Nucleon and nuclear structure from muonic and normal atoms

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Cluster of Excellence PRISMA+

IWHSS 2020, Trieste
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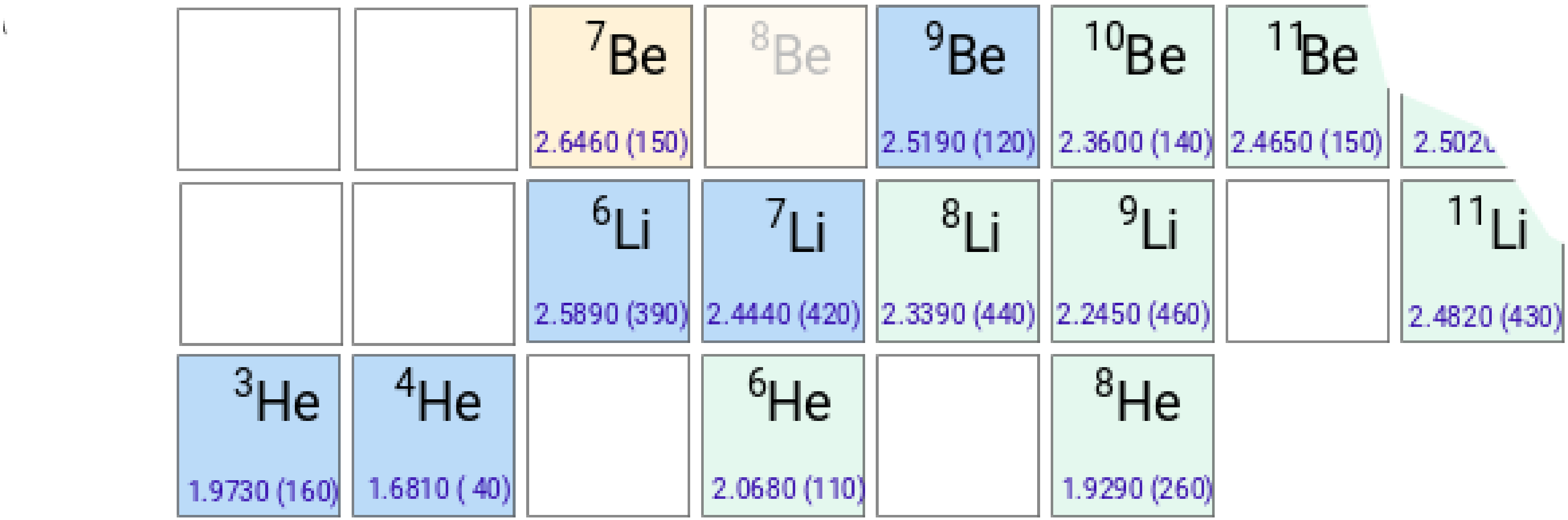
STRONG-2020

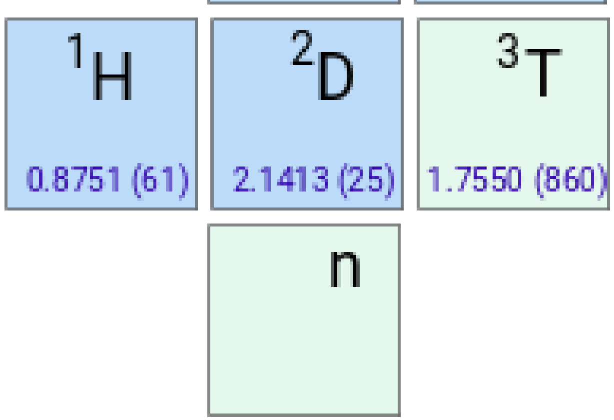
DFG

MPQ

erc

Nuclear radii





Essential input for:

*** Nucleon structure (proton)**

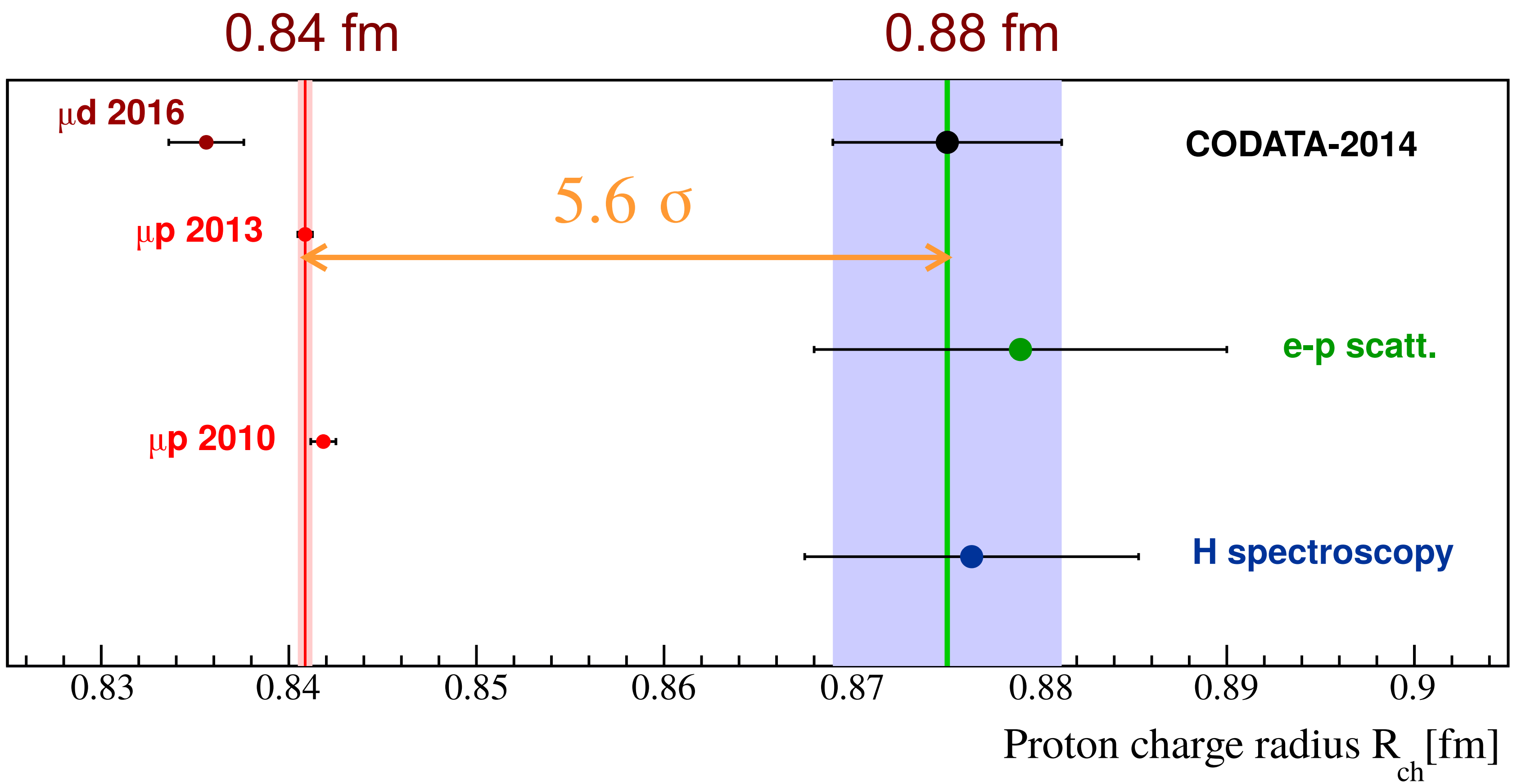
*** Nuclear structure and models**

*** Precision tests of QED and the Standard Model**

*** Fundamental constants (CODATA)**

The “Proton Radius Puzzle”

Measuring R_p using electrons: 0.88 fm (+- 0.7%)
using muons: 0.84 fm (+- 0.05%)



μd 2016: RP et al (CREMA Coll.) Science 353, 669 (2016)
μp 2013: A. Antognini, RP et al (CREMA Coll.) Science 339, 417 (2013)

