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The holographic QCD axion

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Based on

- FB, A. Caddeo, A.L. Cotrone, P. Di Vecchia, A. Marzolla, 1906.12117
- FB, A.L. Cotrone, M. Järvinen, E. Kiritsis, 1906.12132

Plan

- The QCD axion
- Holography
- The model
- Axion couplings to nucleons
- Finite temperature

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The θ-angle in Yang-Mills

- Euclidean Yang-Mills Lagrangian

$$\mathcal{L}_\theta = \frac{1}{2g_{YM}^2} \mathrm{Tr} F_{\mu\nu} F^{\mu\nu} - \frac{i}{32\pi^2} \theta \, \epsilon^{\mu\nu\rho\sigma} \, \mathrm{Tr} F_{\mu\nu} F_{\rho\sigma}$$

- θ term breaks P,T and hence CP.
- θ multiplies topological charge density, whose 4d integral is integer.
- Physics periodic under $\theta \rightarrow \theta + 2\pi$
- Hard problem: need non-perturbative tools to study θ dependence
- Challenging on the Lattice (sign problem: imaginary term)

The θ-angle in QCD

a) With massless quarks: $U(1)_A$ anomaly

$$\psi \to e^{i\alpha\gamma_5}\,\psi \qquad [d\psi][d\bar{\psi}] \to \exp\left(\frac{-i\,\alpha \quad N_f}{32\pi^2}\int d^4x\,\epsilon_{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}\right)[d\psi][d\bar{\psi}]$$

$$\theta \to \theta - 2\alpha N_f$$

- Theta rotated away by chiral rotation.

b) With massive quarks: $\bar{\theta} = \theta + \arg\det \mathrm{M}$

- Neutron Electric Dipole Moment: $d_n \sim \bar{\theta}\,\frac{e m_\pi^2}{m_n^3} \approx 10^{-16}\bar{\theta}\,e\,\mathrm{cm}$
- Experiments: $|d_n| < 2.9 \times 10^{-26} e\,\mathrm{cm}$ so $\bar{\theta} < 10^{-10}$
- **Strong CP problem**: why is $\bar{\theta}$ so tiny?

The QCD axion

- Peccei-Quinn: hidden $U(1)_{PQ}$ chiral symmetry can solve strong CP problem.

- It must be anomalous just like $U(1)_A$ in QCD and spontaneously broken.
- The related pseudo-Goldstone boson is the axion.
- Just like the η', it gets a mass due to the anomaly.
- θ rotated away by PQ-chiral symmetry $\leftrightarrow$ axion potential miminized at $\theta = \langle a \rangle / f_a$

- The axion is a candidate dark matter constituent, if 10^8 GeV $\leq f_a \leq 10^{17}$ GeV

- UV embedding? (Many)
- Can we predict how axion would couple to IR degrees of freedom?

- UV embeddings should guarantee that the PQ mechanism works ("quality of the axion")
- If not, then there could be some residual θ angle.
- This may induce novel CP-odd axion couplings: how to compute in the IR?

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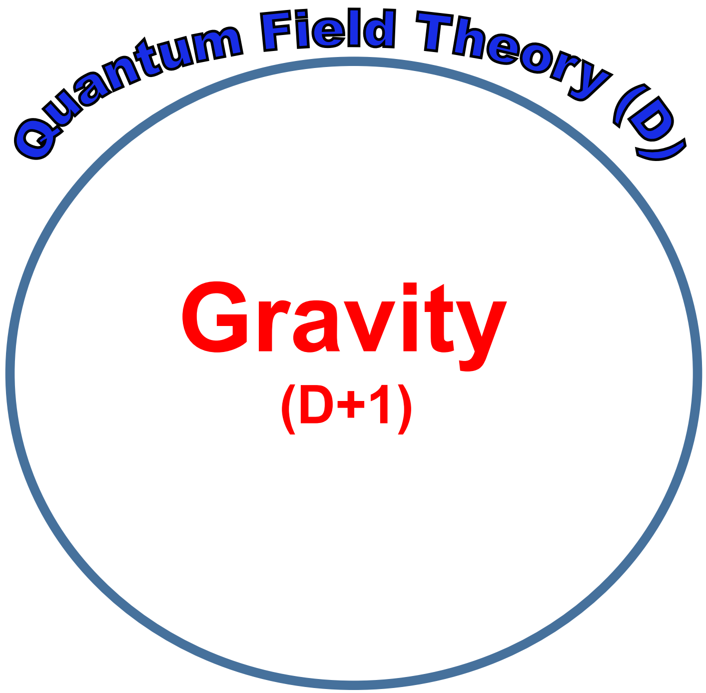
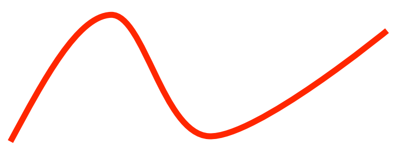
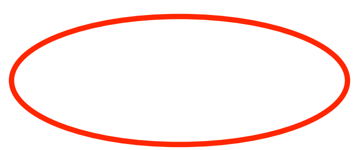
The holographic conjecture

