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B. I. Ermolaev

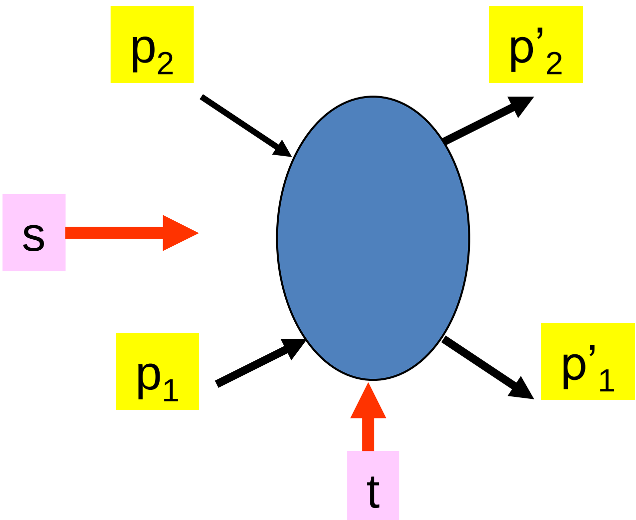
Double-logarithmic contribution to Pomeron and application to photon-photon scattering

talk based on results obtained in collaboration with S.I. Troyan

Classic/phenomenological Pomeron in Regge theory of 60's

Regge theory is based on applying very general concepts such as **Analyticity**, **Unitarity** and **Causality** to predict high-energy asymptotics of scattering amplitudes

Consider amplitude of 2 -> 2 scattering of hadrons at high energies



s, t are the standard Mandelstam variables

$$= A\,(s, t)$$

c.m.f. hadron energy

$$s = (p_1 + p_2)^2 = 4\,E^2$$

$$t = (p'_1 - p_1)^2 \approx -2E^2(1 - \cos\theta)$$

c.m.f. scattering angle

Forward kinematics $\theta \ll 1$ ➔ $s = 4\,E^2 \gg -t = E^2\theta^2$

Analyticity

1. Crossing

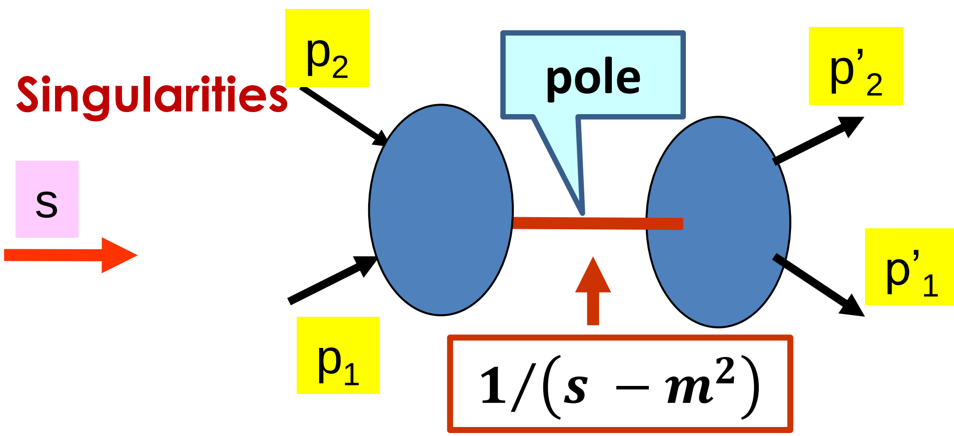
s-channel $a(p_1)\, b(p_2) \rightarrow c(p'_1) d(p'_2)$

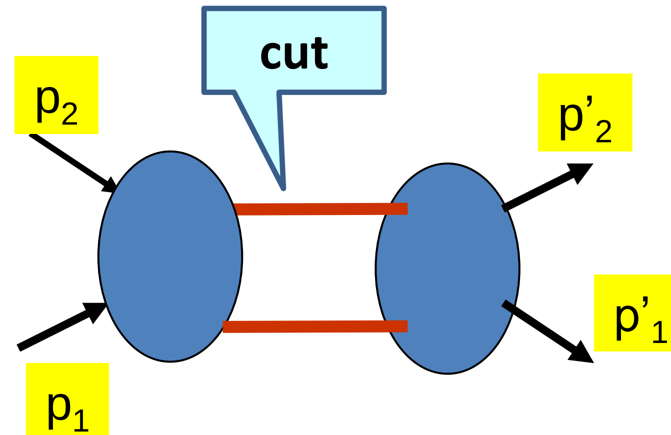
t-channel $a(p_1)\, \bar{c}(p'_1) \rightarrow \bar{b}(p_2) d(p'_2)$

u-channel $a(p_1)\, \bar{d}(p'_2) \rightarrow \bar{b}(p_2) c(p'_1)$

Reactions in all channels can be described with analytical continuation of the scattering amplitudes obtained e.g. in s-channel or in t-channel

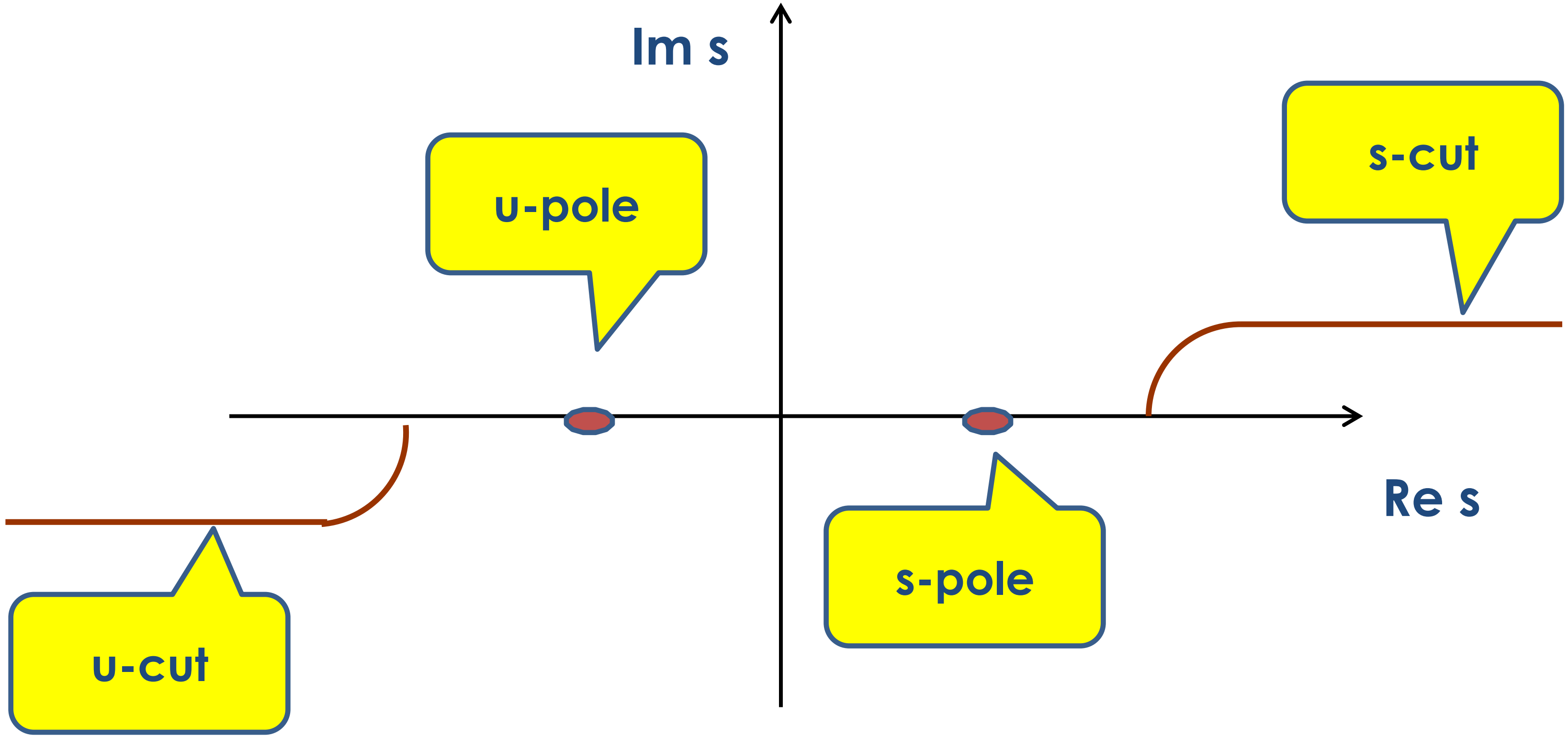
2. Singularities





ASSUMPTION there are no other singularities besides the poles and cuts poles correspond to single-particle intermediate states and cuts correspond to multi-particle states

$$u = -s - t + \sum m^2 < 0$$



Causality

⬇

One can use Dispersion Relations

$$A(s,t) = \sum \frac{c_n}{s - s_n} + \frac{c'_n}{u - u_n} +$$

Poles

$$\frac{1}{\pi} \int_{s_T}^{\infty} ds' \frac{Im_s A(s',t)}{s' - s} + \frac{1}{\pi} \int_{u_T}^{\infty} du' \frac{Im_u A(s',t)}{u' - u}$$

S-cut

U-cut

Onset of Regge theory

Aim: to get expressions for scattering amplitudes at high energies.