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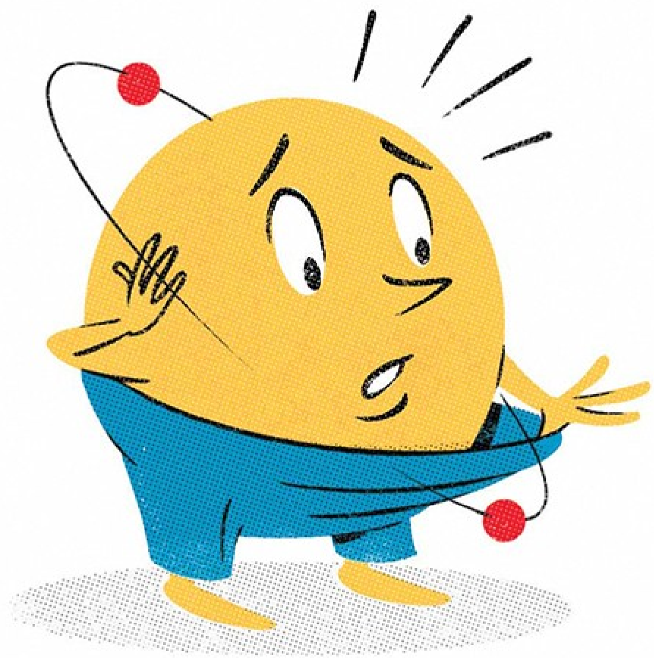
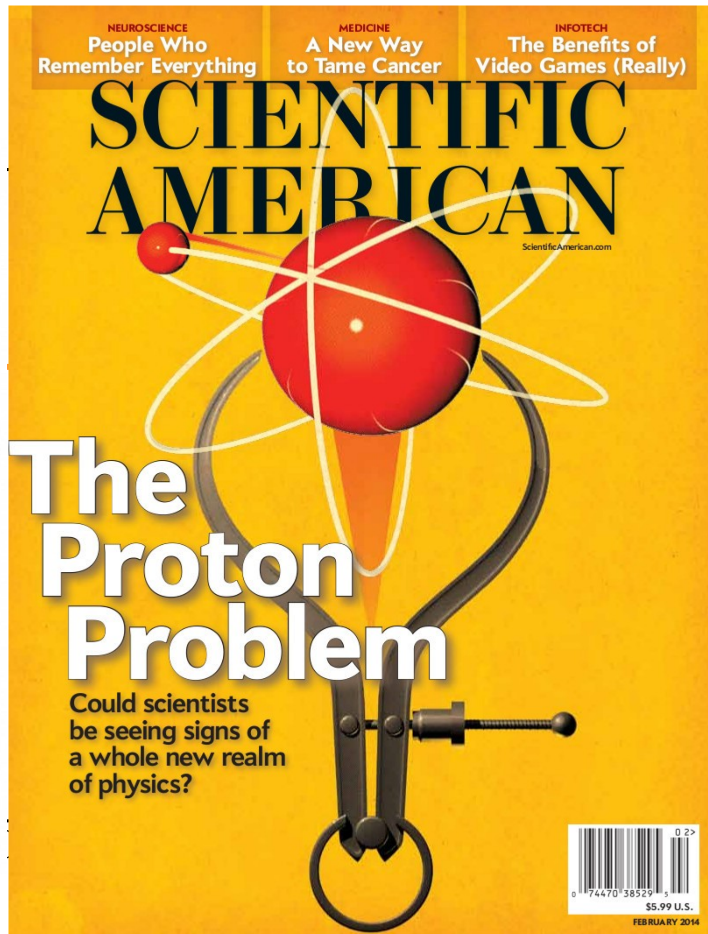
0.84 fm