Quantum Gravity in D+1 dimensions	=	Quantum Field Theory in D dimensions

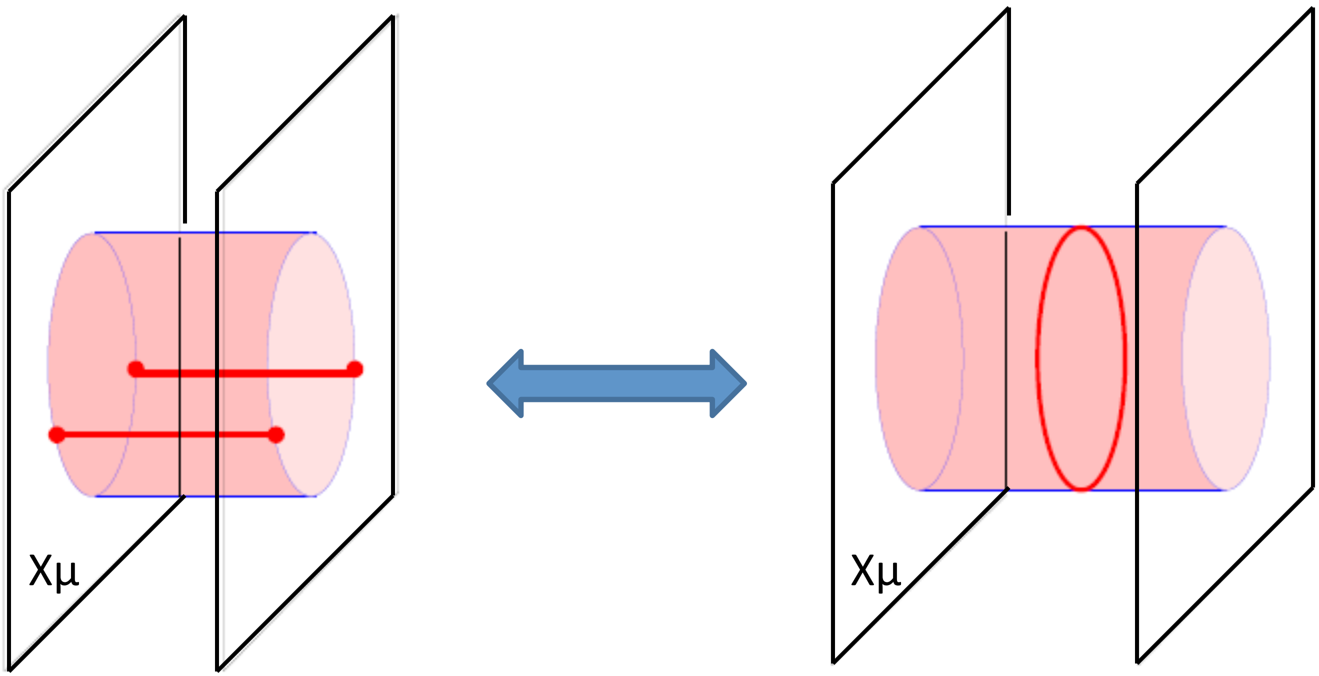


A hint : String theory

- Assumption: fundamental constituents are string-like
- Point particles are different modes of a vibrating string

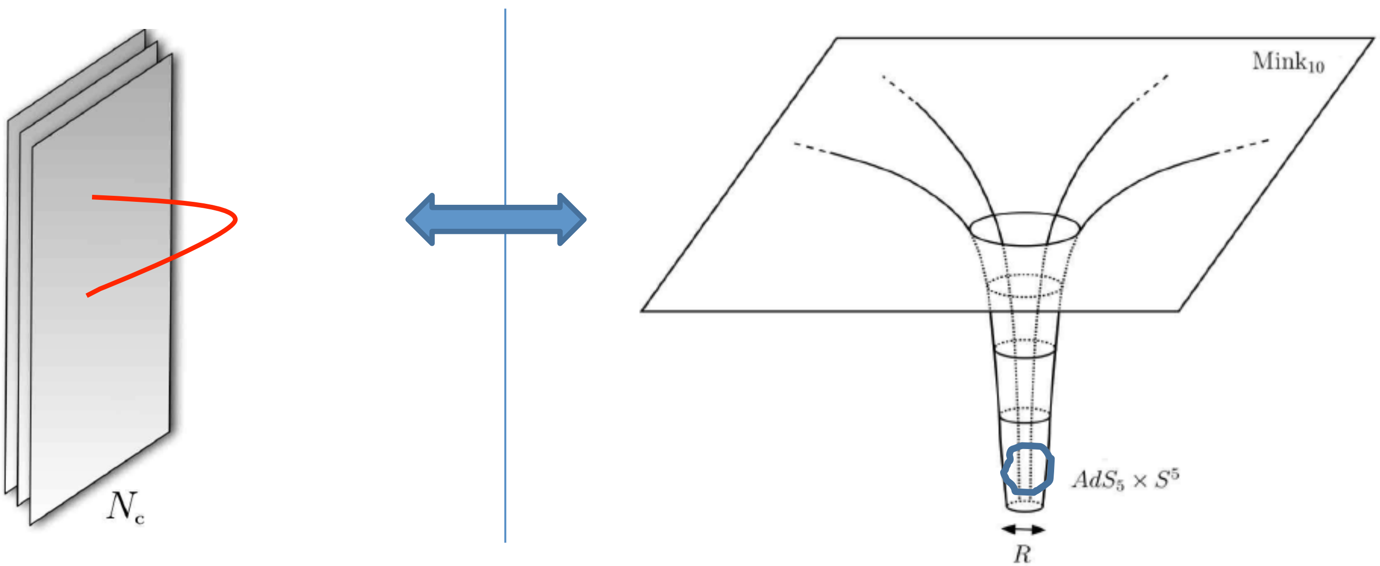
String		Particle
Open		photon (gluon) +…
Closed		graviton +…..

- Open/closed string duality (or: 2 ways of drawing a cylinder)



Open string loop (quantum)	Closed string propagation (classical)
Quantum Field Theory	Theory of gravity
Xμ, u (RG scale)	Xμ, r (extra dimension)

- The dual nature of Dp-branes [Polchinski, 95]

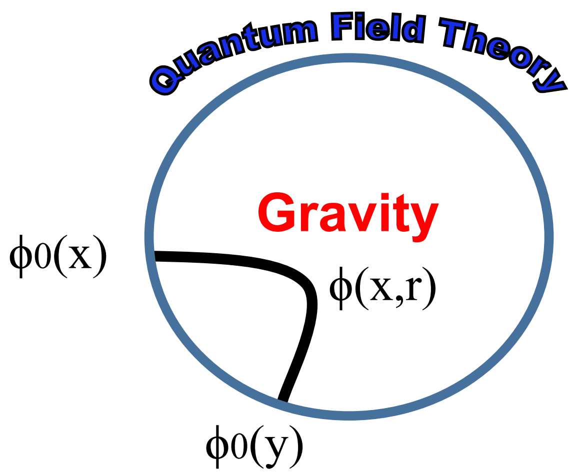


- Taking low energy limit on both sides: two interacting theories [J.M. Maldacena, 97]:
- Left: 4d SU(N_c) susy Yang-Mills **(CFT)**. From open strings on Nc D3-branes
- Right: closed IIB strings (gravity) on **Anti-de-Sitter** 5d background (times S^5)
- $N_c >> 1$, $\lambda = g^2_{YM} N_c >> 1$ in QFT ⟺ Classical theory of gravity

Holography at work

[Maldacena, 97; Witten; Gubser, Klebanov, Polyakov, 98]

- Quantum Field Theory in D dim. ⟷ Gravity in D+1 dim.



$$\boxed{Z_{QFT}[\phi_0] = Z_{QG/String} \approx e^{iS_{gravity}[\phi_0]}\Big|_{\text{“}\phi(x,r)\to\phi_0(x)\text{”}}}$$

- QFT phase (finite T, μ, …) ⟷ Gravity background (black hole, charged bh, …)
- QFT operator ($T_{\mu\nu}$, J_μ, Tr F^2, …) ⟷ Gravity field ($g_{\mu\nu}$, A_μ , ϕ, …)
- Global symmetry (e.g. Chiral) ⟷ Gauge symmetry