Suggestion of T.Regge was to use

Partial waves expansion

Consider scattering amplitude in the t-channel and expand it

$$A(s,t) = \sum_{n=0}^{\infty} (2n+1) a_n(t) P_n(z)$$

Partial waves

Legendre polynomials

Scattering angle

$$-1 < z = \cos\theta \cong s/(t - 4m^2) < 1$$

So, all dependence of *A* on *S* is in the Legendre polynomials and they are well-known

At the first look one can easily obtain asymptotics of ***A*** at large ***s***, using that $P_n(z) \sim (s/t)^n$ at **z >>1**

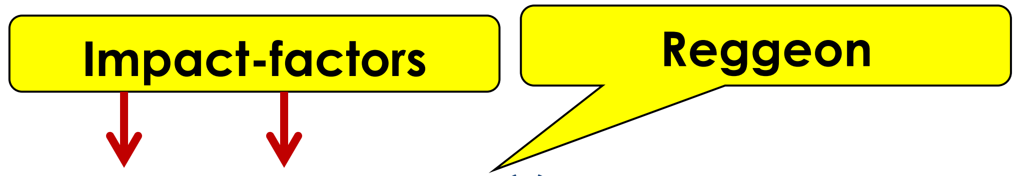
However, the polynomials P_n are defined at |z| <1 only,
They cannot be used in the s-channel where |z|>1.

In addition, P_n are integer functions, without poles and cuts, so the partial waves representation does not correctly describe analytical properties of ***A*** in the s-channel

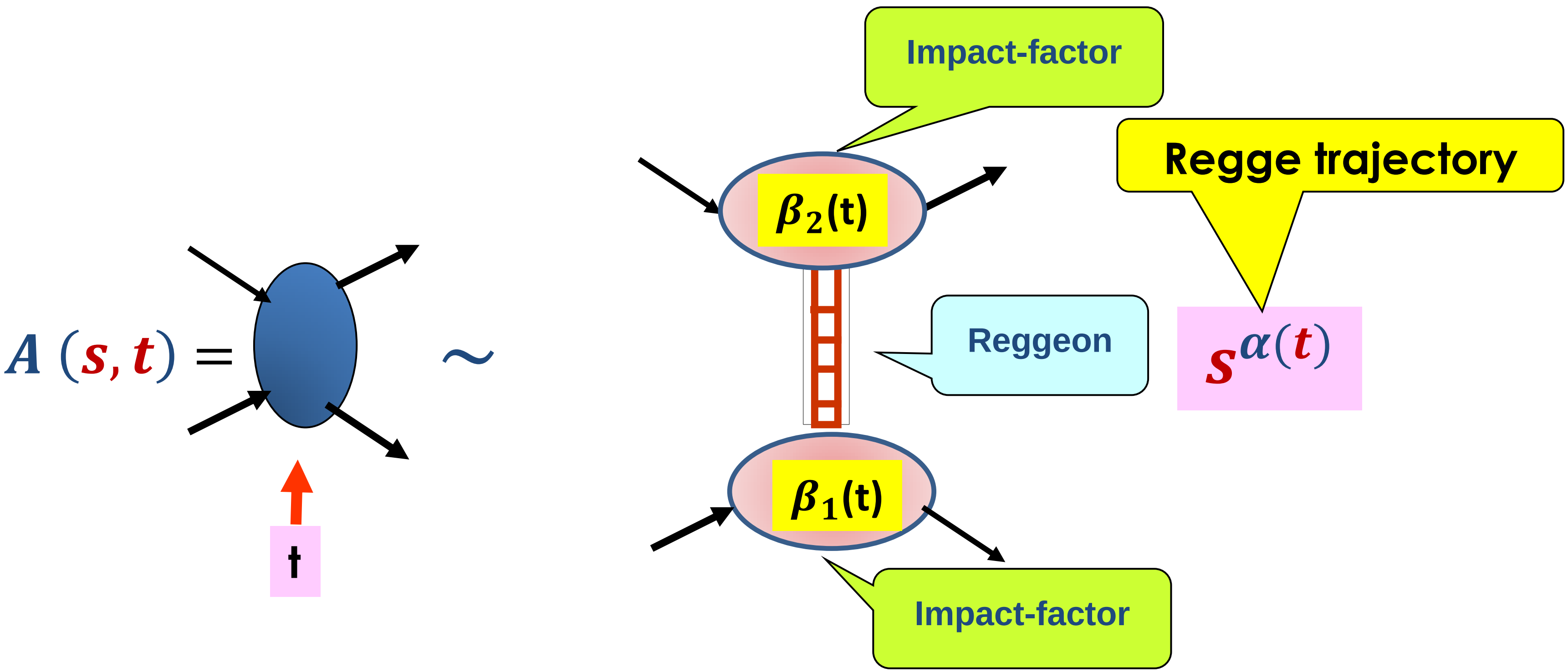
Therefore, the partial waves expansion cannot straightforwardly be used to describe scattering amplitudes in s-channel at large ***s***

WAY OUT: Construct a correct Analytical Continuation of ***A*** from the t-channel to the s-channel
This took serious efforts and eventually the high-energy asymptotics (called often Regge asymptotics) of hadronic scattering amplitudes were obtained

Regge theory predicts that high-energy asymptotics of any scattering amplitude in the forward kinematics is



$$A\,(s,t) \sim \beta_1(t)\,\beta_2(t)\,s^{\alpha(t)}$$



The trajectories are expanded in the series in *t*. However *t* is small, so the linear approximation can be used:

$$\alpha(t) = \alpha(0) + \alpha'(0)\,t$$

intercept

slope

Optical theorem:

$$\sigma_{tot} \sim Im\ A_{el}(s,0)/s \sim (s/s_0)^{\alpha(0)-1}$$

Total cross section

UNITARITY → Froissart-Martin bound

$$\sigma_{tot}(s) \le c\, ln^2 s \rightarrow \alpha(0) \le 1$$

I.Y. Pomeranchuk (1958) suggested existence of Reggeon with intercept = 1

Pomeron $\alpha_P(0) \equiv \alpha(0)_{max} = 1$

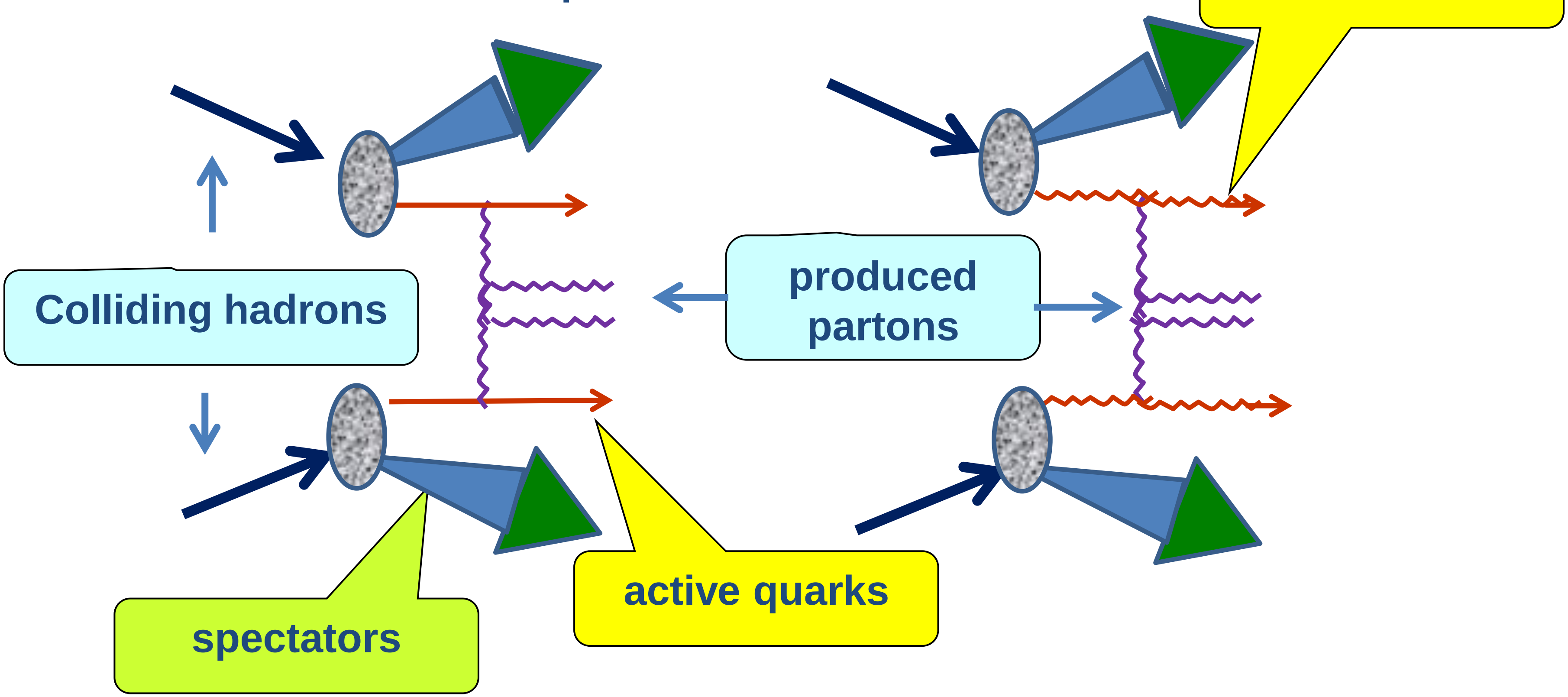
$$\sigma_{tot} \sim (s/s_0)^{\alpha_P(0)-1}$$

