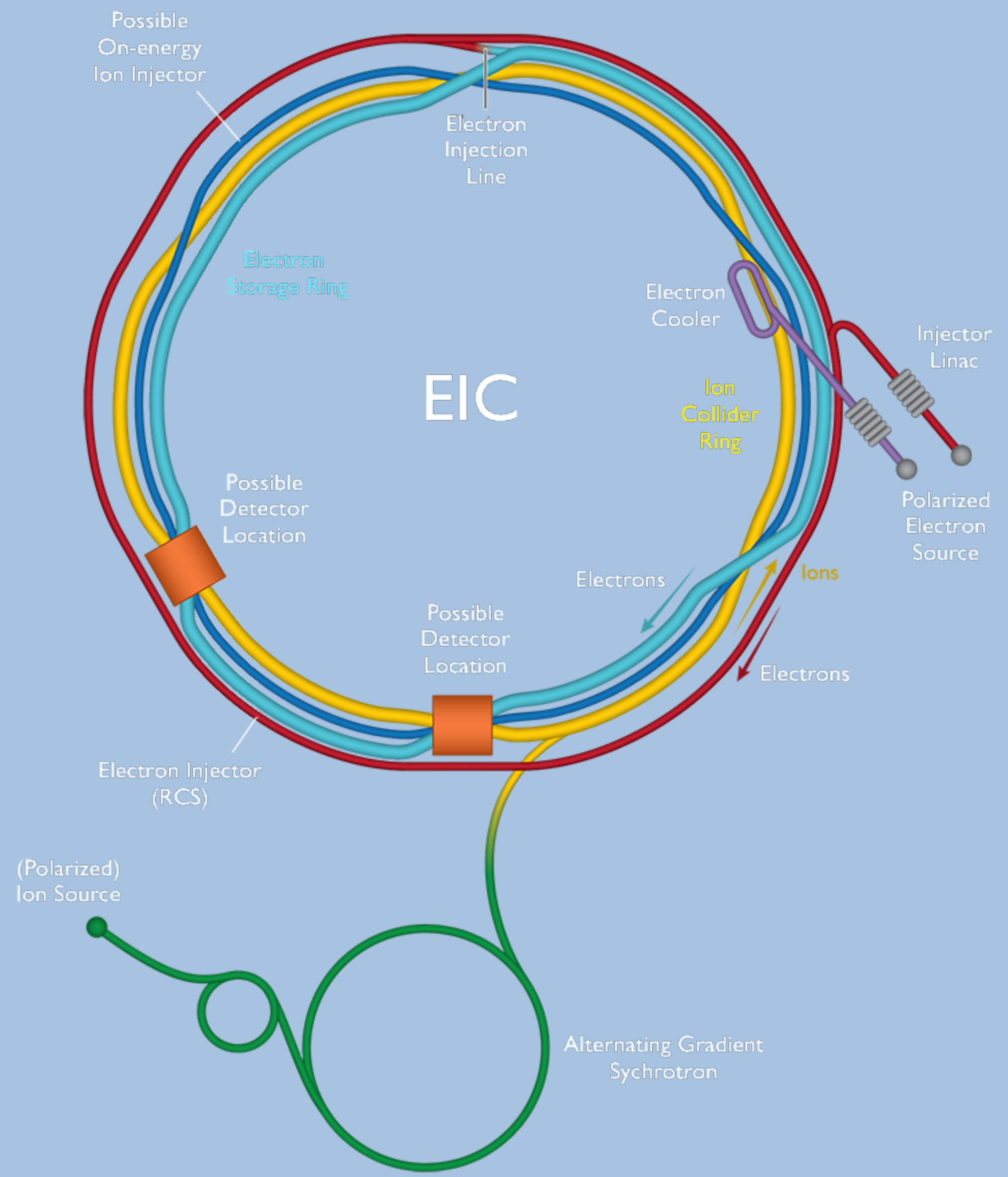

8th Low Emittance Rings Workshop, Frascati, INFN-LNF

Fast dynamic aperture optimization with reversal integration

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Acknowledgements

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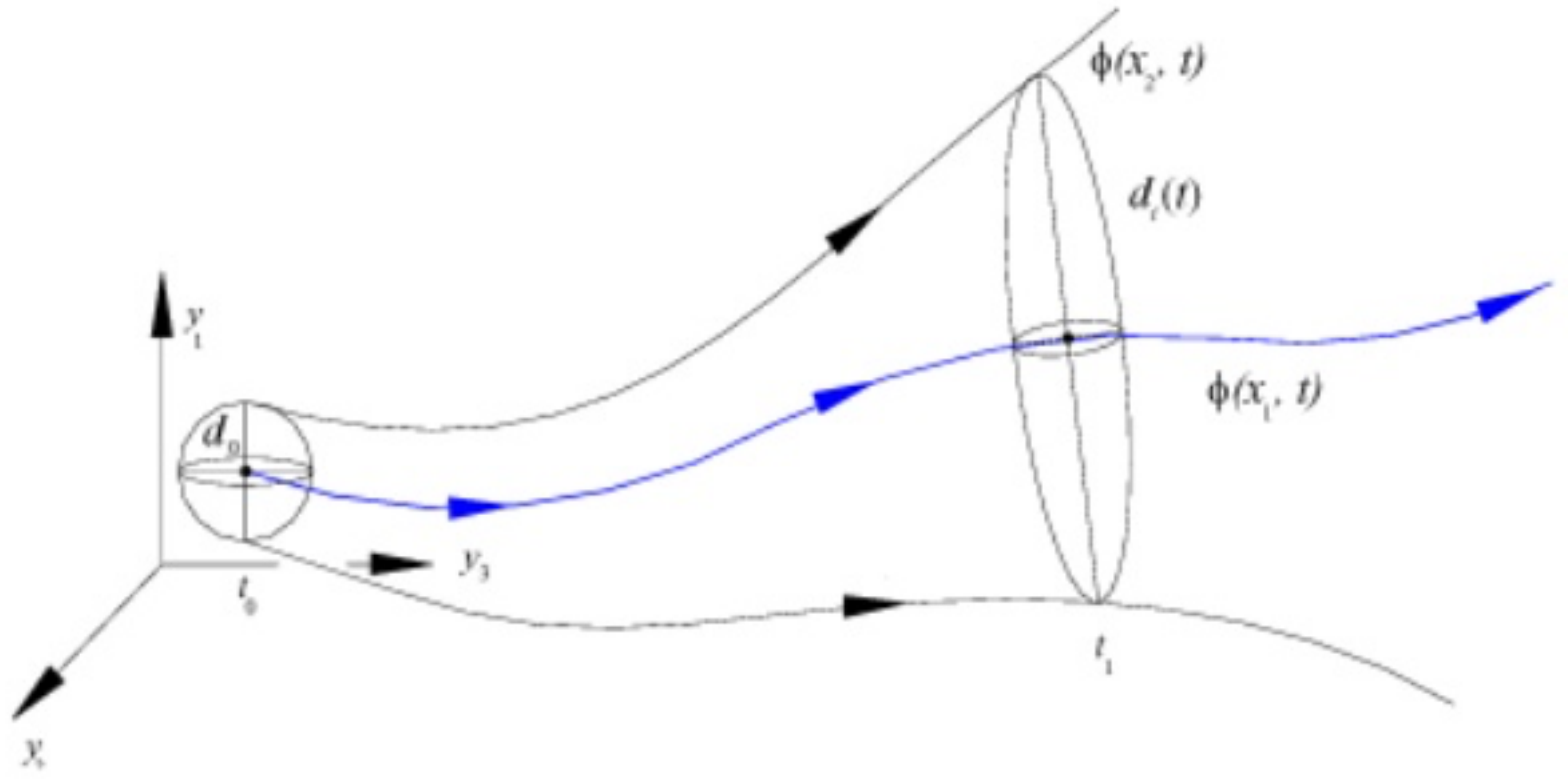
Outline

- Principle
- Proof-of-principle with Hènon map
- Applications of dynamic aperture optimization
  - Dedicated light source rings (NSLS-II, or MBA)
  - Collider ring (BNL-EIC e-ring)
- Summary

Principle

- Old idea: no new physics, empirical method, but effective
- Sensitivity to initial condition: nonlinearity, deterministic, but unpredictable
- Lyapunov exponent: separation growth rate describes the chaos