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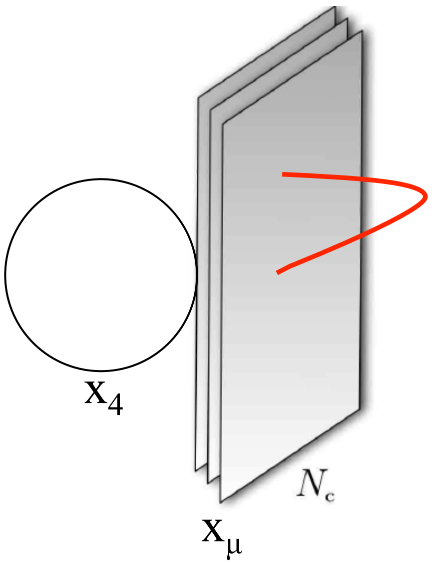
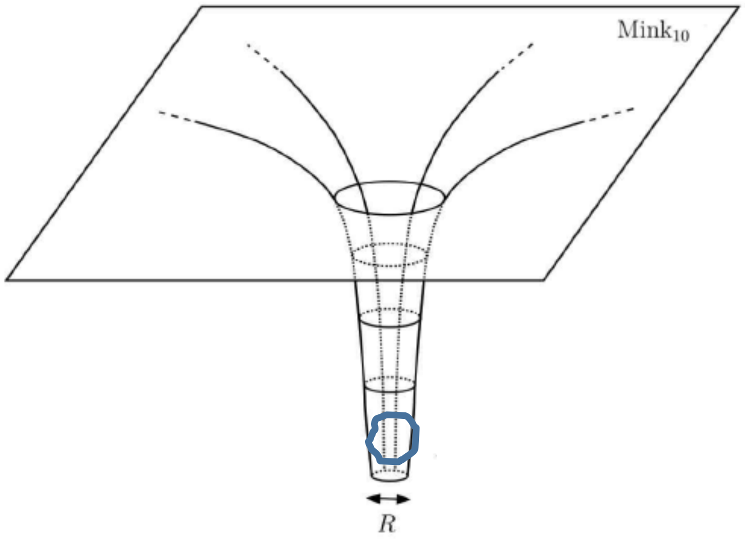
Holographic Yang-Mills [Witten 1998]

$\boxed{\text{SU}(N_c) \text{ Yang-Mills in 3+1 dimensions + massive adjoint KK fields}}$

- N_c D4-branes on circle S^1_{x4}, radius $R_4 = 1/M_{KK}$, antiperiodic fermions.
- Low energy: 4d non-susy $SU(N_c)$ Yang-Mills + massive adjoint KK modes
- Dual description: gravity solution sourced by wrapped D4-branes





Holographic Yang-Mills [Witten 1998]

- Gravity action (closed string description)

$$S = \frac{1}{2k_0^2}\int d^{10}x\sqrt{-g}\left[e^{-2\phi}\left(\mathcal{R}+4(\partial\phi)^2\right)-\frac{1}{2}|F_4|^2-\frac{1}{2}|F_2|^2\right]$$

- Gauge theory action (open string description, IR)

$$S = \frac{1}{8\pi g_s l_s M_{KK}}\int d^4x\ \mathrm{Tr}F_{\mu\nu}F^{\mu\nu} - \frac{i}{32\pi^2 l_s}\int_{S_{x^4}} C_1 \int d^4x\ \epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}F_{\mu\nu}F_{\rho\sigma}$$

$$\boxed{F_2 = d\,C_1} \qquad \boxed{\theta + 2\pi k = \frac{1}{l_s}\int_{S_{x_4}} C_1}$$

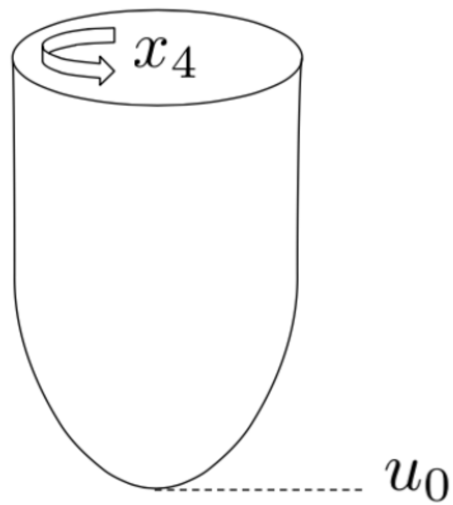
- Classical gravity picture dual to gauge theory at $\lambda_4 \gg 1,\ N_c \gg 1$

$$\lambda_4 = g_{YM}^2 N_c = 2\pi g_s l_s M_{KK} N_c$$

Holographic Yang-Mills [Witten 1998]

$$ds^2 = \left(\frac{u}{R}\right)^{3/2}\left[dx_\mu dx^\mu + f(u)dx_4^2\right] + \left(\frac{u}{R}\right)^{-3/2}\left[\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right]$$

$$e^\phi = g_s\left(\frac{u}{R}\right)^{3/4}, \qquad F_4 = \frac{3N_c}{4\pi}\omega_4\,, \qquad f(u) = 1 - \frac{u_0^3}{u^3}. \qquad R = (\pi g_s N_c)^{1/3} l_s$$



- $x_4 \sim x_4 + 2\pi/M_{KK}$
- (u,x_4) subspace is a cigar
- $g_{00}(u_0) \neq 0$ (regular metric)
- ✓ IR: Confinement and mass gap
- UV: very different from pure YM

$$u_0 = \frac{4R^3}{9}M_{KK}^2$$

- To leading order in θ/Nc treat C_1 as a probe [Witten 1998]:

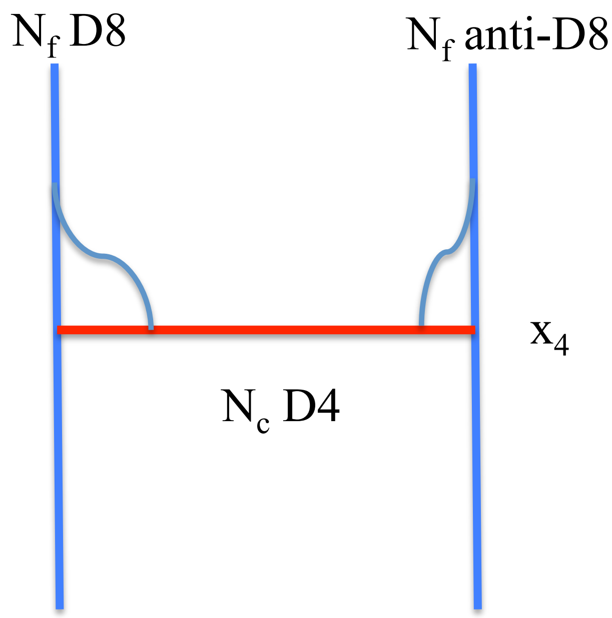
$$C_1 = \frac{\Theta}{l_s g_s} f(U) dx_4\,, \quad \text{where} \quad \Theta \equiv \frac{\lambda}{4\pi^2}\left(\frac{\theta + 2\pi k}{N_c}\right)$$