The New York Times



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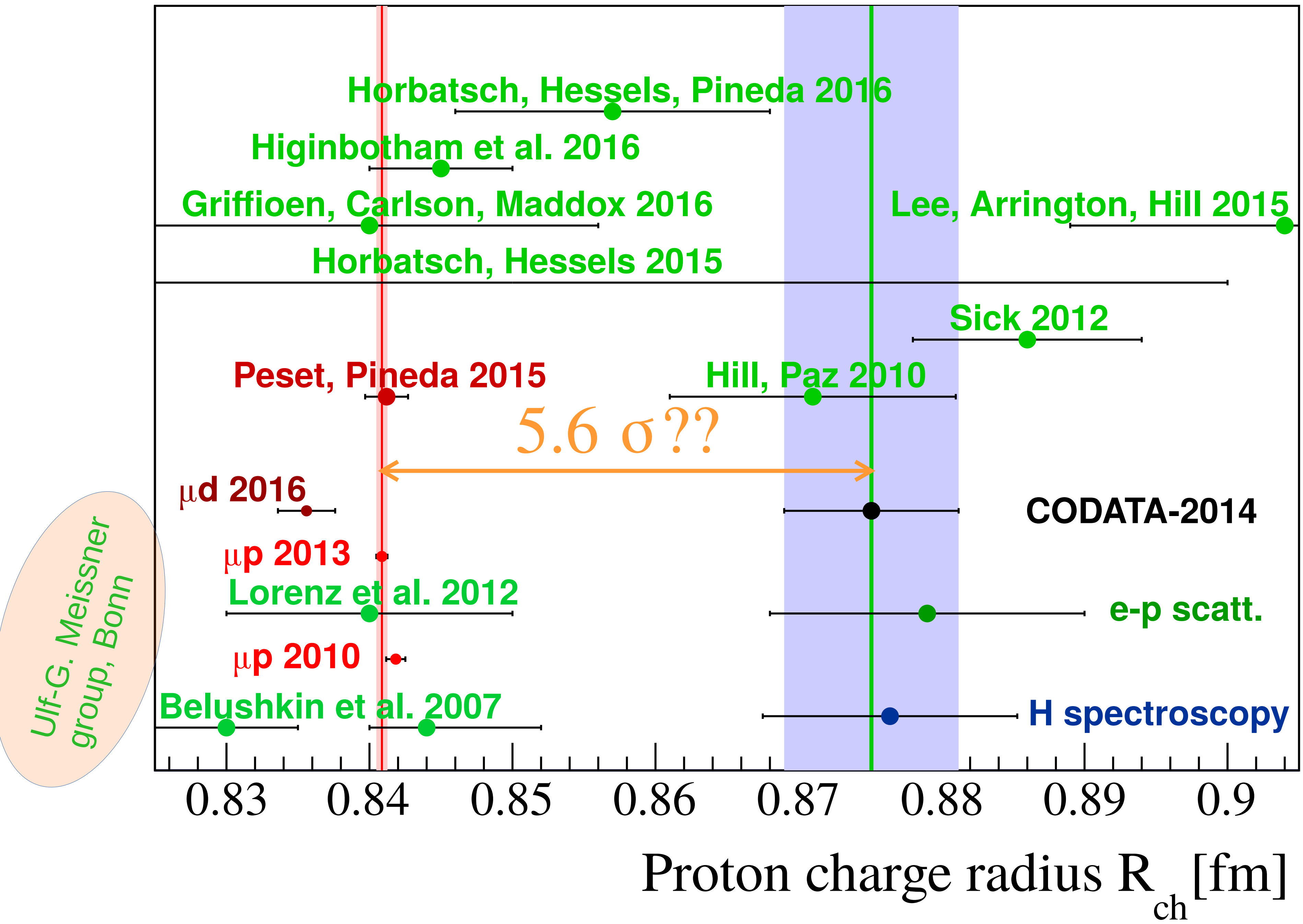
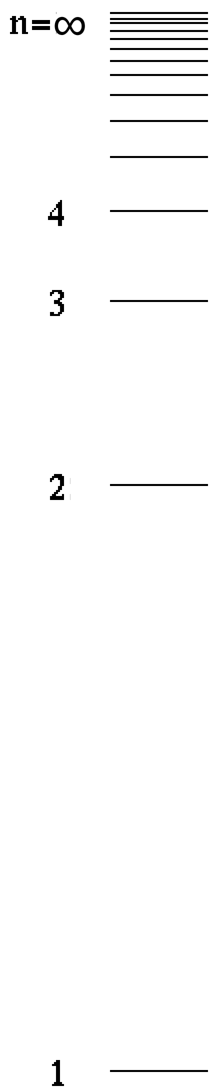
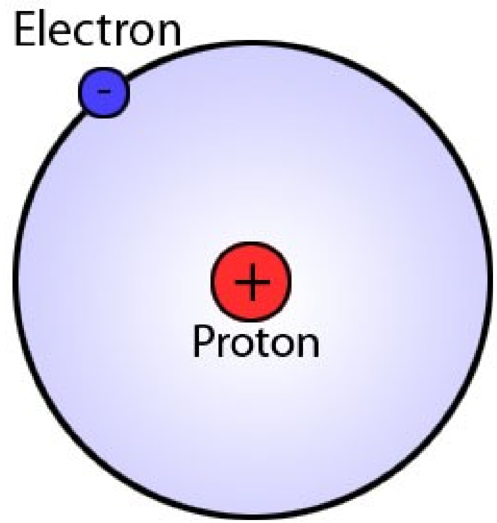
A “Proton Radius **Puzzle**” ??