no power dependency on **s**

Phenomenological Regge theory predicts the high-energy behavior of scattering amplitudes in general but cannot specify numerical values of all parameters it involves and cannot estimate an energy scale since which the asymptotic expressions can be used

Further development of theory of Pomeron was done in QCD where Collisions of hadrons are represented as convolutions of perturbative and non-perturbative objects

QCD description of hadron collisions



There ore more involved scenarios in the literature: multi-parton collisions. They include multiple emission of active partons

Parton-parton scattering is described with Perturbative QCD. It involves total resummations of the perturbative series

We consider amplitudes of the elastic scattering of virtual photons

$$\gamma^*(p)\,\gamma^*(\mathrm{q}) \;\rightarrow\; \gamma^*(p')\,\gamma^*(\mathrm{q}')$$

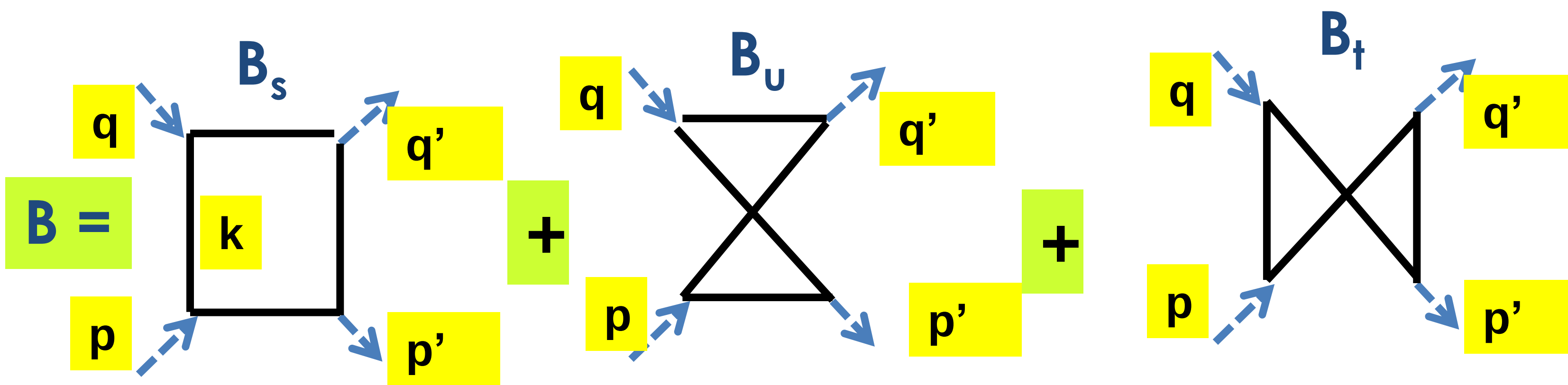
in the forward kinematics

$$s = (p+q)^2 \gg -t = (p-q')^2$$

We presume that all the photons are non-polarized

This process, apart of its experimental importance, is interesting from the theoretical point of view because, in contrast to hadronic reactions, it is free of non-perturbative contributions, so it can be regarded as a test-field for various theoretical approaches

Born (lowest order) approximation:



Only B_s yields double logs of s

We use the standard notations:

$$s = (p+q)^2 \approx -u = (p - q')^2 \gg -t = (p - q')^2 ,$$

$$|p^2| \equiv Q_1^2 \approx |p'^2|, |q^2| \approx |q'^2| \equiv Q_2^2 \gg \mu^2$$