- Theta dependence of free energy from $S_\theta \sim \int d^{10}x |F_2|^2\,, \quad F_2 = dC_1$

Holographic QCD [Sakai,Sugimoto 2004]

Witten's SU(Nc) Yang-Mills + N_f massless fundamental quarks

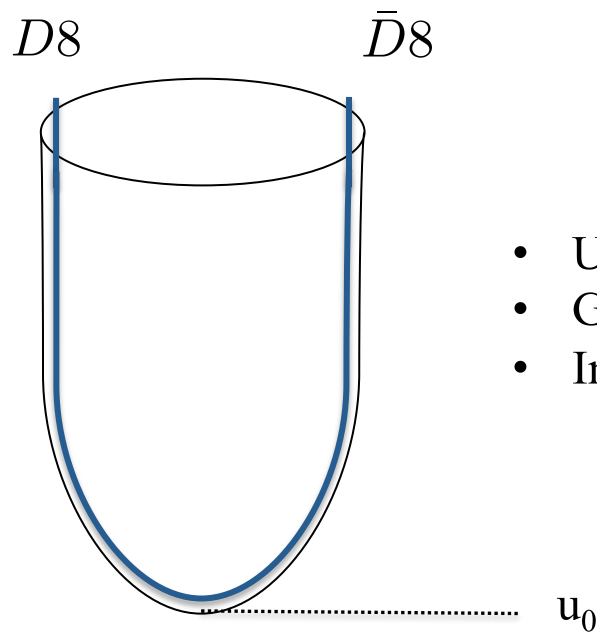
- Nf massless flavors from extra D4-D8 open strings
- $U(Nf)_L \times U(Nf)_R$ gauge symmetry on D8 dual to classical QFT chiral symm.



Holographic QCD [Sakai,Sugimoto 2004]

Witten's SU(Nc) Yang-Mills + N_f massless fundamental quarks

- At strong coupling, replace D4s by dual background.
- If $N_f << N_c$ treat D8-branes as probes.



- $U(N_f)$ gauge theory on D8
- Gauge field fluctuations = **mesons**
- Instanton solutions = **baryons**

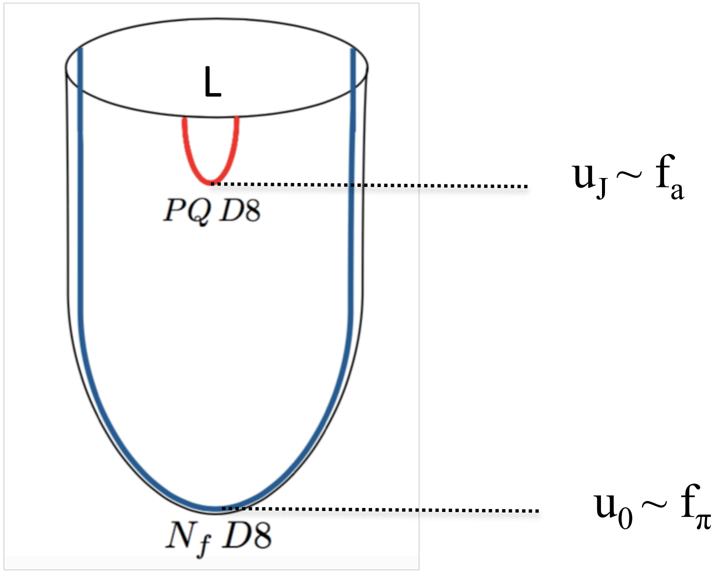
- Chiral symmetry breaking = joining of the two branches
- Pion coupling $f_\pi \sim u_0$

The Holographic QCD axion

[FB, Caddeo, Cotrone, Di Vecchia, Marzolla, ‘19]

Witten’s SU(Nc) Yang-Mills + N_f massless fundamental quarks + **1 extra quark**

- Extra massless quark flavor = extra (Peccei-Quinn) non antipodal D8-anti D8
- Spontaneous breaking of PQ symmetry = joining of extra D8



- Extra parameter $L < \pi R_4$ (or $u_J - u_0$) : related to NJL coupling of extra quark
  [Antonyan, Harvey, Jensen, Kutasov, 06]

- If $L << \pi R_4$, axion coupling $f_a >> f_\pi$

Effective action

$$S_{tot} = S_{WSS} + S_{PQ} + S_\theta + S_{mass}$$

- $$\boxed{S_\theta \sim \int d^{10}x |\tilde{F}_2|^2 \,, \quad d\tilde{F}_2 = \mathrm{Tr}\mathcal{F}^{WSS} \wedge \omega_{WSS} + F^{PQ} \wedge \omega_{PQ}}$$
- S_{mass} : mass term for WSS quarks. From worldsheet instanton between D8
  [Aharony,Kutasov 2008; Hashimoto, Hirayama, Lin, Yee 2008]
- At low energy and integrating over u and S^4: **chiral Lagrangian + axion**
  [Weinberg; … ; Di Vecchia, Rossi, Veneziano,Yankielowicz 2017]

$$\boxed{\mathcal{L}_{\text{eff}} = -\frac{f_\pi^2}{4}\mathrm{Tr}\left[\partial_\mu U \partial^\mu U^\dagger\right] - \frac{1}{2}\partial_\mu a \partial^\mu a + c\,\mathrm{Tr}\left[MU^\dagger + \text{h.c.}\right] - \frac{\chi_{\text{WYM}}}{2}\left(\theta + \frac{\sqrt{2N_f}}{f_\pi}\eta' + \frac{\sqrt{2}}{f_a}a\right)^2}$$

- $U = \mathcal{P}e^{i\int dz \mathcal{A}_z^{WSS}} = e^{\frac{2i}{f_\pi}\left(\pi^a T^a + \frac{\eta'}{\sqrt{2N_f}}\right)}$, $\qquad \boxed{\sqrt{2}\frac{a}{f_a} = \int A_z^{PQ} dz}$
- $-2c = \langle \bar{q}_i q_i \rangle$, $f_\pi^2 = \frac{N_c \lambda}{54\pi^4} M_{\text{KK}}^2 \,, \quad \chi_{\text{WYM}} = \frac{\lambda^3 M_{\text{KK}}^4}{4(3\pi)^6}$
- Axion mass: $m_a^2 = \frac{2}{f_a^2}\chi \,, \quad \chi = \frac{4c\,\chi_{WYM}}{4c + 2\chi_{WYM}\mathrm{Tr}[M^{-1}]}$
- All parameters in terms of λ, N_c, N_f , $R_4 = 1/M_{KK}$, L, M = diag (m_u, m_d , …)