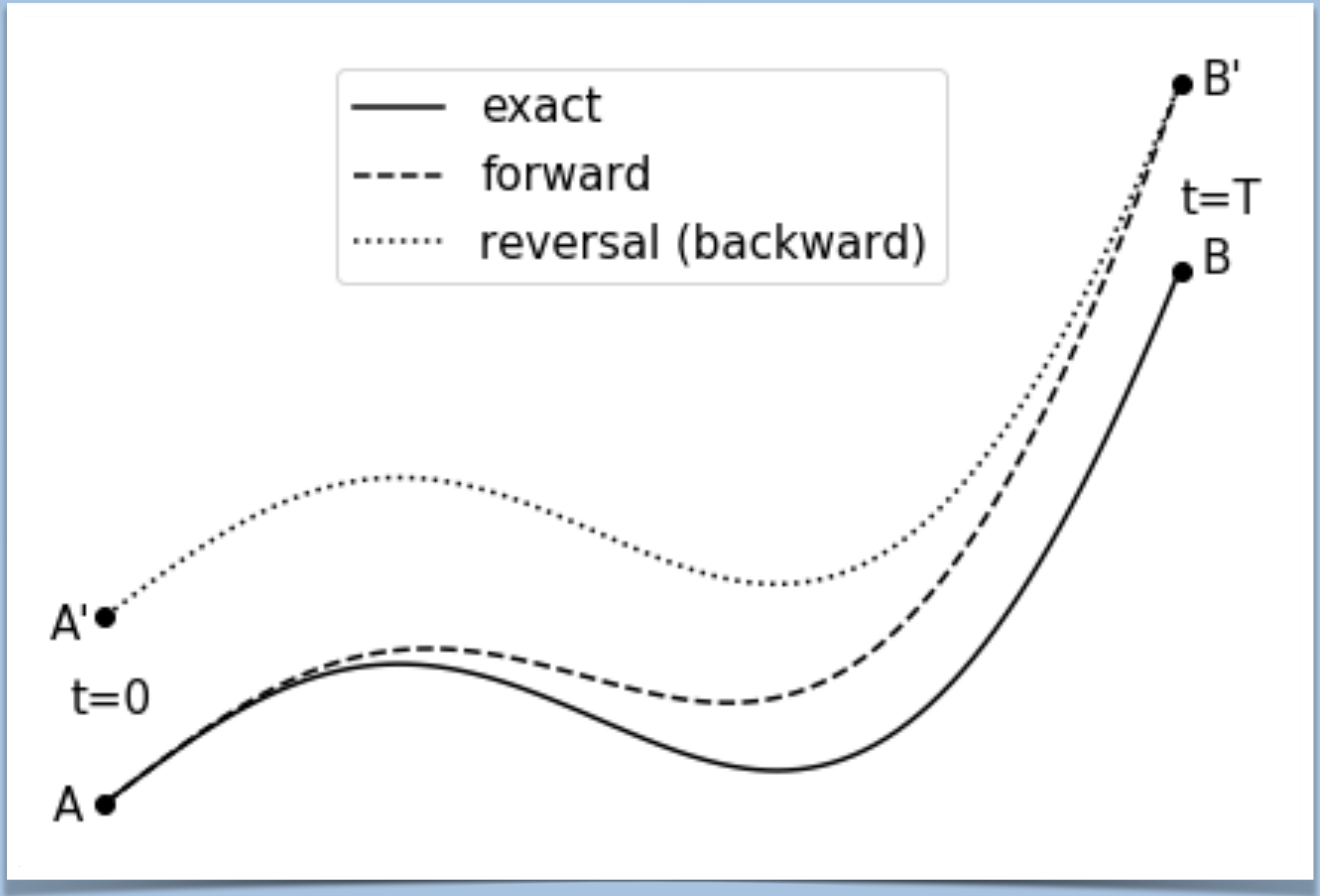
Lyapunov Exponent



- $|\delta \mathbf{Z}(t)| = e^{\lambda t} |\delta \mathbf{Z}(0)|$

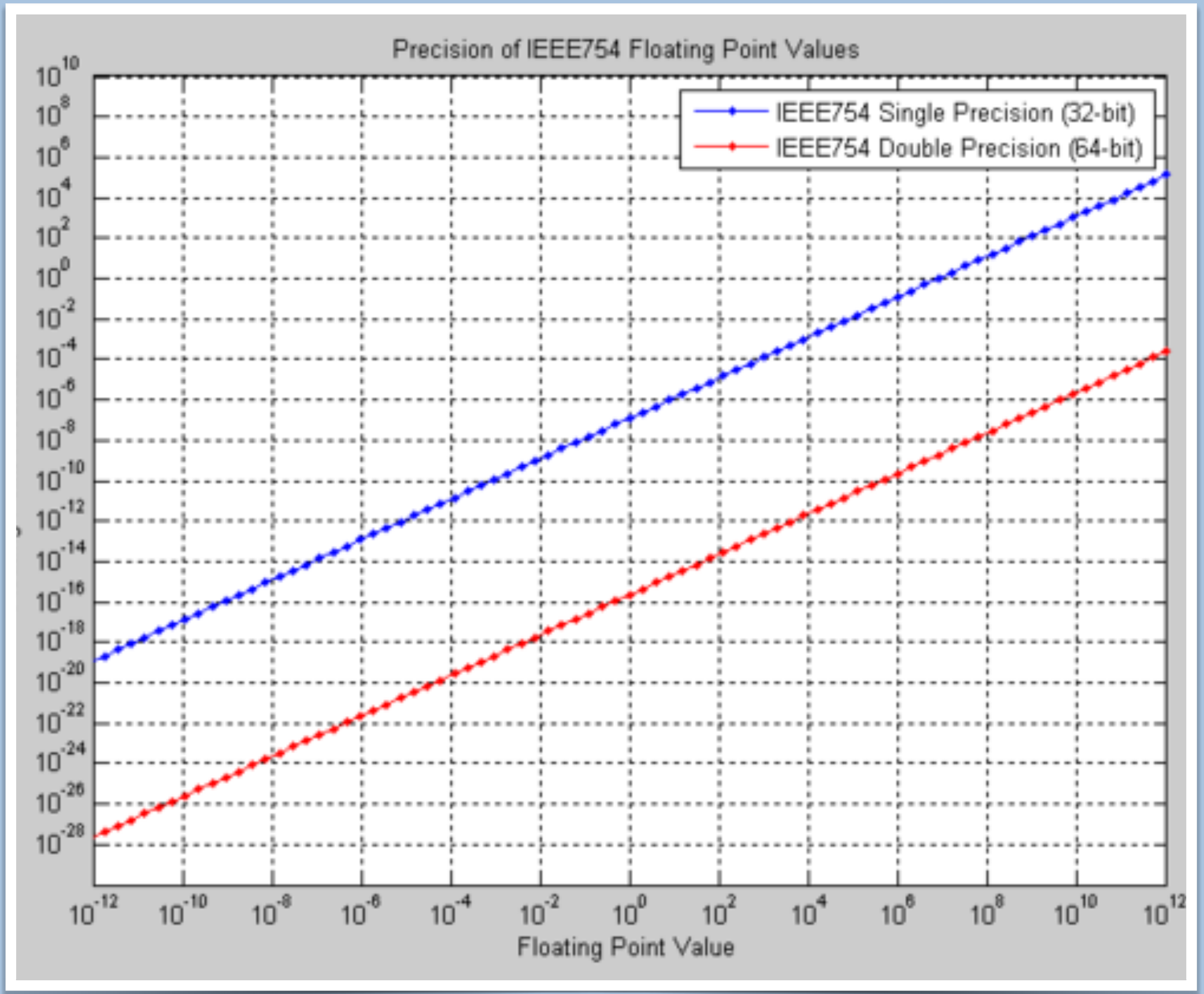
From LE to irreversibility

- Launch two particles, which have a **"small"** difference in their initial conditions
- Two trajectories will separate from each other due to (1) initial difference (2) round-off error (3) chaos
- How to specify a **"small"** value in computer? (see next slide)
- Alternative: Forward-Reversal Integration (FRI)



Round-off errors in computers

- Round-off error of floating point value as defined by IEEE-754 standards depend on:
  - number of bit (computer)
  - absolute value (number)
  - round-off method, ie. floor, ceiling and nearest (software)
- Manually specifying a “small” value at the machine precision is doable, however, complicated



