Isospin breaking in tau input for $(g-2)_\mu$ from Lattice QCD

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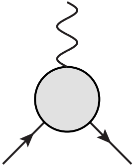
Anomalous magnetic moment



scattering of particle mass m off external photon (μ, q)

$$-ie\left[\gamma_\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q^\nu}{2m}F_2(q^2)\right], g = 2(F_1(0) + F_2(0))$$

$$F_1(0) = 1 \to F_2(0) = a = (g-2)/2$$

A rich history

electron a_e measured in experiment [Kusch, Foley '48]

confirms radiative corrections [Schwinger '48] $\to$ success of QFT

muon a_μ measured in experiement [Columbia exp. '59]

"muon is heavy electron" $\to$ families of leptons

Back to the future

new physics contribution to a: $(a - a^{\mathrm{SM}}) \propto m^2/\Lambda^2_{\mathrm{NP}}$

a_τ experimentally inaccessible, a_μ most promising

CERN

$(g-2)_\mu$ RECAP - II

$(g-2)_\mu$: discrepancy between exp vs theory ($\gtrsim 3\sigma$)
theory error dominated by hadronic physics

5-loop QED	11 658 471.90(0.01)	[Aoyama et al. 2012]
2-loop EW	15.36(0.10)	[Gnendiger et al. 2013]
HVP LO	692.78(2.42)	[KNT19]
HVP NLO	-9.83(0.04)	[KNT19]
HVP NNLO	1.24(0.01)	[KNT19]
HLbL	9.34(2.92)	[Colangelo et al. '17, '18, '19]
BNL E821	11 659 208.9(6.3)	[The g-2 Collab. '06]

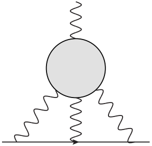
table shows $a_\mu \times 10^{10}$
HLbL = Hadr Ligh-by-Light
HVP = Hadr Vac Pol

$$a_\mu^{\rm exp} - a_\mu^{\rm SM} = 28.0 \overbrace{(2.9)}^{\rm HLbL} \overbrace{(2.4)}^{\rm HVP} \overbrace{(6.3)}^{\rm exp}$$

new exp. Fermilab, J-PARC (improve x4)

$(g-2)_\mu$ RECAP - III

$(g-2)_\mu$: discrepancy between exp vs theory ($\gtrsim 3\sigma$)



can HLbL explain discrepancy?

10.5(2.6) estimated from models [Glasgow consensus]

new exciting results from Lattice [Mainz '18, RBC/UKQCD '18]

new exciting results from dispersive methods
9.34(2.92) [Colangelo et al. '17,'18]

$\to$ preliminary indications HLbL not responsible for discrepancy

$$a_\mu^{\mathrm{HLbL}} = (11.9 \pm 5.3) \times 10^{-10}$$ [RBC/UKQCD Latt18]

$\to$ both methods solid and improvable error estimates
stay tuned for White paper "g-2 theory initiative"

$(g-2)_\mu$ RECAP - IV

$(g-2)_\mu$: discrepancy between exp vs theory ($\gtrsim 3\sigma$)

hadronic contributions dominate the error

HLbL: models, lattice QCD, dispersive method

10% accuracy enough, on a good path

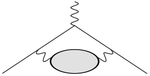
HVP LO: dispersive approach vs lattice QCD

per-mille accuracy required!

Let's focus on Hadronic Vacuum Polarization

1. dispersive approach more precise than lattice
2. alternative data set for dispersive approach: τ
3. isospin-breaking corrections: unde venis?
4. isospin-breaking corrections: quo vadis?

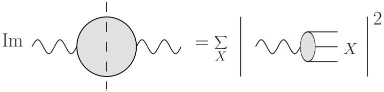
Dispersive integral



$$a_\mu = \frac{\alpha}{\pi} \int \frac{ds}{s} K(s, m_\mu) \frac{\text{Im}\Pi(s)}{\pi}$$ [Brodsky, de Rafael '68]

analyticity $\hat{\Pi}(s) = \Pi(s) - \Pi(0) = \frac{s}{\pi} \int_{4m_\pi^2}^{\infty} dx \frac{\text{Im}\Pi(x)}{x(x - s - i\varepsilon)}$

unitarity



$$\frac{4\pi^2\alpha}{s} \frac{\text{Im}\Pi(s)}{\pi} = \sigma_{e^+e^- \to \gamma^\star \to \text{had}}$$