Hydrogen

Energy levels of hydrogen

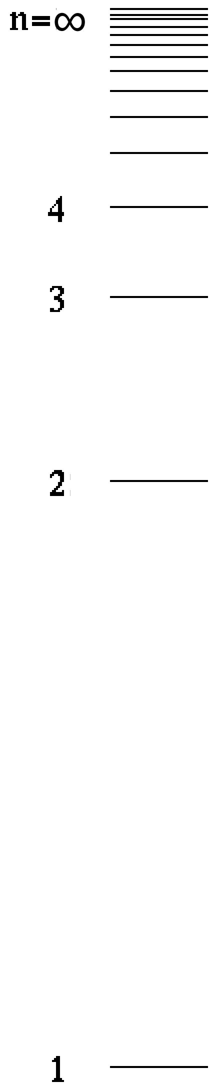




$$E_n \approx -\frac{R_\infty}{n^2}$$

Bohr formula

Energy levels of hydrogen



Rydberg constant

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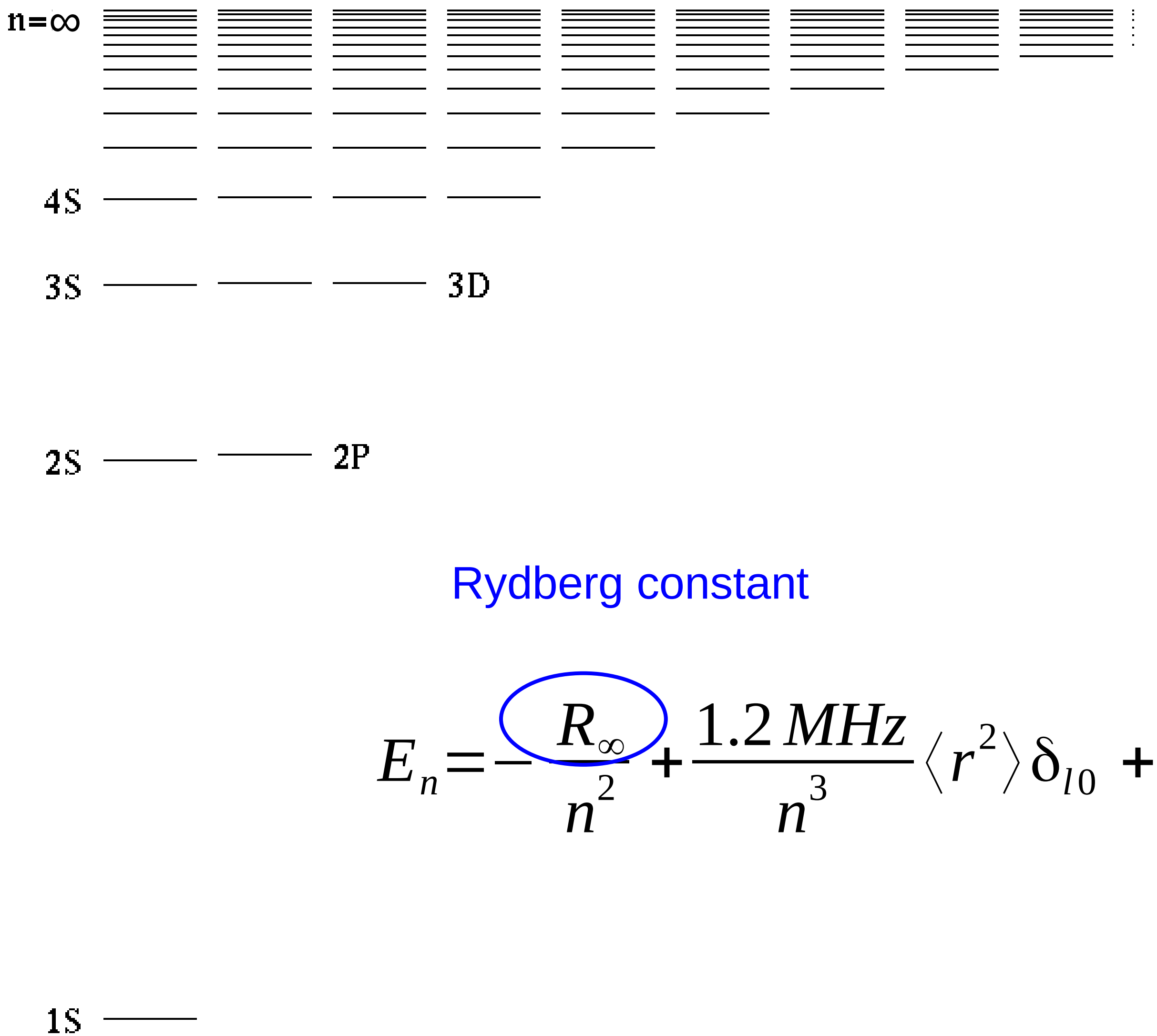
Bohr formula

