

# Unitarity Constraints in BSM: new results in the 331 Models

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COMPOSE-IT

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Sezione di Bologna

based on

AC, M.Ghezzi, G.M.Pruna  
arXiv:2001.08550

G.Corcella, AC, M.Ghezzi, L.Panizzi, G.M.Pruna  
in preparation

# Content

## Unitarity in the SM

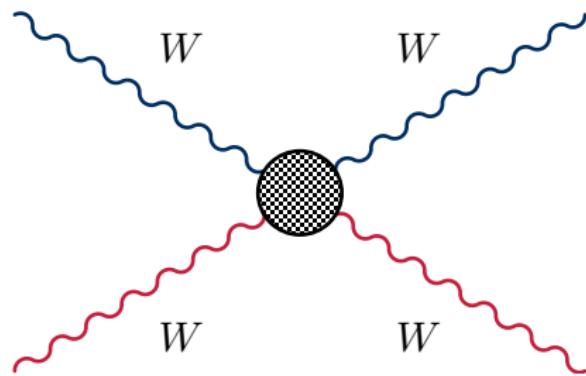
331 Models

Basic Ingredients

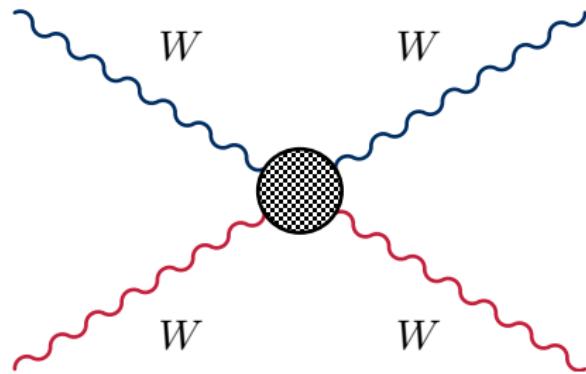
The Scalar Potential: Theoretical Constraints

Preliminary Results

# Higgs in the SM: Unitarity constraint



# Higgs in the SM: Unitarity constraint



$$A(W^+W^- \rightarrow W^+W^-) \xrightarrow{s \gg M_W^2} \frac{1}{v^2} \left[ s + t - \frac{s^2}{s - M_H^2} - \frac{t^2}{t - M_H^2} \right]$$

## Higgs in the SM: Unitarity constraint

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with the optical theorem

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with the optical theorem

$$\begin{aligned}|a_\ell|^2 &= \text{Im}(a_\ell) \Rightarrow [\text{Re}(a_\ell)]^2 + [\text{Im}(a_\ell)]^2 = \text{Im}(a_\ell) \\ &\Rightarrow [\text{Re}(a_\ell)]^2 + [\text{Im}(a_\ell) - \frac{1}{2}]^2 = \frac{1}{4}\end{aligned}$$

## Higgs in the SM: Unitarity constraint

$$\begin{aligned}a_0 &= \frac{1}{16\pi s} \int_s^0 dt |A| \\&= -\frac{M_H^2}{16\pi v^2} \left[ 2 + \frac{M_H^2}{s - M_H^2} - \frac{M_H^2}{s} \log \left( 1 + \frac{s}{M_H^2} \right) \right]\end{aligned}$$

$$a_0 \xrightarrow{s \gg M_H^2} -\frac{M_H^2}{8\pi v^2}$$

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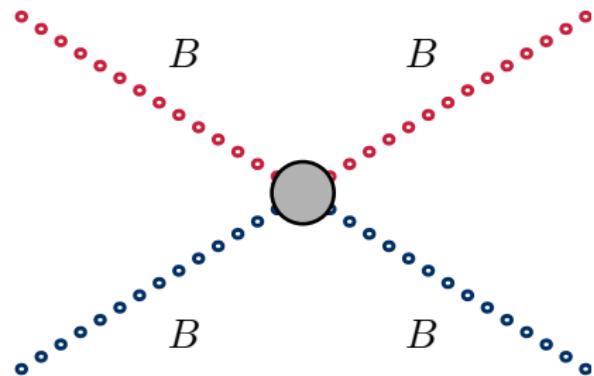
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↓

$$M_H \lesssim 870 \text{ GeV}$$

# Higgs in the SM: Unitarity constraint



$$B = W, Z, H$$

$$\left( W_L^+ W_L^- , \frac{1}{\sqrt{2}} Z_L Z_L , \frac{1}{\sqrt{2}} H H , Z_L H , W_L^+ H , W_L^+ Z_L \right)$$

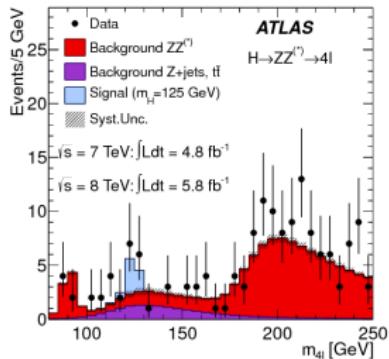
## Higgs in the SM: Unitarity constraint

$$a_0 \propto \frac{M_H^2}{v^2} \begin{pmatrix} 1 & \frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & 0 & 0 & 0 \\ \frac{\sqrt{2}}{4} & \frac{3}{4} & \frac{1}{4} & 0 & 0 & 0 \\ \frac{\sqrt{2}}{4} & \frac{1}{4} & \frac{3}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

⇓

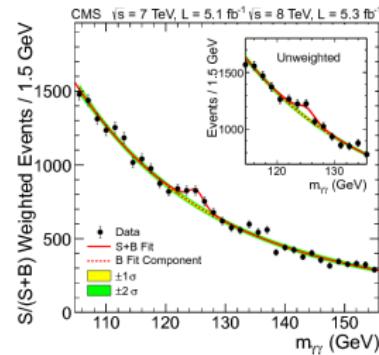
$$M_H \lesssim 710 \text{ GeV}$$

ATLAS



ATLAS Collaboration, Phys.Lett. B716 (2012) 1-29

CMS



CMS Collaboration, Phys.Lett. B716 (2012) 30-61

$$m_H = 125.10 \pm 0.14 \text{ GeV}$$

Unitarity of the SM is experimentally proved!