From round-off to irreversibility

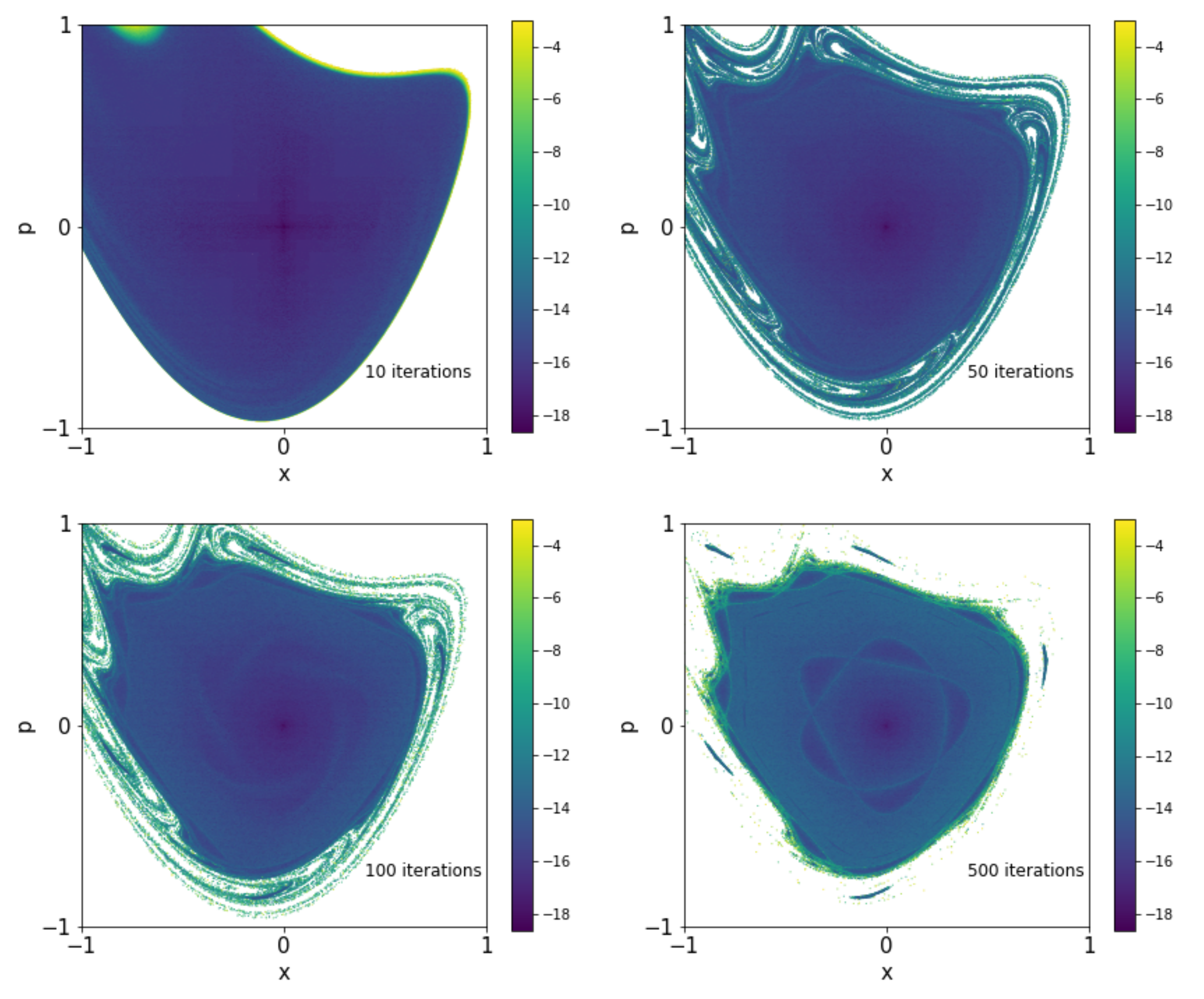
- Laslett (1957): a linear map (no chaos) is numerically irreversible due to round-off error, a "consistent error which grows in direct proportion to the number of iterations executed" by computer
- For a chaotic trajectory, the difference grows exponentially (LE) by scaling the accumulated round-off errors
- Motivation: could we optimize a nonlinear lattice by control the growth rate of irreversibility? (ie. sensitivity to initial condition)

Proof-of-principle with Hènon map

$$\begin{pmatrix} x \\ p \end{pmatrix}_n = \begin{pmatrix} \cos\mu & \sin\mu \\ -\sin\mu & \cos\mu \end{pmatrix} \begin{pmatrix} x \\ p - x^2 \end{pmatrix}_{n-1}$$

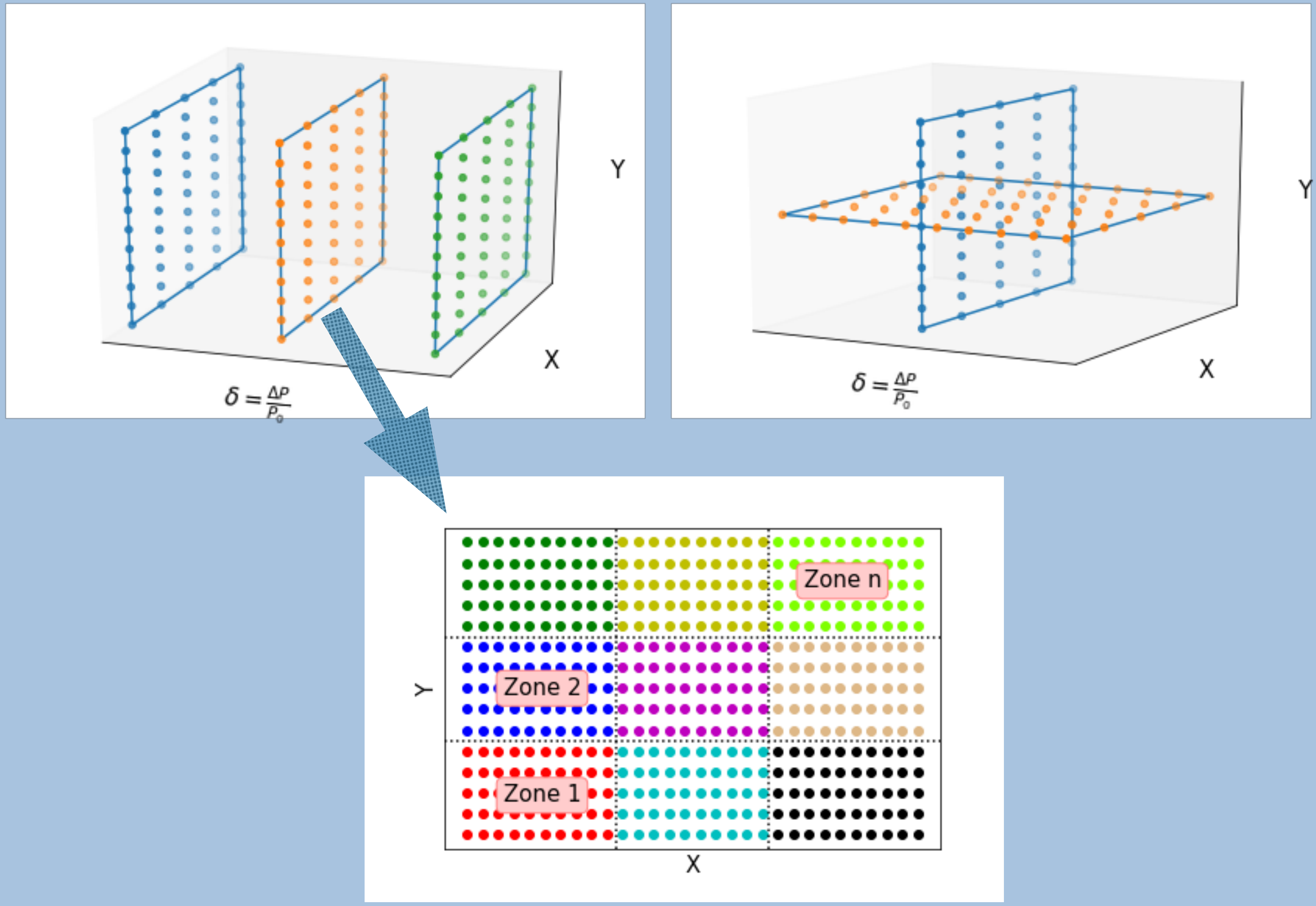
- Is chaos visible with F-R integration?
- How much iterations (turns) is needed to observe chaos?
- Color-bar $\Delta = \log_{10} |z_0 - z_0'|$.

(courtesy Y. Hao)



