The charm and beauty of the Little Bang

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INFN - Sezione di Torino

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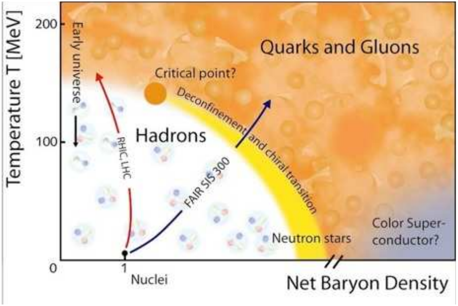
Istituto Nazionale di Fisica Nucleare

Heavy-ion collisions: exploring the QCD phase-diagram



QCD phases identified through the *order parameters*

- Polyakov loop $\langle L \rangle \sim e^{-\beta \Delta F_Q}$: energy cost to add an isolated color charge
- Chiral condensate $\langle \overline{q} q \rangle \sim$ effective mass of a "dressed" quark in a hadron

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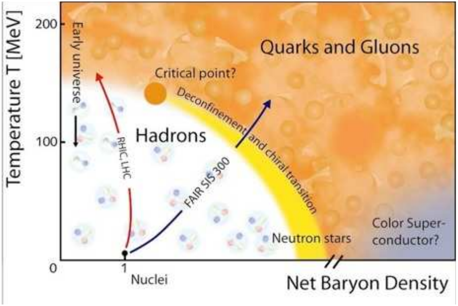

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Heavy-Ion Collision (HIC) experiments performed to study the transition

- From QGP (color deconfinement, chiral symmetry restored)
- to hadronic phase (confined, chiral symmetry broken)

NB $\langle \overline{q}q \rangle \neq 0$ responsible for most of the baryonic mass of the universe: *only* $\sim$*35 MeV of the proton mass from* $m_{u/d} \neq 0$

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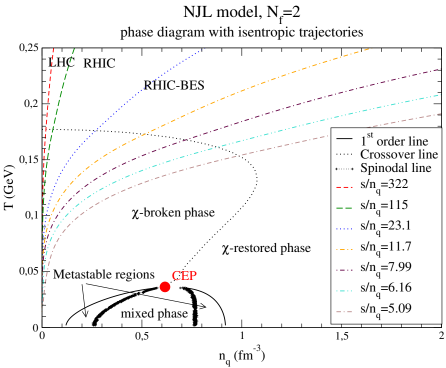

- Region explored at the LHC and highest RHIC energy: *high-T/low-density* (early universe, $n_B/n_\gamma \sim 10^{-9}$)
- Higher baryon-density region accessible at lower $\sqrt{s_{\rm NN}}$ (Beam-Energy Scan at RHIC)

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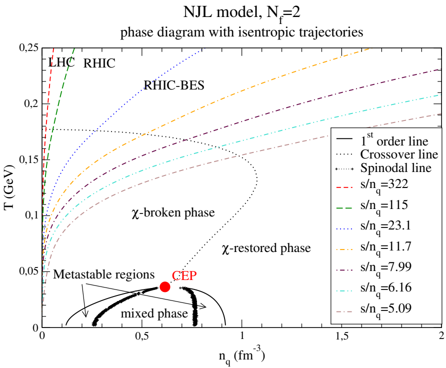

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Is there a Critical End-Point in the QCD phase diagram?

QCD at high temperature: expectations

Based on *asymptotic freedom*, for $T \gg \Lambda_{QCD}$ hot-QCD matter should behave like a non-interacting plasma of massless quarks (the ones for which $m_q \ll T$) and gluons. In such a regime T is the only scale μ at which evaluating the gauge coupling, for which one has

$$\lim_{T/\Lambda_{QCD} \to \infty} g(\mu \sim T) = 0$$

Hence one expects the asymptotic *Stefan-Boltzmann* behaviour

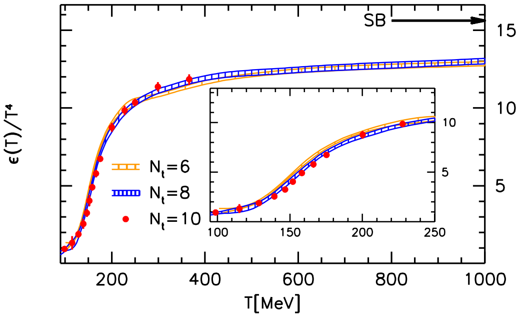
$$\epsilon = \frac{\pi^2}{30} \left[g_{\rm gluon} + \frac{7}{8} g_{\rm quark} \right] T^4,$$

where

$$g_{\rm gluon} = \underbrace{2 \times (N_c^2 - 1)}_{\text{pol.} \times \text{col.}} \quad \text{and} \quad g_{\rm quark} = \underbrace{2 \times 2 \times N_c \times N_f}_{q/\overline{q} \times \text{spin} \times \text{col.} \times \text{flav.}}$$

QCD at high temperature: lattice results

Continuum-extrapolated ($a \to 0$) lattice-QCD simulations with realistic quark masses now available (W.B. Collab. [JHEP 1011 (2010) 077])



Rapid rise in the energy density suggesting a change in the *number of active degrees of freedom* (hadrons $\to$ partons):

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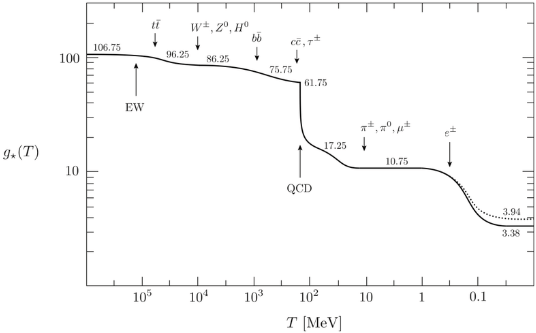
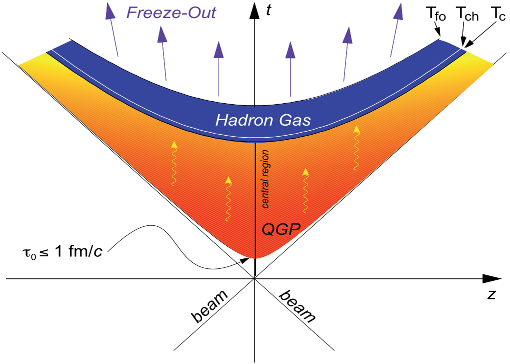


Rapid rise in the energy density suggesting a change in the *number of active degrees of freedom* (hadrons $\to$ partons):
the most dramatic drop experienced by the early universe in which

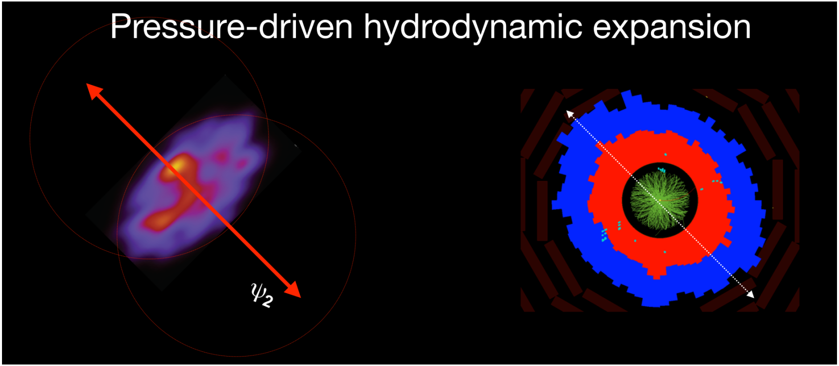
$$H^2 = \frac{8\pi G}{3}\epsilon_{\rm rel} = \frac{8\pi G}{3}\left(\frac{\pi^2}{30} g_* T^4\right)$$

Heavy-ion collisions: a cartoon of space-time evolution



- Soft probes (low-p_T hadrons): collective behavior of the *medium*;
- Hard probes (high-p_T particles, heavy quarks, quarkonia): produced in *hard pQCD processes* in the initial stage, allow to perform a tomography of the medium

A medium displaying a collective behavior



$$(\epsilon + P)\frac{dv^i}{dt} \underset{v \ll c}{=} -\frac{\partial P}{\partial x^i}$$

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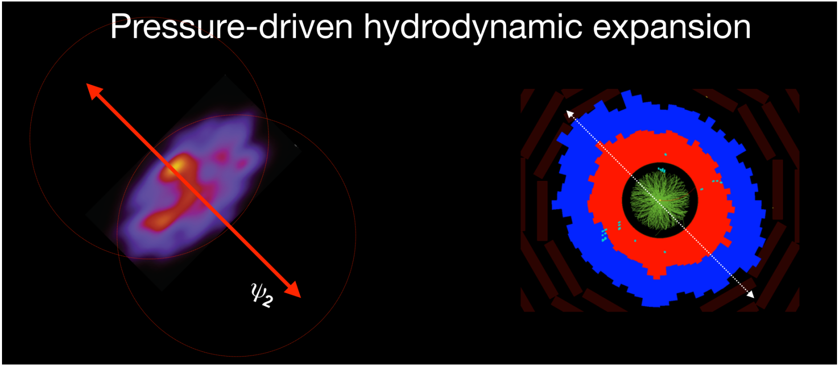

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NB picture relying on the condition $\lambda_{\rm mfp} \ll L$

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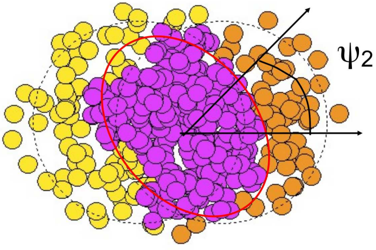
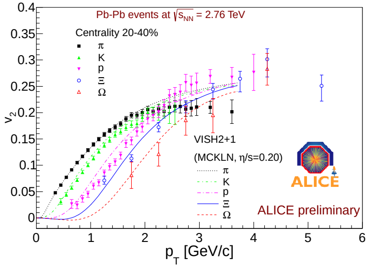


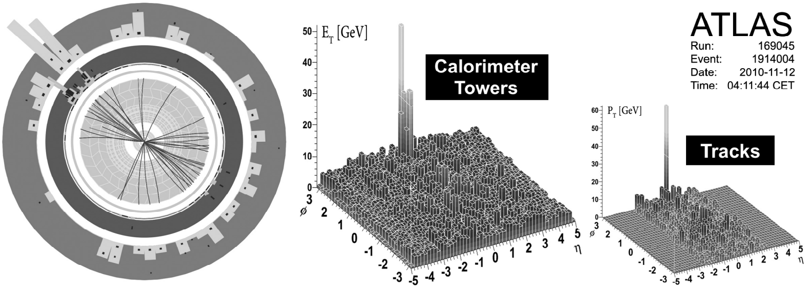


Anisotropic azimuthal distribution of hadrons as a response to pressure gradients quantified by the *Fourier coefficients* v_n

$$\frac{dN}{d\phi} = \frac{N_0}{2\pi}\left(1 + 2\sum_n v_n \cos[n(\phi - \psi_n)] + \ldots\right)$$

$$v_n \equiv \langle \cos[n(\phi - \psi_n)] \rangle$$

A medium inducing energy-loss to colored probes



Strong unbalance of di-jet events, visible at the level of the event-display itself, without any analysis: jet-quenching

