted by small gap undulators.
- On-axis injection (i.e. swap-out or longitudinal injection) will always work for rings operating on the coupling resonance – see for example: C. Du, et al., „Studies of round beam at HEPS storage ring by driving linear difference resonance“, Nucl. Instrum. And Methods, A976 (2020) 16426
- Transverse off-axis injection is feasible with the right choice of coupling strength and tune shift with amplitude so that only particles with small oscillation amplitudes are fully coupled.

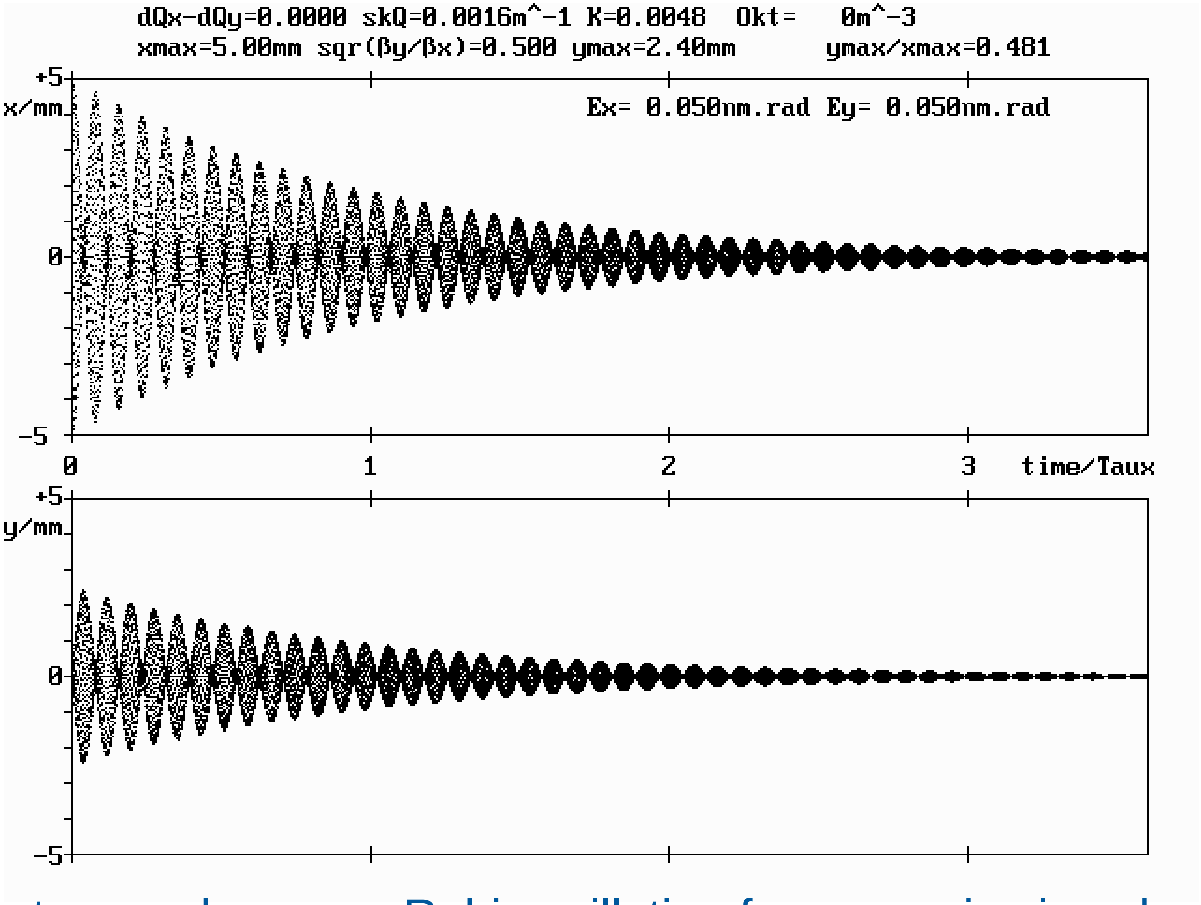
- Damping leads to a reduction of the oscillation from the injection and small coupling leads to a slow momentum exchange.




$$\Omega_{Rabi} \cdot T_0 = \sqrt{K^2 + (2\pi\Delta Q)^2}$$

- The coupling is too small and can not fully overcome the damping – the steady-state emittance ratio is 0.75 instead of the desired 1 for a round beam and emittance variations will occur with small coupling changes induced by IDs.