Applications to DA optimization at NSLS-II

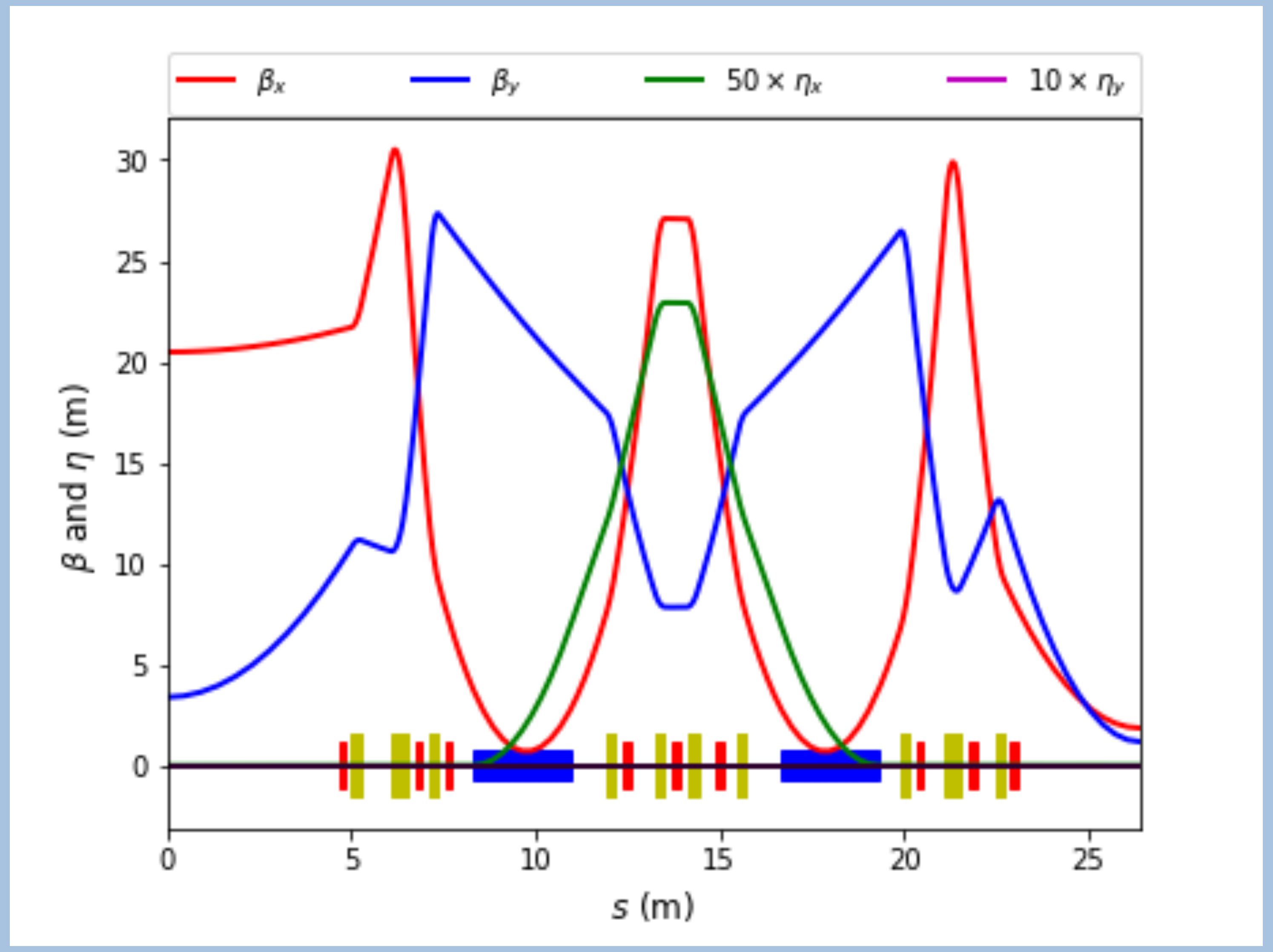
- Method: by tuning nonlinear knobs (sextupole, ~~octuple~~) to reduce the chaos within the desired DA
- Implementation: **slicing** and **dividing** the volume of DA
- Using multi-objective optimizer



"Early" chaos indictor: "fast"

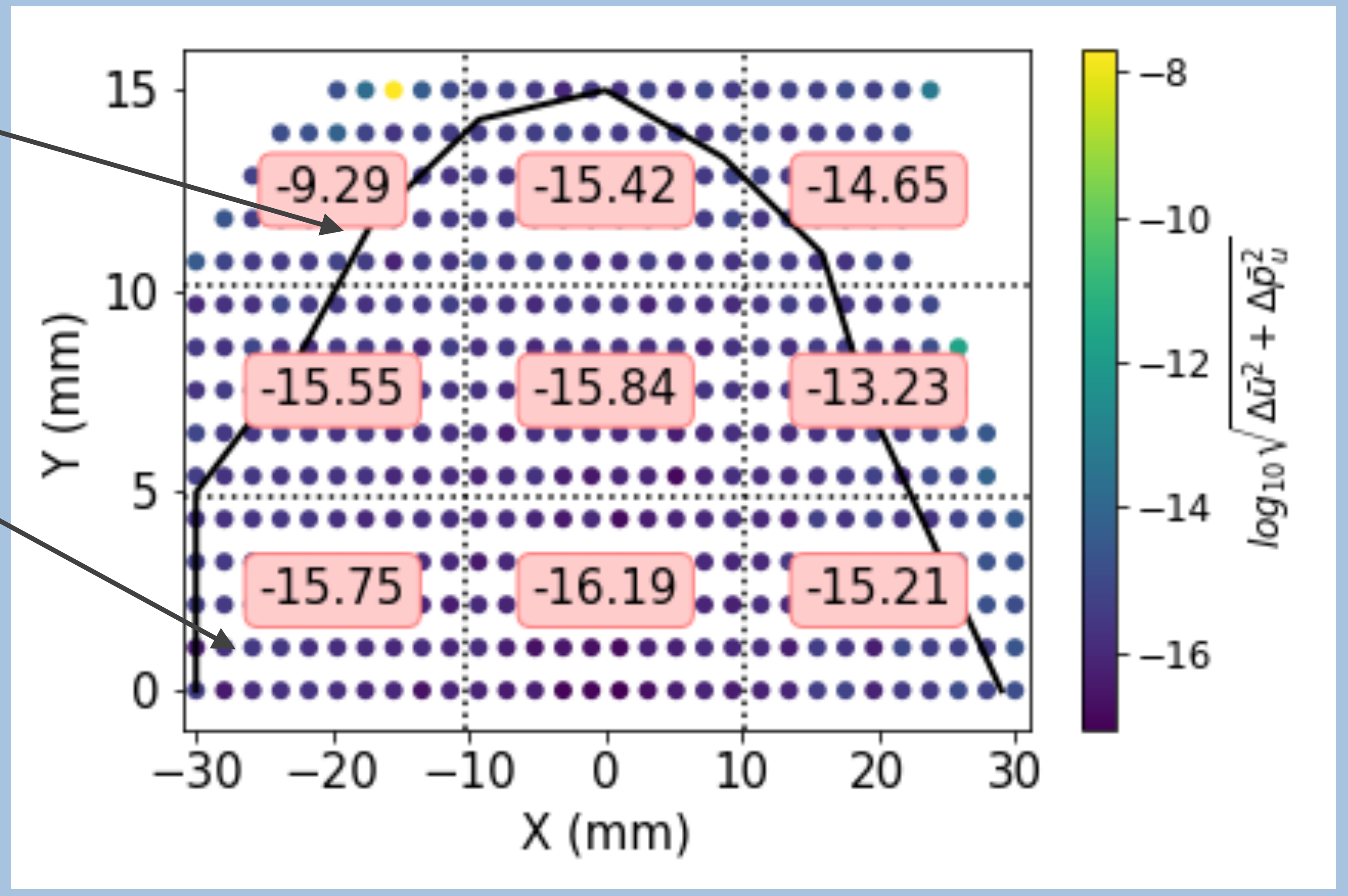
- NSLS-II lattice: **1 turn (15 supercells)** can provide visible difference
- We don't attempt to extract the profile of DA with 1-turn F-R integration.
- Rule out the uncompetitive candidates (smaller DA⟺larger chaos) during optimization
- Color-bar $\Delta = \log_{10}\sqrt{\Delta\bar{x}^2 + \Delta\bar{p}_x^2 + \Delta\bar{y}^2 + \Delta\bar{p}_y^2}$