Quarks



Leptons



Any need for BSM?

## Any need for BSM?

- ◆ dark matter
  - ◆ neutrino masses
  - ◆ EWPT
  - ◆ matter-antimatter asymmetry
  - ◆ hierarchy problem
  - ◆ strong CP problem
  - ◆  $n_{Q_f} = n_{L_f} = 3$
  - ◆ hierarchy in fermion masses
- ...

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The Scalar Potential: Theoretical Constraints

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# Extended Gauge Group

$$SU(3)_c \times SU(3)_L \times U(1)_X$$

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SU(3) has two diagonal generators



$$\mathbb{Q} = \mathbb{T}_3 + \beta_Q \mathbb{T}_8 + X \mathbb{I}$$

# Field Content

$$Q_1 = \begin{pmatrix} u \\ d \\ D \end{pmatrix}, \quad Q_2 = \begin{pmatrix} c \\ s \\ S \end{pmatrix}, \quad Q_{1,2} \in (3, 3, \textcolor{red}{X}_{Q_{1,2}})$$

$$Q_3 = \begin{pmatrix} b \\ t \\ T \end{pmatrix}, \quad Q_3 \in (3, \bar{3}, \textcolor{red}{X}_{Q_3})$$

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$$Q_3 = \begin{pmatrix} b \\ t \\ T \end{pmatrix}, \quad Q_3 \in (3, \bar{3}, X_{Q_3})$$

$$L = \begin{pmatrix} l \\ \nu_l \\ E_l \end{pmatrix}, \quad l \in (1, \bar{3}, X_L), \quad l = e, \mu, \tau$$

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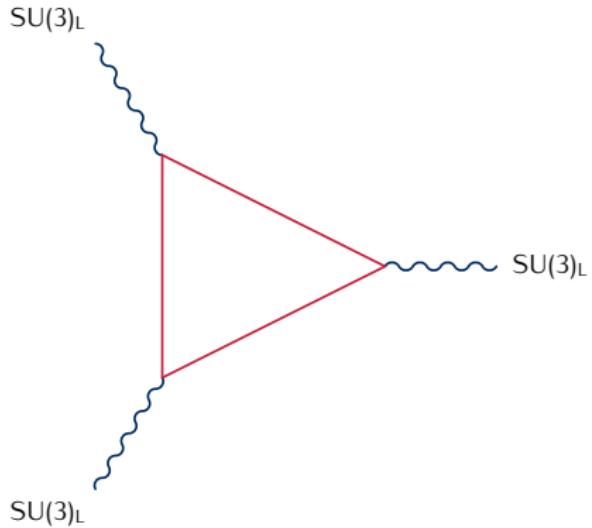
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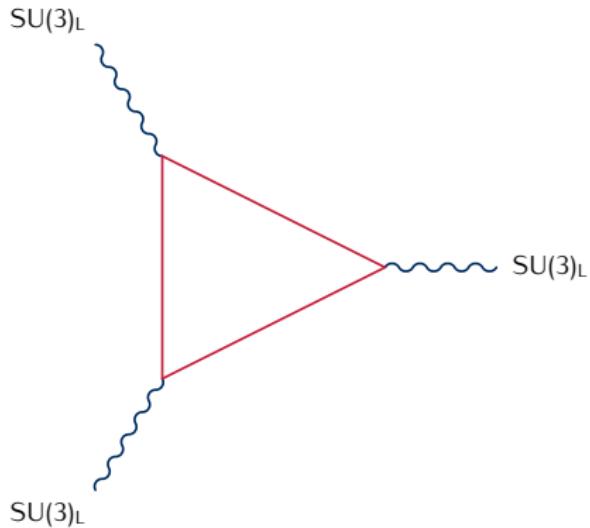
$$\chi = \begin{pmatrix} \chi^A \\ \chi^B \\ \chi^0 \end{pmatrix} \in (1, 3, \textcolor{red}{X}_\chi), \quad \rho = \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^- \end{pmatrix} \in (1, 3, \textcolor{red}{X}_\rho), \quad \eta = \begin{pmatrix} \eta^0 \\ \eta^- \\ \eta^- \end{pmatrix} \in (1, 3, \textcolor{red}{X}_\eta)$$

$$Q^A = \frac{1}{2} + \frac{\sqrt{3}}{2} \beta_Q, \quad Q^B = -\frac{1}{2} + \frac{\sqrt{3}}{2} \beta_Q$$

## Anomaly Cancellation: the $SU(3)_L$ example



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$$Q_1 = +3 \times 3_c$$

$$Q_2 = +3 \times 3_c$$

$$Q_3 = -3 \times 3_c$$

$$L = -3 \times 3_f$$

$$n_{Q_f} = n_{L_f} = 3 \kappa$$

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From  $SU(3)_L \times U(1)_X$  to  $U(1)_{em}$

$SU(3)_L \times U(1)_X$

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$SU(3)_L \times U(1)_X$

$$\langle \chi \rangle$$
$$\Downarrow$$

$SU(2)_L \times U(1)_Y$

From  $SU(3)_L \times U(1)_X$  to  $U(1)_{\text{em}}$

$SU(3)_L \times U(1)_X$

$\langle \chi \rangle$   
↓

$SU(2)_L \times U(1)_Y$

$\langle \eta \rangle, \langle \rho \rangle$   
↓

$U(1)_{\text{em}}$

From  $SU(3)_L \times U(1)_X$  to  $U(1)_{\text{em}}$

$SU(3)_L \times U(1)_X$

$W_1, \dots, W_8, B_X$

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$SU(2)_L \times U(1)_Y$

$W_1, W_2, W_3, B_Y, Z', Y^{\pm A}, V^{\pm B}$

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↓

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↓

$U(1)_{em}$

$\gamma, Z, W^\pm, Z', Y^{\pm A}, V^{\pm B}$

# The Scalar Potential

The ( $\beta_Q$ -invariant) potential is

$$\begin{aligned} V = & m_1 \rho^* \rho + m_2 \eta^* \eta + m_3 \chi^* \chi \\ & + \lambda_1 (\rho^* \rho)^2 + \lambda_2 (\eta^* \eta)^2 + \lambda_3 (\chi^* \chi)^2 \\ & + \lambda_{12} \rho^* \rho \eta^* \eta + \lambda_{13} \rho^* \rho \chi^* \chi + \lambda_{23} \eta^* \eta \chi^* \chi \\ & + \zeta_{12} \rho^* \eta \eta^* \rho + \zeta_{13} \rho^* \chi \chi^* \rho + \zeta_{23} \eta^* \chi \chi^* \eta \\ & + \sqrt{2} f_{\rho \eta \chi} (\rho \eta \chi + h.c.) \end{aligned}$$