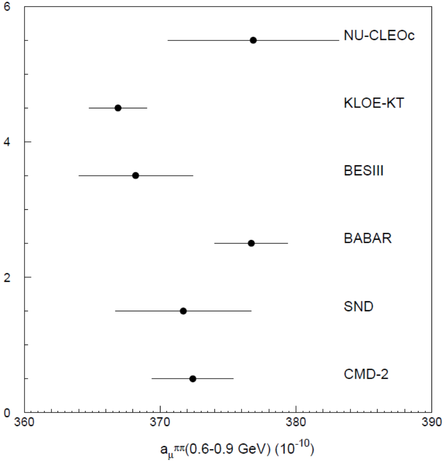
At present $O(30)$ channels: $\pi^0\gamma, \pi^+\pi^-, 3\pi, 4\pi, K^+K^-, \cdots$

$K(s, m_\mu) \to$ $\pi^+\pi^-$ dominates due to ρ resonance

$\pi\pi$ channel is $\sim 70\%$ of signal and $\sim 70\%$ of error

Some problems

[Davier '18]



KLOE vs Babar

most precise exp. disagree on cross-sections in $\pi\pi$ channel

averaging of cross-sections before dispersive integral $\rightarrow$ error of

$$3 \times 10^{-10}$$

difference of a_μ after dispersive integral as systematic error $\rightarrow$

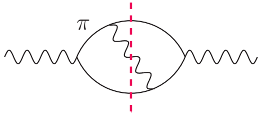
$$10 \times 10^{-10}$$

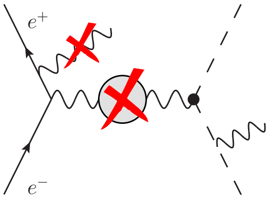
Seattle '19: agree to disagree, new dispersive error 5×10^{-10}

Radiative events

per-mille accuracy goal:

$\sigma_{\pi^+\pi^-(\gamma)}$: contains $\pi\pi$ and $\pi\pi\gamma$ $\leftarrow$





$-$ remove Initial state radiation (ISR)
$-$ undress photon (remove VP)
$+$ leave final photon (FSR)
$=$ $\sigma^{\text{bare}}_{\pi^+\pi^-(\gamma)}$

(C invariance, ISR FSR factorize)

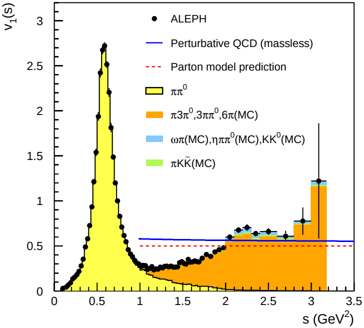
experiments do (most of) it for us

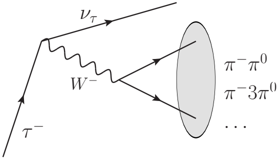
We introduce spectral function $v_0(s) = \dfrac{s}{4\pi\alpha^2}\sigma^{\text{bare}}_{\pi^+\pi^-(\gamma)}(s)$

$v_0(s)$ used in dispersive integral for a_μ

define pion form factor $v_0 = c\text{FSR}\beta_0^3|F_\pi^0|^2$

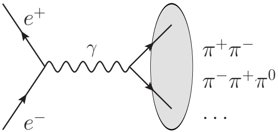
Motivations for τ





$V - A$ current

Final states $I = 1$ charged



EM current

Final states $I = 0, 1$ neutral

τ data can improve $a_\mu[\pi\pi]$
$\rightarrow 72\%$ of total Hadronic LO

or $a_\mu^{ee} \neq a^\tau \rightarrow$ NP [Cirigliano et al '18]

Isospin Corrections

Restriction to $e^+e^- \to \pi^+\pi^-$ and $\tau^- \to \pi^-\pi^0\,\nu_\tau$

$$v_0(s) = \frac{s}{4\pi\alpha^2}\sigma_{\pi^+\pi^-(\gamma)}(s)$$

$$v_-(s) = \frac{m_\tau^2}{6|V_{ud}|^2}\frac{\mathcal{B}_{\pi\pi^0}}{\mathcal{B}_e}\frac{1}{N_{\pi\pi^0}}\frac{dN_{\pi\pi^0}}{ds}\left(1-\frac{s}{m_\tau^2}\right)^{-1}\left(1+\frac{2s}{m_\tau^2}\right)^{-1}\frac{1}{S_{\mathrm{EW}}}$$

Isospin correction $v_0 = R_{\mathrm{IB}} v_-$ $\quad R_{\mathrm{IB}} = \frac{\mathrm{FSR}}{G_{\mathrm{EM}}}\frac{\beta_0^3|F_\pi^0|^2}{\beta_-^3|F_\pi^-|^2}$ [Alemani et al. '98]