Linear action: J



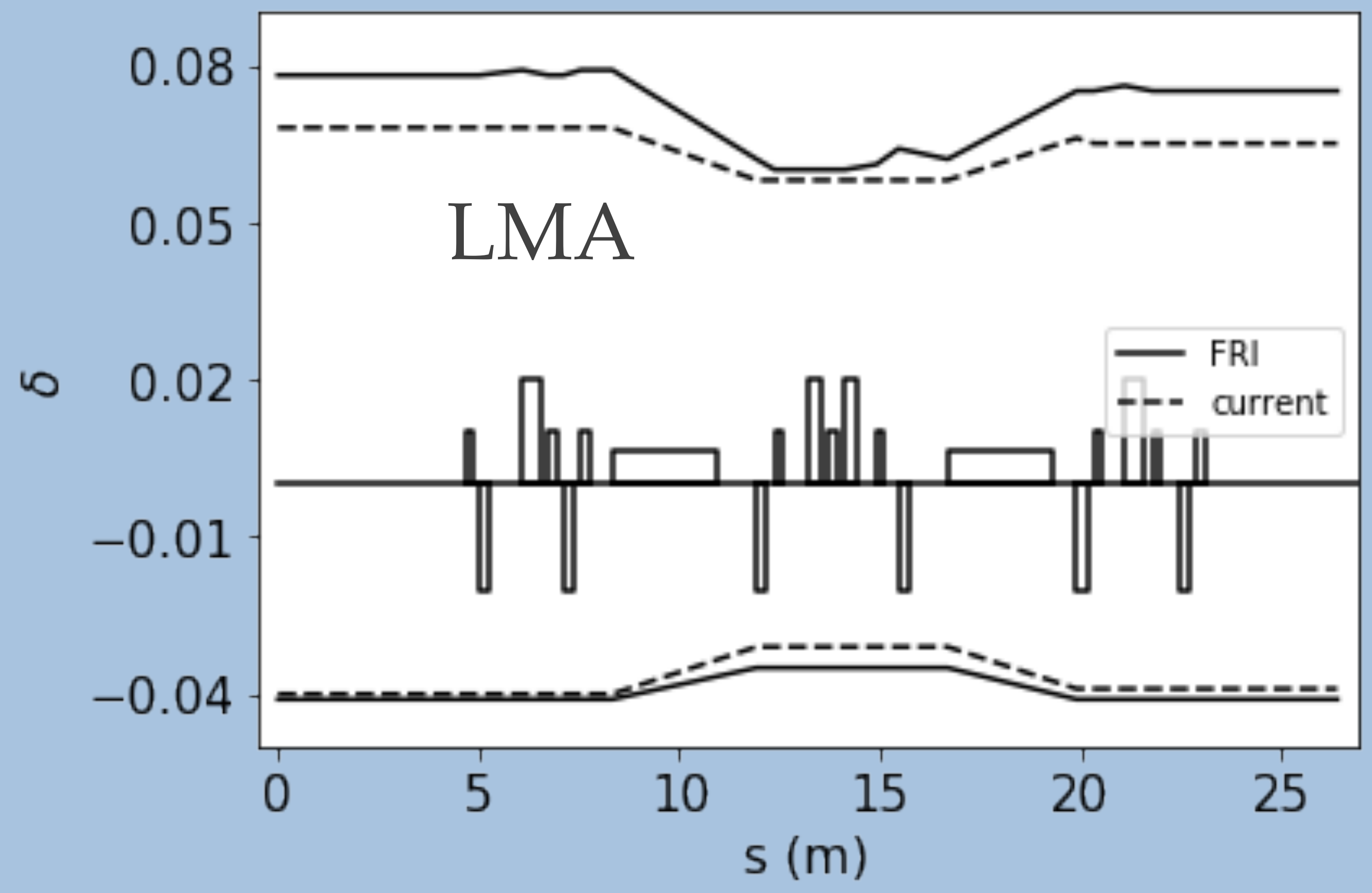
Line: 1,024 turns DA

Dots: 1 turn FRI

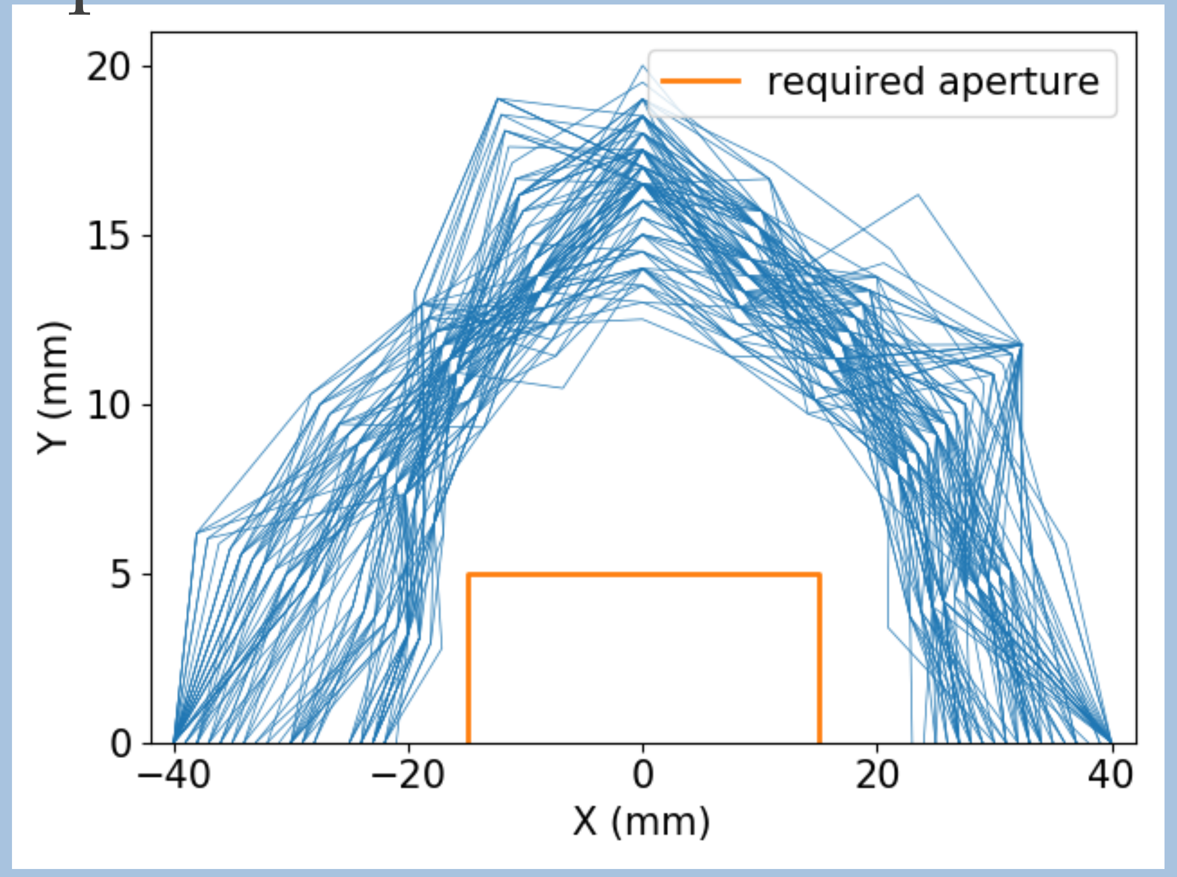


MOGA optimization results

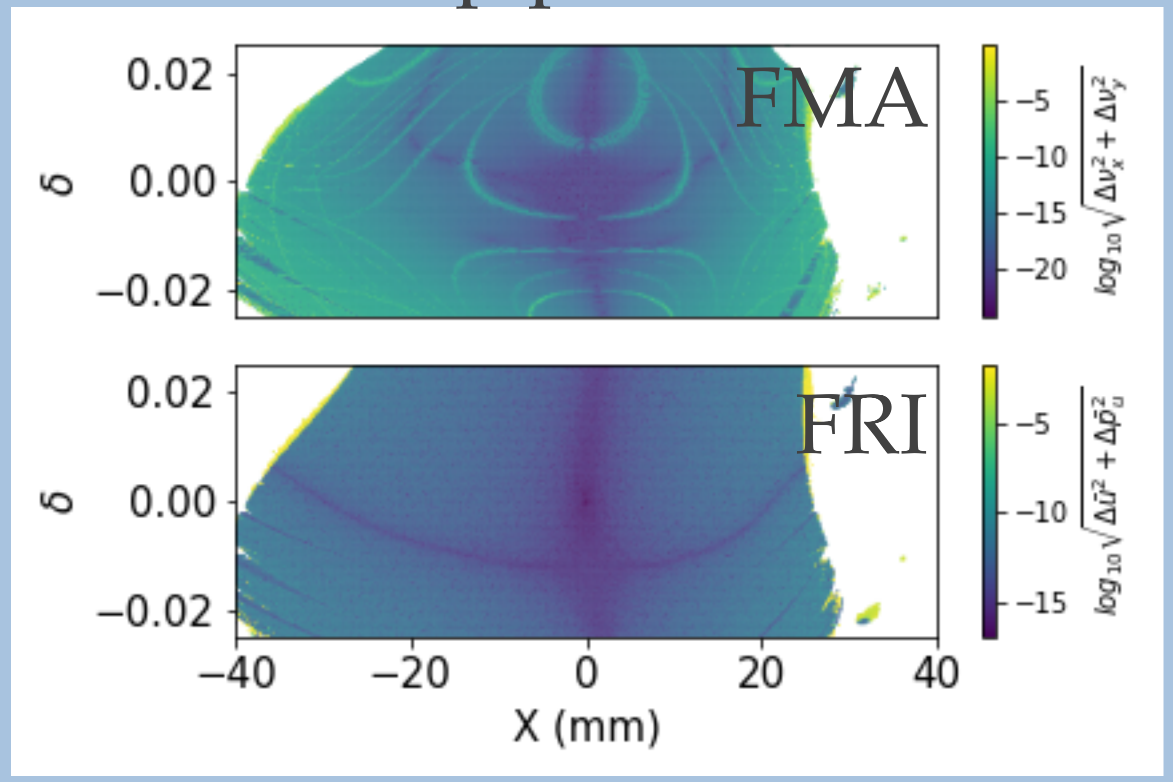
- Many solutions can meet the requirement
- An elite solution has been confirmed by a multi-turn tracking, frequency map analysis (FMA), and a beam test at the NSLS-II ring
- A multi-bend achromat lattice for 4th-gen. LS has been tested