$$[m_i] = M^2$$

$$[f_{\rho \eta \chi}] = M^1$$

$$[\lambda_j] = [\zeta_k] = M^0$$

# Massive Scalar States

$$\chi = \begin{pmatrix} \chi^A \\ \chi^B \\ \chi^0 \end{pmatrix}, \quad \rho = \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^- \end{pmatrix}, \quad \eta = \begin{pmatrix} \eta^0 \\ \eta^- \\ \eta^- \end{pmatrix}$$

#3 neutral scalar, #1 neutral pseudoscalar ( $G_Z, G_{Z'}$ ),  
#1 singly-charged ( $G_W$ ),  
#1 A-charged ( $G_{Y^A}$ ), #1 B-charged ( $G_{V^B}$ )

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diagonalisation



$$\begin{aligned} m_{h_i} &= f_{h_i}(\vec{\lambda}, \vec{\zeta}, f_{\rho\eta\chi}, v_j) \\ m_a &= f_a(\vec{\lambda}, \vec{\zeta}, f_{\rho\eta\chi}, v_j) \\ m_{h^{\pm Q}} &= f_{h^{\pm Q}}(\vec{\lambda}, \vec{\zeta}, f_{\rho\eta\chi}, v_j) \end{aligned}$$

"inverse" diagonalization



$$\begin{aligned} \lambda &= F_\lambda(m_{h_i}, m_a, m_{h^{\pm Q}}, v_j, \alpha_k) \\ f_{\rho\eta\chi} &= F_{f_{\rho\eta\chi}}(m_{h_i}, m_a, m_{h^{\pm Q}}, v_j, \alpha_k) \\ \zeta &= F_\zeta(m_{h_i}, m_a, m_{h^{\pm Q}}, v_j, \alpha_k) \end{aligned}$$

# Trading the Parameter: Explicit Example

$$\begin{aligned}\lambda_1 = & -\frac{m_{a_1}^2 \tan^2 \beta}{2v^2} + m_{h_1}^2 \frac{c_2^2 c_3^2 \sec^2 \beta}{2v^2} \\ & + m_{h_2}^2 \frac{\sec^2 \beta (s_1 s_2 c_3 - c_1 s_3)^2}{2v^2} \\ & + m_{h_3}^2 \frac{\sec^2 \beta (c_1 s_2 c_3 + s_1 s_3)^2}{2v^2} + O\left(\frac{m}{v_X}\right)\end{aligned}$$

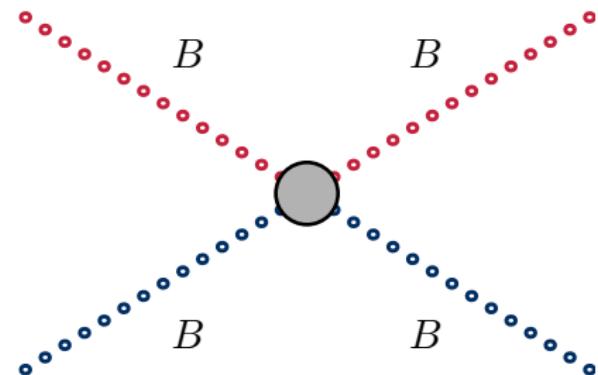
$$\begin{aligned}\lambda_2 = & -\frac{m_{a_1}^2 \cot^2 \beta}{2v^2} + m_{h_1}^2 \frac{c_2^2 s_3^2 \csc^2 \beta}{2v^2} \\ & + m_{h_2}^2 \frac{\csc^2 \beta (s_1^2 s_2^2 s_3^2 + 2s_1 c_1 s_2 c_3 s_3 + c_1^2 c_3^2)}{2v^2} \\ & + m_{h_3}^2 \frac{\csc^2 \beta (c_1 s_2 (c_1 s_2 s_3^2 - 2s_1 c_3 s_3) + s_1^2 c_3^2)}{2v^2} + O\left(\frac{m}{v_X}\right)\end{aligned}$$

$$\begin{aligned}\lambda_{12} = & \frac{m_{a_1}^2}{v^2} + m_{h_1}^2 \frac{c_2^2 s_3 c_3 \csc \beta \sec \beta}{v^2} \\ & + m_{h_2}^2 \frac{\csc \beta \sec \beta}{4v^2} \left( 4c_1 s_1 s_2 (c_3^2 - s_3^2) \right. \\ & \left. - c_3 s_3 (2s_1^2 (c_2^2 - s_2^2) + 6c_1 s_1 + 1) \right) \\ & - m_{h_3}^2 \frac{\csc \beta \sec \beta}{4v^2} \left( 4c_1 s_1 s_2 (c_3^2 - s_3^2) \right. \\ & \left. + c_3 s_3 (2c_1^2 (c_2^2 - s_2^2) - 3(c_1^2 - s_1^2) + 1) \right) + O\left(\frac{m}{v_X}\right)\end{aligned}$$