- Stronger coupling leads to faster momentum exchange and the vertical amplitude approaches the theoretical limit given by $y/x \leq \sqrt{\beta_y/\beta_x}$.



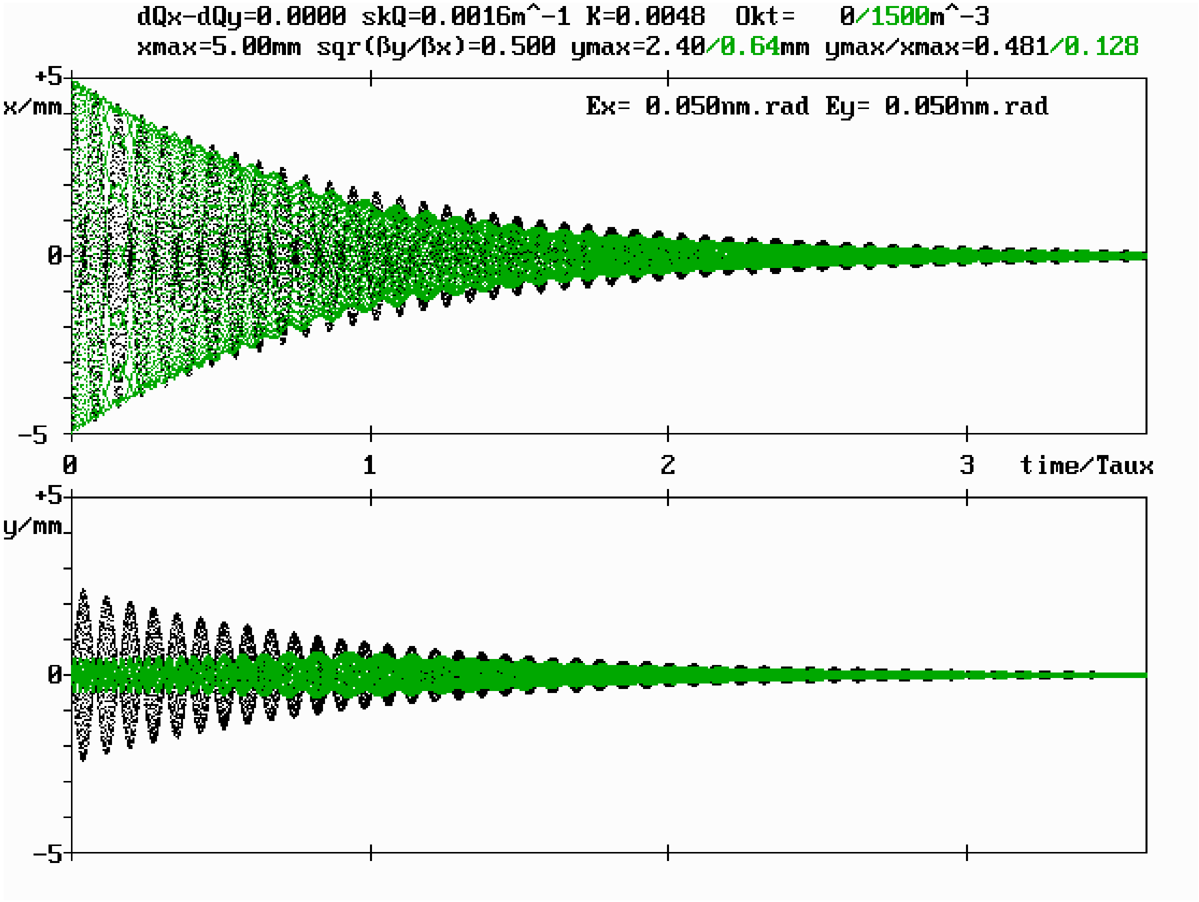

- The momentum exchange or Rabi oscillation frequency is given by:

$$\Omega_{Rabi} \cdot T_0 = \sqrt{K^2 + (2\pi\Delta Q)^2}$$

and must be fast compared to the damping time.