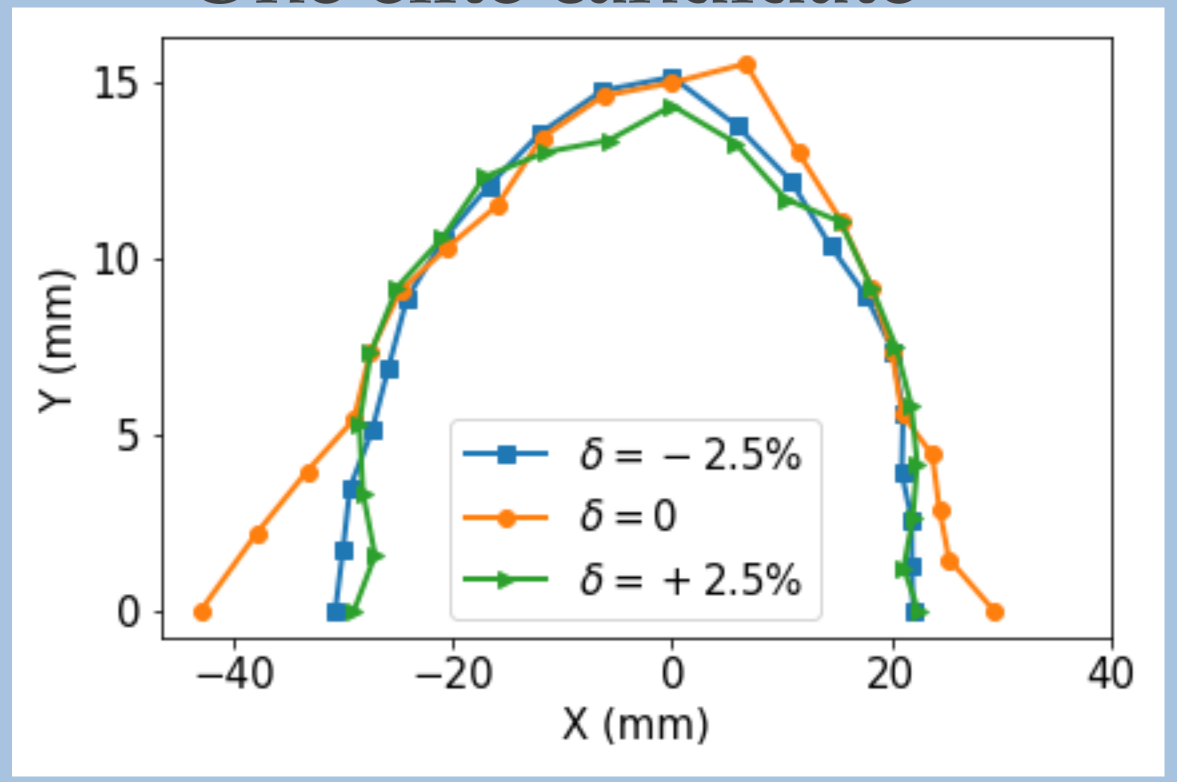
Top 100 candidates from MOGA



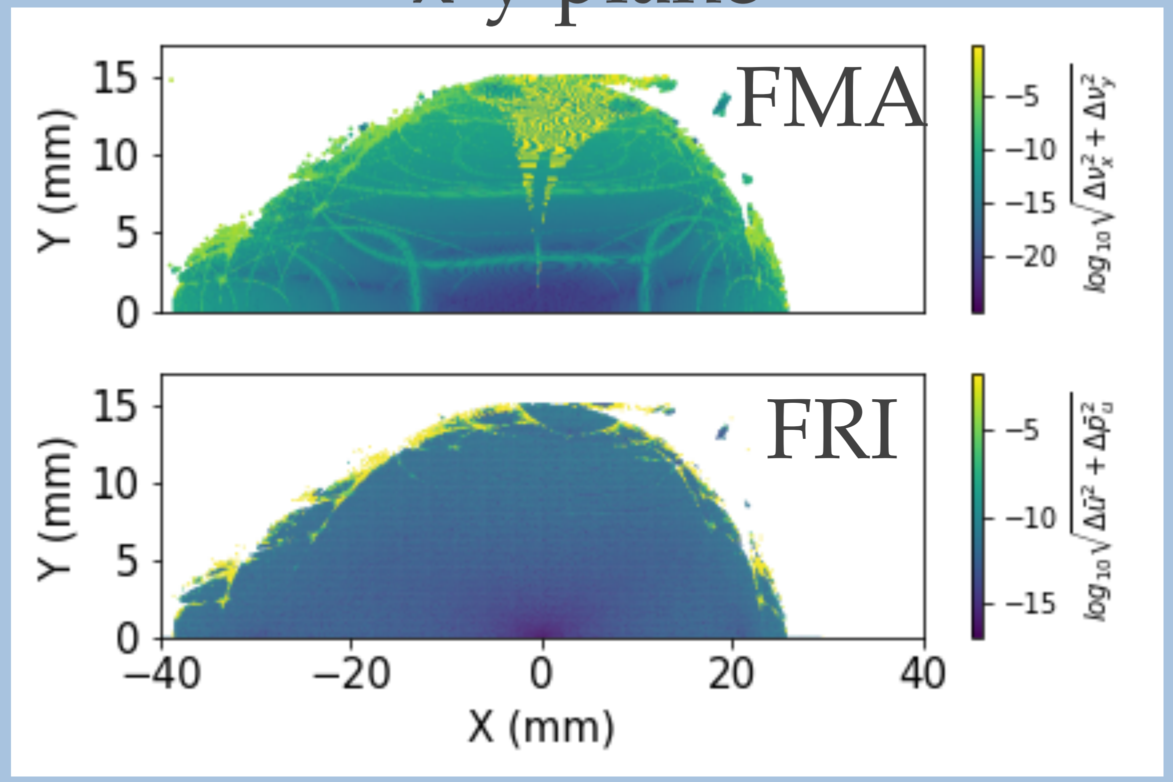
x-dp plane



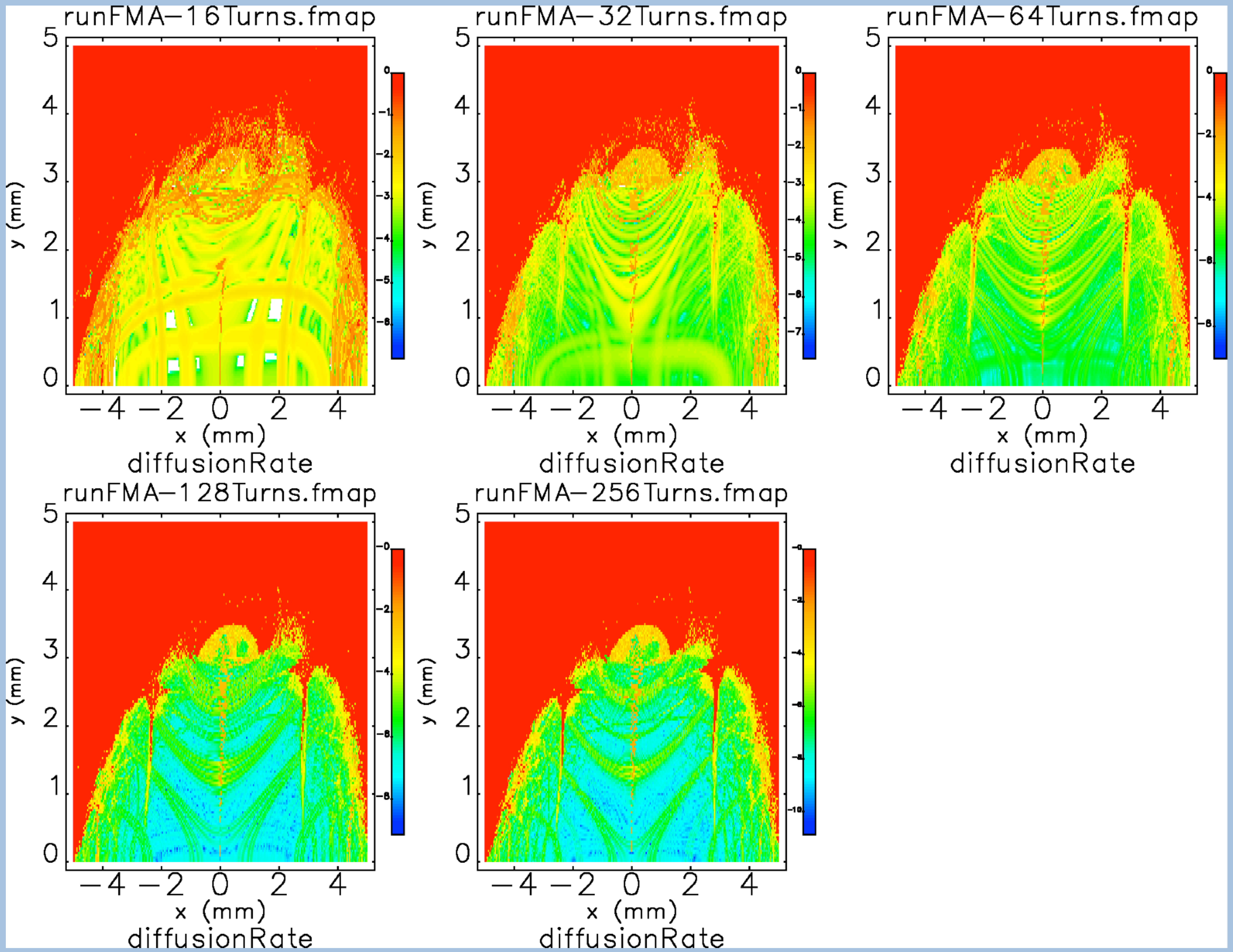
One elite candidate