0. S_{EW} electro-weak radiative correct. [Marciano, Sirlin '88][Braaten, Li '90]

1. Final State Radiation of $\pi^+\pi^-$ system [Schwinger '89][Drees, Hikasa '90]

2. G_{EM} (long distance) radiative corrections in τ decays
Chiral Resonance Theory [Cirigliano et al. '01, '02]
Meson Dominance [Flores-Talpa et al. '06, '07]

3. Phase Space ($\beta_{0,-}$) due to $(m_{\pi^\pm} - m_{\pi^0})$

Long distance QED - I

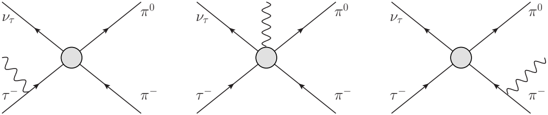
At low energies relevant degrees of freedom are mesons

Chiral Perturbation Theory [Cirigliano et al. ’01, ’02]

Meson dominance model [Flores-Talpa et al. ’06, ’07]

Corrections casted in one function $v_-(s) \to v_-(s) G_{\mathrm{EM}}(s)$

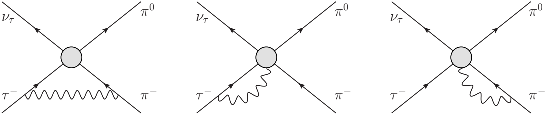
Real photon corrections



Real + virtual

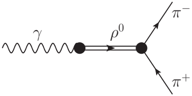
$\to$ IR divergences cancel

Virtual photon corrections



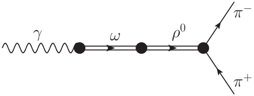
Pion form factors

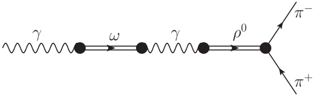
$$F_\pi^0(s) \propto \frac{m_\rho^2}{D_\rho(s)}$$



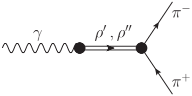
[Gounaris, Sakurai '68]
[Kühn, Santamaria '90]

$$\times \left[1 + \delta_{\rho\omega} \frac{s}{D_\omega(s)}\right]$$

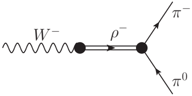




$$+ \frac{m_X^2}{D_X(s)} \qquad X = \rho' \, , \rho''$$



$$F_\pi^-(s) \propto \frac{m_{\rho^-}^2}{D_{\rho^-}(s)} + (\rho' \, , \rho'')$$



Sources of IB breaking in phenomenological models

$m_{\rho^0} \neq m_{\rho^\pm}$, $\Gamma_{\rho^0} \neq \Gamma_{\rho^\pm}$, $m_{\pi^0} \neq m_{\pi^\pm}$

$\rho - \omega$ mixing $\delta_{\rho\omega} \simeq O(m_\mathrm{u} - m_\mathrm{d}) + O(e^2)$

Status

$a_\mu^{\text{HVP,LO}}[\pi\pi, ee] = 503.51(3.5) \times 10^{-10}$ with $E \in [2m_\pi, 1.8 \text{ GeV}]$

$a_\mu^{\text{HVP,LO}}[\pi\pi, \tau] = 531.3(3.3) \times 10^{-10}$

$\Delta a_\mu[\pi\pi, \tau] = -12.0(2.6)$ [Cirigliano et al.]

$\Delta a_\mu[\pi\pi, \tau] = -16.1(1.8)$ [Davier et al. '09]

(≈ -10 due to S_{EW}, rest R_{IB})

$a_\mu[\tau]$: { model dependence; e^+e^- data more precise } = abandoned

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Additional $\rho\gamma$ mixing correction [Jegerlehner, Szafron '11]

partly accounted in $m_{\rho^0} - m_{\rho^-}$ in [Davier et al. '09]

$a_\mu[\pi\pi, ee] = 385.2(1.5)$ with $E \in [0.582 - 0.975]$ GeV

$a_\mu[\pi\pi, \tau] = 386.0(2.4)$ after R_{IB}

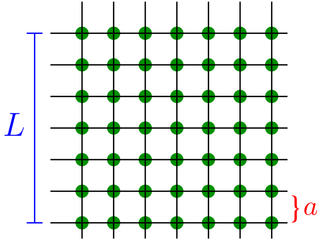
Lattice field theories

lattice spacing $a \to$ regulate UV divergences

finite size $L \to$ infrared regulator

Continuum theory $a \to 0$, $L \to \infty$

Euclidean metric $\to$ Boltzman interpretation of path integral



$$\langle O \rangle = \mathcal{Z}^{-1} \int [DU] e^{-S[U]} O(U) \approx \frac{1}{N} \sum_{i=1}^{N} O[U_i]$$