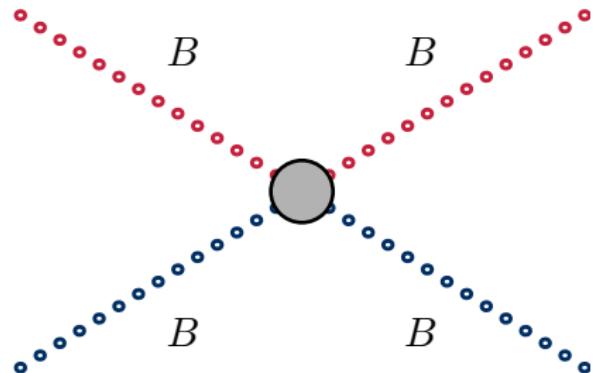
$$\begin{aligned}\zeta_{12} = & \frac{2}{v^2} \left( m_{h_1^\pm}^2 - m_{a_1}^2 \right) + O\left(\frac{m}{v_X}\right) \\ \lambda_3 = \lambda_{13} = \lambda_{23} = \zeta_{13} = \zeta_{23} = & O\left(\frac{m}{v_X}\right)\end{aligned}$$

perturbativity of the couplings requires certain degeneracy among masses

# Unitarity of the 331 Models



# Unitarity of the 331 Models



$$B = Z, W^\pm, Z', Y^{\pm A}, V^{\pm B}, h_i, a_1, h^\pm, h^{\pm A}, h^{\pm B}$$

$$\begin{aligned} Q &= 0, 1, 2, Q^A, Q^B, Q^A + 1, Q^B + 1, Q^A - 1, Q^B - 1, \\ &Q^A + Q^B, Q^A - Q^B, 2Q^A, 2Q^B \end{aligned}$$

# Unitarity of the 331 Models

perturbative unitarity  
↓

$$\frac{1}{2} > \lambda_{\max} = \max \text{eig}[\mathcal{S}]$$

$\mathcal{S}$  matrix of all  $2 \rightarrow 2$  bosonic amplitudes (in the  $s \rightarrow \infty$  limit)

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$\mathcal{S}$  matrix of all  $2 \rightarrow 2$  bosonic amplitudes (in the  $s \rightarrow \infty$  limit)

to be addressed numerically, the largest matrix is **30×30**  
 $(\mathcal{Q} = 0)$

# BFB in the 331 Models

boundedness from below of the scalar potential



analysis of the highest powers of the fields

# BFB in the 331 Models

boundedness from below of the scalar potential



analysis of the highest powers of the fields

not-so-easy in multi-Higgs models, done for multi-doublets

Hadeler(1983); Klimenko(1984); Ivanov(2018); Maniatis(2006);  
Degee(2012); Kannike(2012); Maniatis(2015); Kannike(2016);  
Faro(2019);

# BFB in the 331 Models

$$\Phi_i = \sqrt{r_i} e^{i \gamma_i} \begin{pmatrix} \sin a_i \cos b_i \\ e^{i \beta_i} \sin a_i \sin b_i \\ e^{i \alpha_i} \cos a_i \end{pmatrix}$$

$$V^{(4)} = V_R + \zeta'_{12} \tau_{12} + \zeta'_{13} \tau_{13} + \zeta'_{23} \tau_{23} = V_R + V_A,$$

$$\begin{aligned} V_R &= \lambda_1(\rho^* \rho)^2 + \lambda_2(\eta^* \eta)^2 + \lambda_3(\chi^* \chi)^2 \\ &\quad + \lambda'_{12} \rho^* \rho \eta^* \eta + \lambda'_{13} \rho^* \rho \chi^* \chi + \lambda'_{23} \eta^* \eta \chi^* \chi \end{aligned}$$

$$\tau_{ij} = \left( \Phi_i^\dagger \Phi_i \right) \left( \Phi_j^\dagger \Phi_j \right) - \left( \Phi_i^\dagger \Phi_j \right) \left( \Phi_j^\dagger \Phi_i \right)$$

## BFB in the 331 Models

BFB  $\equiv$  co-positivity of  $Q_{ij}$

$$V^{(4)} = Q_{ij} r_i r_j$$

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co-positivity of a generic rank-3 matrix  $A$  is

$$A_{ii} \geq 0, \quad \text{with } i = 1, 2, 3,$$

$$\mathring{A}_{ij} \equiv \sqrt{A_{ii} A_{jj}} + A_{ij} \geq 0, \quad \text{with } i, j = 1, 2, 3,$$

$$\begin{aligned} & \sqrt{A_{11} A_{22} A_{33}} + A_{12} \sqrt{A_{33}} + A_{13} \sqrt{A_{22}} + A_{23} \sqrt{A_{11}} \\ & + \sqrt{2\mathring{A}_{12}\mathring{A}_{13}\mathring{A}_{23}} \geq 0 \end{aligned}$$

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# C<sup>2</sup> G P<sup>2</sup> Collaboration

analytical "inverse" diagonalization + scan over *physical*  
masses, vevs and rotation angles + pheno constraint on  
lightest neutral scalar + perturbative unitarity constraint + ...