The axion coupling f_a

$$f_a^2 = \frac{N_c \lambda}{16\pi^3} \frac{J^3(b)}{I(b)} \frac{1}{M_{\rm KK} L^3}\,, \qquad b \equiv \frac{u_0}{u_J}$$

$$J(b) = \frac{2}{3}\sqrt{1-b^3}\int_0^1 dy \frac{y^{\frac{1}{2}}}{(1-b^3 y)\sqrt{1-b^3 y-(1-b^3)y^{\frac{8}{3}}}}$$

$$I(b) = \int_0^1 dy \, \frac{y^{-\frac{1}{2}}}{\sqrt{1-b^3 y-(1-b^3)y^{\frac{8}{3}}}}$$

$$L = J(b) R^{\frac{3}{2}} u_J^{-\frac{1}{2}}$$

- Tune b so that $10^9 \lesssim \frac{f_a}{f_\pi} \lesssim 10^{18} \quad : \quad 10^{-24} \lesssim b \lesssim 10^{-12}$
- Within this interval $f_a^2 \approx 0.1534 \frac{N_c \lambda}{16\pi^3} \frac{1}{M_{\rm KK} L^3}$

Comments on the model

- PQ symmetry = axial U(1) acting on extra quark, NOT on SM quarks.
- Extra quark condensation (U-shaped PQ brane) breaks it.
- PQ and WSS branes are distant: quarks and extra quark interact only through gauge sector. Since we work at large N interactions are suppressed.

- Hence our axion model fits in the KSVZ class [Kim; Shifman, Vainshtein, Zakharov, '80]
- Thus axion coupling to nucleons only receive IR contributions.

- Electromagnetic current in WSS: weakly gauging of U(1) in U(N_f)
- As such it pertains to WSS D-branes: the PQ quarks are uncharged.

- Hence electromagnetic interactions of the axion just come from mixing with neutral pion and η', with no UV contribution.

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Axion couplings to nucleons

[FB, Cotrone, Kiritsis, Yarvinen, '19]

- Focus on $N_f=2$ case. $M=\mathrm{diag}\,(m_u, m_d)$. $N=(p, n)$ nucleon field.
- Compute CP even (c_N) and CP odd ($\bar{c}_N$) axion-nucleon couplings

$$\delta\mathcal{L}_{aNN,\text{der}} = -\frac{\partial_\mu a}{\sqrt{2} f_a} c_N \bar{N}\gamma^\mu\gamma^5 N\,, \quad \delta\mathcal{L}_{aNN,\text{non-der}} = \bar{c}_N a \bar{N} N$$

- Ingredient 1: in WSS model, **nucleons = instanton** solutions for $\mathcal{F}_{WSS}$
- Nucleon mass scales like N_c : a non relativistic limit can be taken

- Ingredient 2: neglecting η'- π mixing, mass eigenstates to $O(1/f_a)$ are

$$\hat{\eta}' = \eta' + \frac{\chi f_\pi}{4c}\mathrm{Tr}[M^{-1}]\frac{\sqrt{2}a}{f_a}\,,$$

$$\hat{\pi}^a = \pi^a + \frac{\chi f_\pi}{4c}\mathrm{Tr}[\tau^a M^{-1}]\frac{\sqrt{2}a}{f_a}\,,$$

$$\hat{a} = a - \frac{\chi f_\pi}{4c}\mathrm{Tr}[M^{-1}]\frac{\sqrt{2}\eta'}{f_a} - \frac{\chi f_\pi}{4c}\mathrm{Tr}[\tau^a M^{-1}]\frac{\sqrt{2}\pi^a}{f_a}$$

Axion couplings to nucleons

- CP-even nucleon-meson couplings computed in WSS [Hashimoto, Sakai, Sugimoto 08]

$$\delta\mathcal{L} = i g_{\eta' NN} \eta' \bar{N}\gamma_5 N + i g_{\pi NN} \pi^a \bar{N}\gamma_5 \tau^a N$$

- Non-derivative CP-even couplings can be traded for derivative ones at large nucleon mass

$$i\partial_\mu \phi \bar{N}\gamma^\mu\gamma^5 N \approx 2 m_N \phi \bar{N}\gamma^5 N$$

- Due to mixing: CP even axion couplings from meson-nucleon ones. In chiral limit:

$$\boxed{\begin{aligned} c_p &\approx -\frac{1}{2}\hat{g}_A - \frac{1}{2} g_A \frac{m_d - m_u}{m_d + m_u} \\ c_n &\approx -\frac{1}{2}\hat{g}_A + \frac{1}{2} g_A \frac{m_d - m_u}{m_d + m_u} \end{aligned}}$$

$$\hat{g}_A \approx \frac{27}{2\lambda}$$

$$g_A \approx \frac{2}{\pi}\sqrt{\frac{2}{15}} N_c$$

- $g_{\eta' NN} = \frac{m_N}{f_\pi}\hat{g}_A \,, \quad g_{\pi NN} = \frac{m_N}{f_\pi} g_A$ Goldberger-Treiman relations

- c_p, c_n just as in KSVZ class.

Axion couplings to nucleons

- CP-odd axion nucleon couplings: $\delta\mathcal{L}_{aNN,\text{non-der}} = \bar{c}_N a\bar{N}N$
- Consider the quark mass contribution to the nucleon mass term

$$\delta\mathcal{L}_M = c\mathrm{Tr}\left[M e^{-i\frac{\theta}{N_f}}\left(U_{cl} - \mathbb{1}_2\right) + h.c.\right]$$

- U_{cl} classical instanton solution describing a nucleon [Hata,Sakai,Sugimoto,Yamato 07]

$$U_{cl} = \exp\left[i\pi\frac{\vec{\tau}\cdot\vec{x}}{|\vec{x}|}\left(1 - \frac{1}{\sqrt{1+\rho^2/|\vec{x}|^2}}\right)\right]$$

- Analogous to Skyrme hedgehog. ρ is the instanton radius, $\rho_{cl}^2 \sim \lambda^{-1}$

- $\delta M_N(\theta) = -\int d^3x\,\delta\mathcal{L}_M \approx 0.032\frac{m_\pi^2 N_c^{3/2}}{f_\pi}\cos\left(\frac{\theta}{2}\right)$