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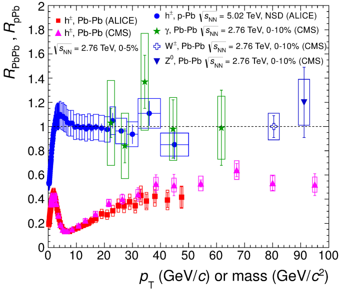

Medium-induced suppression of high-momentum hadrons and jets quantified through the *nuclear modification factor*

$$R_{AA} \equiv \frac{\left(dN^h/dp_T\right)^{AA}}{\langle N_{\rm coll}\rangle \left(dN^h/dp_T\right)^{pp}}$$

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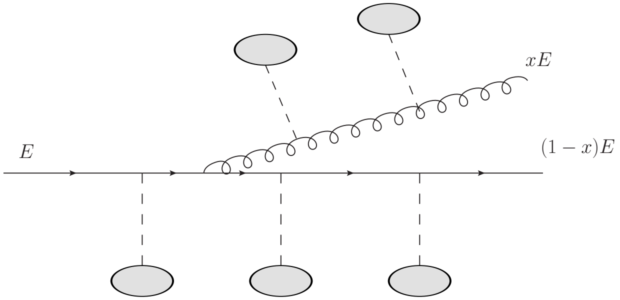

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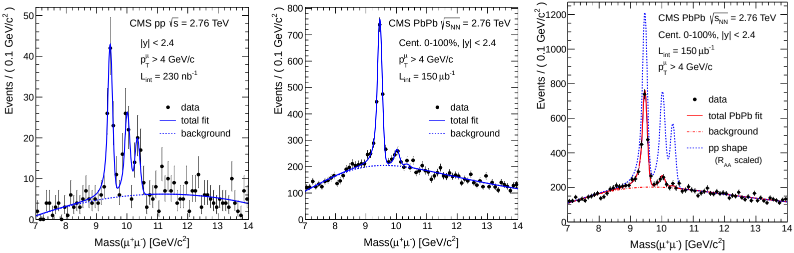
interpreted as energy carried away by radiated gluons

A medium screening the $Q\overline{Q}$ interaction



Suppression of Υ production in Pb-Pb collisions at the LHC, in particular its excited (weaker binding, larger radius!) states.

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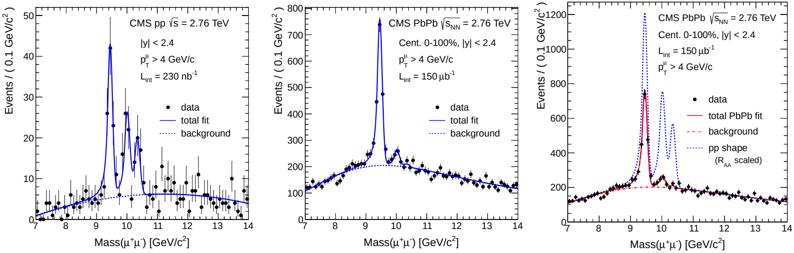

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In first approximation, Debye screening of the $Q\overline{Q}$ interaction[1]:

$$V_{Q\overline{Q}}(r) = -C_F \frac{\alpha_s}{r} \quad \longrightarrow \quad -C_F \frac{\alpha_s}{r} e^{-m_D r}$$

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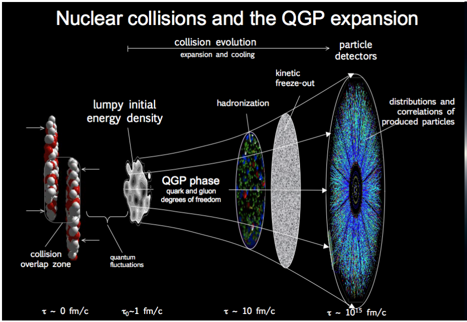
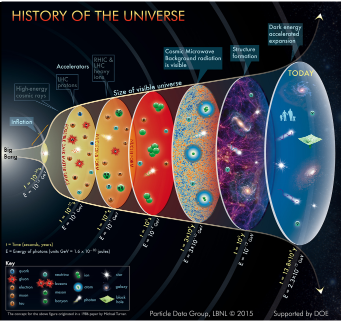
Little Bang vs Big Bang





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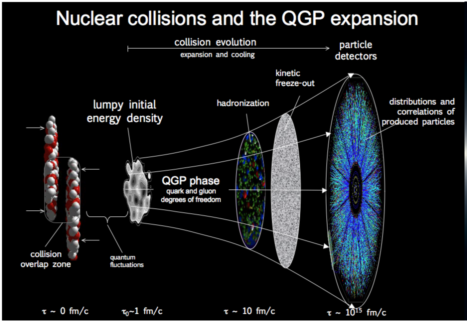
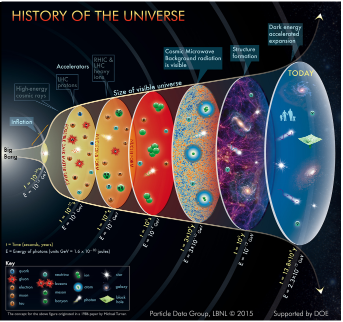
Little Bang vs Big Bang





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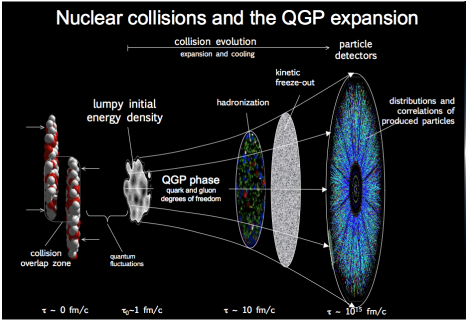

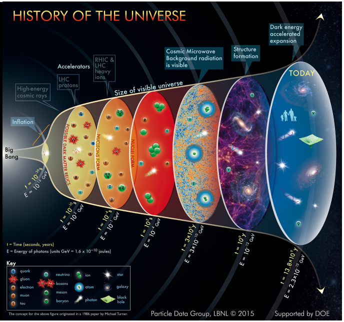

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- Expansion of the universe governed by the equations of the gravitational field. In nuclear collisions gravity does not play any role, expansion of the fireball driven by pressure gradients;
- QGP produced in nuclear collisions has a much shorter lifetime (10^{-22}s vs 10^{-6}s) and a much more violent expansion (with deep consequences!).

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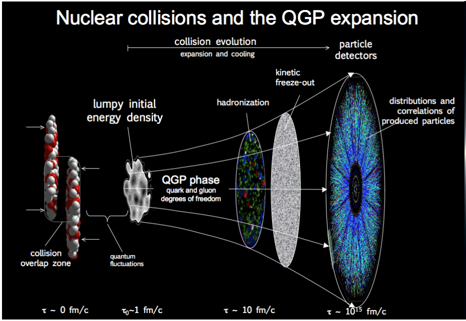
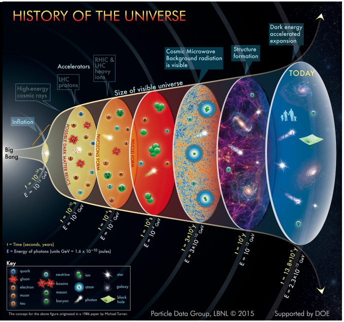




To be more precise, compare the expansion rates:

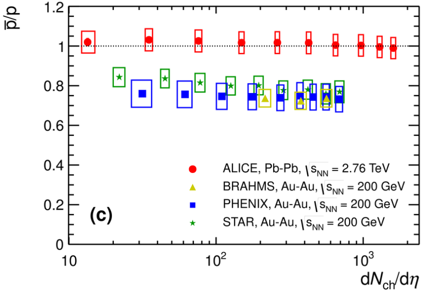
- Radiation-dominated universe

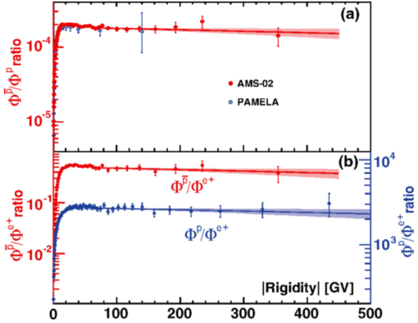
$$a \sim t^{1/2} \quad \longrightarrow \quad \dot{a} \sim \frac{1}{2} a^{-1/2} \quad H \equiv \frac{\dot{a}}{a} = \frac{1}{2t} \sim 10^6 \, \mathrm{s}^{-1}$$

- QGP in HIC's undergoing longitudinal expansion $v^z = z/t$

$$\theta \equiv \partial_\mu u^\mu \underset{z \to 0}{\sim} \frac{1}{t} \sim 10^{22} \, \mathrm{s}^{-1}$$

Matter vs Antimatter in Little and Big Bang





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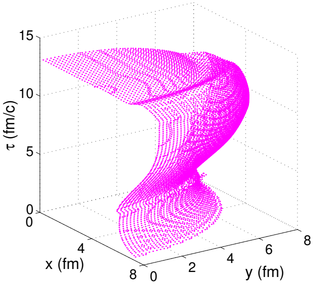

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$$p + \overline{p} \leftrightarrow \#\mathrm{pions},$$

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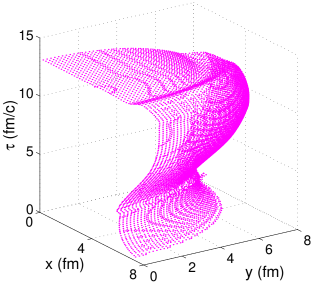

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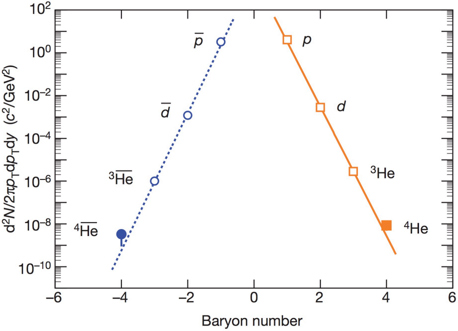
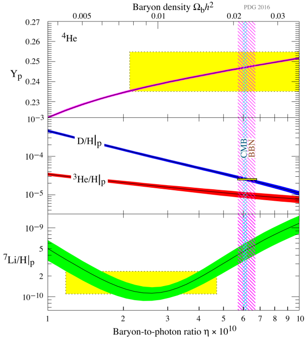
$$\lambda^{\mathrm{ann}}_{\mathrm{mfp}} = \frac{1}{n_p \sigma^{\mathrm{in}}_{p\overline{p}}} \quad \mathrm{with} \quad n \approx 10^{-2}\mathrm{fm}^{-3} \quad \longrightarrow \quad \lambda^{\mathrm{ann}}_{\mathrm{mfp}} \approx 30\mathrm{fm} \gg L$$