in collaboration with *G. Corcella, M. Ghezzi, L. Panizzi and G.M. Pruna*

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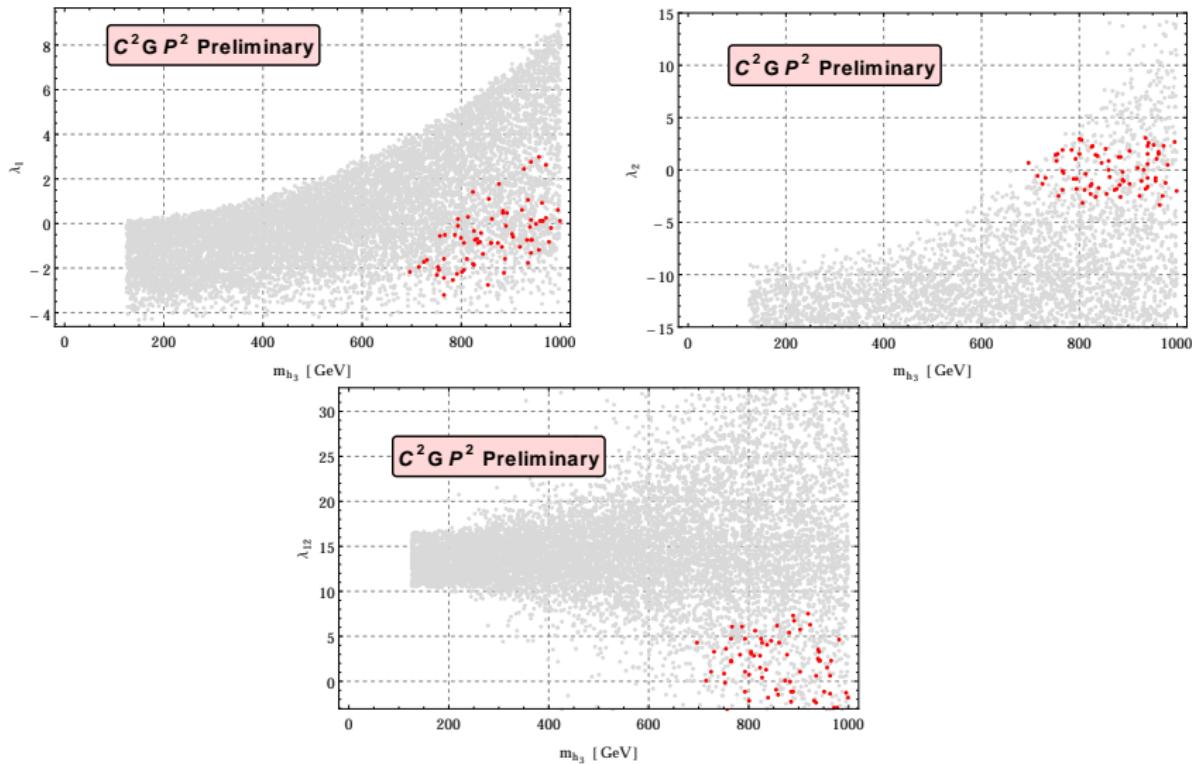
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perturbative unitarity  
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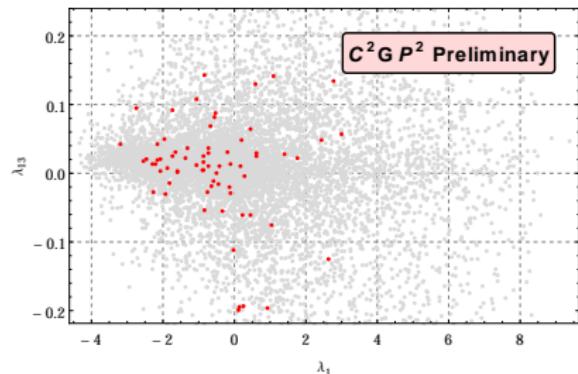
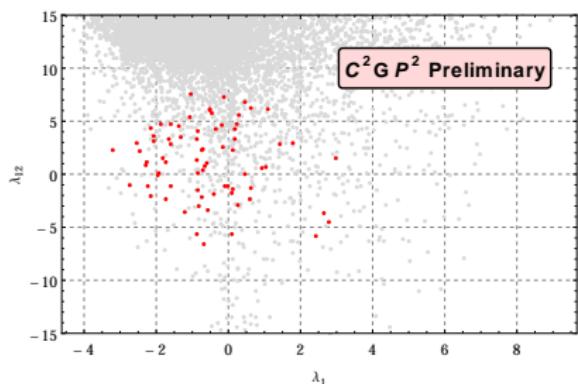
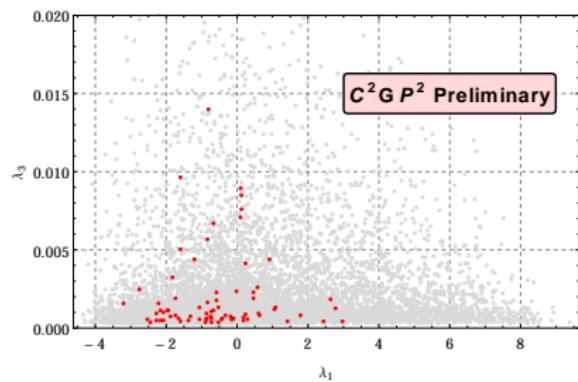
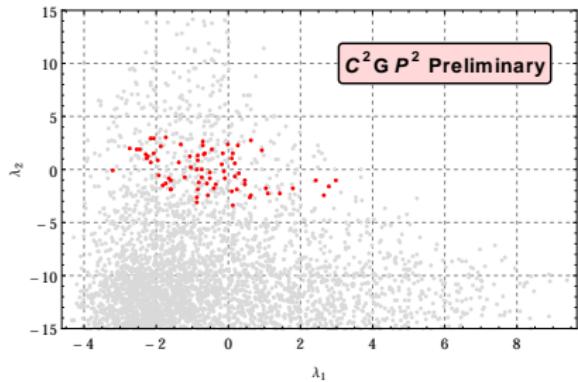
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# $C^2 G P^2$ Collaboration Preliminary Results



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# Conclusions

- ◆ SM issues: dark matter, neutrino masses ... → needs to be improved
- ◆ 331 model(s) explain the observed number of fermion families ( $n_{Q_f} = n_{L_f} = 3\kappa$ )
- ◆ 331 model is phenomenologically appealing as embrace various different scenarios
- ◆ analysis of the scalar potential is  $\beta_Q$ -independent (theoretical constraints: unitarity, perturbativity, BFB NEW!!!)
- ◆ forthcoming results on exotic quark phenomenology ( $C^2 G$   
 $P^2$ )



Thanks

# BACKUP

# Minimization Conditions

$$\rho^0 = \frac{1}{\sqrt{2}}v_\rho + \frac{1}{\sqrt{2}}(\operatorname{Re} \rho^0 + i \operatorname{Im} \rho^0)$$

$$\eta^0 = \frac{1}{\sqrt{2}}v_\eta + \frac{1}{\sqrt{2}}(\operatorname{Re} \eta^0 + i \operatorname{Im} \eta^0)$$

$$\chi^0 = \frac{1}{\sqrt{2}}v_\chi + \frac{1}{\sqrt{2}}(\operatorname{Re} \chi^0 + i \operatorname{Im} \chi^0)$$