We neglect quark masses and introduce IR cut-off μ^2

IR cut-off

Sudakov parametrization

$$k = -\alpha q' + \beta p' + k_{\perp}$$

light-cone vectors

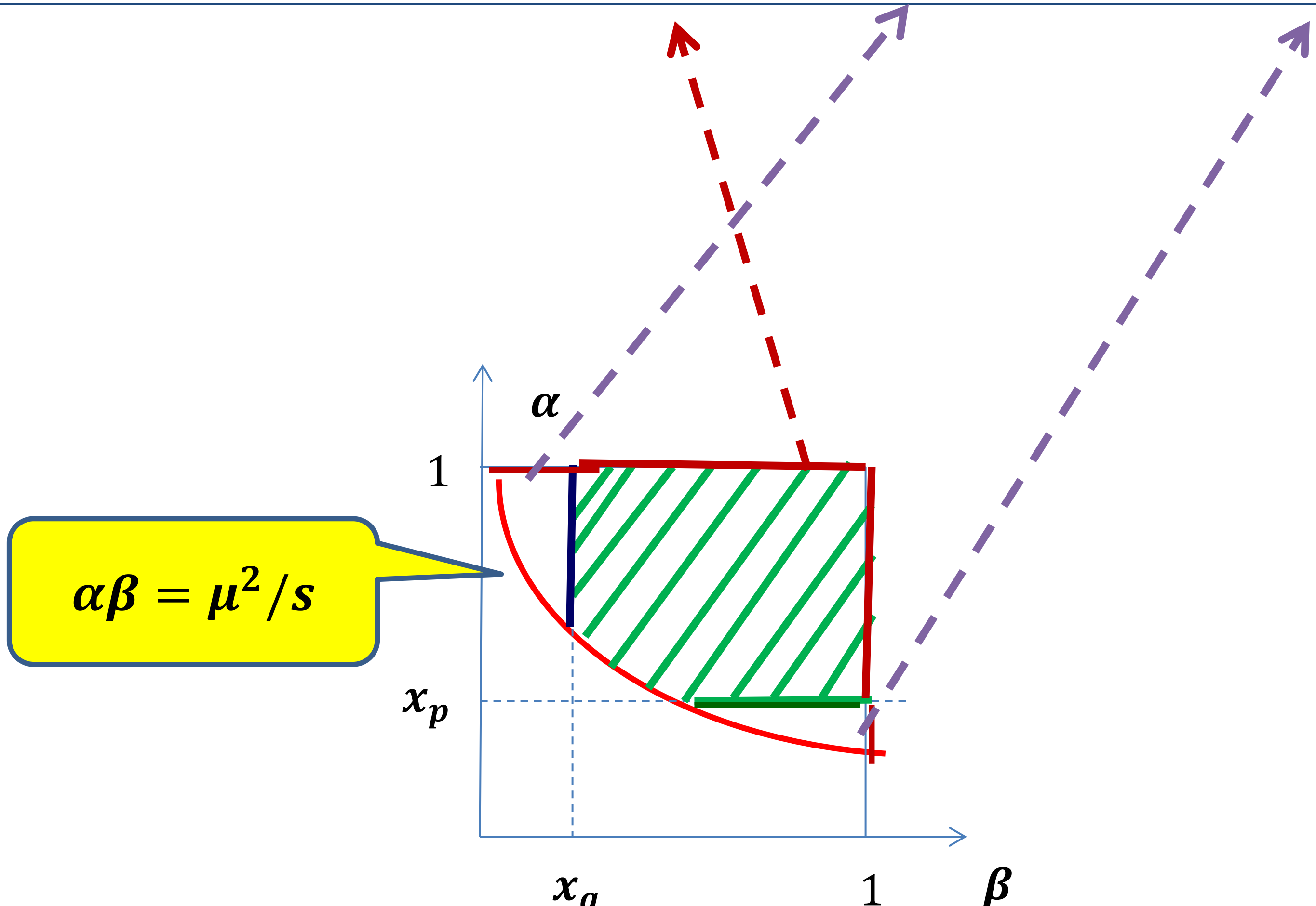
$$q' = q - x_q p, \; p' = p - x_p q$$

$$x_q = Q_2^2/s, \; x_p = Q_1^2/s$$

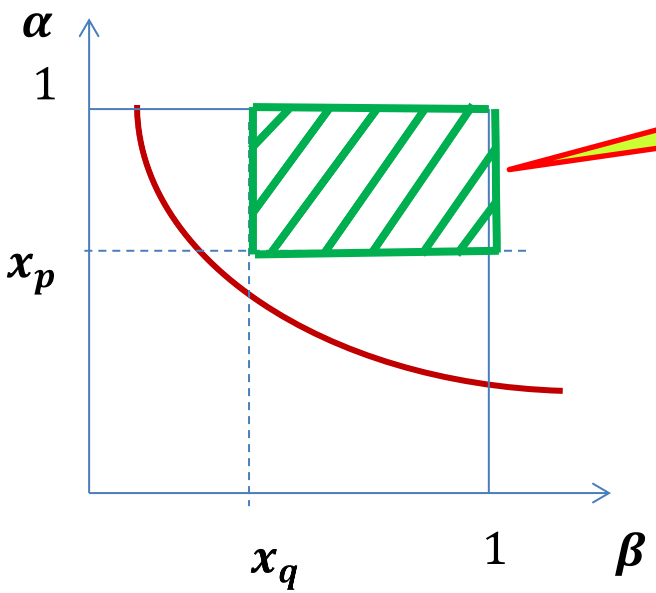
There are two kinematic regions in the Born approximation :

Moderately virtual photons: $s\,\mu^2 \gg Q_1{}^2 Q_2{}^2$

$$B_s = -\left(e^4/16\pi^2\right)\left[ln^2\left(-s/\mu^2\right) - ln^2\left(Q_2{}^2/\mu^2\right) - ln^2\left(Q_1{}^2/\mu^2\right)\right]$$



Deeply virtual photons $s\,\mu^2 \ll Q_1{}^2 Q_2{}^2$



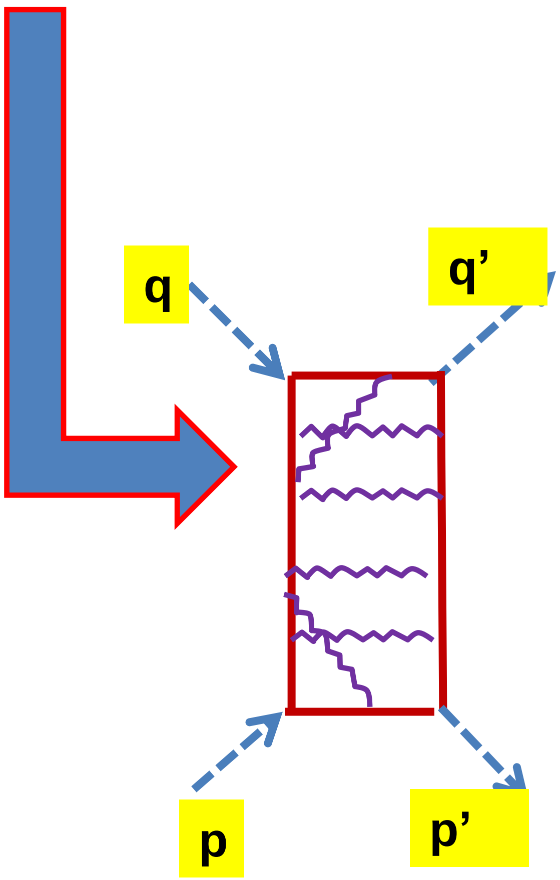
Integration region does not involve μ^2

$$B_s = -\left(e^4/16\pi^2\right)\left[\,2\, ln\left(-s/Q_1{}^2\right) ln\left(-s/Q_2{}^2\right)\right]$$

IR- stable: does not depend on μ^2

Beyond the Born approximation

Step 1 overall Amplitudes of photon- photon scattering via quark loop in DLA



Bartels-Lublinsky 2003

Explicit DLA expressions for such amplitudes at $t = 0$ (collinear scattering) obtained with several different approaches

Ermolaev-Ivanov-Troyan 2017

Generalization to $s \gg -t \neq 0$ and accounting for the running coupling

We used InfraRed Evolution Equations (IREE)
This method was invented by L.N. Lipatov and applied to both Gravity (Lipatov, 1982) and quark-quark elastic scattering (Kirschner-Lipatov, 1982)

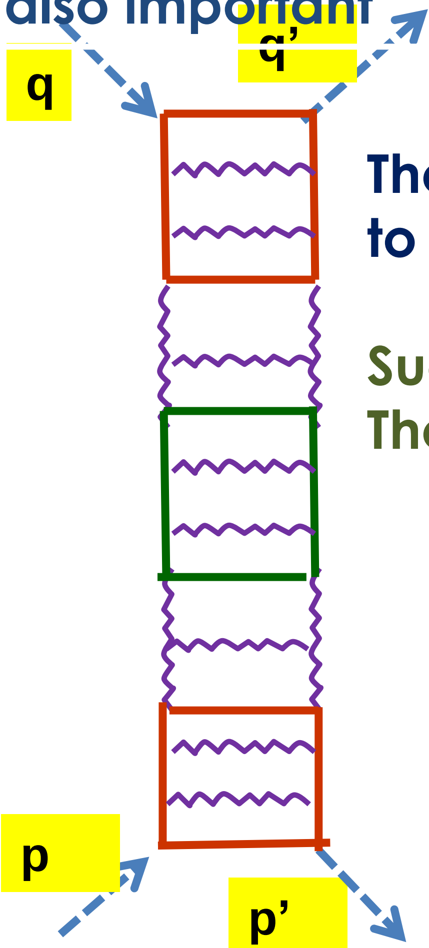
After that IREE approach has been applied to many problems of QED, QCD and EW interactions

STEP 2: photon scattering via mix of quark and gluon loops

Ermolaev-Troyan 2017

First, there are ladder graphs with quark and gluon rungs

Then there are non-ladder graphs which are not depicted though also important

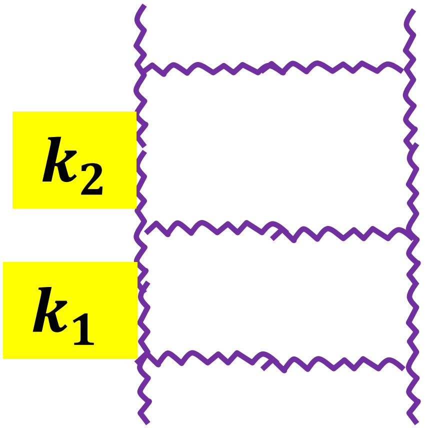


The aim is to calculate all involved Feynman graphs to all orders in α_s

Such resummations can be done approximately only. There are several methods available in the literature

These methods deal with the same Feynman graphs but account for different contributions of the graphs, so each method is tailored for a specific kinematics. Selection of most important terms in each kinematics is achieved by imposing **ORDERINGS**

Sudakov parametrization



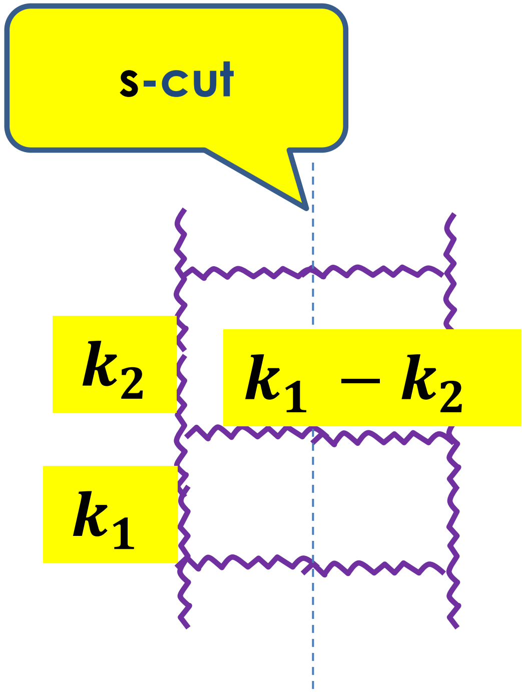
$$k_i = \alpha_i q' + \beta_i p' + k_{i\perp}$$

p' and *q'* are light-cone momenta. They are made of the external moments *p* and *q*

$$p' = p - q\left(p^2/w\right) \quad q' = q + p\,x$$

$$w = 2pq, \qquad Q^2 = -q^2, \quad x = Q^2/w$$

$$k^2{}_i = w\alpha\beta - k^2{}_{i\perp} \approx -k^2{}_{i\perp}$$



are used to integrate the delta-functions corresponding to the cut

$$k_i = \alpha_i\, q' + \beta_i\, p' + k_{i\perp}$$

$$\delta\left((k_1 - k_2)^2\right)$$

The orderings impose a certain hierarchy on the remaining longitudinal β_i and transverse $k_{i\perp}$ Sudakov variables