Protons and antiprotons decouple immediately after the QCD transition

Little-Bang vs Big-Bang nucleosynthesis





- LBN: yields of light nuclei (and antinuclei!) decreaseas as A increases (fig. from STAR Coll., Nature 473, 353356(2011));
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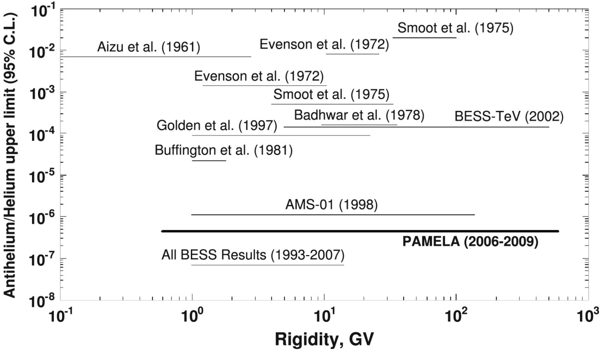

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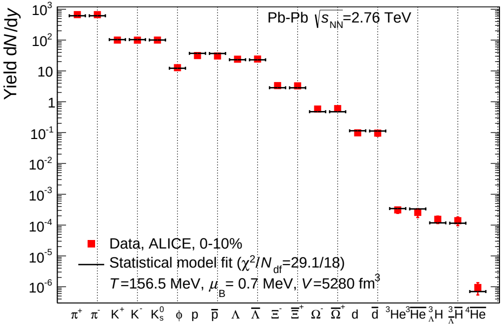

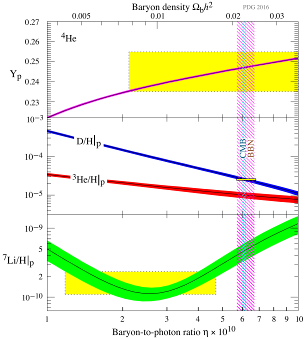

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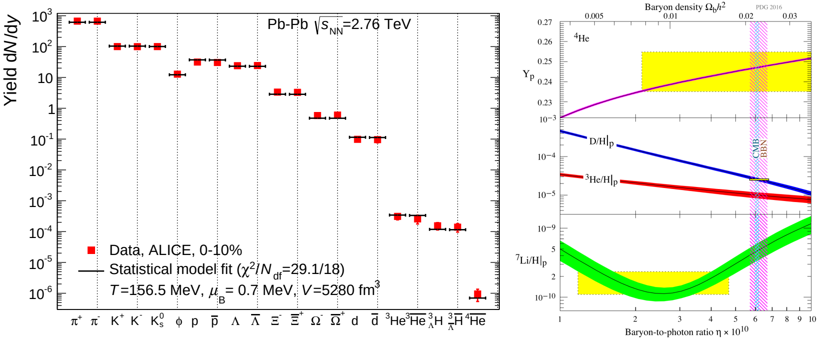

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- LBN: light-nucleus yields effectively frozen at the same chemical freeze-out temperature $T \approx 155$ MeV as the other hadrons;
- BBN: photons remain in thermal equilibrium with the plasma and continuously destroy deuteron as soos as it is formed (deuteron bottleneck)

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NB At high-p_T the interest in heavy flavor is no longer related to thermalization, but to the study of the mass and color charge dependence of jet-quenching (not addressed in this talk)

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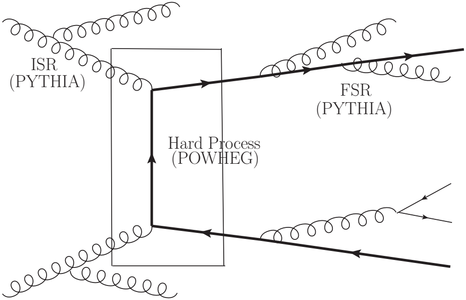
  - However, *a source of systematic uncertainty for studies of parton-medium interaction*;
- Hadronic rescattering (e.g. $D\pi \to D\pi$), from effective Lagrangians, but no experimental data the on relevant cross-sections

The initial hard production



- A convenient automated tool to simulate the initial $Q\overline{Q}$ production (the POWHEG-BOX package[2]) interfaces the output of a NLO event-generator for the hard process with a parton-shower describing the Initial and Final State Radiation and modeling other *non-perturbative processes* (intrinsic k_T, MPI, hadronizazion)
- This provides a *fully exclusive information on the final state*

[2] Alioli et al., JHEP 1006 (2010) 043

HQ chemical equilibration in the Little and Big Bang

The rate of approach of HQ's to chemical equilibrium is given by[3]

$$\Gamma_{\rm chem} \underset{M \gg T}{\approx} \frac{g^4 C_F}{8\pi M^2}\left(2C_F - \frac{N_c}{2} + N_f\right)\left(\frac{MT}{2\pi}\right)^{3/2} e^{-M/T}$$

At the initial $T_0 \approx 0.5$ GeV one gets for charm $\Gamma_{\rm chem} \approx 0.015\ {\rm fm}^{-1}$, i.e. $\tau_{\rm chem} \approx 65\ {\rm fm/c}$,

[3] D. Bodeker and M. Laine, JHEP 1207 (2012) 130

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$$\theta_0 = 1/\tau_0 \approx 2{\rm fm}^{-1} \quad \longrightarrow \quad \Gamma_{\rm chem} \ll \theta_0$$

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$$H = 1/2t \sim 10^6 s^{-1} \quad \longrightarrow \quad \Gamma_{\rm chem} \gg H$$

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NB, since $T \sim \tau^{-1/3}$, during the expansion $\Gamma_{\rm chem} \sim \tau^{-1/2} e^{-\alpha\tau^{1/3}}$

[3] D. Bodeker and M. Laine, JHEP 1207 (2012) 130

How many $Q\overline{Q}$ pairs in HIC's?

Rapidity density of $Q\overline{Q}$ pairs in AA collisions estimated rescaling the pp result by the number of binary nucleon-nucleon collisions

$$\frac{dN^{Q\overline{Q}}}{dy} = \langle N_{\rm coll} \rangle \frac{1}{\sigma^{\rm in}} \frac{d\sigma^{Q\overline{Q}}}{dy}$$

[4] FONLL calculation, M. Cacciari et al.

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For 0-5% most central Pb-Pb collisions at the LHC one gets[4]

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The initial ($\tau_0 \approx 0.5$ fm/c, $T_0 \approx 0.5$ GeV) density given by pQCD and at equilibium is

$$n^{Q\overline{Q}}_{\rm pQCD} = \frac{dN^{Q\overline{Q}}}{d\vec{x}} \approx \frac{1}{\pi R^2_{Pb}} \frac{1}{\tau_0} \frac{dN^{Q\overline{Q}}}{dy} \quad \text{vs} \quad n^{Q\overline{Q}}_{\rm therm} = (2s+1)N_c\left(\frac{MT}{2\pi}\right)^{3/2} e^{-M/T}$$

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One has:

$$n_{\rm pQCD}^{c\bar{c}} \approx 0.179\,{\rm fm}^{-3} \quad \text{vs} \quad n_{\rm therm}^{c\bar{c}} \approx 1.539\,{\rm fm}^{-3}$$

$$n_{\rm pQCD}^{b\bar{b}} \approx 0.011,{\rm fm}^{-3} \quad \text{vs} \quad n_{\rm therm}^{c\bar{c}} \approx 0.012\,{\rm fm}^{-3}$$

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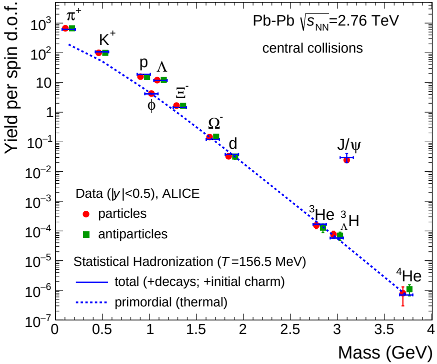

HQ number is conserved during the evolution: at hadronization charm is overpopulated with respect to the other hadrons at chemical equilibrium (figure from A. Andronic et al., Phys.Lett. B797 (2019) 134836)

Transport theory: general setup

Transport theory: the Boltzmann equation

Time evolution of HQ phase-space distribution $f_Q(t, \boldsymbol{x}, \boldsymbol{p})$:

$$\frac{d}{dt} f_Q(t, \boldsymbol{x}, \boldsymbol{p}) = C[f_Q]$$

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$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \boldsymbol{v}\frac{\partial}{\partial \boldsymbol{x}} + \boldsymbol{F}\frac{\partial}{\partial \boldsymbol{p}}$$

Neglecting $\boldsymbol{x}$-dependence and mean fields: $\partial_t f_Q(t, \boldsymbol{p}) = C[f_Q]$

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- Collision integral:

$$C[f_Q] = \int d\boldsymbol{k} [\underbrace{w(\boldsymbol{p}+\boldsymbol{k}, \boldsymbol{k}) f_Q(\boldsymbol{p}+\boldsymbol{k})}_{\text{gain term}} - \underbrace{w(\boldsymbol{p}, \boldsymbol{k}) f_Q(\boldsymbol{p})}_{\text{loss term}}]$$

  $w(\boldsymbol{p}, \boldsymbol{k})$: HQ transition rate $\boldsymbol{p} \to \boldsymbol{p} - \boldsymbol{k}$

From Boltzmann to Fokker-Planck

Expanding the collision integral for *small momentum exchange*[5] (Landau)

$$C[f_Q] \approx \int d\boldsymbol{k} \left[k^i \frac{\partial}{\partial p^i} + \frac{1}{2} k^i k^j \frac{\partial^2}{\partial p^i \partial p^j} \right] [w(\boldsymbol{p}, \boldsymbol{k}) f_Q(t, \boldsymbol{p})]$$

[5] B. Svetitsky, PRD 37, 2484 (1988)

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The Boltzmann equation reduces to the *Fokker-Planck equation*

$$\frac{\partial}{\partial t} f_Q(t, \boldsymbol{p}) = \frac{\partial}{\partial p^i} \left\{ A^i(\boldsymbol{p}) f_Q(t, \boldsymbol{p}) + \frac{\partial}{\partial p^j} [B^{ij}(\boldsymbol{p}) f_Q(t, \boldsymbol{p})] \right\}$$

where

$$A^i(\boldsymbol{p}) = \int d\boldsymbol{k}\, k^i w(\boldsymbol{p}, \boldsymbol{k}) \longrightarrow \underbrace{A^i(\boldsymbol{p}) = A(p)\, p^i}_{\text{friction}}$$

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Problem reduced to the *evaluation of three transport coefficients*,
directly derived from the scattering matrix

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Approach to equilibrium in the FP equation

The FP equation can be viewed as a continuity equation for the phase-space distribution of the kind $\partial_t \rho(t,\vec{p}) + \vec{\nabla}_p \cdot \vec{J}(t,\vec{p}) = 0$

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admitting a steady solution $f_{\rm eq}(p) \equiv e^{-E_p/T}$ when the current vanishes:

$$A^i(\vec{p})f_{\rm eq}(p) = -\frac{\partial B^{ij}(\vec{p})}{\partial p^j}f_{\rm eq}(p) - B^{ij}(\boldsymbol{p})\frac{\partial f_{\rm eq}(p)}{\partial p^j}.$$

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One gets

$$A(p) p^i = \frac{B_1(p)}{TE_p} p^i - \frac{\partial}{\partial p^j} \left[\delta^{ij} B_0(p) + \hat{p}^i \hat{p}^j (B_1(p) - B_0(p)) \right],$$

leading to the Einstein *fluctuation-dissipation* relation

$$A(p) = \frac{B_1(p)}{TE_p} - \left[\frac{1}{p} \frac{\partial B_1(p)}{\partial p} + \frac{d-1}{p^2} (B_1(p) - B_0(p)) \right],$$

quite involved due to the *momentum dependence* of the transport coefficients (*measured* HQ's are relativistic particles!)