Minimization conditions ( $\frac{\partial V}{\partial \Phi}|_{\Phi=0} = 0$ ) are

$$m_1 v_\rho + \lambda_1 v_\rho^3 + \frac{\lambda_{12}}{2} v_\rho v_\eta^2 - f_{\rho\eta\chi} v_\eta v_\chi + \frac{\lambda_{13}}{2} v_\rho v_\chi^2 = 0$$

$$m_2 v_\eta + \lambda_2 v_\eta^3 + \frac{\lambda_{12}}{2} v_\rho^2 v_\eta - f_{\rho\eta\chi} v_\rho v_\chi + \frac{\lambda_{23}}{2} v_\eta v_\chi^2 = 0$$

$$m_3 v_\chi + \lambda_3 v_\chi^3 + \frac{\lambda_{13}}{2} v_\rho^2 v_\chi - f_{\rho\eta\chi} v_\rho v_\eta + \frac{\lambda_{23}}{2} v_\eta^2 v_\chi = 0$$

# Scalars

CP-even neutral scalars mix

$$h_i = \mathcal{R}_{ij}^S H_j$$

$$\vec{H} = (\text{Re } \rho^0, \text{Re } \eta^0, \text{Re } \chi^0), \vec{h} = (h_1, h_2, h_3)$$

$$f_{\rho\eta\chi} = \kappa v_\chi$$

$$\beta = \tan^{-1} v_\eta / v_\rho, v = \sqrt{v_\eta^2 + v_\rho^2}.$$

$$m_h^2 = \begin{pmatrix} \kappa \tan \beta v_\chi^2 + 2\lambda_1 v^2 \cos^2 \beta & \lambda_{12} v^2 \cos \beta \sin \beta - \kappa v_\chi^2 & v_\chi v (\lambda_{13} \cos \beta - \kappa \sin \beta) \\ \lambda_{12} v^2 \cos \beta \sin \beta - \kappa v_\chi^2 & \kappa \cot \beta v_\chi^2 + 2\lambda_2 v^2 \sin^2 \beta & v_\chi v (\lambda_{23} \sin \beta - \kappa \cos \beta) \\ v_\chi v (\lambda_{13} \cos \beta - \kappa \sin \beta) & v_\chi v (\lambda_{23} \sin \beta - \kappa \cos \beta) & 2\lambda_3 v_\chi^2 + \kappa v^2 \cos \beta \sin \beta \end{pmatrix}$$

# Pseudocalar

CP-odd neutral scalars mix

$$a_i = \mathcal{R}_{ij}^P A_j$$

$$\vec{A} = (\text{Im } \rho^0, \text{Im } \eta^0, \text{Im } \chi^0), \vec{a} = (a_{G_Z}, a_{G_{Z'}}, a_1).$$

$$m_a^2 = \begin{pmatrix} \kappa v_X^2 \tan \beta & \kappa v_X^2 & \kappa v_X v \sin \beta \\ \kappa v_X^2 & \kappa v_X^2 \cot \beta & \kappa v_X v \cos \beta \\ \kappa v_X v \sin \beta & \kappa v_X v \cos \beta & \kappa v^2 \cos \beta \sin \beta \end{pmatrix}$$

↓

$$m_{a_1}^2 = \kappa(v_X^2 \csc \beta \sec \beta + v^2 \cos \beta \sin \beta)$$

# Singly-Charged State

Singly-charged states mix

$$h_i^- = \mathcal{R}_{ij}^C H_j^-$$

$$\vec{H}^- = ((\rho^+)^*, \eta^-), \vec{h}^- = (h_{G_W}^-, h_1^-)$$

$$m_{h^\pm}^2 = \begin{pmatrix} \kappa \tan \beta v_\chi^2 + \frac{1}{2} \zeta_{12} v^2 \sin^2 \beta & \kappa v_\chi^2 + \frac{1}{2} \zeta_{12} v^2 \cos \beta \sin \beta \\ \kappa v_\chi^2 + \frac{1}{2} \zeta_{12} v^2 \cos \beta \sin \beta & \kappa \cot \beta v_\chi^2 + \frac{1}{2} \zeta_{12} v^2 \cos^2 \beta \end{pmatrix}$$

⇓

$$m_{h_1^\pm}^2 = \frac{1}{2} \zeta_{12} v^2 + \kappa v_\chi^2 \csc \beta \sec \beta$$

# A-Charged State

A-charged states mix

$$h_i^A = \mathcal{R}_{ij}^A H_j^A$$

$$\vec{H}^A = ((\eta^{-A})^*, \chi^A), \vec{h}^A = (h_{G_{V^A}}^A, h_1^A)$$

$$m_{h^{\pm A}}^2 = \begin{pmatrix} \frac{1}{2} v_\chi^2 (\zeta_{23} + 2\kappa \cot \beta) & \frac{1}{2} v_\chi v (2\kappa \cos \beta + \zeta_{23} \sin \beta) \\ \frac{1}{2} v_\chi v (2\kappa \cos \beta + \zeta_{23} \sin \beta) & \frac{1}{2} v^2 \sin \beta (2\kappa \cos \beta + \zeta_{23} \sin \beta) \end{pmatrix}$$



$$m_{h_1^{\pm A}}^2 = \frac{1}{4} (\zeta_{23} + 2\kappa \cot \beta) (2v_\chi^2 + v^2 - v^2 \cos 2\beta)$$

# B-Charged State

$B$ -charged states mix

$$h_i^B = \mathcal{R}_{ij}^B H_j^B$$

$$\vec{H}^B = ((\rho^{-B})^*, \chi^B), \vec{h}^B = (h_{G_{V^B}}^B, h_1^B)$$

$$m_{h^{\pm B}}^2 = \begin{pmatrix} \frac{1}{2} v_\chi^2 (\zeta_{13} + 2\kappa \tan \beta) & \frac{1}{2} v_\chi v (2\kappa \sin \beta + \zeta_{13} \cos \beta) \\ \frac{1}{2} v_\chi v (2\kappa \sin \beta + \zeta_{13} \cos \beta) & \frac{1}{2} v^2 \cos \beta (2\kappa \sin \beta + \zeta_{13} \cos \beta) \end{pmatrix}$$



$$m_{h_1^{\pm B}}^2 = \frac{1}{4} (\zeta_{13} + 2\kappa \tan \beta) (2v_\chi^2 + v^2 - v^2 \cos 2\beta)$$