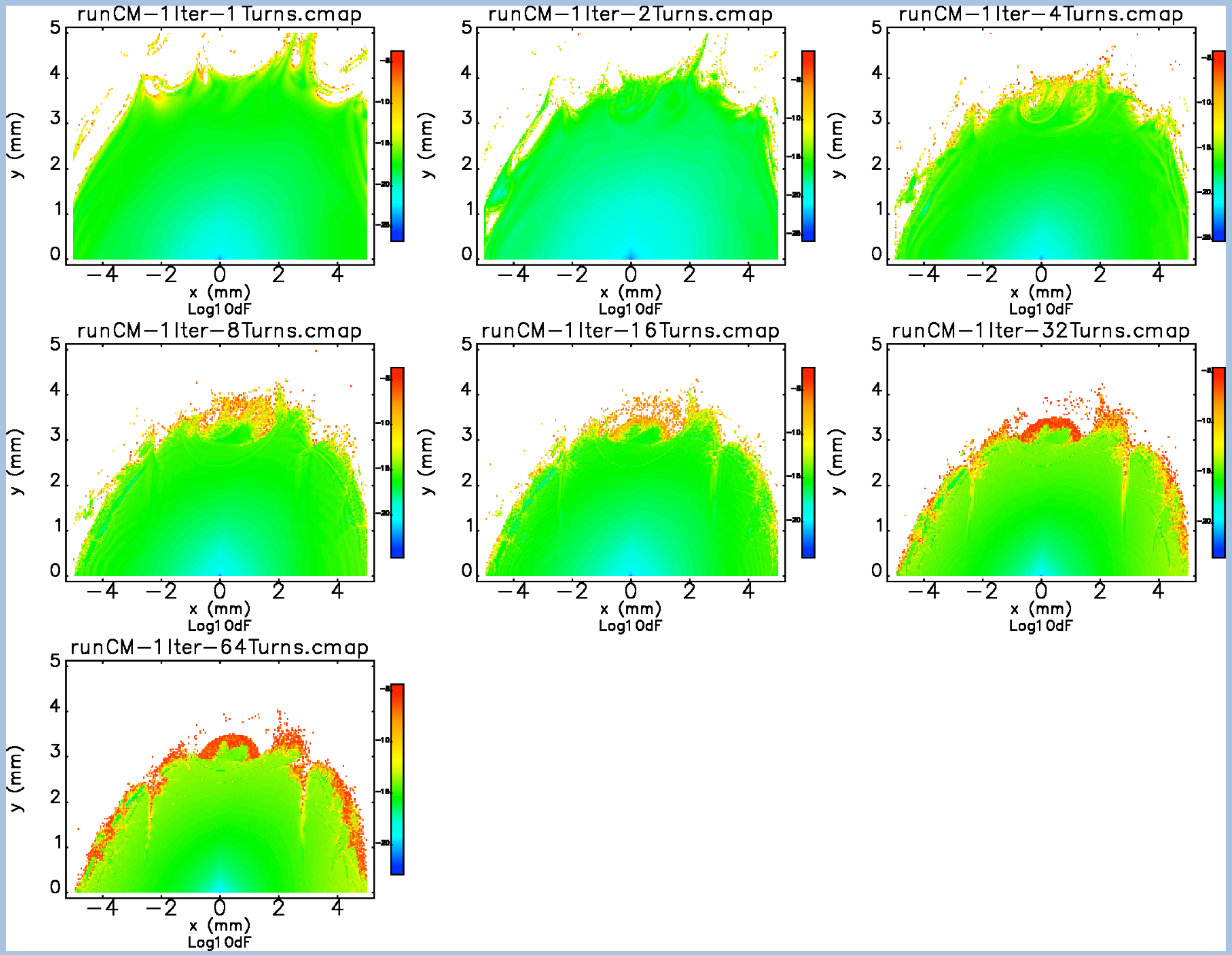


x-y plane



FMA vs FRI for APS-U 7BA lattice

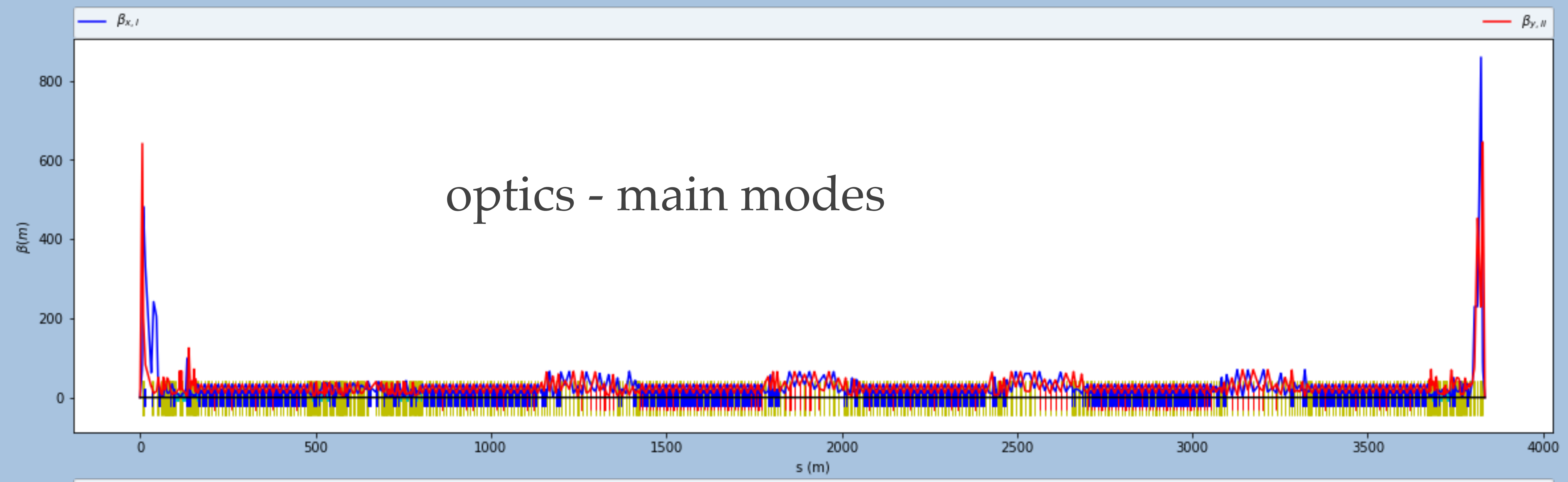


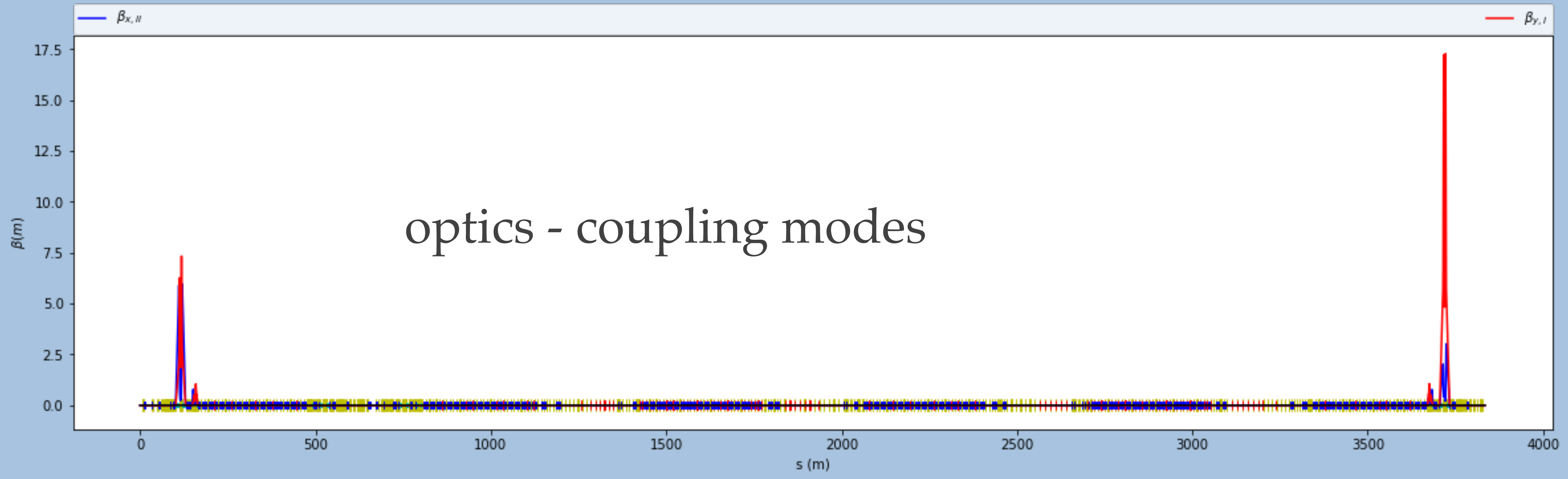


(courtesy M. Borland)

BNL-EIC e-ring (10GeV 60Deg)