Very high dimensional integral $\to$ Monte-Carlo methods

Markov Chain of gauge field configs $U_0 \to U_1 \to \cdots \to U_N$

Details of calculation

Our calculation: Domain Wall Fermion ensemble $N_f = 2+1$

$a^{-1} \simeq 1.73$ GeV $\simeq 0.11$ fm, $L \approx 5.4$ fm

$a^{-1} \simeq 1.01$ GeV $\simeq 0.19$ fm, $L \approx 4.6, 6.1, 9.12$ fm

$a^{-1} \simeq 1.43$ GeV $\simeq 0.14$ fm, $L \approx 4.5$ fm

Diagrammatic expansion to $O(\alpha)$ and $O(m_u - m_d)$ [RM123]

e.g. $\langle O \rangle_{\mathrm{QCD+QED}} = \langle O_0 \rangle_{\mathrm{QCD}} + \alpha \langle O_1 \rangle_{\mathrm{QCD}} + O(\alpha^2)$

QED_L and QED_∞: remove zero-modes of photon [Hayakawa, Uno '08]

hadronic scheme at $O(\alpha)$ and $O(m_u - m_d)$: [Blum et al. '18]

- Ω^- mass $\rightarrow a$ latt.spacing
- $m_{\pi^\pm} - m_{\pi^0}$ and $m_{\pi^\pm} \rightarrow m_u\,, m_d$
- $m_{K^\pm} \rightarrow m_s$

Local vector current $\rightarrow Z_V$

a_μ ON THE LATTICE - I

$a_\mu = 4\alpha^2 \int dQ^2 K(Q^2)[\Pi(Q^2) - \Pi(0)]$ (Q^2 euclidean) [Blum '03]

$\Pi_{\mu\nu}(Q^2) = \int d^4x e^{iQ\cdot x} \langle j^\gamma_\mu(x) j^\gamma_\nu(0)\rangle$ on the lattice

small $Q^2 \lesssim m_\mu^2$ very difficult

Time-momentum representation [Bernecker, Meyer, '11]

$G^\gamma(t) = \frac{1}{3}\sum_k \int d\vec{x}\ \langle j^\gamma_k(x) j^\gamma_k(0)\rangle$, $[\Pi(Q^2) - \Pi(0)] = \int dt\, G^\gamma(t) f(t, Q^2)$

$$a_\mu = 4\alpha^2 \int dt\, w(t)\, G^\gamma(t)\,,$$ $w(t)$ muon kernel (weights)

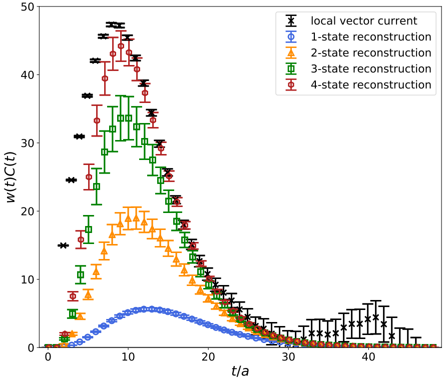
more natural to study G^γ in euclidean time
spectral decomposition (reconstruction)

a_μ ON THE LATTICE - II

$$G(t) = \sum_n e^{-E_n t} |\langle n|\hat{j}_\mu|0\rangle|^2 \quad t \gg 0\,, G(t) \approx \sum_n^N e^{-E_n t} |\langle n|\hat{j}_\mu|0\rangle|^2$$

dedicated calculation to resolve lowest N states

$\rightarrow$ partially cured signal-to-noise growth



[MB, Meyer, Lehner, Izubuchi PoS '19]

naive full sum

$\delta a_\mu = 38 \times 10{-}10$

truncated sum (bounding method)

$\delta a_\mu = 16 \times 10{-}10$

3-state reconstruction

$\delta a_\mu = \ \ 5 \times 10{-}10$

area $= a_\mu$

Contribution to a_μ

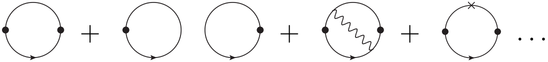
Time-momentum representation [Bernecker, Meyer, '11]

$$G^\gamma(t) = \frac{1}{3}\sum_k \int d\vec{x}\, \langle j_k^\gamma(x) j_k^\gamma(0)\rangle \quad \rightarrow \quad a_\mu = 4\alpha^2 \sum_t w_t G^\gamma(t)$$