# Yukawa Interactions: Quark Sector

$$\begin{aligned}\mathcal{L}_{q,triplet}^{Yuk.} = & \left( y_d^1 Q_1 \rho^* d_R + y_d^2 Q_2 \rho^* s_R + y_d^3 Q_3 \eta b_R^* \right. \\ & + y_u^1 Q_1 \eta^* u_R^* + y_u^2 Q_2 \eta^* c_R^* + y_u^3 Q_3 \rho t_R^* \\ & \left. + y_E^1 Q_1 \chi^* D_R^* + y_E^2 Q_2 \chi^* S_R^* + y_E^3 Q_3 \chi T_R^* \right) + \text{h.c.}\end{aligned}$$

$$v_\chi \gg v_{\eta, \rho}$$

$\Downarrow$

$$m_{D,S,T} = \mathcal{O}(TeV) \text{ if } y_E^i \sim 1$$

# Yukawa Interactions: Lepton Sector

$$\begin{aligned}\mathcal{L}_{I, \text{triplet}}^{Yuk} &= G_{ab}^\rho (l_a^i \epsilon^{\alpha\beta} l_b^j) \rho^{*k} \epsilon^{ijk} + \text{h.c.} \\ &= G_{ab}^\rho l_a^i \cdot l_b^j \rho^{*k} \epsilon^{ijk} + \text{h.c.}\end{aligned}$$

$a$  and  $b$  are flavour indices

$\alpha$  and  $\beta$  are Weyl indices ( $l_a^i \cdot l_b^j \equiv l_{a\alpha}^i \epsilon^{\alpha\beta} l_{b\beta}^j$ )

$i, j, k = 1, 2, 3$ , are  $SU(3)_L$  indices

$l_a^i \cdot l_b^j \rho^{*k} \epsilon^{ijk}$  is antisymmetric



$G_{ab}^\rho$  has to be antisymmetric

# Yukawa Interactions: Lepton Sector

$$\mathcal{L}_{I,\text{sextet}}^{\text{Yuk.}} = G_{ab}^\sigma I_a^i \cdot I_b^j \sigma_{i,j}^*$$

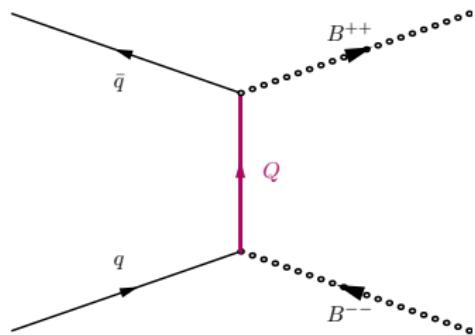
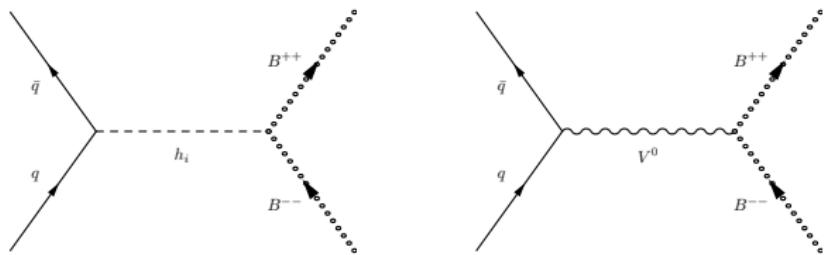
with

$$\sigma = \begin{pmatrix} \sigma_1^{++} & \sigma_1^+/\sqrt{2} & \sigma^0/\sqrt{2} \\ \sigma_1^+/\sqrt{2} & \sigma_1^0 & \sigma_2^-/\sqrt{2} \\ \sigma^0/\sqrt{2} & \sigma_2^-/\sqrt{2} & \sigma_2^{--} \end{pmatrix} \in (1, 6, 0)$$

$G_{ab}^\sigma$  is symmetric

$H^{\pm\pm} \rightarrow I^\pm I^\pm$  allowed ( $\rho \not\supset \rho^{\pm\pm}$ )

# $B^{\pm\pm}$ @ the LHC



# Signal & Backgrounds at 13 TeV

## Benchmark Point

$$m_{Y^{\pm\pm}} \simeq m_{H^{\pm\pm}} \sim 870 \text{ GeV}$$

$$Br(Y^{\pm\pm} \rightarrow l^\pm l^\pm) = Br(H^{\pm\pm} \rightarrow l^\pm l^\pm) = \frac{1}{3}$$

## SIGNAL

$$pp \rightarrow Y^{++} Y^{--} (H^{++} H^{--}) \rightarrow (l^+ l^+) (l^- l^-) \quad l = e, \mu$$

$$\sigma(pp \rightarrow YY \rightarrow 4l) \simeq 4.3 \text{ fb} \quad \sigma(pp \rightarrow HH \rightarrow 4l) \simeq 0.3 \text{ fb}$$

## BACKGROUNDS

$$pp \rightarrow ZZ \rightarrow (l^+ l^-) (l^+ l^-)$$

$$\sigma(pp \rightarrow ZZ \rightarrow 4l) \simeq 6.1 \text{ fb}$$

## Number of Events (13 TeV and $\mathcal{L}=300 \text{ fb}^{-1}$ )

Defining the significance  $s$  to discriminate a signal  $S$  from a background  $B$  as

$$\sigma_S = \frac{S}{\sqrt{B + \sigma_B^2}},$$

$\sigma_B$  systematic error on B ( $\sigma_B \simeq 0.1B$ )