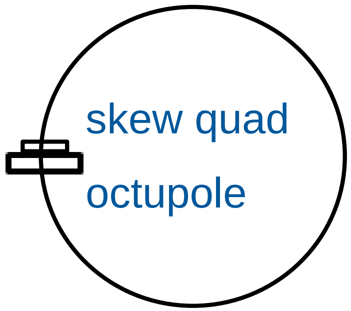
# III. Injecting Single Particle on Coupling Resonance

- Tune shift with amplitude from an octupole magnet can shift the transverse tunes away from the resonance and the momentum exchange sets in at smaller amplitudes.




- For efficient off-axis injection into round beams - balance between coupling strength and tune shift with amplitude is needed

- Diagram showing the final steady-state emittances (horizontal lines) and vertical oscillation amplitudes after an electron was injected with an amplitude of 5 mm as function of octupole strength (B·ρ=1 Tm) or tuneshift with amplitude.

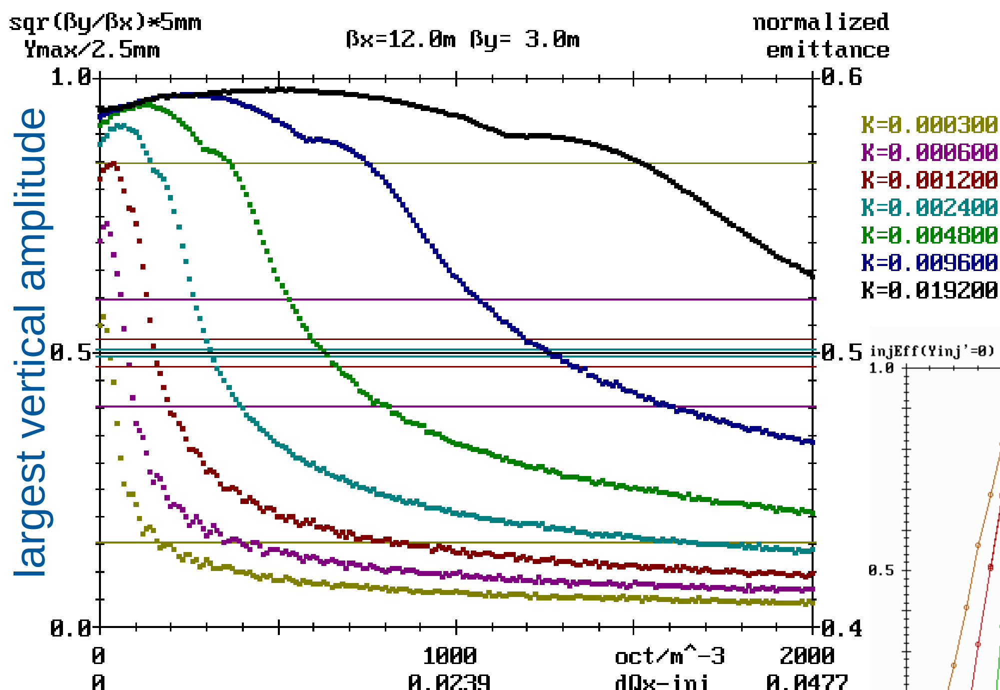


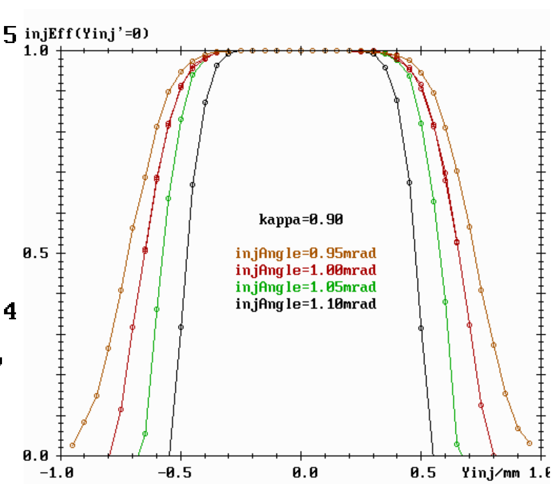

Tune shift with amplitude from a single octupole magnet:

$$\Delta Q_x / J_x = \frac{L_{oct} \cdot \beta_x^2}{16\pi \cdot B \cdot \rho} \frac{\partial^3 B_y}{\partial x^3}$$