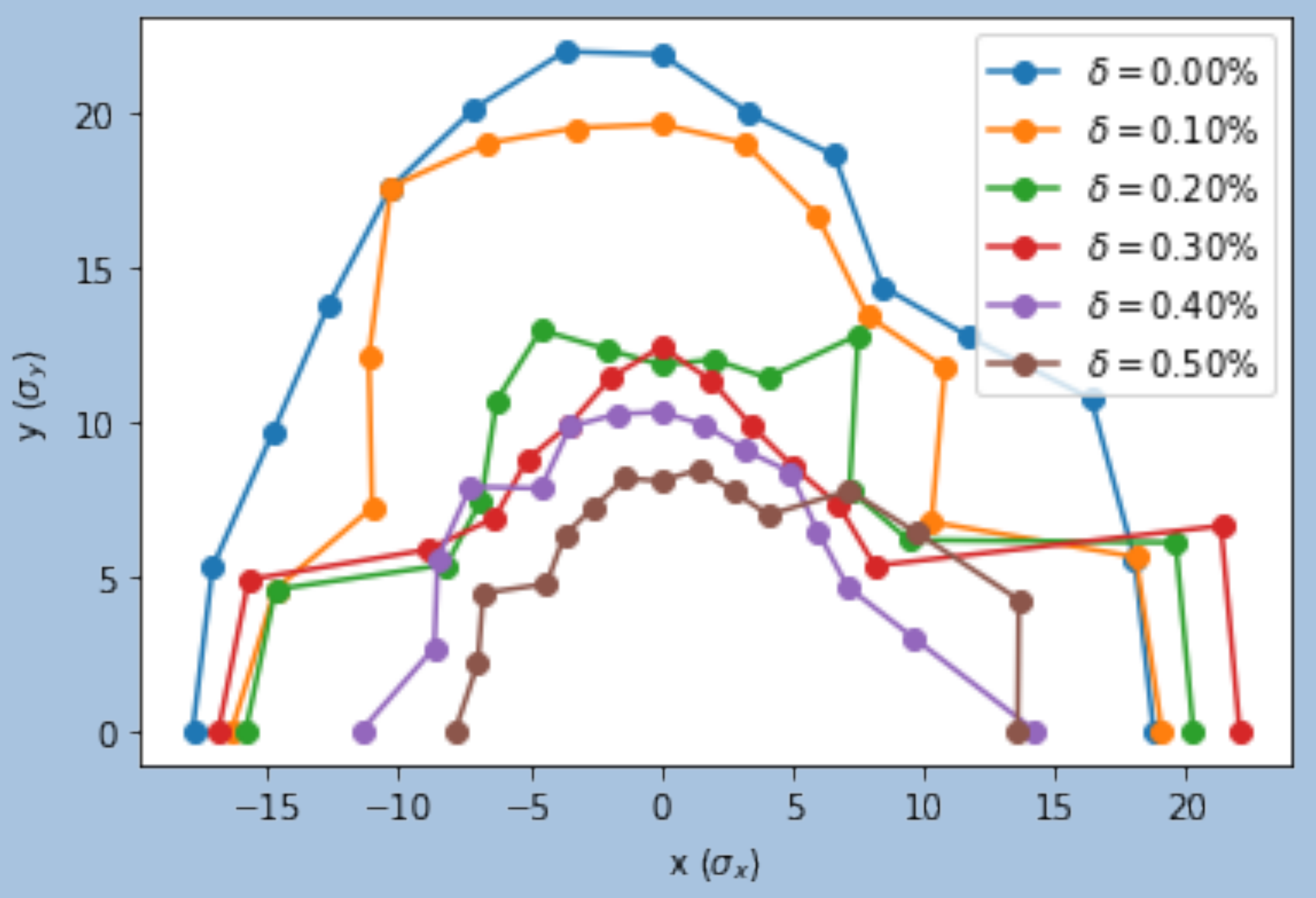
- Challenges:
  - Large ring: impractical if the optimizer driven by multi-turns (N>100?) tracking
  - High dimension parameter space: 36 families sextupoles
  - low periodicity: 1



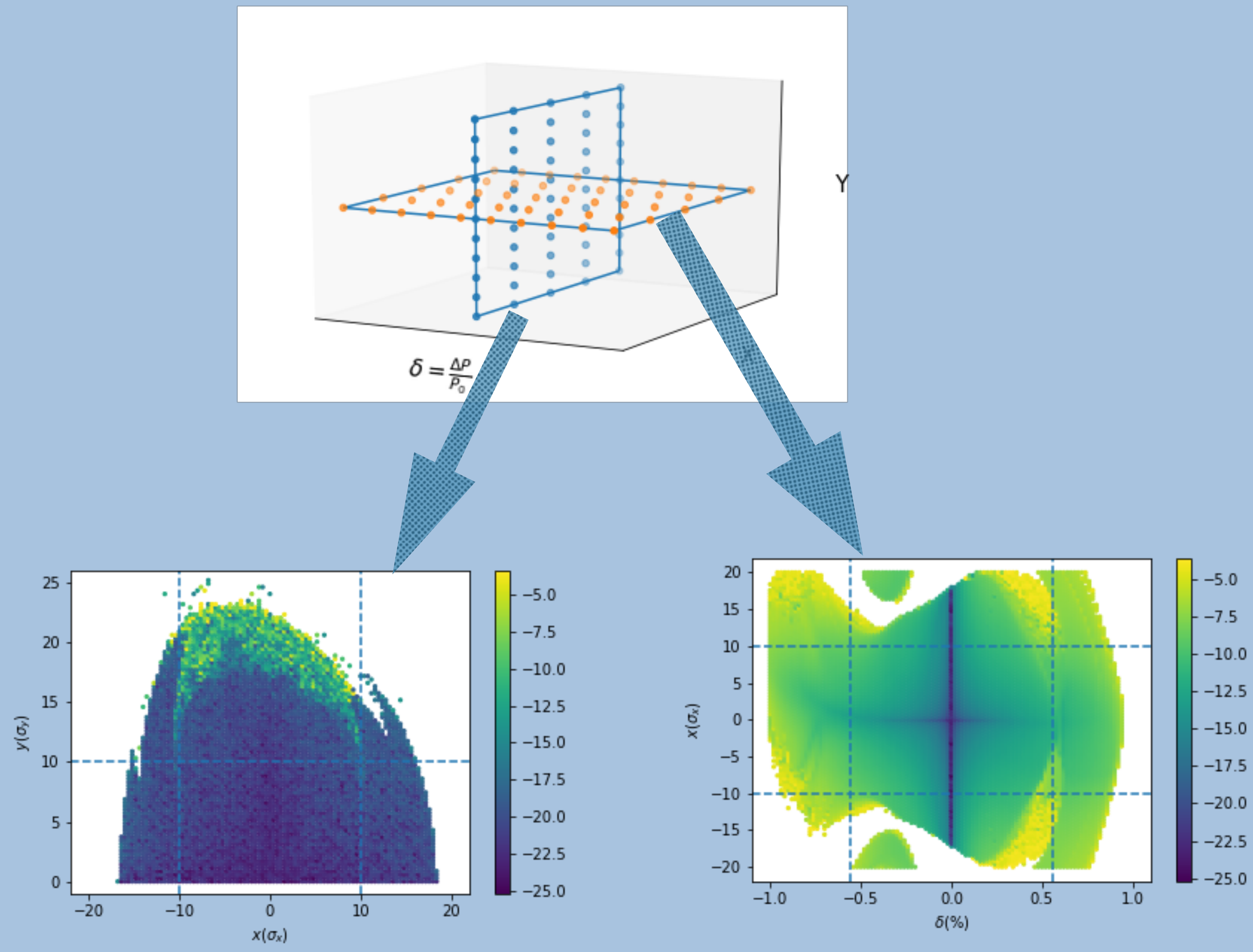


Different slicing method

- A large momentum acceptance is needed, which is difficult for FODO lattice
- 16 turns 5D tracking is used for optimization
- 1024 turns 6D tracking is used to confirm the DA at with synchro-betatron coupling



6D tracking



5D tracking

Implementation at ELEGANT

Thanks to M. Borland
Since Version 2019.4

7.11 chaos_map

- type: major action command.
- function: compute chaos map from tracking. Note that the number of turns tracked is set by the `run_control` command.
- can use parallel resources (`Pelegant`)
- Command syntax, including use of equations and subcommands, is discussed in 7.2.
- NB: this feature is new in 2019.4 and somewhat experimental. Please report problems on the forum.

```
&chaos_map
    STRING output = NULL;
    double xmin =  -0.1;
    double xmax =  0.1;
    double ymin =  1e-6;
    double ymax =  0.1;
    double delta_min = 0;
    double delta_max = 0;
    long nx = 20;
    long ny = 21;
    long ndelta = 1;
    long forward_backward = 0;
    double change_x = 1e-6;
    double change_y = 1e-6;
    long verbosity = 1;
&end
```

← Mode switch between the forward-only and F-R integrations (for `forward_backward`)

← Manually input a "**small**" difference. Not recommended) (for `change_x`, `change_y`)

Summary

- A early chaos indicator using forward reversal integration has been demonstrated to optimize DA
- It is fast because only a very few turns is needed to rule out uncompetitive candidates
- It has been integrated into the ELEGANT code (different magnets, fringe field, undulator - kickmap, RFC, etc. have been implemented)