The relativistic Langevin equation

The Fokker-Planck equation can be recast into a form suitable to follow the dynamics of each individual quark arising from the pQCD Monte Carlo simulation of the initial $Q\overline{Q}$ production: the Langevin equation

$$\frac{\Delta p^i}{\Delta t} = -\underbrace{\eta_D(p)p^i}_{\text{determ.}} + \underbrace{\xi^i(t)}_{\text{stochastic}} \,,$$

with the properties of the noise encoded in

$$\langle \xi^i(\boldsymbol{p}_t) \rangle = 0 \quad \langle \xi^i(\boldsymbol{p}_t)\xi^j(\boldsymbol{p}_{t'}) \rangle = b^{ij}(\boldsymbol{p}) \frac{\delta_{tt'}}{\Delta t} \quad b^{ij}(\boldsymbol{p}) \equiv \kappa_L(p)\hat{p}^i\hat{p}^j + \kappa_T(p)(\delta^{ij} - \hat{p}^i\hat{p}^j)$$

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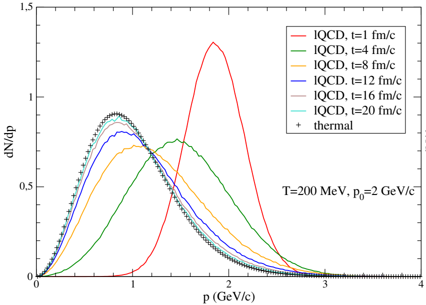
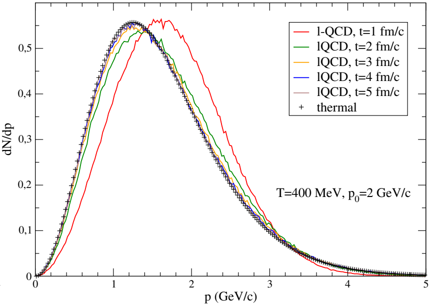
Transport coefficients related to the FP ones:

- *Momentum diffusion*: $\kappa_T(p) = 2B_0(p)$ and $\kappa_L(p) = 2B_1(p)$
- *Friction* term, in the *Ito pre-point discretization scheme*,

$$\eta_D^{\text{Ito}}(p) = A(p) = \frac{B_1(p)}{TE_p} - \left[\frac{1}{p}\frac{\partial B_1(p)}{\partial p} + \frac{d-1}{p^2}(B_1(p) - B_0(p))\right]$$

Consistency check I: thermalization in a static medium





(Test with a sample of c quarks with $p_0 = 2$ GeV/c).
For $t \gg 1/\eta_D$ one approaches a relativistic Maxwell-Jüttner distribution

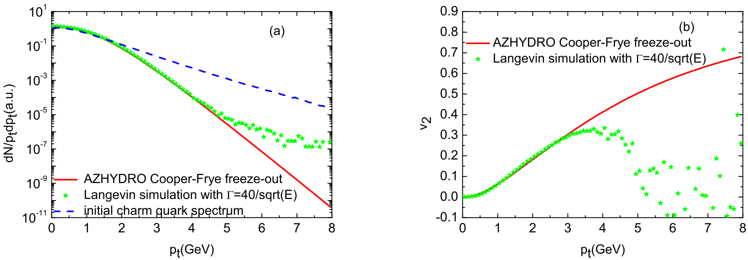
$$f_{\rm MJ}(p) \equiv \frac{e^{-E_p/T}}{4\pi M^2 T\, K_2(M/T)}, \qquad \text{with} \quad \int d^3p\, f_{\rm MJ}(p) = 1$$

The larger κ ($\kappa \sim T^3$), the faster the approach to thermalization.

Consistency check II: thermalization in a static medium

In the limit of large transport coefficients heavy quarks should reach local thermal equilibrium and decouple from the medium as the other light particles, according to the Cooper-Frye formula:

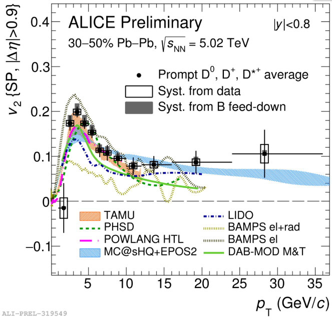
$$E(dN/d^3p) = \int_{\Sigma_{\rm fo}} \frac{p^\mu \cdot d\Sigma_\mu}{(2\pi)^3} \exp[-p\cdot u/T_{\rm fo}]$$

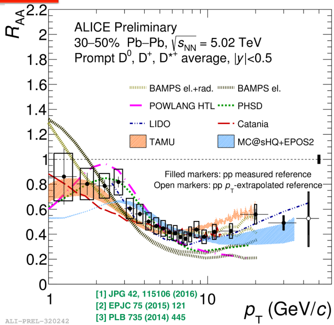


This was verified to be actually the case (M. He, R.J. Fries and R. Rapp, PRC 86, 014903).

Theory-to-data comparison: a snapshot of recent results

New





$$v_n \equiv \langle \cos[n(\phi - \Psi_n)] \rangle \qquad R_{\mathrm{AA}} \equiv \frac{dN/dp_T|_{\mathrm{AA}}}{\langle N_{\mathrm{coll}} \rangle \, dN/dp_T|_{\mathrm{pp}}}$$

In spite of their large mass, also the D-mesons turn out to be quenched and to have a sizable v_2. Does also charm reach local thermal equilibrium? Transport calculations are challenged to consistently reproduce this rich phenomenology.

What do we want to learn? A bit of history...

Theory and experimental verification of brownian motion by Einstein (1905) and Perrin (1909)

From the vertical distribution of an emulsion

$$n(z) = n_0 e^{-(Mg/K_B T)z}$$

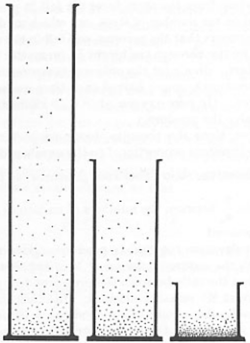


FIG. 3.

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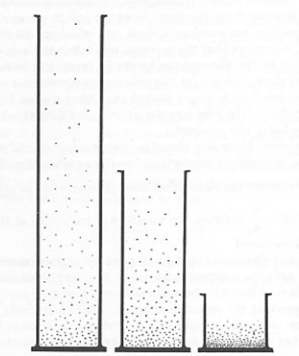

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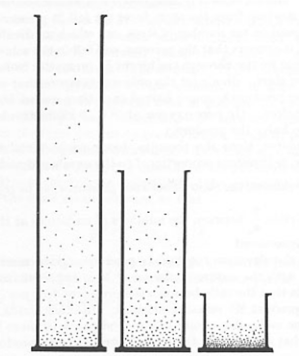

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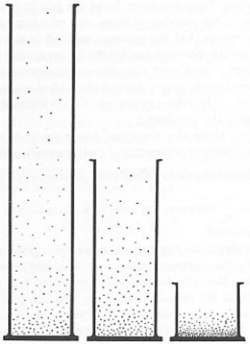

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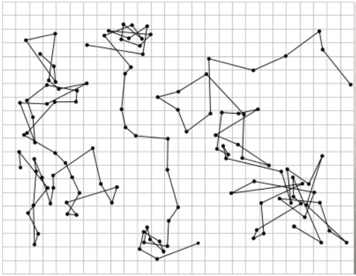
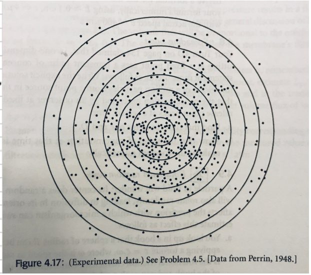
One gets an expression for the diffusion coefficient

$$D = \frac{K_B T}{6\pi a \eta}$$

What do we want to learn? A bit of history...





From the random walk of the emulsion particles (follow the motion along one direction!) one extracts the diffusion coefficient

$$< x^2 > \underset{t\to\infty}{\sim} 2Dt$$

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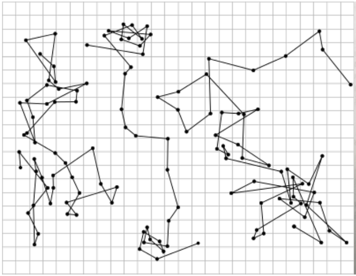

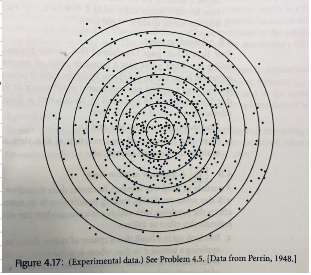

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and from Einstein formula one estimates the Avogadro number:

$$\mathcal{N}_A K_B \equiv \mathcal{R} \quad \longrightarrow \quad \mathcal{N}_A = \frac{\mathcal{R}\,T}{6\pi a\,\eta\,D}$$

Perrin obtained the values $\mathcal{N}_A \approx 5.5 - 7.2 \cdot 10^{23}$.