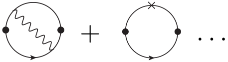
Isospin decomposition of u, d current

$$j_\mu^\gamma = \frac{i}{6}\left(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d\right) + \frac{i}{2}\left(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\right) = j_\mu^{(0)} + j_\mu^{(1)}$$

$$G_{00}^\gamma \leftarrow \langle j_k^{(0)}(x) j_k^{(0)}(0)\rangle =$$



$$G_{01}^\gamma \leftarrow \langle j_k^{(0)}(x) j_k^{(1)}(0)\rangle =$$



$$G_{11}^\gamma \leftarrow \langle j_k^{(1)}(x) j_k^{(1)}(0)\rangle =$$

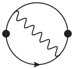
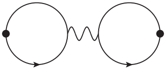
Decompose $a_\mu = a_\mu^{(0,0)} + a_\mu^{(0,1)} + a_\mu^{(1,1)}$

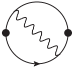
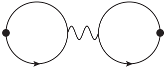
Neutral vs Charged

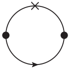
$$\tfrac{i}{2}\left(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\right),\begin{bmatrix} I=1 \\ I_3=0 \end{bmatrix} \quad \to \quad j_\mu^{(1,-)} = \tfrac{i}{\sqrt{2}}\left(\bar{u}\gamma_\mu d\right),\begin{bmatrix} I=1 \\ I_3=-1 \end{bmatrix}$$

Isospin 1 charged correlator $G^W_{11} = \frac{1}{3}\sum_k \int d\vec{x}\, \langle j_k^{(1,+)}(x) j_k^{(1,-)}(0)\rangle$

$\delta G^{(1,1)} \equiv G^\gamma_{11} - G^W_{11}$ [MB et al.' PoS '18]

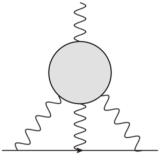
$$= Z_V^4 (4\pi\alpha)\frac{(Q_\mathrm{u}-Q_\mathrm{d})^4}{4}\Big[$$  $+$  $\Big]$

$$G^\gamma_{01} = Z_V^4 \frac{(Q_u^2-Q_d^2)^2}{2}(4\pi\alpha)\Big[$$  $+ 2\times$  $+$  $+\dots\Big]$

$$+Z_V^2\frac{Q_u^2-Q_d^2}{2}(m_u-m_d)\Big[2\times$$  $+\dots\Big]$

$\dots$ = subleading diagrams currently not included

Synergy - I



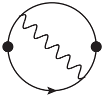
from QCD we need a 4-point function $f(x, y, z, t)$:

known kernel with details of photons and muon line

1 pair of point sources (x, y), sum over z, t exact at sink

stochastic sampling over (x, y) (based on $|x - y|$)

Successfull strategy: x10 error reduction [RBC '16]



from QCD we need a 4-point function $f(x, y, z, t)$:

$(g - 2)_\mu$ kernel + photon propagator

Similar problem $\rightarrow$ re-use HLbL point sources!

The RBC & UKQCD collaborations

BNL and BNL/RBRC

Yasumichi Aoki (KEK)
Taku Izubuchi
Yong-Chull Jang
Chulwoo Jung
Meifeng Lin
Aaron Meyer
Hiroshi Ohki
Shigemi Ohta (KEK)
Amarjit Soni

UC Boulder

Oliver Witzel

CERN

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Christopher Kelly
Bob Mawhinney
Masaaki Tomii
Jiqun Tu
Bigeng Wang
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Dan Hoying (BNL)
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Felix Erben
Vera Gülpers
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Julia Kettle
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Fionn Ó hÓgáin
Antonin Portelli
Tobias Tsang
Andrew Yong
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Nils Asmussen
Jonathan Flynn
Ryan Hill
Andreas Jüttner
James Richings
Chris Sachrajda

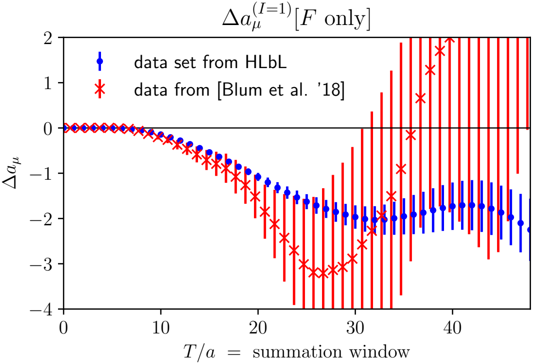
Stony Brook University

Jun-Sik Yoo
Sergey Syritsyn (RBRC)