They are used to construct evolution equations

DGLAP ordering

Factorization scale

$\beta_1 \sim \beta_2 \sim \ldots \sim \beta_n \ldots \sim 1$

$\mu^2 < k^2{}_{1\perp} < k^2{}_{1\perp} < \ldots < k^2{}_{n\perp} < Q^2$

No logs of x accounted for

logs of Q^2 resummed

Integrations over longitudinal and transverse momenta are unrelated

DGLAP ordering sums logs of Q^2 and does not sum logs of x, so it can be used in kinematics $x \sim 1,\ Q^2 \gg \mu^2$ where logs of x are small and logs of Q^2 are great

DGLAP universally applies to both unpolarized and spin-dependent processes

BFKL ordering

$\mu^2 \sim k^2{}_{1\perp} \sim k^2{}_{1\perp} \sim \ldots \sim k^2{}_{n\perp}$

$1 \gg \beta_1 \gg \beta_2 \sim \gg \ldots \gg \beta_n$

No logs of Q^2 accounted for

logs of x resummed

BFKL sums leading logs of x in kinematics $x \ll 1$

It can be applied to unpolarized reactions only

Integrations over longitudinal and transverse momenta are unrelated

Double-Logarithmic ordering Gorshkov-Gribov-Frolov-Lipatov

$$1 \gg \beta_1 \gg \beta_2 \gg \ldots \gg \beta_n$$

$$k^2{}_{1\perp}/\beta_1 < k^2{}_{2\perp}/\beta_2 < \ldots < k^2{}_{n\perp}/\beta_n$$

DL of x resumed

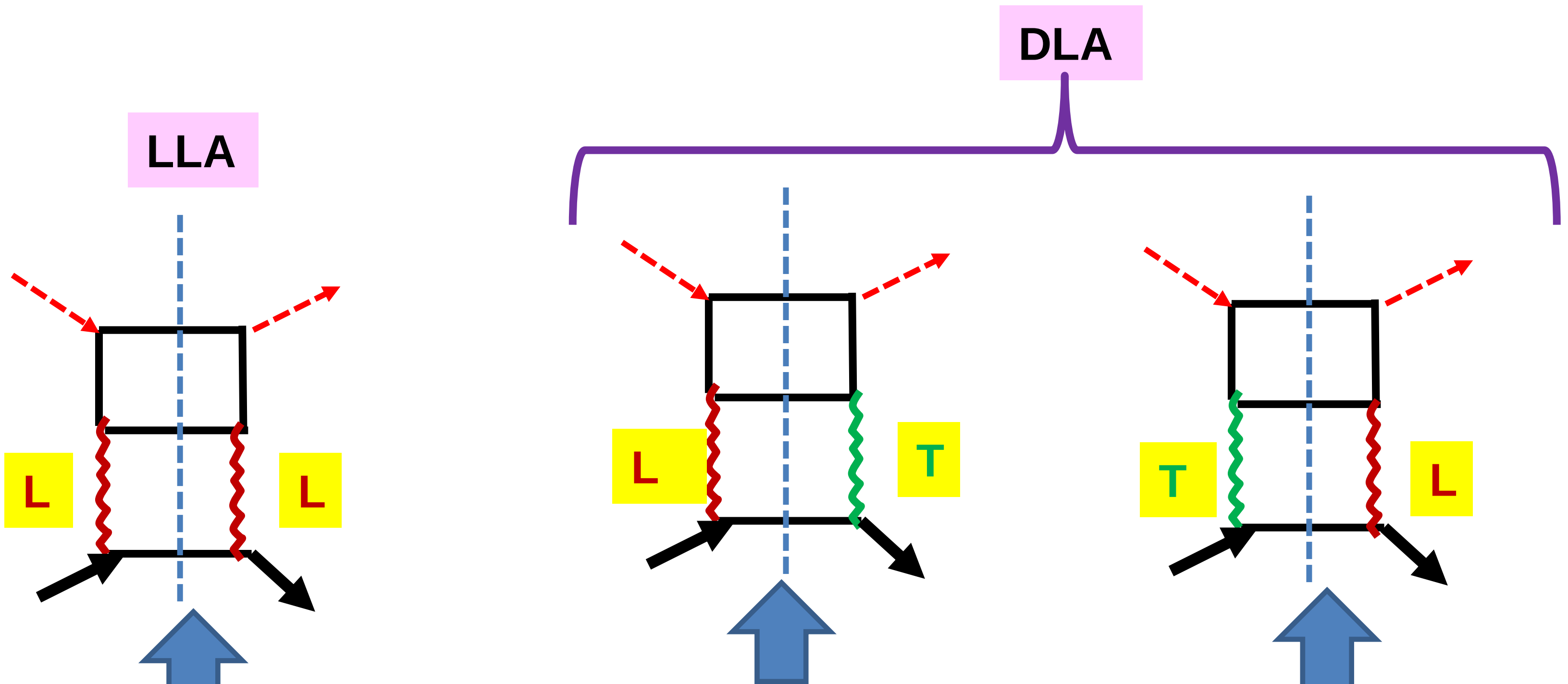
DL of Q^2 resumed

Integrations over longitudinal and transverse momenta are related

Double-logarithmic approximation (DLA) universally applies to both unpolarized and spin-dependent processes at small x and large Q^2

$$x \ll 1,\ Q^2 \gg \mu^2$$

LLA and DLA deal with the same graphs but account for different polarizations of the ladder (vertical) gluons. For instance, such difference for the graphs below is



Both gluons have longitudinal polarizations (L)

One of the gluons has longitudinal (L) polarization while the other has transverse (T)

It is convenient to use the Mellin transform in both DLA and LLA calculations

$$A^{(\pm)}(s/\mu^2) = \int_{-i\infty}^{i\infty} \frac{d\,\omega}{2\pi i} \left(\frac{s}{\mu^2}\right)^{\omega} \xi^{(\pm)}(\omega)\, f^{(\pm)}{}_0(\omega)$$

signature factor

$$\xi^{(\pm)} = -\left(1 \pm e^{-i\pi\omega}\right)/2$$

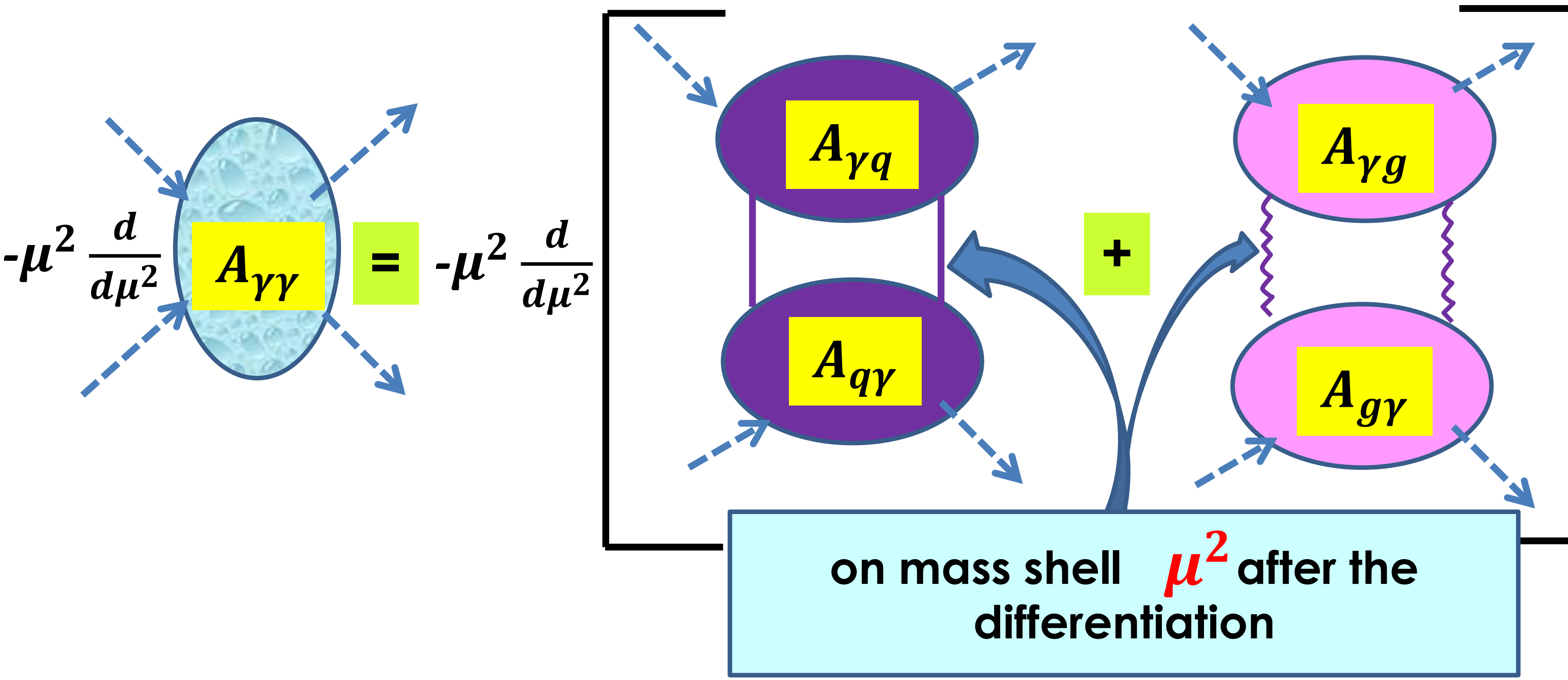
logarithmic variables:

$$\rho = ln\left(s/\mu^2\right) \quad y_1 = ln\left(Q_1{}^2/\mu^2\right) \quad y_2 = ln\left(Q_2{}^2/\mu^2\right)$$

$$A(s, Q_1{}^2, Q_2{}^2) = - \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} e^{\omega\rho} \quad F(\omega, y_1, y_2)$$

IREEs for $A_{\gamma\gamma}(s, {Q_1}^2, {Q_2}^2)$ involve auxiliary amplitudes

$A_{\gamma q}(s, {Q_1}^2)$, $A_{\gamma g}(s, {Q_2}^2)$



Applying the standard Feynman rules , arrive at IREEs in analytic form
They look simpler in the Melin space

In analytic form the l.h.s of the IREE is

$$-\mu^2 \frac{d}{d\mu^2} A(s, {Q_1}^2, {Q_2}^2) = \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} e^{\omega\rho} \left[\omega + \frac{\partial}{\partial y_1} + \frac{\partial}{\partial y_2}\right] F(\omega, y_1, y_2)$$

The l.h.s. is the same for both Moderately and Deeply Virtual photons but r.h.s. are differerent:

Moderately virtual photons $s\,\mu^2 \gg {Q_1}^2 {Q_2}^2$

$$\left[\omega + \frac{\partial}{\partial y_1} + \frac{\partial}{\partial y_2}\right] F^{(M)}{}_{\gamma\gamma}(\omega, y_1, y_2) = \frac{1}{8\pi^2} F_{\gamma q}(\omega, y_1) F_{q\gamma}(\omega, y_2) + \frac{1}{8\pi^2} F_{\gamma g}(\omega, y_1) F_{g\gamma}(\omega, y_2)$$

Deeply virtual photons $s\,\mu^2 \ll {Q_1}^2 {Q_2}^2$

$$\left[\omega + \frac{\partial}{\partial y_1} + \frac{\partial}{\partial y_2}\right] F^{(D)}{}_{\gamma\gamma}(\omega, y_1, y_2) = 0$$

Solving these equations allows us to express amplitudes $A_{\gamma\gamma}(s, {Q_1}^2, {Q_2}^2)$
In terms of the auxiliary amplitudes

For instance, solution for deeply virtual photons $s\,\mu^2 \gg {Q_1}^2{Q_2}^2$

$$A^{(M)}{}_{\gamma\gamma}(\omega, y_1, y_2) = \frac{1}{8\,\pi^2}\int_{-i\infty}^{i\infty}\frac{d\,\omega}{2\pi i}\left(\frac{s}{\sqrt{{Q_1}^2{Q_1}^2}}\right)^{\omega}\sum_{r=q,g}\left[{}^1/_\omega\, f_{\gamma r}(\omega) f_{r\gamma}(\omega) + f_{r\gamma}(\omega)\int_o^\eta dz F_{\gamma r}(\omega, z) + {}^1/_2\int_\eta^{2\rho-\xi} dz\; e^{\omega z/2} F_{\gamma r}(\omega, z) F_{r\gamma}(\omega, z)\right]$$

where the symmetrical variables are used

$\xi = y_1 + y_2$,

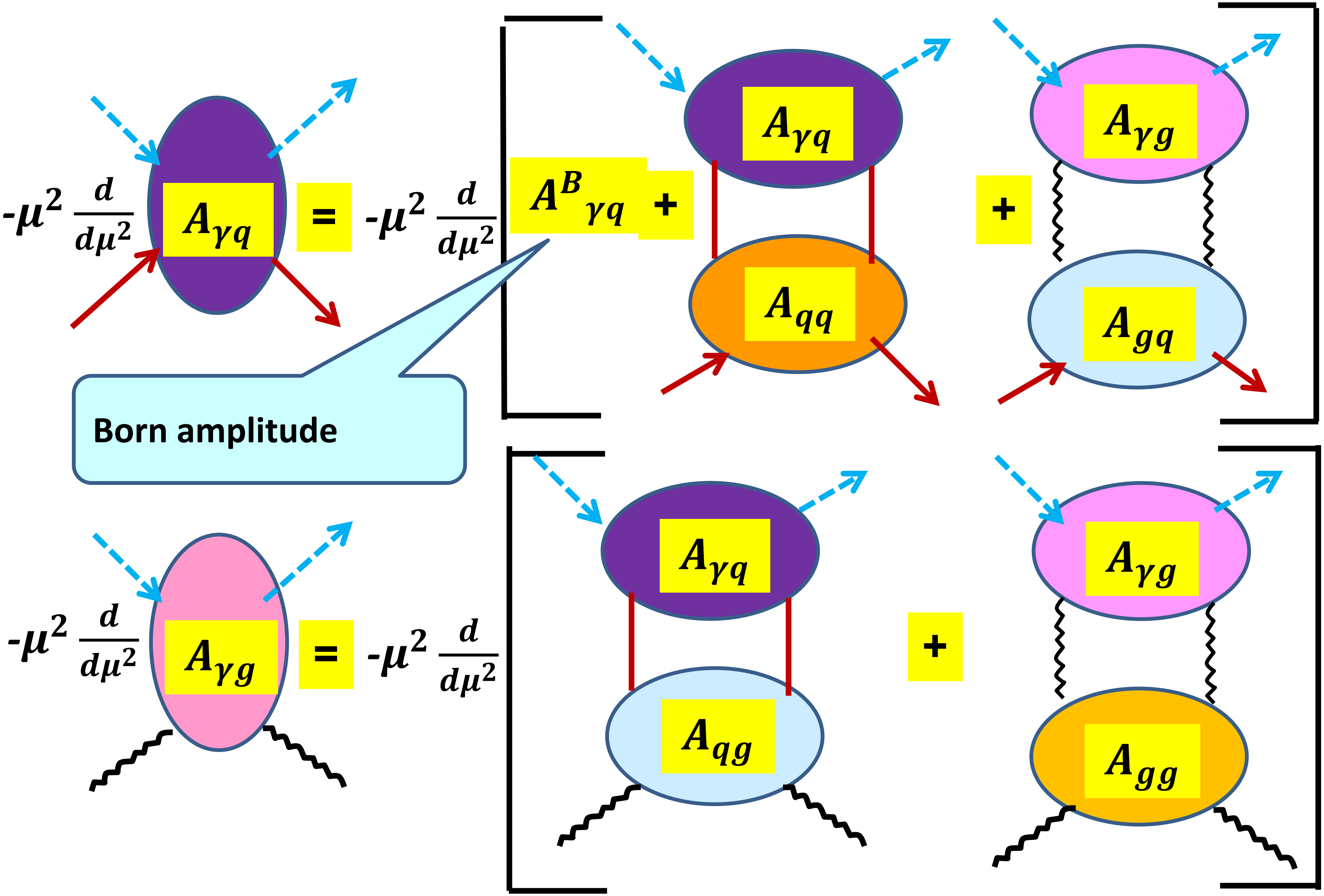
$\eta == |y_1 - y_2|$

Auxiliary amplitudes $F_{\gamma r}(\omega, z)$, $f_{\gamma r}(\omega)$ describe photon-parton scattering