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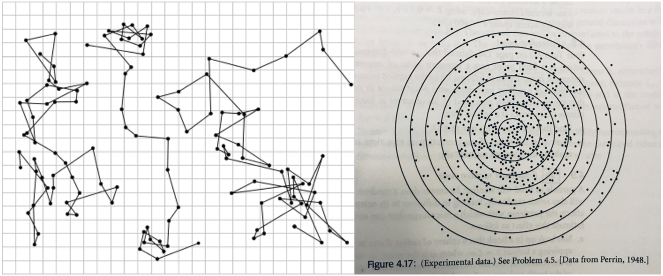

Figure 4.17: (Experimental data.) See Problem 4.5. [Data from Perrin, 1948.]

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Perrin obtained the values $\mathcal{N}_A \approx 5.5 - 7.2 \cdot 10^{23}$. We would like to derive HQ transport coefficients in the QGP with a comparable precision!

HQ transport coefficients: non-perturbative definition

One consider the non-relativistic limit of the Langevin equation for a HQ

$$\frac{dp^i}{dt} = -\eta_D p^i + \xi^i(t), \quad \text{with} \quad \langle \xi^i(t)\xi^j(t')\rangle = \delta^{ij}\delta(t-t')\kappa$$

in which the strength of the noise is given by a single number, the momentum-diffusion coefficient κ. Hence, in the $p \to 0$ limit:

$$\kappa = \frac{1}{3}\int_{-\infty}^{+\infty} dt \langle \xi^i(t)\xi^i(0)\rangle_{\rm HQ} \approx \frac{1}{3}\int_{-\infty}^{+\infty} dt\, \underbrace{\langle F^i(t)F^i(0)\rangle_{\rm HQ}}_{\equiv D^>(t)},$$

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For a static ($M=\infty$) HQ the force is due to the color-electric field:

$$\boldsymbol{F}(t) = \int d\boldsymbol{x}\, Q^\dagger(t,\boldsymbol{x}) t^a Q(t,\boldsymbol{x}) \boldsymbol{E}^a(t,\boldsymbol{x})$$

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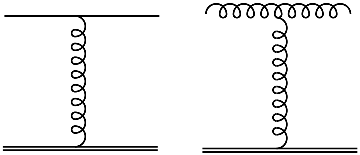
The above non-perturbative definition, referring to the $M \to \infty$ limit, is the starting point for a thermal-field-theory evaluation based on

- weak-coupling calculations (up to NLO);
- gauge-gravity duality ($\mathcal{N} = 4$ SYM)
- lattice-QCD simulations

HQ momentum diffusion: weak-coupling calculation



- HQ momentum diffusion due to scattering with light quarks and gluons

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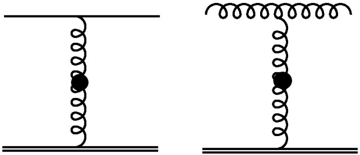

- HQ momentum diffusion due to scattering with light quarks and gluons
- Already the tree-level result actually contains higher-order (all order!) corrections due to the screening of the interaction

$$\frac{1}{\vec{q}^2} \longrightarrow \frac{1}{\vec{q}^2 + m_D^2} \quad \text{with} \quad m_D \sim gT$$

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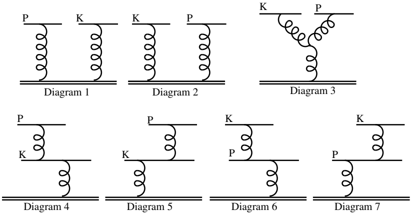

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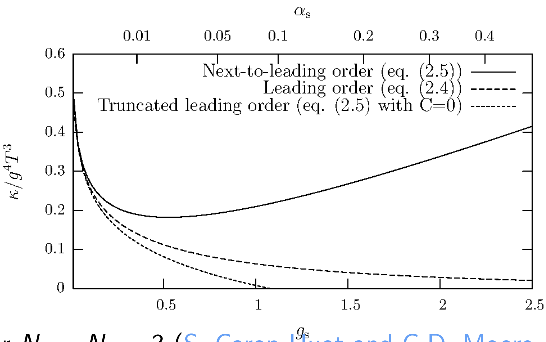
$$\frac{1}{\vec{q}^2} \quad \longrightarrow \quad \frac{1}{\vec{q}^2 + m_D^2} \quad \text{with} \quad m_D \sim gT$$

- Further $\mathcal{O}(g)$ corrections to κ arise from overlapping scatterings. Having a total scattering rate $\sim g^2 T$ and the duration of a single scattering $\sim 1/q \sim 1/gT$ entails that a fraction $\mathcal{O}(g)$ of scattering events overlap with each other (see diagrams)

HQ momentum diffusion: weak-coupling calculation



One gets, for $N_f = N_c = 3$ (S. Caron-Huot and G.D. Moore, JHEP 0802 (2008) 081),

$$\kappa = \frac{16\pi}{3}\alpha_s^2 T^3 \left(\ln\frac{1}{g} + 0.07428 + 1.9026g + \mathcal{O}(g^2) \right)$$

- For realistic values of the coupling $\alpha_s \sim 0.3$ NLO corrections to κ are large!
- NLO result limited to the $M = \infty$ case

HQ momentum diffusion: lattice-QCD

Getting the HQ momentum-diffusion coefficient requires to evaluate

$$\kappa = \frac{1}{3}\int_{-\infty}^{+\infty} dt \langle \xi^i(t)\xi^i(0)\rangle_{\rm HQ} = \frac{1}{3}\int_{-\infty}^{+\infty} dt\, \underbrace{\langle F^i(t)F^i(0)\rangle_{\rm HQ}}_{\equiv D^>(t)}$$

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From the lattice one can get only the euclidean correlator ($t = -i\tau$)

$$D_E(\tau) = -\frac{\langle {\rm Re\,Tr}[U(\beta,\tau)gE^i(\tau,\mathbf{0})U(\tau,0)gE^i(0,\mathbf{0})]\rangle}{\langle {\rm Re\,Tr}[U(\beta,0)]\rangle}$$

How to proceed?

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How to proceed? κ comes from the $\omega \to 0$ limit of the FT of $D^>$. In a thermal ensemble $\sigma(\omega) \equiv D^>(\omega) - D^<(\omega) = (1 - e^{-\beta\omega})D^>(\omega)$, so that

$$\kappa \equiv \lim_{\omega\to 0}\frac{D^>(\omega)}{3} = \lim_{\omega\to 0}\frac{1}{3}\frac{\sigma(\omega)}{1-e^{-\beta\omega}} \underset{\omega\to 0}{\sim} \frac{1}{3}\frac{T}{\omega}\sigma(\omega)$$

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From $D_E(\tau)$ one extracts the spectral density according to

$$D_E(\tau) = \int_0^{+\infty} \frac{d\omega}{2\pi}\frac{\cosh(\tau - \beta/2)}{\sinh(\beta\omega/2)}\sigma(\omega)$$

HQ momentum diffusion: lattice-QCD

The direct extraction of the spectral density from the euclidean correlator

$$D_E(\tau) = \int_0^{+\infty} \frac{d\omega}{2\pi} \frac{\cosh(\tau - \beta/2)}{\sinh(\beta\omega/2)} \sigma(\omega)$$

is a ill-posed problem, since the latter is known for a limited set (~ 20) of points $D_E(\tau_i)$, and one wishes to obtain a fine scan of the the spectral function $\sigma(\omega_j)$. A direct χ^2-fit is not applicable.

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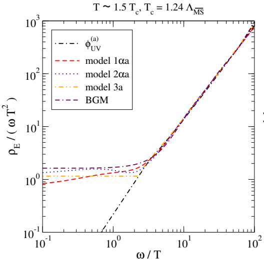
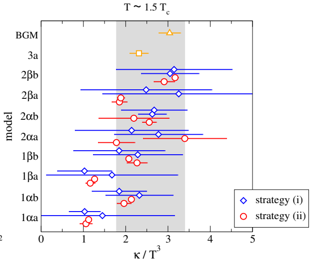
- Bayesian techniques (Maximum Entropy Method)
- Theory-guided ansatz for the behaviour of $\sigma(\omega)$ to constrain its functional form (A. Francis *et al.*, PRD 92 (2015), 116003)





From the different ansatz on the functional form of $\sigma(\omega)$ one gets a systematic uncertainty band:

$$\kappa/T^3 \approx 1.8 - 3.4$$

Collisional broadening in the non-static case

In the case of experimental interest HQ's have a large but finite mass and most of the p_T-bins for which data are available refer to quite fast, or even relativistic, HF hadrons: extending the estimates for the HQ transport coefficients to finite momentum is mandatory to provide theoretical predictions relevant for the experiment.

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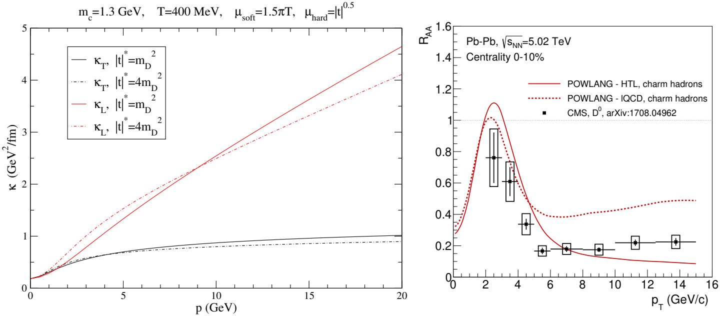
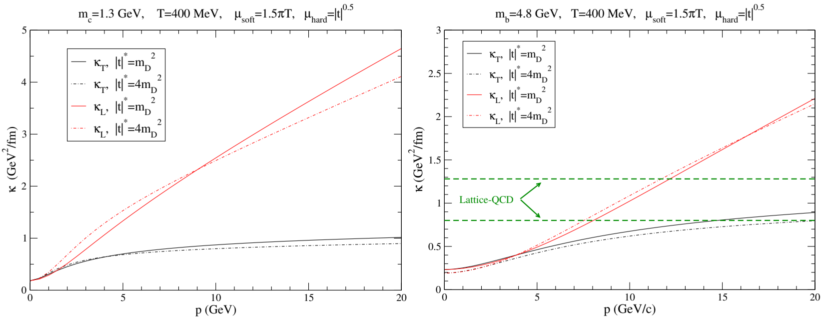


For the same hydro background, simulations with momentum dependent transport coefficients $\kappa_{T/L}$ (left panel: weak-coupling HTL calculation) leads to quite different D-meson p_T-distributions wrt to the static lattice-QCD results (A.B. *et al.*, JHEP 1802 (2018) 043).