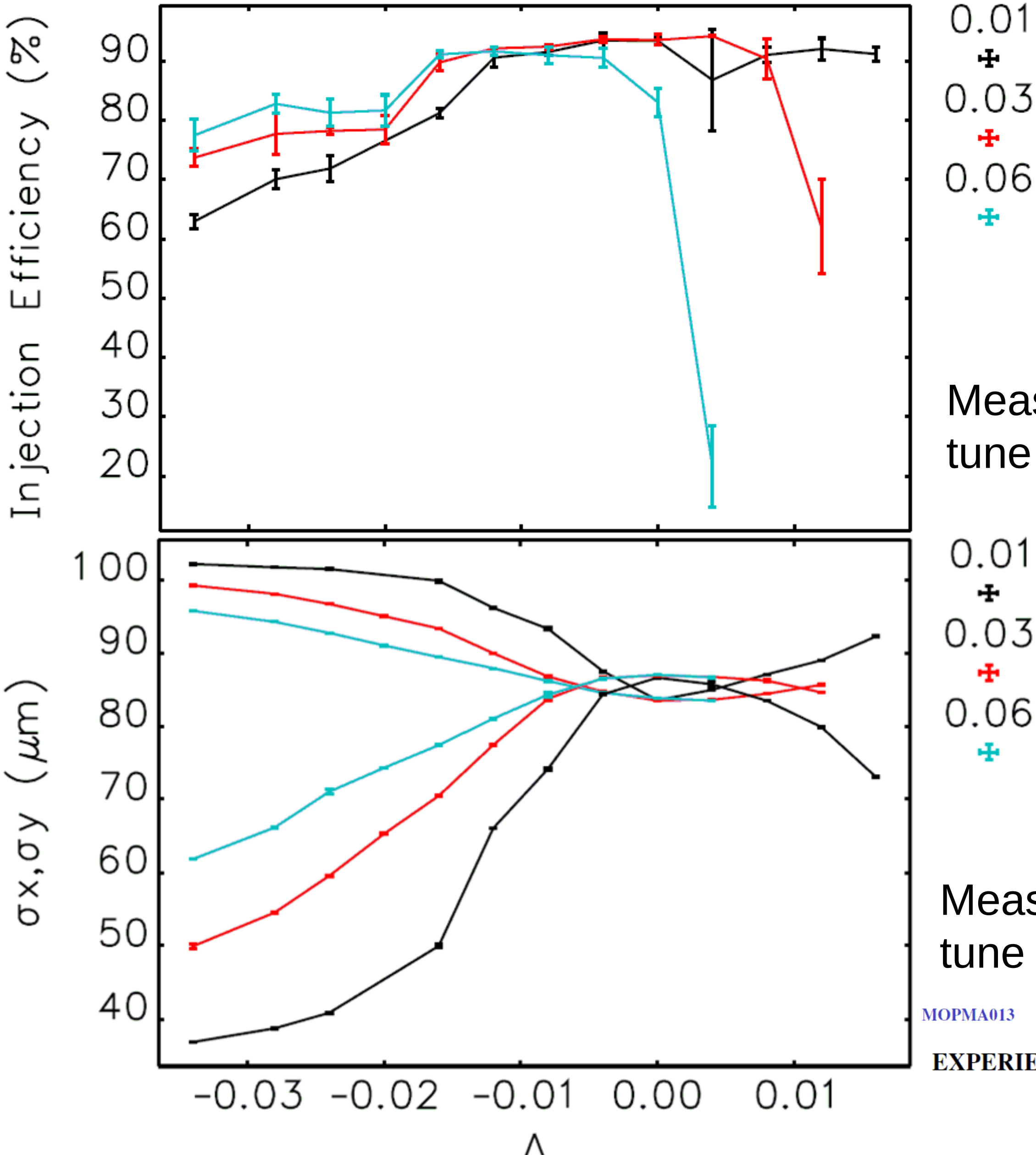
$$J_x = (5mm)^2/(2 \cdot \beta_x)$$

with $\alpha_x$=0




- Round beams require K≥0.0025 (magenta, green, blue and black curves) and larger and larger tuneshifts with amplitudes in order to keep vertical oscillations small.
- Requires a corresponding resonance-free tune window with good injection efficiency




$$|\kappa| = \frac{1}{2\pi B\rho}\frac{\partial B_x}{\partial x} L \sqrt{\beta_x\beta_y}$$

Measured top-up injection efficiency vs. tune separation $\Delta$ at different $\kappa$ (legend)

Measured beam size (raw data) vs. tune separation $\Delta$ at different $\kappa$ (legend)