$Q^2 \gg \mu^2$

$Q^2 \sim \mu^2$

IREE for auxiliary amplitudes



Applying the standard Feynman rules , arrive at IREEs in analytic form

$$\left[\omega + \frac{\partial}{\partial y}\right] F_{\gamma q}(\omega, y) = \frac{1}{8\pi^2} F_{\gamma q}(\omega, y) f_{qq}(\omega) + \frac{1}{8\pi^2} F_{\gamma g}(\omega, y) f_{gq}(\omega)$$

$$\left[\omega + \frac{\partial}{\partial y}\right] F_{\gamma g}(\omega, y) = \frac{1}{8\pi^2} F_{\gamma q}(\omega, y) f_{qg}(\omega) + \frac{1}{8\pi^2} F_{\gamma g}(\omega, y) f_{gg}(\omega)$$

IREEs involve new objects: parton-parton amplitudes

$f_{ik}(\omega)$

Eqs are linear, so it is easy to find general solutions in terms of the parton-parton amplitudes.
It is more convenient to use

$$h_{ik}(\omega) = \left(1/8\pi^2\right) f_{ik}(\omega)$$

Particular solution are specified with the matching

$$F_{\gamma q}(\omega, y = 0) = f_{\gamma q}(\omega)$$

amplitudes of photon-parton scattering when y =0. They are unknown and must be calculated independently

IREEs for them are algebraic because *y=0*

$$\omega f_{\gamma q}(\omega) = a_{\gamma q} + f_{\gamma q}(\omega) h_{qq}(\omega) + f_{\gamma g}(\omega) h_{gq}(\omega)$$

$$\omega f_{\gamma g}(\omega) = f_{\gamma q}(\omega) h_{qg}(\omega) + f_{\gamma g}(\omega) h_{gg}(\omega)$$

with $a_{\gamma q} = \alpha/2\,\pi$,

Solution to these equations allows us to express the auxiliary amplitudes in terms of the parton-parton amplitudes

IREE for the parton-parton amplitudes

$$\omega h_{qq} = b_{qq} + h_{qq} h_{qq} + h_{qg} h_{gq}$$
$$\omega h_{qg} = b_{qg} + h_{qq} h_{qg} + h_{qg} h_{gg}$$
$$\omega h_{gq} = b_{gq} + h_{gq} h_{qq} + h_{gg} h_{gq}$$
$$\omega h_{gg} = b_{gg} + h_{gq} h_{qg} + h_{gg} h_{gg}$$

$$b_{ik}(\omega) = a_{ik}(\omega) + V_{ik}(\omega)$$

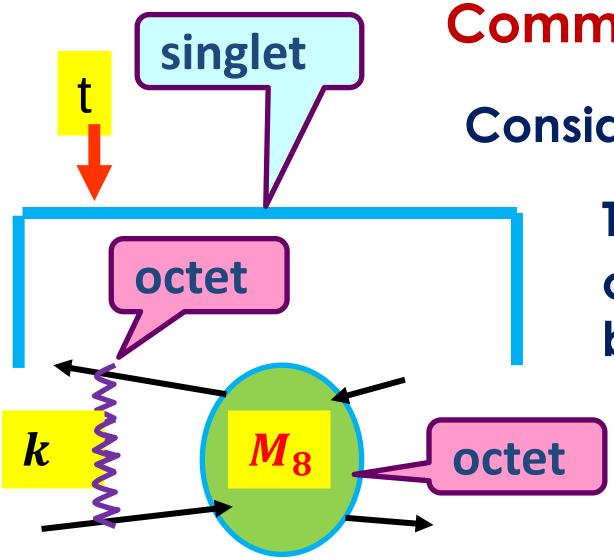
Born contributions. They are independent of ω when QCD coupling is fixed but depend on it when the coupling is running

Contributions of the color octet (non-ladder graphs)

$a_{ik}(\omega)$ coincide with analogous factors for g_1 singlet, see e.g. Bartels-Ermolaev-Ryskin (1996)

However such coincidence does not take place for $V_{ik}(\omega)$

Comment on the color octets $V_{ik}(\omega)$



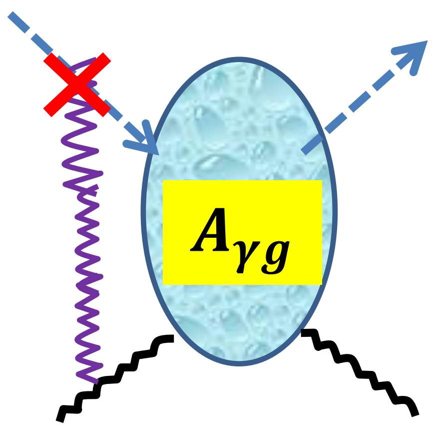
Consider a term of IREE with the factorized gluon.

The gluon belongs to the vector representation of the color group SU(3). Hence, the amplitude M_8 belongs to the octet representation too

$$8 \otimes 8 = 1 \oplus 8_s \oplus 8_A \oplus \ \dots$$

We have to compose IREEs for the parton-parton octet amplitudes.
It is done absolutely in the same way

Fortunately, we need not octet components for photon-parton amplitudes:
Photon-gluon vertices are absent in the QCD Lagrangian



Fixed QCD coupling

$$a_{qq} = (\alpha_s/2\pi)\, C_F\,, \qquad a_{qg} = (\alpha_s/\pi) C_F,$$
$$a_{gq} = -(\alpha_s/2\pi)\, n_f\,, \qquad a_{gg} = (\alpha_s/\pi)\, 2N$$

Running QCD coupling

One cannot use the DGLAP parametrization $\alpha_s = \alpha_s(Q^2)$
based on the parametrization
in every ladder rung $\alpha_s = \alpha_s({k_T}^2)$
because it fails at small x (in the high-energy limit)

In this case the more adequate parametrization should be used.

$$A(\omega) = (1/b)\left[\frac{\eta}{\eta^2+\pi^2} - \int_0^\infty dz \frac{e^{-z\omega}}{(z+\eta)^2+\pi^2}\right]$$

Ermolaev-Greco-Troyan

k is time-like momentum

$$A'(\omega) = (1/b)\left[\frac{1}{\eta} - \int_0^\infty dz \frac{e^{-z\omega}}{(z+\eta)^2}\right]$$

k is space-like momentum

where $\eta = ln(\mu^2/\Lambda_{QCD}{}^2)$, $b = (11N - 2\,n_f)/(12\,\pi)$

Principle of Minimal Sensitivity (P.M.Stevenson) was used to specify the scale of μ

It leads to the following expressions

$$a_{qq} = (A(\omega)/2\pi)\ C_F\,, \qquad a_{qg} = (A'(\omega)/\pi)C_F,$$
$$a_{gq} = -(A'(\omega)/2\pi)\ n_f\,, \qquad a_{gg} = (A(\omega)/\pi)\ 2N$$

The scale of α_s for NLO BFKL Pomeron intercept was set (Brodsky-Fadin-Kim-Lipatov-Pivovarov) with using both Principle of Minimal sensitivity and Principle of Maximum Conformality (Brodsky-Di Giustino)

The system of non-linear algebraic equations for can be solved exactly for both the case of fixed and running QCD coupling. Explicit expressions for the parton-parton amplitudes h_{ik} are

$$h_{qq} = \frac{1}{2}\left[\omega - Z - \frac{b_{gg} - b_{qq}}{Z}\right] \qquad h_{qg} = \frac{b_{qg}}{Z}$$

$$h_{gq} = \frac{b_{gq}}{Z} \qquad h_{gg} = \frac{1}{2}\left[\omega - Z + \frac{b_{gg} - b_{qq}}{Z}\right]$$

where

$$Z = \frac{1}{\sqrt{2}}\left[U + W\right]$$

$$U = \omega^2 - 2\left(b_{qq} + b_{gg}\right)$$

$$W = \left[\left(\omega^2 - 2b_{gg} - 2b_{qq}\right)^2 - 4\left(b_{gg} - b_{qq}\right)^2 - 16 b_{qg} b_{gq}\right]^{1/2}$$

Substituting them in expressions for auxiliary amplitudes $F_{\gamma q}$ $F_{\gamma q}$ and then in expressions for photon-photon amplitudes, we arrive at explicit expressions for $A^{(M)}{}_{\gamma\gamma}$, $A^{(D)}{}_{\gamma\gamma}$

Substituting the explicit expressions for parton-parton amplitudes into the expressions for the auxiliary photon-parton amplitudes and then substituting the latter into the expressions for the photon-photon amplitudes we arrive at the explicit expressions to them