Collisional broadening in the non-static case



Weak-coupling calculation with resummation of medium effects for soft collisions (W.M. Alberico et al., EPJC 73 (2013) 2481):

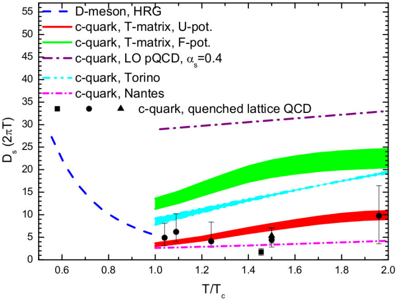
- strong momentum dependence for charm quarks
- milder momentum dependende for beauty, with $\kappa_L \approx \kappa_T$ up to 5 GeV

From momentum broadening to spatial diffusion

In the *non-relativistic* limit an excess of HQ's initially placed at the origin will diffuse according to

$$\langle \vec{x}^2(t) \rangle \underset{t\to\infty}{\sim} 6D_s t \quad \text{with} \quad D_s = \frac{2T^2}{\kappa}.$$

For a strongly interacting system spatial diffusion is very small! Theory calculations for D_s have been collected (F. Prino and R. Rapp, JPG 43 (2016) 093002) and are often used by the experimentalists to summarize the difference among the various models (BUT momentum dependence, not captured by D_s, is important!)



- lattice-QCD

$$(2\pi T)D_s^{lQCD} \approx 3.7 - 7$$

- $\mathcal{N} = 4$ SYM:

$$(2\pi T)D_s^{SYM} = \frac{4}{\sqrt{g_{SYM}^2 N_c}} \approx 1.2$$

for $N_c = 3$ and $\alpha_{SYM} = \alpha_s = 0.3$.

From quarks to hadrons

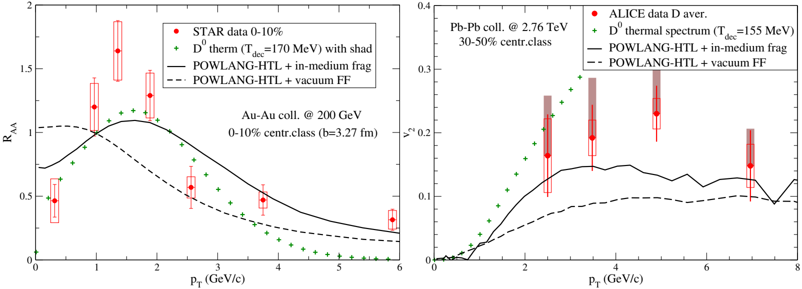
In the presence of a medium, rather then fragmenting like in the vacuum (e.g. $c \to cg \to c\overline{q}q$), HQ's can hadronize by recombining with light thermal quarks (or even *diquarks*) from the medium. This has been implemented in several ways in the literature:

- $2 \to 1$ (or $3 \to 1$ for baryon production) coalescence of partons close in phase-space: $Q + \overline{q} \to M$
- String formation: $Q + \overline{q} \to \mathrm{string} \to \mathrm{hadrons}$
- Resonance formation/decay $Q + \overline{q} \to M^\star \to Q + \overline{q}$

In-medium hadronization may affect the R_{AA} and v_2 of final D-mesons due to the *collective (radial and elliptic) flow* of light quarks. Furthermore, it can change the HF hadrochemistry, leading for instance to and enhanced productions of strange particles (D_s) and baryons (Λ_c): no need to excite heavy $s\overline{s}$ or diquark-antidiquark pairs from the vacuum as in elementary collisions, a lot of thermal partons available nearby! Selected results will be shown in the following.

From quarks to hadrons: *kinematic* effect on R_{AA} and v_2

Experimental D-meson data show a peak in the R_{AA} and a sizable v_2 one would like to interpret as a signal of *charm radial flow and thermalization* (green crosses: kinetic equilibrium, decoupling from FO hypersurface)



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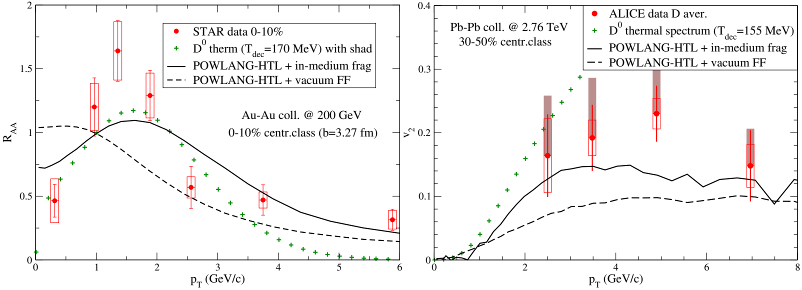
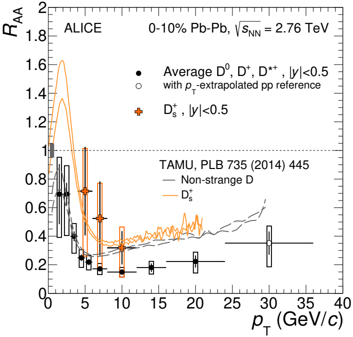


However, *comparing transport results with/without the boost* due to $u^{\mu}_{\rm fluid}$, at least part of the effect might be due to the radial and elliptic flow of the light partons from the medium picked-up at hadronization (POWLANG results A.B. et al., in EPJC 75 (2015) 3, 121).

From quarks to hadrons: HF hadrochemistry

The abundance of strange quarks in the plasma can lead e.g. to an enhanced production of D_s mesons wrt p-p collisions via $c + \bar{s} \to D_s$



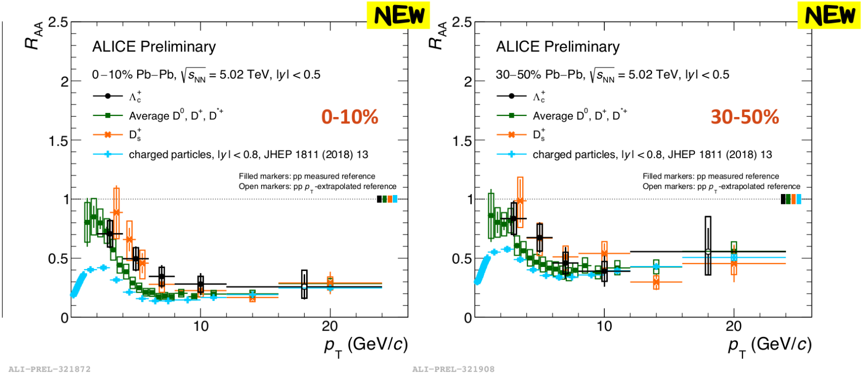
ALICE data for D and D_s mesons (JHEP 1603 (2016) 082) compared with TAMU-model predictions (M- He et al., PLB 735 (2014) 445)

Langevin transport simulation in the QGP + hadronization modeled via

$$\left(\partial_t + \vec{v}\cdot\vec{\nabla}\right) F_M(t,\vec{x},\vec{p}) = -\underbrace{(\Gamma/\gamma_p)F_M(t,\vec{x},\vec{p})}_{M\to Q+\bar{q}} + \underbrace{\beta(t,\vec{x},\vec{p})}_{Q+\bar{q}\to M}$$

$$\text{with} \quad \sigma(s) = \frac{4\pi}{k^2}\frac{(\Gamma m)^2}{(s-m^2)^2+(\Gamma m)^2}$$

From quarks to hadrons: HF hadrochemistry

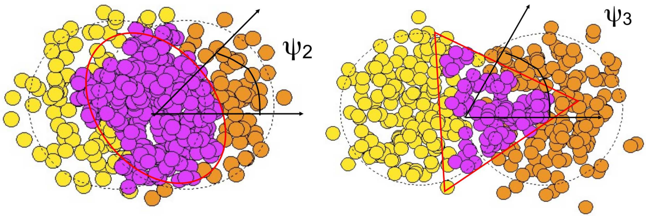


Also data on Λ_c baryon in HIC's now available

Some recent developments

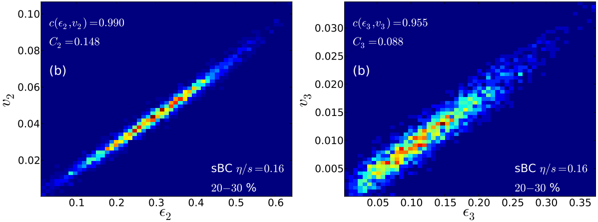
- Event-by-event fluctuations: odd harmonics (v_3) and event-shape engineering;
- Directed flow v_1: access to initial conditions, thermalization and magnetic field?

Event-by-event fluctuations



- The random distribution of nucleons can lead to different geometric deformations (elliptic, triangular...) for the same impact parameter. Odd anisotropies (triangular, pentagonal...) can only arise from EBE fluctuations;

Event-by-event fluctuations



- The random distribution of nucleons can lead to different geometric deformations (elliptic, triangular...) for the same impact parameter. Odd anisotropies (triangular, pentagonal...) can only arise from EBE fluctuations;
- One observes, for *light hadrons*, that $v_n \sim \epsilon_n$ for $n = 2, 3$: anisotropy of particle distribution proportional to geometric eccentricity.

Event-by-event fluctuations and odd flow-harmonics

The study of odd flow-harmonics (v_3, v_5) in AA collisions requires a modeling of initial-state event-by-event fluctuations. We perform a Glauber-MC sampling of the initial conditions, each one characterized by a *complex eccentricity*

$$s(\boldsymbol{x}) = \frac{K}{2\pi\sigma^2}\sum_{i=1}^{N_{\rm coll}} \exp\left[-\frac{(\boldsymbol{x}-\boldsymbol{x}_i)^2}{2\sigma^2}\right] \quad \longrightarrow \quad \epsilon_m e^{im\Psi_m} \equiv -\frac{\left\{r^2 e^{im\phi}\right\}}{\left\{r^2\right\}}$$

with orientation and modulus given by

$$\Psi_m = \frac{1}{m}\,\mathrm{atan2}\left(-\{r^2 \sin(m\phi)\}, -\{r^2\cos(m\phi)\}\right)$$

$$\epsilon_m = \frac{\sqrt{\{r_\perp^2\cos(m\phi)\}^2 + \{r_\perp^2\sin(m\phi)\}^2}}{\{r_\perp^2\}} = -\frac{\{r^2\cos[m(\phi-\Psi_m)]\}}{\{r^2\}}$$

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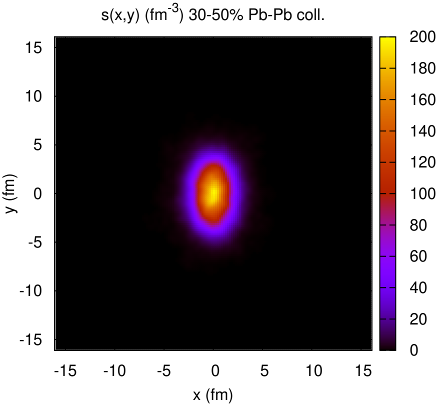
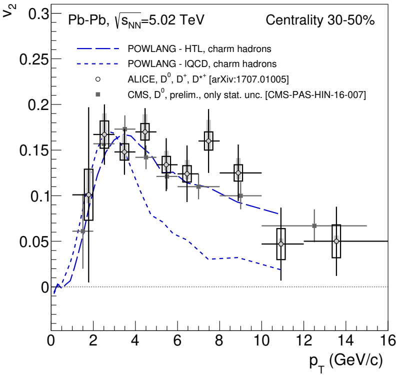
Exploiting the fact that, on an event-by-event basis, for $m = 2, 3$ $v_m \sim \epsilon_m$ one can again consider an *average background* obtained summing all the events of a given centrality class, each one rotated by its *event-plane* angle ψ_m, depending on the harmonic one is considering.