SYNERGY - II

Stat. improvements from data of HLbL project [Phys.Rev.Lett. 118 (2017)]

contribution of diagram F to pure $I = 1$ part of Δa_μ



$O(1000)$ point-src per conf.
$5 \cdot 10^5$ combinations
80 configurations

$\times 4$ reduction in error

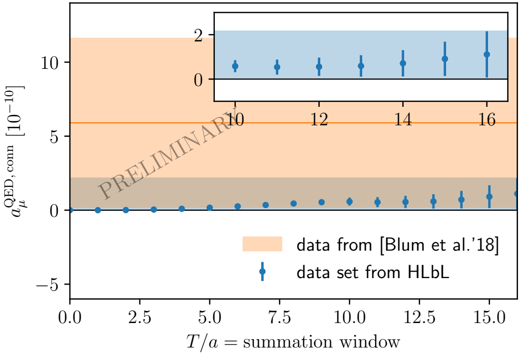
finite volume errs relevant
$\rightarrow$ dedicated study

data from [Blum et al. '18]: $O(500)$ point-src per conf.
76 configurations

Synergy - III

Stat. improvements from data of HLbL project [Phys.Rev.Lett. 118 (2017)]

contribution of diagrams V, S to a_μ



$O(2000)$ point-src per conf.

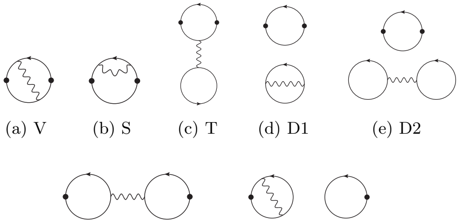
~ 3000 combinations

$O(10)$ configurations

$\times 4$ reduction in error

expected QED conn. error $\leq 3 \times 10^{-10} \rightarrow$ matches target

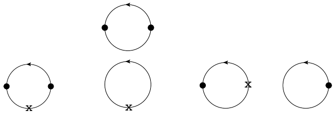
Synergy - IV



(a) V (b) S (c) T (d) D1 (e) D2

(f) F (g) D3

[Blum et. al. '18]



(a) M (b) R (c) O

Presently only leading diagrams are computed $V\,,F\,,S\,,M$ [Blum et al. '18]

- same diagrams for isospin-breaking in τ spectral functions
- improvement in precision beneficial to both $(g-2)_\mu$ and τ
- preliminary numbers for $SU(3)$ and $1/N_c$ suppressed diagrams

Last slide, then Plots!

Restriction to $2\pi \to$ neglect pure $I = 0$ part $a_\mu^{(0,0)}[\pi^0\gamma, 3\pi, \ldots]$

$$\text{Lattice: } \Delta a_\mu[\pi\pi,\tau] = 4\alpha^2 \sum_t w_t \times \left[G_{01}^{\gamma}(t) + G_{11}^{\gamma}(t) - G_{11}^{W}(t)\right]$$

$$\text{Pheno: } \Delta a_\mu[\pi\pi,\tau] = \int_{4m_\pi^2}^{m_\tau^2} ds K(s) \left[v_0(s) - v_-(s)\right]$$

Conversion to Euclidean time for direct comparison

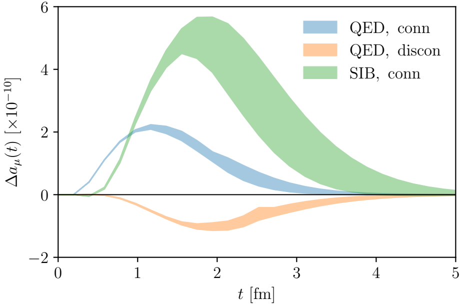
$$\Delta a_\mu[\pi\pi,\tau] = 4\alpha^2 \sum_t w_t \times \left\{ \frac{1}{12\pi^2} \int d\omega\, \omega^2 e^{-\omega t} \left[R_{\mathrm{IB}}(\omega^2) - 1\right] v_-(\omega^2) \right\}$$

Lattice fully inclusive

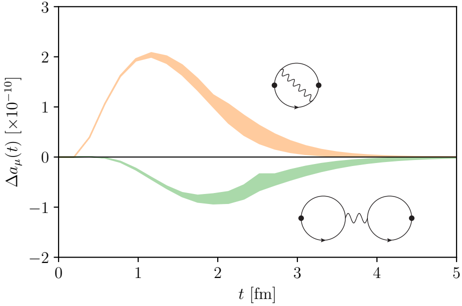
- manipulate $G(t)$ (e.g. Backus-Gilbert) to implement cut $E < m_\tau$
- include additional channels in v_0/v_-
  - effects above ~ 1 GeV suppressed by (muon) kernel
  - preliminary: smaller than current precision for Δa_μ
    - additional investigations on the way

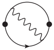
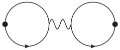
Lattice: Preliminary results - I

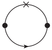
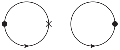
$\Delta a_\mu \to G_{01} + \delta G_{11}$:



Pure $I = 1$ only $O(\alpha)$ terms:



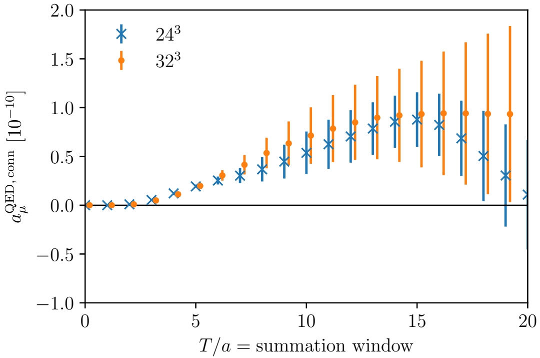
$V =$  $F =$  $S =$ 

$M =$  $O =$  relevant, negative, neglected

Systematic errors

$a_\mu^{\mathrm{QED,conn}} = V + 2S$

FV study at coarse
$a^{-1} \sim 1$ GeV



Finite volume errors

empirical observation: diagrams may have largish FV errors

cancellation of FV effects in physical combinations

similar observation in ChPT, e.g. [Bijnens, Portelli '19]

Dilemma

I am interested in comparing integrands beyond integrals

I have computed correlation functions in Euclidean time

To be or not to be Euclidean

1. leave lattice as it is, convert experiment to Euclidean time
   well-posed problem, simple Laplace trafo

2. spectral reconstruction from lattice data [Hansen, Lupo, Tantalo '19]
   ill-posed problem, not needed for integrals like a_μ

let's do the comparison in Euclidean time

Calculation incomplete, what follows mostly qualitative!

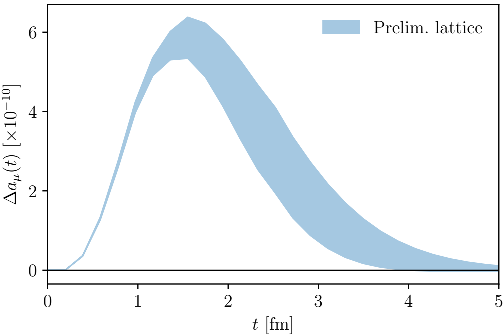
Lattice: Preliminary results - II

Study integrand in euclidean time $\rightarrow$ as important as integral

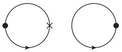
direct comparison
Lattice vs. EFT+Pheno

1. validate previous estimates of $R_{\rm IB}$
2. study neutral/charged ρ and ω properties

Preliminary lattice (full) calculation: $G_{01}^{\gamma} + \delta G$

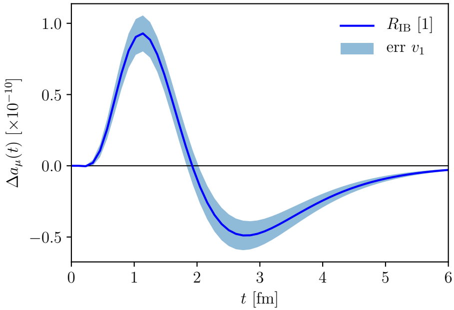


Not included:

1.  relevant
2. sub-leading $1/N_c$, $SU(N_f)$
3. finite-volume errors
4. discretization errors

Model calculations

Preliminary (using $G^{\pi}_{\rm EM}$ and without $S_{\rm EW}$)



Data from private comm. with F. Jegelehner

[1] = [Jegelehner, Szafron '11]

depends on ρ^0 and ρ^- masses/widths

requires $G^{\pi}_{\rm EM}$ to compare with lattice

resembles lattice results qualitative agreement

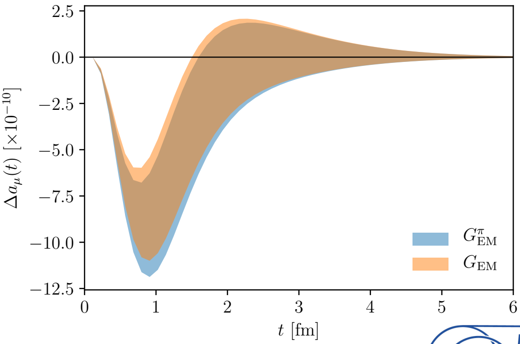
Experimental results

$$\Delta a_\mu(t) = 4\alpha^2 \sum_t w_t \Big\{ \int ds\, h(s,t)\, \Big[v_0(s) - \frac{v_1(s)}{G_{\rm EM}(s)} \Big] \Big\}$$

v_0 BaBar, v_1 Aleph

lattice suggests a different answer

preliminary GEM^π

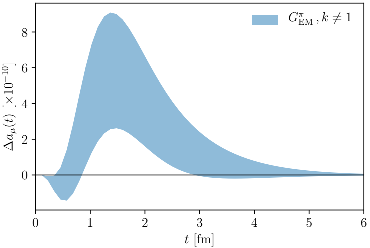


$v_1 \to k v_1$

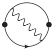
$k = 1$ Standard Model

$k \neq 1$ BSM (SMEFT)