Axion couplings to nucleons

- CP-odd axion nucleon couplings: $\delta\mathcal{L}_{aNN,\text{non-der}} = \bar{c}_N a\bar{N}N$
- [If there is any residual θ angle]. In chiral limit, from mixing, get

$$\bar{c}_p \approx -\frac{1}{2}\bar{g}_{\eta' NN}\frac{f_\pi}{f_a} - \frac{1}{2}\bar{g}_{\pi NN}\frac{f_\pi}{f_a}\frac{m_d - m_u}{m_d + m_u}$$
$$\bar{c}_n \approx -\frac{1}{2}\bar{g}_{\eta' NN}\frac{f_\pi}{f_a} + \frac{1}{2}\bar{g}_{\pi NN}\frac{f_\pi}{f_a}\frac{m_d - m_u}{m_d + m_u}$$

$$\bar{g}_{\eta' NN} \approx -0.016\frac{m_\pi^2 N_c^{3/2}}{f_\pi^2}\theta$$
$$\bar{g}_{\pi NN} \approx 0.062\frac{m_\pi^2\sqrt{N_c}}{f_\pi^2\lambda}\theta$$

- Computing mass term on instanton solution get, to first order in m_d-m_u

$$\bar{c}_p \approx \frac{1}{4}\sigma_N\frac{\theta}{f_a} - \frac{1}{8}(M_n - M_p)_{\text{str.}}\frac{\theta}{f_a}\,,$$
$$\bar{c}_n \approx \frac{1}{4}\sigma_N\frac{\theta}{f_a} + \frac{1}{8}(M_n - M_p)_{\text{str.}}\frac{\theta}{f_a}$$

- $\sigma_N = \delta M_N(0)$: pion-nucleon sigma term (quark mass contribution to nucleon mass)
- Cfr [Moody, Wilczek 1984]

Axion couplings to nucleons

- CP-odd axion nucleon couplings: $\delta\mathcal{L}_{aNN,\text{non-der}} = \bar{c}_N a\bar{N}N$
- Computed in holoQCD and in Skyrme as functions of model parameters.
- Compare with estimates from previous (CL) formulae, using value of sigma term from lattice [ETCM coll. 2019] or from pionic atoms [pheno: Meissner 2017]. $(M_n\text{-}M_p)_{str.}$ from [Borsanyi et al 2015]

	CL + lattice	CL + pheno	Skyrme	Holography
$\bar{c}_n$	9.4(9)	13.0(1.1)	27(5)	21(4)
$\bar{c}_p$	8.8(9)	12.4(1.1)	27(5)	20(4)

[in MeV θ/f_a units]

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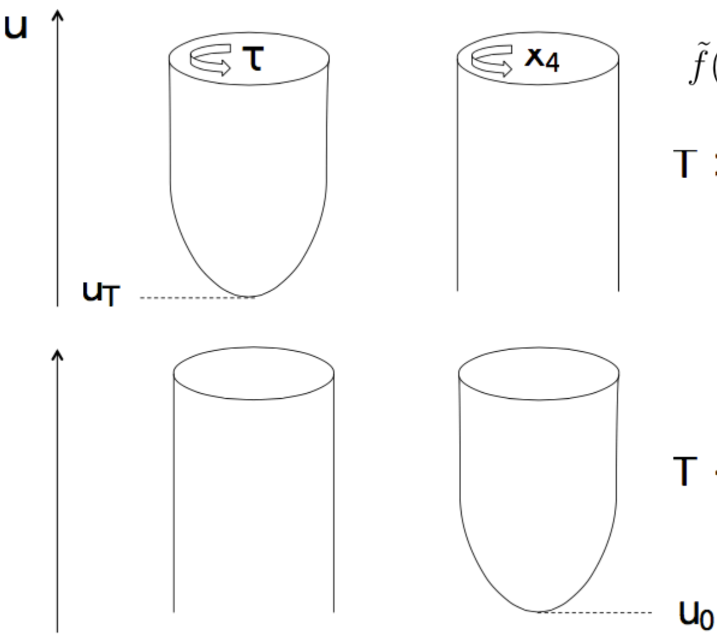
Holographic YM at Finite temperature

Two possible gravity solutions, with Euclidean time circle of length 1/T

$$ds^2 = \left(\frac{u}{R}\right)^{3/2}\left[\tilde{f}(u)dx_0^2 + dx_a dx^a + dx_4^2\right] + \left(\frac{u}{R}\right)^{-3/2}\left[\frac{du^2}{\tilde{f}(u)} + u^2 d\Omega_4^2\right],$$



$$\tilde{f}(u) = 1 - \frac{u_T^3}{u^3}.$$

$T > T_c$

- black hole solution
- $g_{00}(u_T) = 0$: deconfinement
- no theta dependence to leading order

$$\boxed{C_1 \sim \theta dx_4, \;\; F_2 = 0 \quad [(u, x_4) : \text{cylinder}]}$$

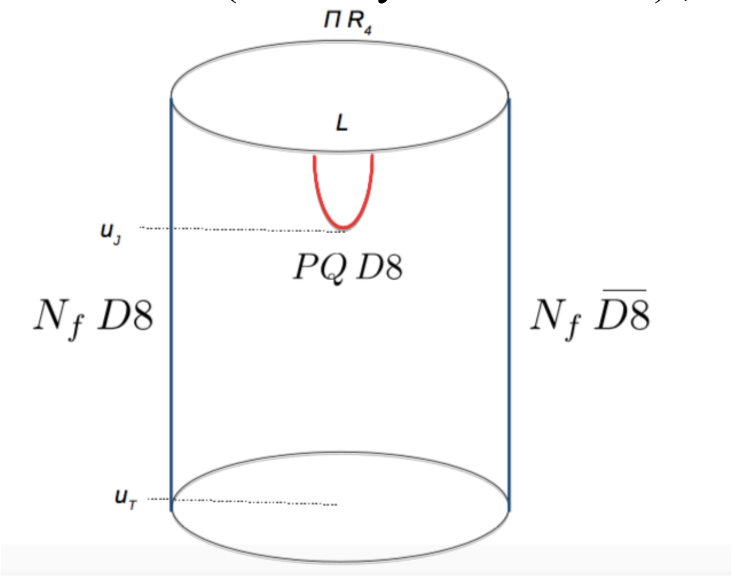
$T < T_c$

- Euclidean version of T=0 one
- Theta-dependence
- Confinement

- At $T_c = \frac{M_{KK}}{2\pi}$ first order phase transition

Holographic YM at Finite temperature

- If $T_c < T \ll f_a$ QCD mesons melt (chiral symm. restored) , while our axion survives



- We will compute the temperature dependence of the axion mass in the unflavored case
- Hence we can compute χ_{WYM} (T)

- Since $F_2=0$ at $T>T_c$, classical gravity approximation gives χ_{WYM} (T)=0
- Need stringy instanton corrections to gravity action [Green, Gutperle, Vanhove 97]

Holographic YM at Finite temperature

- θ-dependence in deconfined phase can be recovered including instanton corrections

- Instanton = Euclidean D0-brane wrapped on the x_4 circle

$$S_{D0} = \frac{1}{l_s}\int e^{-\phi}\sqrt{g_{44}}dx_4 - \frac{i}{l_s}\int C_1 = \frac{8\pi^2}{g_{\rm YM}^2} - i\theta$$

- Instanton corrections to the IIA supergravity action on a circle are known
- They can be deduced from M-theory on a torus [Green, Gutperle, Vanhove 97]

- Compute on-shell gravity action including instanton corrections.