MOPMA013 Proceedings of IPAC2015, Richmond, VA, USA

**EXPERIENCE WITH ROUND BEAM OPERATION AT THE ADVANCED PHOTON SOURCE** *

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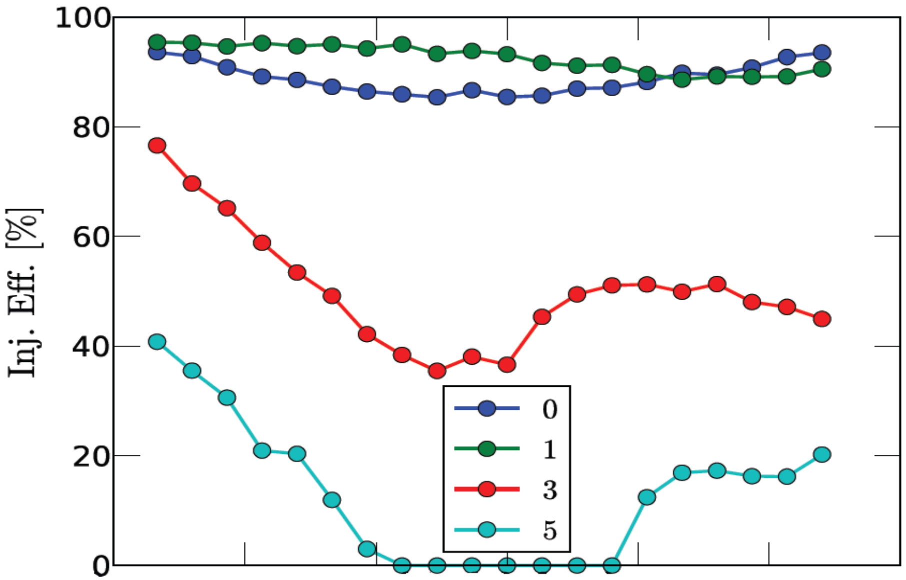
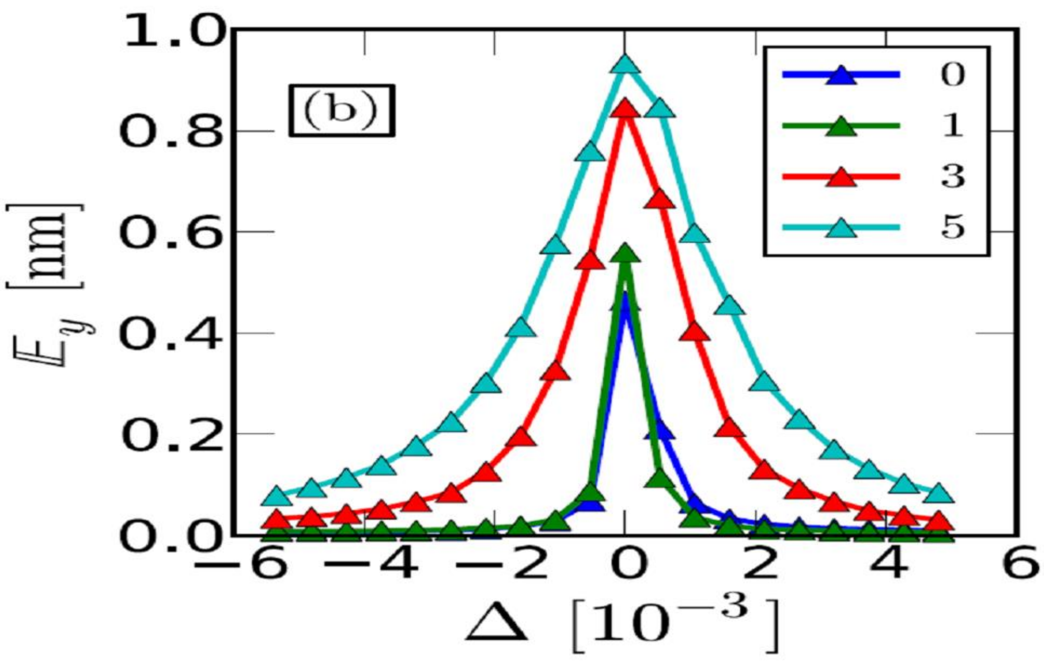
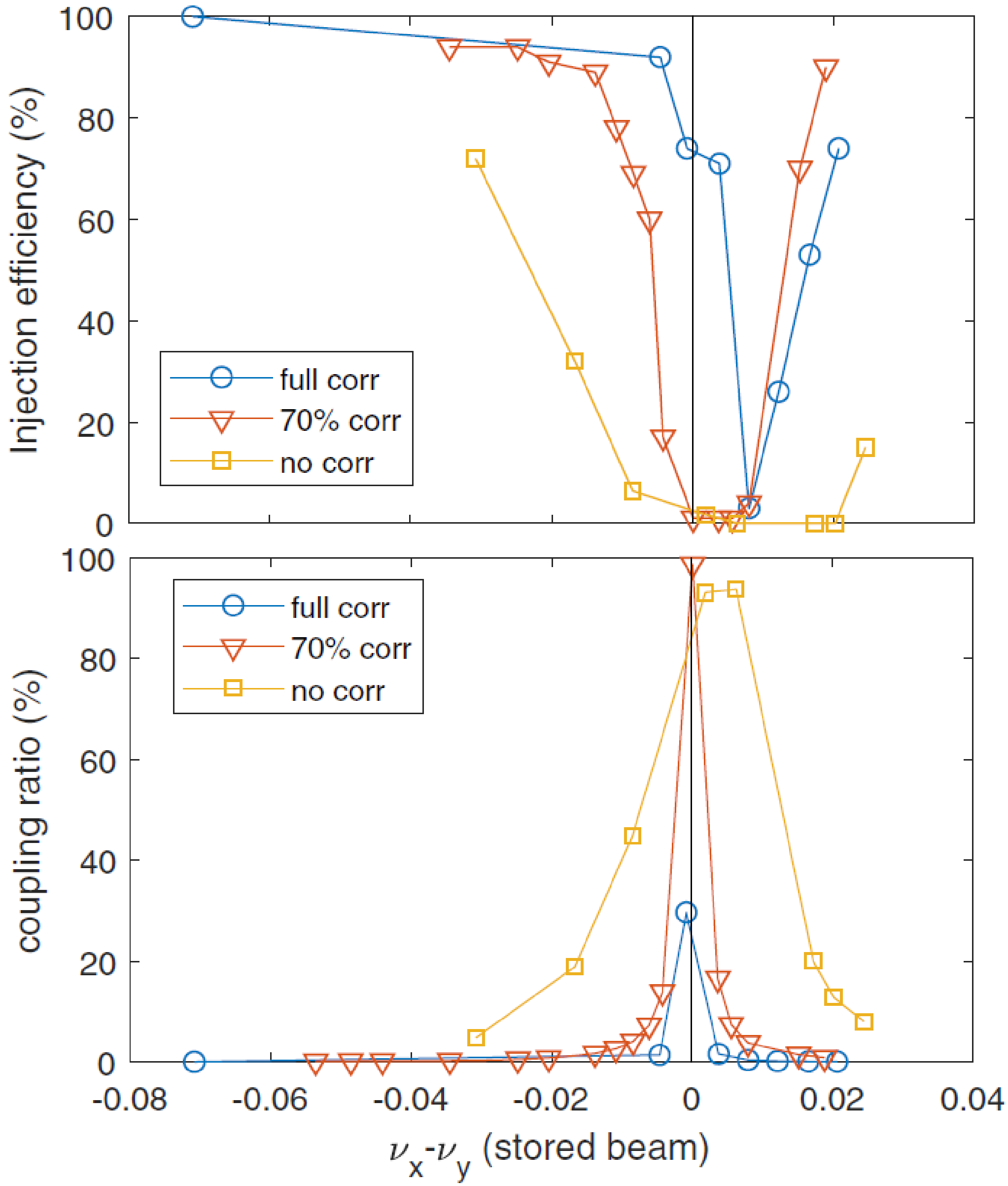
# III. Round Beam Studies at NSLS-II (Y. Hidaka, et al., IPAC2018, TUPMK018)