Event-by-event fluctuations and odd flow-harmonics





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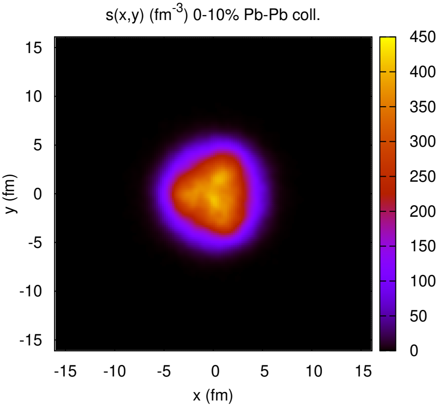

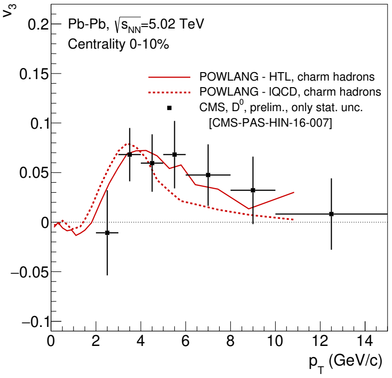

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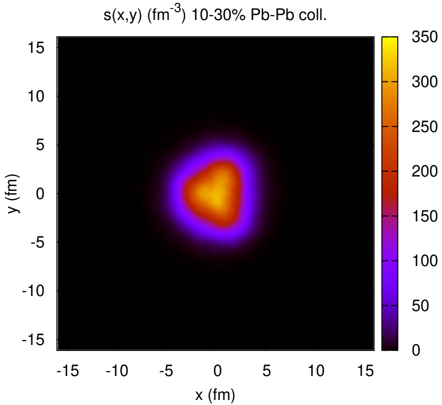

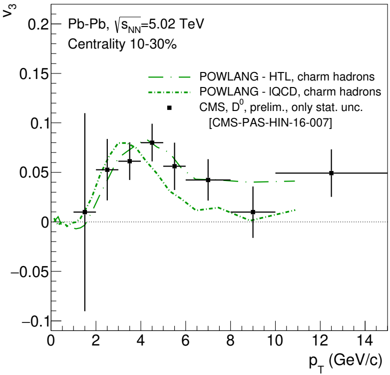

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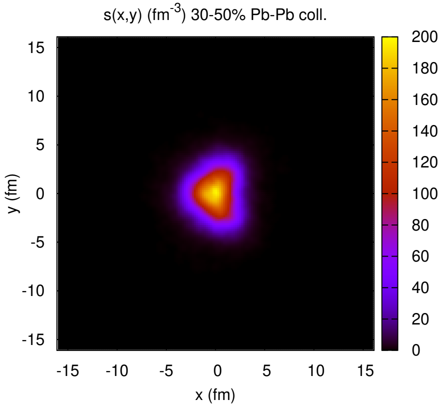

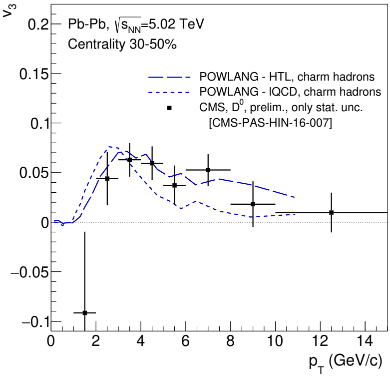

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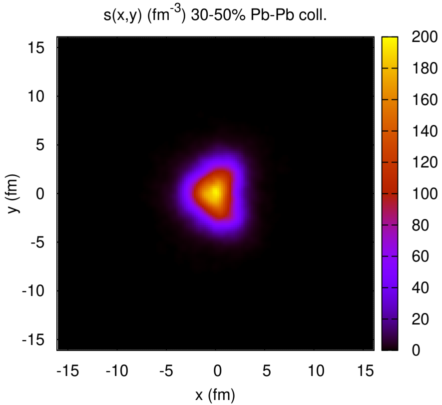
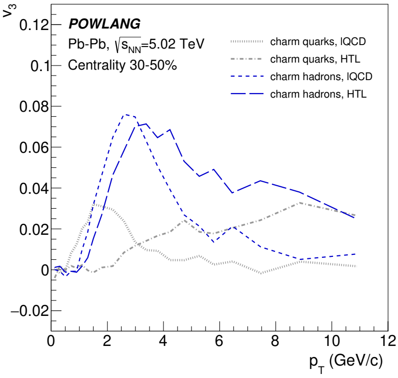
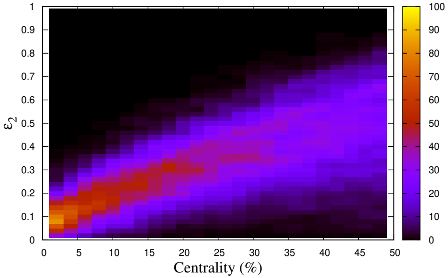
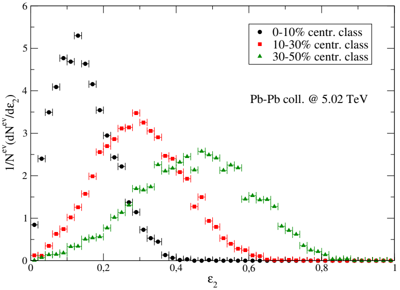




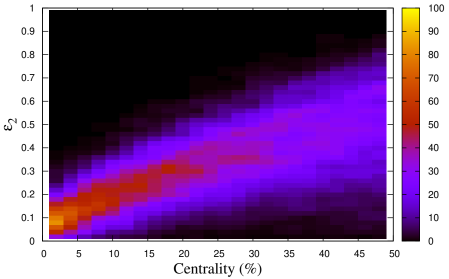
- CMS and ALICE data for *D*-meson $v_{2,3}$ satisfactory described (A.B. *et al.*, JHEP 1802 (2018) 043);
- Recombination with light quarks at hadronization provides a relevant contribution to the *D*-meson v_n;

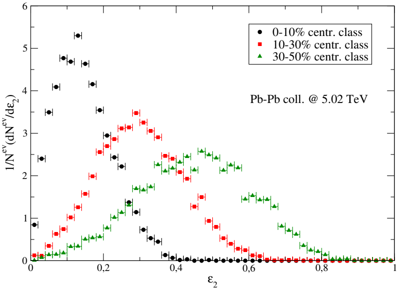




Very broad eccentricity distribution within a given centrality class!

Event-shape-engineering

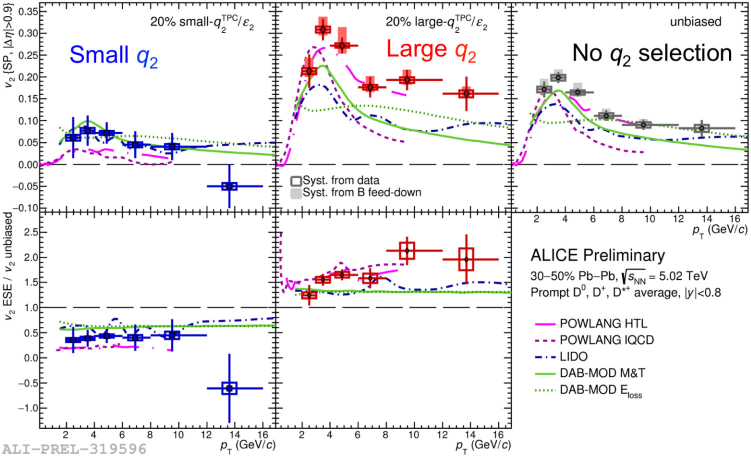




Very broad eccentricity distribution within a given centrality class!
One selects events of similar centrality, but very different initial eccentricity ϵ_2 (th.) or average elliptic flow of light hadrons q_2 (exp.)

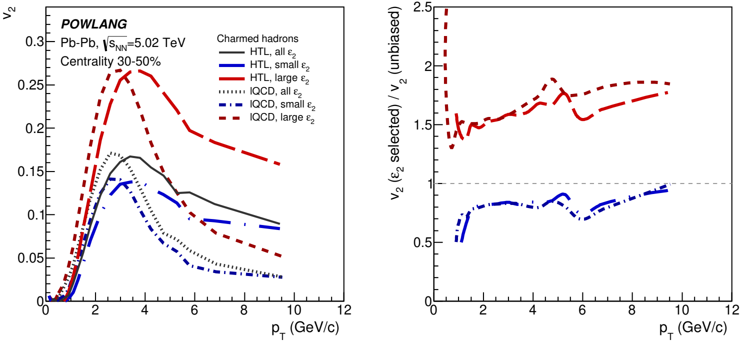
$$\epsilon_2 = \frac{\sqrt{\{r_\perp^2 \cos(2\phi)\}^2 + \{r_\perp^2 \sin(2\phi)\}^2}}{\{r_\perp^2\}} \qquad \text{Glauber} - \text{MC}$$

$$q_{2x} = \sum_{i=1}^{M} \cos(2\phi_i)/M \quad q_{2y} = \sum_{i=1}^{M} \sin(2\phi_i)/M \qquad \text{detected hadrons}$$



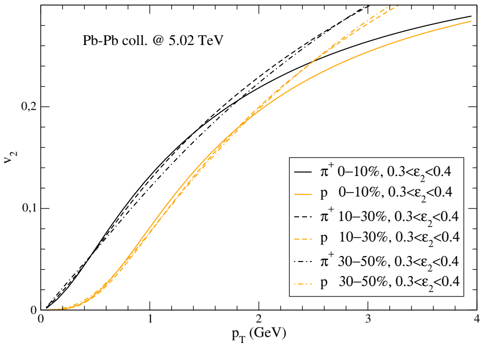
Various transport models reproduce quite well the ratio $v_2^{\mathrm{ESE}} / v_2^{\mathrm{unbiased}}$

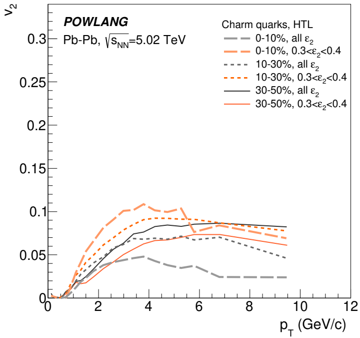
Event-shape-engineering: a deeper insight



Both v_2^{ESE} and v_2^{unbiased} are affected by the strength of the HQ-medium interaction, but the ratio $v_2^{\mathrm{ESE}}/v_2^{\mathrm{unbiased}}$ of charm hadrons displays only a mild dependence on the HQ transport coefficients (A.B. *et al.*, Eur.Phys.J. C79 (2019) no.6, 494).