Axion mass at T > T_c

[FB, Caddeo, Cotrone, Di Vecchia, Marzolla, 19]

- Topological susceptibility and hence the axion mass
- $$\boxed{\chi_{\rm WYM}(T) = \frac{3285\pi^{3/2}}{42}\left(\frac{4\pi}{3}\right)^4 \frac{\sqrt{N_c}}{\sqrt{\lambda_{\rm eff}(T)}}\, T^4 e^{-\frac{8\pi^2}{g^2_{\rm YM}}}}$$ $$\lambda_{\rm eff}(T) \equiv g^2_{\rm YM} N_c \frac{T^2}{M^2_{\rm KK}} \equiv \lambda_{\rm YM}\frac{T^2}{M^2_{\rm KK}}$$
- $m_a^2(T) f_a^2 = 2\chi_{WYM}(T) \sim M_{KK} T^3$

- Axion mass increases with T
- Strong difference with Yang-Mills instanton gas.

- Aside result: also computed for N=4 SYM: $\chi_{\rm SYM}(T) = \frac{15}{128}\pi^{3/2}\sqrt{N_c}\, T^4 e^{-\frac{8\pi^2}{g^2_{\rm YM}}}$

Conclusions

- A top-down holographic model of a composite axion in the KSVZ class
- Effective Lagrangian coincides with the axion-dressed chiral QCD one at large N

- $f_a/f_\pi \approx R_4/L$ can be made parametrically large.
- UV completion under control, higher dimensional model in string theory.

- Analytic predictions on the (strongly coupled) IR physics.
- Derivative and non-derivative axion-nucleon couplings computed.

- Axion mass at high temperature: very different behavior from QCD instanton gas.

- Future plans: axion cosmology, gravitational waves…

Thank you for your time

NJL model from D4-D8 setup

[Antonyan, Harvey, Jensen, Kutasov, 06]

In the D4-D8 setup q_L and q_R are separated in the x_4 direction.
They live in the 3+1 dimensional D4-D8 intersection but the gauge field they interchange is 4+1 dimensional (before compactifying x4)

$$\mathcal{S} = \int d^5x \left[-\frac{1}{4g_5^2} F_{MN}^2 + \delta(x^4 + \frac{L}{2}) q_L^\dagger \bar{\sigma}^\mu (i\partial_\mu + A_\mu) q_L + \delta(x^4 - \frac{L}{2}) q_R^\dagger \sigma^\mu (i\partial_\mu + A_\mu) q_R \right]$$

At weak coupling q_L and q_R interact via a (non local) one (five dimensional) gluon exchange. Integrating out the 5d gauge field in the single gluon exchange approx

$$\mathcal{S}_{\text{eff}} = i \int d^4x \left(q_L^\dagger \bar{\sigma}^\mu \partial_\mu q_L + q_R^\dagger \sigma^\mu \partial_\mu q_R \right) + \frac{g_5^2}{4\pi^2} \int d^4x d^4y G(x-y, L) \left[q_L^\dagger(x) \cdot q_R(y) \right] \left[q_R^\dagger(y) \cdot q_L(x) \right]$$

$G(x, x_4)$ 5d scalar propagator

A (non local) NJL model: $\mathcal{L}_{\text{int}} = G q_L^\dagger \cdot q_R q_R^\dagger \cdot q_L$

NJL: for $G>G_c$ chiral symmetry breaking i.e. $\langle q_L^\dagger \cdot q_R \rangle$ condensate.

The QCD axion

- At energies below f_a and EW-breaking scale, axion dependent Lagrangian reads

$$\mathcal{L}_a = \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu} + \frac{1}{4}\,a\,g^0_{a\gamma\gamma}F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{\partial_\mu a}{2f_a}\,j^\mu_{a,0}$$

- $j^\mu_{a,0} = c^0_q\bar{q}\gamma^\mu\gamma_5 q$, G is SU(3) field strength, F is U(1) electromagnetic field strenght.

Axion couplings to nucleons

- Hence $\delta\mathcal{L} = -\delta M_N(\theta)\bar{N}N$ in nucleon effective Lagrangian.
- Using $\langle a\rangle \sim f_a\,\theta$ and expanding around θ=0 we get (chiral limit, N_f=2)

$$\boxed{\bar{g}_{aNN} = \frac{1}{N_f^2}\frac{\theta}{f_a}\delta M_N(0)} \approx -\frac{1}{2}\bar{g}_{\eta' NN}\frac{f_\pi}{f_a} \qquad \bar{g}_{\eta' NN} \approx -0.016\frac{m_\pi^2 N_c^{3/2}}{f_\pi^2}\theta$$

- [Cfr. Moody, Wilczek 1984]
- We also computed $\bar{g}_{\pi NN}$ considering 1/N corrections to instanton solution

$$\bar{g}_{\pi NN} \approx 0.062\frac{m_\pi^2\sqrt{N_c}}{f_\pi^2\lambda}\theta$$

- Since $f_\pi^2 \sim N_c$, we see that $\bar{g}_{\eta' NN} \sim N_c^{1/2}$ and $\bar{g}_{\pi NN} \sim N_c^{-1/2}$
- From axion-meson mixing we thus get the CP-odd axion-nucleon couplings

$$\bar{c}_p \approx -\frac{1}{2}\bar{g}_{\eta' NN}\frac{f_\pi}{f_a} - \frac{1}{2}\bar{g}_{\pi NN}\frac{f_\pi}{f_a}\frac{m_d - m_u}{m_d + m_u}$$
$$\bar{c}_n \approx -\frac{1}{2}\bar{g}_{\eta' NN}\frac{f_\pi}{f_a} + \frac{1}{2}\bar{g}_{\pi NN}\frac{f_\pi}{f_a}\frac{m_d - m_u}{m_d + m_u}$$