- At NSLS-II the injection efficiency (off-axis injection) drops for coupling strengths needed for full emittance sharing.
- The round beam condition is incompatable with the existing amplitude dependent tuneshifts.



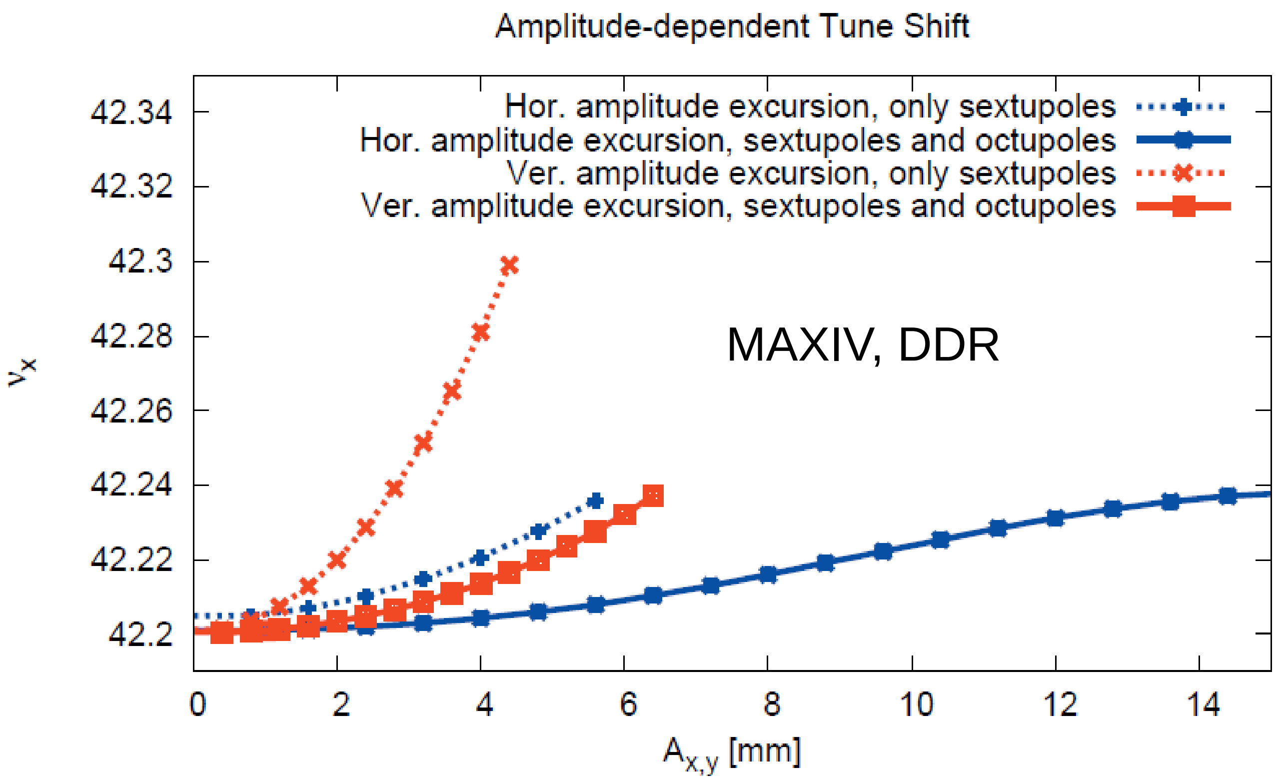



- At SPEAR 3 the injection efficiency (off-axis injection) drops for coupling strengths needed for full emittance sharing.
- The round beam condition is incompatable with the existing amplitude dependent tuneshifts.

T. Zhang and X. Huang, „Off-axis injection for storage rings with full coupling“, Phys. Rev. Accel. Beams 21, 084002 (2018)

- More experimental injection studies are needed, in combination with measurements of the tuneshift with amplitudes.

MAXIV would be a candidate to try off-axis injection with a round beam.



# III. SUMMARY – Injection of Round Beams

- The linear coupling theory has been extended and includes the damping and diffusion due to the emission of synchrotron radiation. This theory explains why there might be only partial emittance exchange and not emittance sharing on the coupling resonance.

- Injecting round beams is superior to injecting flat beams usually delivered by synchrotrons. An emittance exchange before extraction is significantly better than operating the booster on the coupling resonance.

- Injecting beams in storage rings operating in round beam mode is straight forward for on-axis injection schemes (swap-out or longitudinal injection).
- Transverse off-axis injection is feasible with carefully chosen coupling strength and amplitude dependent tune shift.

- I found 3 experimental studies on this subject and only the APS results show that this is indeed possible. More investigations on this subject are highly desirable and should be accompanied by measurements of the tune shift with amplitude.

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**Thank you for your attention**