Event-shape-engineering: a deeper insight

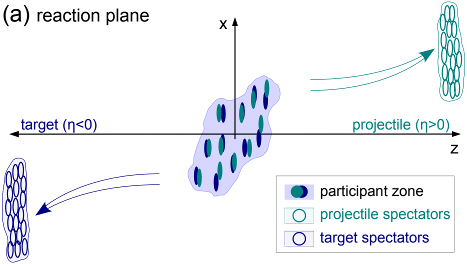




A complementary approach would consist in selecting events of similar eccentricity, but belonging to different centrality class:

- Light hadrons display a very similar flow, independent from centrality;
- The incomplete thermalization of charm quarks leads to lower values of v_2 going from more central to more peripheral events

HF directed flow: initial tilted geometry



- Participant nucleons tend to deposit more energy along the direction of their motion $\longrightarrow$ tilted geometry of the fireball;

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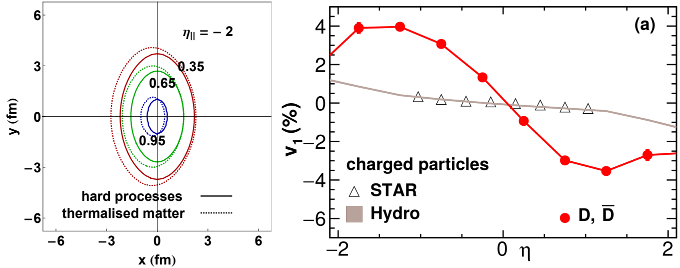
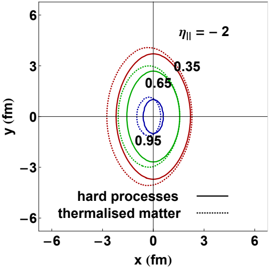
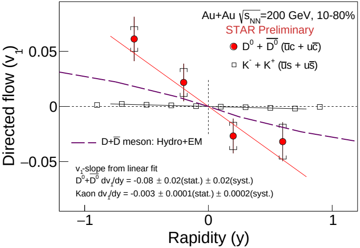


- Participant nucleons tend to deposit more energy along the direction of their motion $\longrightarrow$ tilted geometry of the fireball;
- HQ's on the other hand are distributed according to $n_{\rm coll}(\vec{x}_\perp)$, with no F/B asymmetry, longitudinal position fixed by their initial rapidity

This leads, for non zero rapidity, to a sizable D-meson directed flow v_1, much larger then the one of light hadrons (S. Chatterjee and P. Bozek, PRL 120 (2018), 192301).





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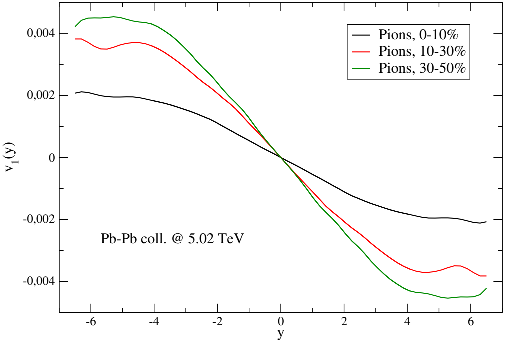
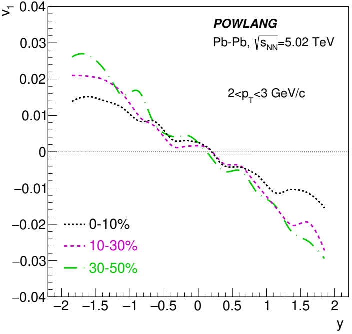
This leads, for non zero rapidity, to a sizable *D*-meson directed flow v_1, much larger then the one of light hadrons (S. Chatterjee and P. Bozek, PRL 120 (2018), 192301). Notably, $v_1^D \approx 0$ both in the case of no interaction and in the case of full thermalization of HQ's with the medium: $v_1^D \gg v_1^{\rm light}$ potentially provides a rich information!

HF directed flow: work in progress





Much larger v_1 signal for D mesons than for light hadrons!

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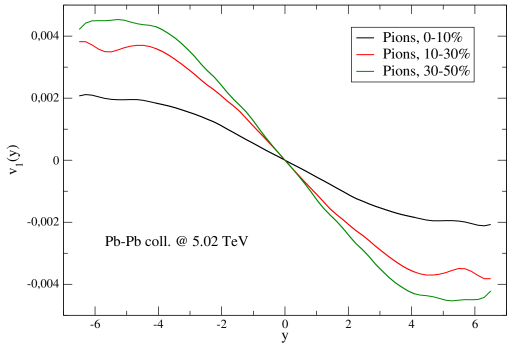

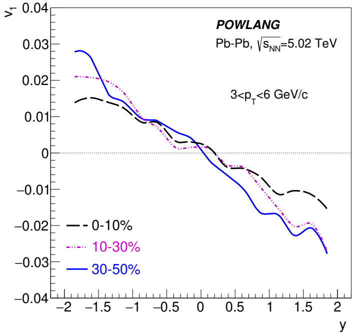

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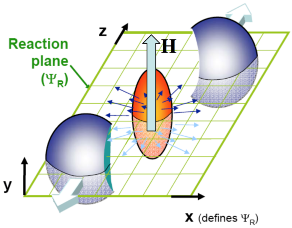
HF directed flow: signature of the EM field?



Colliding nuclei generate a huge initial magnetic field $B \sim 10^{15}$ T

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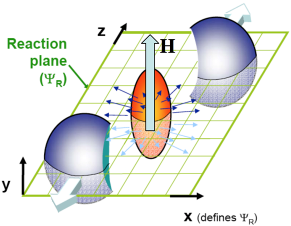

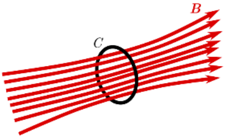

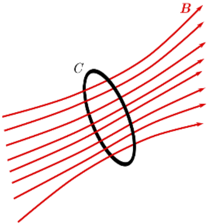

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- If the produced medium is an ideal conductor ($\sigma_E \to \infty$) this field decays very slowly (Alfvén theorem, flux-freezing);

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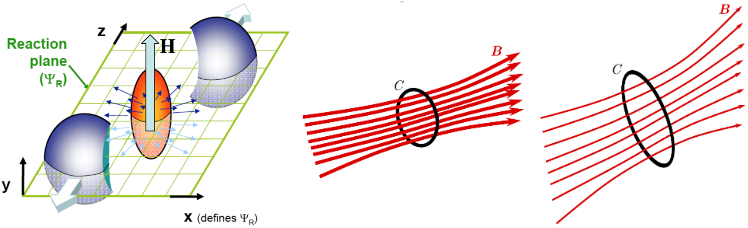

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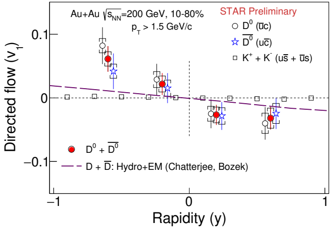

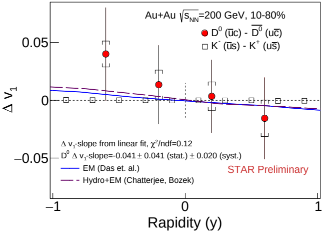

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The Langevin equation can be corrected to account for the Lorentz force:

$$\Delta \vec{p}/\Delta t = -\eta_D \vec{p} + \vec{\xi} + Q(\vec{E} + \vec{v} \times \vec{B})$$

This could lead to a different v_1 for D^0 and $\overline{D}^0$, which could be explained as due to the EM interaction in the QGP phase (S. Chatterjee and P. Bozek arXiv:1804.04893, S.K. Das *et al.*, Phys.Lett. B768 (2017) 260-264)

Summary and outlook

- Solid first-principle theory calculations still limited to a range of masses ($M \to \infty$) and/or couplings ($g \ll 1$) of limited experimental relevance, although some consistent semi-quantitative information (e.g. for κ) can be in any case obtained;

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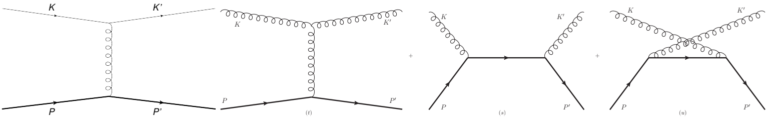
- Wait for beauty measurement at low p_T to have a safe framework to extract transport coefficients

Back-up slides



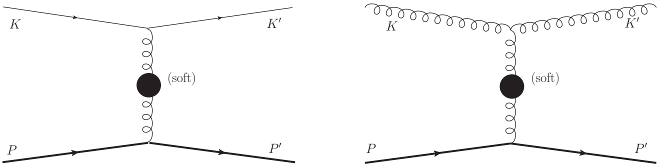
$$\kappa_T^{g/q(\mathrm{hard})} = \frac{1}{2}\frac{1}{2E}\int_k \frac{n_{B/F}(k)}{2k}\int_{k'}\frac{1 \pm n_{B/F}(k')}{2k'}\int_{p'}\frac{1}{2E'}\theta(|t|-|t|^*)\times$$
$$\times\,(2\pi)^4\delta^{(4)}(P+K-P'-K')\left|\overline{\mathcal{M}}_{g/q}(s,t)\right|^2 q_T^2$$

$$\kappa_L^{g/q(\mathrm{hard})} = \frac{1}{2E}\int_k \frac{n_{B/F}(k)}{2k}\int_{k'}\frac{1 \pm n_{B/F}(k')}{2k'}\int_{p'}\frac{1}{2E'}\theta(|t|-|t|^*)\times$$
$$\times\,(2\pi)^4\delta^{(4)}(P+K-P'-K')\left|\overline{\mathcal{M}}_{g/q}(s,t)\right|^2 q_L^2$$

where: $(|t| \equiv q^2 - \omega^2)$.

NB At high momentum also Compton-like diagrams give a non-negligible contribution ($\neq$ static calculation)

Transport coefficients $\kappa_{T/L}(p)$: soft contribution



When the exchanged 4-momentum is **soft** the t-channel gluon feels the presence of the medium and requires **resummation**.
The *blob* represents the *dressed gluon propagator*, which has longitudinal and transverse components:

$$\Delta_L(z,q) = \frac{-1}{q^2 + \Pi_L(z,q)}, \quad \Delta_T(z,q) = \frac{-1}{z^2 - q^2 - \Pi_T(z,q)},$$

where *medium effects* are embedded in the HTL gluon self-energy.
NB In the corresponding static calculation only longitudinal gluon exchange, dressed simply by a Debye mass, without any energy and momentum dependence