These expressions are complicated, so we do not write them

Instead, let us focus on the high-energy asymptotics of amplitudes

$$A_{\gamma\gamma}(s, Q_1{}^2, Q_2{}^2)$$

The standard mathematical tool to calculate asymptotics at $s \to \infty$ is Saddle-Point method:

Saddle-Point method:

$$A(s) = \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} e^{\omega\rho} F(\omega) = \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} e^{\omega\rho + \Psi(\omega)}$$

We remind that $\rho = ln\left(s/\mu^2\right)$

Asymptotics: At $s \rightarrow \infty$, $A \sim \frac{e^{\Psi(\omega_0)}}{\sqrt{2\pi\Psi''(\omega_0)}} \left(\frac{s}{\mu^2}\right)^{\omega_0}$

$\rho + \Psi'(\omega_0) = 0$

$\rho \rightarrow \infty$

$\Psi'(\omega) \rightarrow -\infty$ when $\omega \rightarrow \omega_0$

Singularity point

There can be many singularities but the most important is the rightmost singularity. It can be a pole or a branching point of $\Psi'(\omega)$

Using explicit expression for $\Psi'(\omega)$ we conclude that ω_0 is the rightmost branching point . It corresponds to the largest root of

$$\left(\omega^2 - 2b_{gg} - 2b_{qq}\right)^2 - 4\left(b_{gg} - b_{qq}\right)^2 - 16 b_{qg} b_{gq} = 0$$

Stationary point equation

This equation can be solved analytically when QCD coupling is fixed. However if it runs, only numerical solution can be obtained

Asymptotics of light-by-light amplitudes is of the Regge form:

impact-factors

Reggeon

$$A_{\gamma\gamma} \sim \bar{A}_{\gamma\gamma} = \frac{N}{ln^{3/2}(s/\mu^2)} \left(\frac{s}{\sqrt{{Q_1}^2{Q_1}^2}}\right)^{\omega_0}$$

This Reggeon has vacuum quantum numbers, so potentially it can be a new, DL contribution to Pomeron. It has nothing in common with BFKL Pomeron where Leading Logs are accounted for

We define $x = \left(\sqrt{{Q_1}^2{Q_1}^2}\right)/s$ and we will write down the asymptotics in the BFKL manner:

$$\bar{A}_{\gamma\gamma} = x^{-(1+\Delta)}$$

BFKL Pomeron is asymptotics of the SL series:

$$\left(\frac{1}{x}\right)\left[1 + c_1(\alpha_s ln(1/x)) + c_2(\alpha_s ln(1/x))^2 + c_3(\alpha_s ln(1/x))^3 + \cdots\right]$$

Asymptotics $x^{-\omega_0}$

$\omega_0 = 1 + \Delta$

comes from resummation

comes from the overall factor 1/x

DL Pomeron is asymptotics of the DL series:

$$1 + c'_1\left(\alpha_s ln^2(1/x)\right) + c'_2\left(\alpha_s ln^2(1/x)\right)^2 + c'_3\left(\alpha_s ln^2(1/x)\right)^3 + \cdots$$

The factor 1/x is absent, so the whole ω_0 comes entirely from calculations

For comparison with BFKL, it is convenient to introduce

$$\Delta \equiv \omega_0 - 1$$

We calculate the intercept for several particular cases:

1. QCD coupling is fixed

Then the equation for the stationary point is algebraic and can be solved analytically:

$$\omega_{0_{fix}} = (\alpha_s/\pi)^{1/2}\left[4N + C_F + \sqrt{(4N - C_F)^2 - 8n_f C_F}\right]^{1/2}$$

α_s=0.24

Ermolaev-Greco-Troyan on basis of Principle of Minimal Sensitivity

A. quark contributions neglected, i.e. purely gluonic Pomeron

Δ_{fix}= 0.35

close to LO BFKL intercept

$\Delta_{LO\ BFKL}$=0.34

B. both gluon and quark contributions accounted for Δ_{fix}= 0.29

2. Accounting for the running α_s effects

C. Purely gluonic Pomeron $\Delta = 0.25$

D. Both gluon and quark contributions are taken into account

$\Delta = 0.066$ ← Close to NLO BFKL intercept

↕

$\Delta_{NLO\ BFKL}$ =0.08

We think that there is no a physical reason whatsoever for DL intercepts be close to BFKL ones and consider it as coincidence

OBSERVATION: The higher accuracy, the smaller the Pomeron intercept

Δ_{fix}= 0.35 — Fixed coupling, gluons only — Hard Pomeron

Δ_{fix}= 0.29 — Fixed coupling, gluons and quarks

$\Delta = 0.25$ — Coupling runs, gluons only

$\Delta = 0.07$ — Coupling runs, gluons and quarks — Soft Pomeron

SUGGESTION: Increase of accuracy converts hard Pomeron into Soft one, so eventually the intercept could go down to zero, which would restore Unitarity

This tendency is seen for both BFKL and DL Pomerons

Applicability region of Regge asymptotics

Regge asymptotics are given by simple and elegant expressions. Let us fix their applicability region

We introduce $R_{as} = \bar{A}_{\gamma\gamma} / A_{\gamma\gamma}$

and study its x-dependence at fixed Q^2 e.g. at $Q^2 = 10$ GeV^2

NB The asymptotics reliably represents the parent amplitude $A_{\gamma\gamma}$ when R_{as} is close to 1.

$R_{as} = 0.9$ x = 10^{-6}

$R_{as} = 0.67$ x = 10^{-4}

$R_{as} = 0.5$ x = 10^{-3}

Appicability region for DL Pomeron

x $< x_{max}$ = 10^{-6}

$$s > s_{min} = 10^6 \sqrt{{Q_1}^2 {Q_2}^2}$$

Does not depend on the specific process but is the same for all reactions involving DL Pomeron

At larger values $x > x_{max} = 10^{-6}$ the parent amplitudes should be used instead of Regge asymptotics .
However the asymptotics are often used outside their applicability region. It leads to various misconceptions. We consid[...] important ones:

Model intercept

1. Necessity of phenomenological/model hard Pomerons to fit exp data

Let us denote a parent amplitude *M*
By definition $x_{max}{}^{-\omega_0} \approx M$ with $x_{max} = 10^{-6}$

The model Regge expression representing *M* at $x_1 > x_{max}$ is $x_1{}^{-\sigma}$
Therefore $x_1{}^{-\sigma} \approx M$ and $x_{max}{}^{-\omega_0} \approx x_1{}^{-\sigma}$

Choosing $x_1 = 10^{-4}$ we arrive at

$$\sigma = \frac{6}{4}\omega_0 = \frac{3}{2} 1.07 = 1.6$$

Usage of asymptotics outside their Applicability region causes hard Pomeron

soft

hard

2. Necessity of spin-dependent Pomerons

Asymptotics of spin-dependent structure function g_1 is also of the Regge type with the non-vacuum intercept $\omega_0^{(-)} = 0.87$

Ermolaev-Greco-Troyan

Let a model Reggeon with intercept $\sigma^{(-)}$ mimic g_1 at 10^{-4}

$$\sigma^{(-)} = \frac{6}{4}\,\omega_0^{(-)} = \frac{3}{2}\,0.87 = 1.3$$

Non-vacuum Reggeon

Hard Pomeron

Mimicking parent amplitudes by Regge-like asymptotics inevitably leads to introducing model hard Pomerons, which is totally groundless

Such transformations of Pomerons can be wrongly interpreted as dependence of the intercepts on x or Q^2

CONCLUSIONS

We have obtained explicit expressions for light-by-light scattering amplitudes $A_{\gamma\gamma}$ in DLA, with fixed and running QCD coupling

Applying Saddle-Point method to these expressions, we arrive at the Regge asymptotics, with the Reggeon bearing the vacuum quantum numbers. So, it a new, DL contribution to Pomeron.

Altough intercepts of DL Pomeron are not far from the ones of BFKL and both of them are supercritical, DL Pomeron has nothing in common with BFKL Pomeron which is
asymptotics of total resummation of single-logarithmic contributions while we deal with double logarithms.

Value of the DL Pomeron intercept monotonically decreases with increase of accuracy of calculations. It tempts us to suggest that the further increase of accuracy should make the intercept be= 1, which would agree with the Froissart-Martin bound i.e. with Unitarity

Explicit expressions for scattering amplitudes are quite complicated. In contrast, their Regge asymptotics are represent by simple exponential expressions, so they are often used instead of the parent amplitudes. Comparing $A_{\gamma\gamma}$ to their asymptotics , we fixed the applicability region of the high-energy asymptotics of $A_{\gamma\gamma}$

We found that usage of the asymptotics outside their applicability region inevitably leads to introducing hard Pomeron(s)

We think that Interference of BFKL and DL Pomerons contributions to different reactions should be examined in detail

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