[Cirigliano et al. '18]

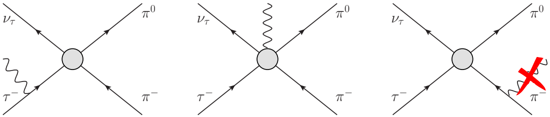


Towards a comparison

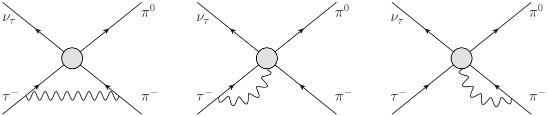
Lattice contains $\pi^0\pi^-\gamma$ states $\rightarrow$ 

Re-evaluation of $G_{\rm EM} \rightarrow G_{\rm EM}^{\pi}$ [in collab. with Cirigliano]

Real photon corrections



Virtual photon corrections



$G_{\rm EM}^{\pi}$ w/o $\pi^0\pi^-\gamma$ FSR

$\dfrac{v_-}{G_{\rm EM}^{\pi}}$ w $\pi^0\pi^-\gamma$ FSR

Outlook

use arbitrary kernels with desired properties [with M. Gonzales-Alonso]

- even stronger suppression of neglected channels at high energies
- suppression of short distances (cutoff effects)
- suppression of long distances (noise)

map other spectral functions to the corresponding correlators

- e.g. $K^\star$ channel in vector-vector correlator

Eventually proper calculation is isospin-breaking corrections of $\pi\pi$ form factors

Conclusions

These are exciting times for $(g-2)_\mu$:

1% goal for lattice results to be expected soon

QED+SIB crucial to reach target uncertainty

As a bi-product we get $\Delta a_\mu[\tau]$:

1. first lattice calculation of $\Delta a_\mu[\tau]$ almost complete
2. tests/checks previous calculations

   comparing v_- with experiment requires G^{π}_{EM}

   study G^{γ}_{01} alone $\to$ $\rho\omega$ mixing; $\delta G^{(1,1)}$ alone $\to$ ρ^0 vs ρ^-
3. possibly sensitive to new physics

Thanks for your attention

$\rho\gamma$ MIXING - I

Gounaris-Sakurai based on VMD model w/o EM gauge invariance

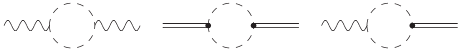
- generation of a photon mass

$+$ based on phase shift (proper pion rescattering behavior)

widely used: e.g. PDG estimates of m_ρ, Γ_ρ

VMD model with gauge-invariance [Kroll, Lee, Zumino '67]

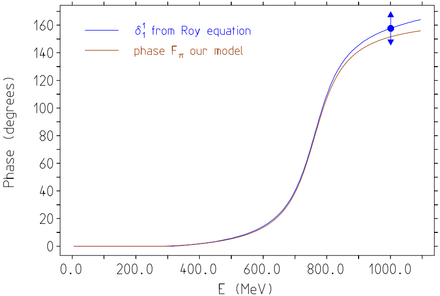
at 1-loop s-dependent mass matrix [Jegerlehner, Szafron '11]

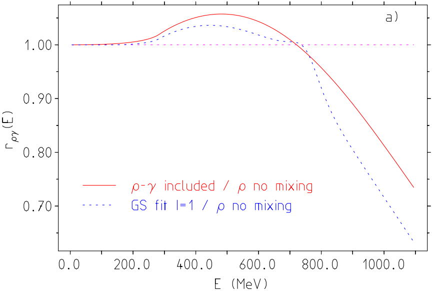


limits of validity pion-loop? high enough energy must break down

$\rho\gamma$ MIXING - II

[Jegerlehner, Szafron '11]





30% correction at 1 GeV

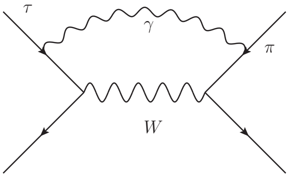
δ_1^1 in good agreement $E < 800$ MeV

perhaps restrict the $\rho\gamma$ below 800 MeV?

Radiative corrections

Some QED corrections computed in Chiral PT [Cirigliano et al. '01]

e.g. photon exchange between τ and hadrons



relevant to compare lattice data vs v_-

is current precision enough?

alternative calculation from lattice possible [Giusti et al. '17]