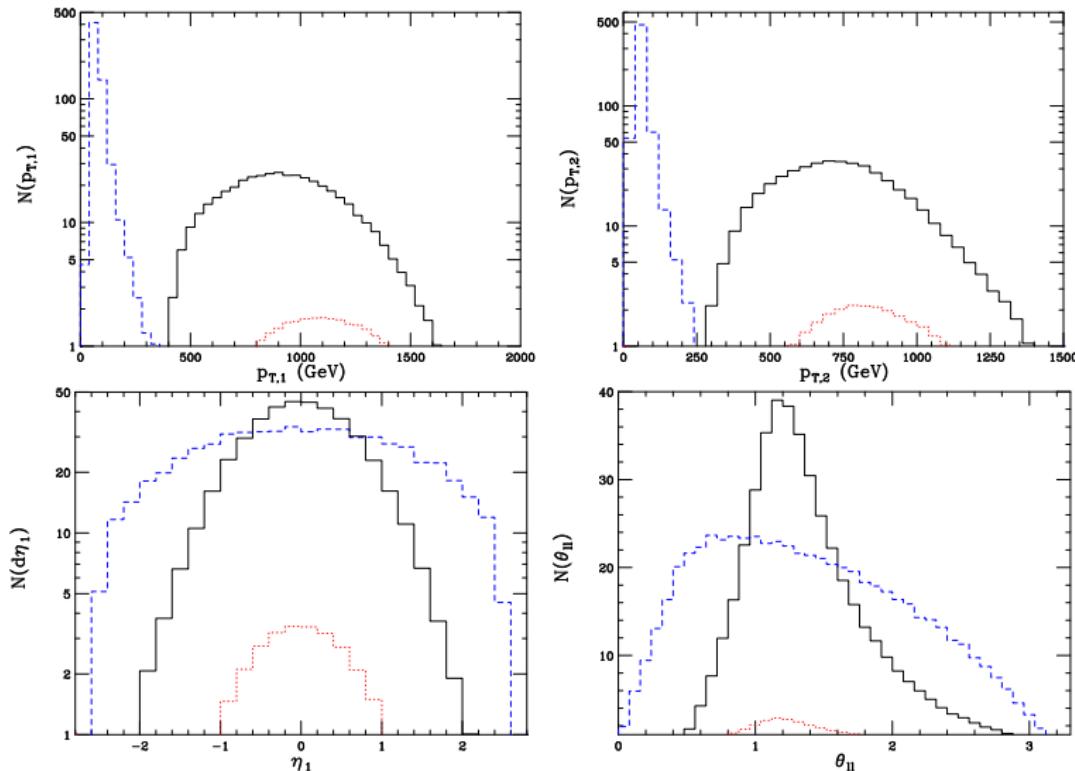
$$N(YY) \simeq 1302, \ N(HH) \simeq 120, \ N(ZZ) \simeq 1836$$



$$\sigma_{YY} \simeq 6.9, \ \sigma_{HH}^{B_{SM}} = 0.6, \ \sigma_{HH}^{B_{YY}} = 0.9$$

arXiv:1806.04536 [hep-ph]

# Distributions



Background, Scalar, Vector

# Trinification

$$[SU(3)]^3 \equiv SU(3)_c \times SU(3)_L \times SU(3)_R$$

....maximal subgroup of  $E_6$

## Trinification: Field Content

$$H = \begin{pmatrix} h_{11}^0 & h_{12}^+ & h_{13}^+ \\ h_{21}^- & h_{22}^0 & h_{23}^0 \\ h_{31}^- & h_{32}^0 & h_{33}^0 \end{pmatrix} \in (\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3})$$

$$L = \begin{pmatrix} L_1^1 & E^- & e^- \\ E^+ & L_2^2 & \nu \\ e^+ & \hat{\nu} & L_3^3 \end{pmatrix} \in (\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3})$$

$$Q_L = \begin{pmatrix} u_L \\ d_L \\ D_L \end{pmatrix} \in (\bar{\mathbf{3}}, \mathbf{3}, \mathbf{1})$$

$$Q_R = \left( \begin{array}{ccc} \bar{u}_R & \bar{d}_R & \bar{D}_R \end{array} \right) \in (\mathbf{3}, \mathbf{1}, \bar{\mathbf{3}})$$

$$\mathbb{Q}^{\text{em}} = \mathbb{T}_L^3 + \mathbb{T}_R^3 + \frac{1}{\sqrt{3}} \mathbb{T}_L^8 + \frac{1}{\sqrt{3}} \mathbb{T}_R^8$$

$$SU(3)^3 \rightarrow \dots \rightarrow \mathcal{G}_{SM} \rightarrow SU(3)_c \times U(1)_{\text{em}}$$

$$\langle H_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 & 0 & 0 \\ 0 & b_1 & 0 \\ 0 & 0 & M_1 \end{pmatrix}, \quad \langle H_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_2 & 0 & 0 \\ 0 & b_2 & b_3 \\ 0 & M & M_2 \end{pmatrix}$$

$$\begin{aligned} SU(3)^3 &\xrightarrow{M_i} SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\ &\xrightarrow{M} \mathcal{G}_{SM} \\ &\xrightarrow{v_i, b_j} SU(3)_c \times U(1)_{\text{em}} \end{aligned}$$

$$(\sqrt{2} G_F)^{1/2} \sim v_i, \quad b_j < M < M_i \sim m_{GUT}$$

Hetzel, Stech, Phys.Rev.D91 (2015) 055026

# Not-Exotic...but Heavy!

$$m_{D_i} = \frac{1}{\sqrt{2}} M_{1,2} Y_{Q_i} \lesssim m_{GUT}$$



# $\mathcal{G}_{331} \subset [SU(3)]^3$ : Spontaneous Symmetry Breaking Chain

$$SU(3) \rightarrow SU(2)_a \times U(1)_b$$



$$3 \rightarrow 2_b + 1_{-2b}$$

$$SU(2)_a \rightarrow U(1)_a$$



$$2_b + 1_{-2b} \rightarrow 1_{a,b} + 1_{-a,b} + 1_{0,-2b}$$

Thus, when  $SU(3)_R$  breaks into  $U(1)_a \otimes U(1)_b$  the following branching rule applies:

$$3_R \rightarrow (a)(b) + (-a)(b) + (0)(-2b)$$