Axion couplings to nucleons

- In the WSS model we can express CP-odd couplings in terms of CP-even ones

$$\bar{c}_p = \frac{\theta}{f_a}\frac{1}{16\sqrt{3}\pi^2\epsilon^{3/2}}\frac{m_\pi^2}{f_\pi}\left[18\gamma(c_n-c_p)^{3/2}-\hat{\gamma}\pi\epsilon^2(c_n-c_p)^{1/2}(c_n+c_p)\right] , \qquad \epsilon = \frac{(m_d-m_u)}{m_d+m_u)}$$

$$\bar{c}_n = \frac{\theta}{f_a}\frac{1}{16\sqrt{3}\pi^2\epsilon^{3/2}}\frac{m_\pi^2}{f_\pi}\left[18\gamma(c_n-c_p)^{3/2}+\hat{\gamma}\pi\epsilon^2(c_n-c_p)^{1/2}(c_n+c_p)\right] . \qquad \gamma\sim 1.10 \quad , \quad \hat{\gamma}\sim 1.05$$

- Fixing the physical parameters and the CP-even couplings as computed in [Villadoro et al 15]

$$m_\pi = 134.98 \text{ MeV} , \qquad f_\pi = 92.21(14) \text{ MeV} , \qquad \epsilon = 0.35(3) ,$$
$$c_p = -0.47(3) , \qquad c_n = -0.02(3) .$$

- We get the following estimates for the CP-odd couplings

$$\bar{c}_p = \theta\left(\frac{22(4)\text{ MeV}}{f_a}\right) , \qquad \bar{c}_n = \theta\left(\frac{19(4)\text{ MeV}}{f_a}\right)$$

Axion couplings to nucleons

- We performed novel analogous computations in the Skyrme model!
- In particular we computed for the first time the CP odd coupling $\bar{g}_{\pi NN}$
- $\bar{g}_{\pi NN} \sim N_c^{-1/2}$ (as in WSS) confirming expectations in [Riggs, Schnitzer, 92]
- An average of the results in Skyrme and WSS model at $N_f=2$ gives our best estimate

$$\bar{c}_p = \theta \left(\frac{24(6)\ \mathrm{MeV}}{f_a} \right)$$
$$\bar{c}_n = \theta \left(\frac{23(5)\ \mathrm{MeV}}{f_a} \right)$$

- We also considered the axion-nucleon couplings at small N using standard effective field theory (to one loop) [cfr. Gutsche et al, 2016].
- Loop corrections affect above results at the level of 30% at most.

Holographic YM at Finite temperature

- θ-dependence in deconfined phase can be recovered including instanton corrections
- Instanton = Euclidean D0-brane wrapped on the x_4 circle

$$S_{D0} = \frac{1}{l_s}\int e^{-\phi}\sqrt{g_{44}}dx_4 - \frac{i}{l_s}\int C_1 = \frac{8\pi^2}{g^2_{\rm YM}} - i\theta$$

- Instanton corrections to the IIA supergravity action on a circle are known
- They can be deduced from M-theory on a torus [Green, Gutperle, Vanhove 97]

- Notice that WYM background can be obtained from AdS_7x S^4 black hole solution

$$ds^2 = G_{MN}dx^M dx^N = \frac{y^2}{R^2}\left[-f(y)dt^2 + \sum_{i=1}^{4} dx_i^2 + dx_{10}^2\right] + \frac{4R^2}{f(y)y^2}dy^2 + R^2 d\Omega_4^2, \qquad f(y) = 1 - y_0^6/y^6$$

- torus $S_{x_{10}} \times S_{x_4}$ radii $R_4 = M_{\rm KK}^{-1}$ and $R_{10} = g_s l_s$

Holographic YM at Finite temperature

- Quartic corrections to 11d sugra action on a torus

$$\delta S = -\frac{1}{\kappa_{11}^{2/3}} \int d^{11}x\sqrt{-G}\, W \left[\frac{2\pi^2}{3} + \mathcal{V}_2^{-3/2} f(\rho,\bar{\rho})\right]$$

- W in terms of the Weyl tensor $W = C^{hmnk} C_{pmnq} C_h^{\ rsp} C^q_{\ rsk} + \frac{1}{2} C^{hkmn} C_{pqmn} C_h^{\ rsp} C^q_{\ rsk}$

- $\mathcal{V}_2$ related to the volume of the torus $V_T = \kappa_{11}^{4/9} \mathcal{V}_2 = \int dx_4 dx_{10} \sqrt{G_{(2)}}$

- Modular function up to one-instanton corrections

$$f(\rho,\bar{\rho}) = 2\zeta(3)\rho_2^{3/2} + \frac{2\pi^2}{3}(\rho_2)^{-1/2} + 4\pi(e^{2\pi i\rho} + e^{-2\pi i\bar{\rho}}) + \cdots$$

$$\rho \equiv \rho_1 + i\rho_2 = (2\pi)^{-1}\theta + 4\pi i g_{\mathrm{YM}}^{-2} \sim S_{D_0}$$

- On AdS$_7$xS^4 black hole $W = \frac{3285}{64R^8} \frac{y_0^{24}}{y^{24}}$ [Gubser,Klebanov,Tseytlin 98]

Holographic YM at Finite temperature

- Compute on-shell value of the quartic correction to the action
- By holographic map this is related to a correction to the QFT free energy.

- The θ-correction reads [FB, Caddeo, Cotrone, Di Vecchia, Marzolla, 19]

$$\delta f_\theta = -\frac{8760}{7}\left(\frac{2\pi}{3}\right)^4 M_{KK} T^3 \pi^{3/2} \frac{\sqrt{N_c}}{\sqrt{\lambda_{YM}}} e^{-\frac{8\pi^2}{g_{YM}^2}} \cos\theta$$

- Thus the WYM topological susceptibility and hence the axion mass

$$\boxed{\chi_{\rm WYM}(T) = \frac{3285\pi^{3/2}}{42}\left(\frac{4\pi}{3}\right)^4 \frac{\sqrt{N_c}}{\sqrt{\lambda_{\rm eff}(T)}} T^4 e^{-\frac{8\pi^2}{g_{\rm YM}^2}}} \qquad m_a^2(T) f_a^2 = 2\chi_{WYM}(T) \sim M_{KK} T^3$$

- $\lambda_{\rm eff}(T) \equiv g_{\rm YM}^2 N_c \frac{T^2}{M_{\rm KK}^2} \equiv \lambda_{\rm YM}\frac{T^2}{M_{\rm KK}^2}$

- Axion mass increases with T and T-dependence of f_a (computed) negligible
- Strong difference with Yang-Mills inst. gas, due to asymptotic freedom, χ decreases

- We also computed χ(T) in *N*=4 SYM ! $\chi_{\rm SYM}(T) = \frac{15}{128}\pi^{3/2}\sqrt{N_c}\,T^4 e^{-\frac{8\pi^2}{g_{\rm YM}^2}}$