Energy levels of hydrogen

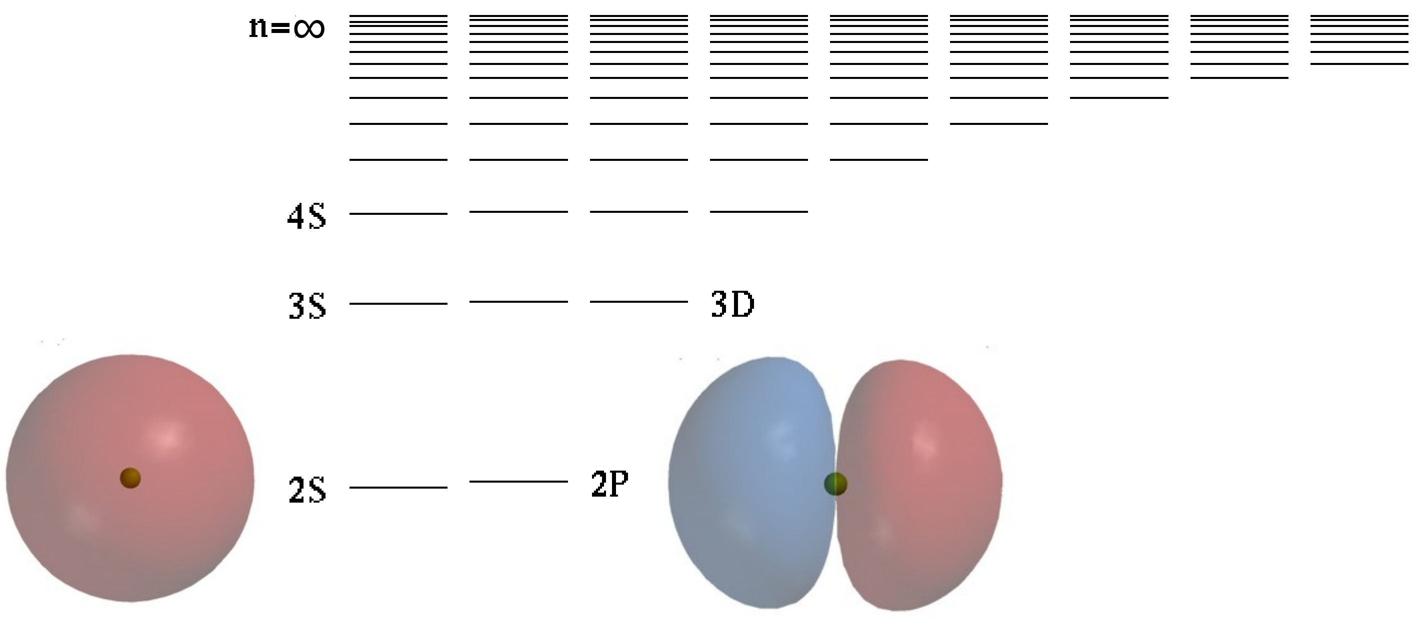


Rydberg constant

$$E_n = -\frac{R_\infty}{n^2} + \frac{1.2\,MHz}{n^3}\langle r^2\rangle \delta_{l0} + \Delta(n, l, j)$$

RP et al., Metrologia 54, L1 (2017)

Energy levels of hydrogen



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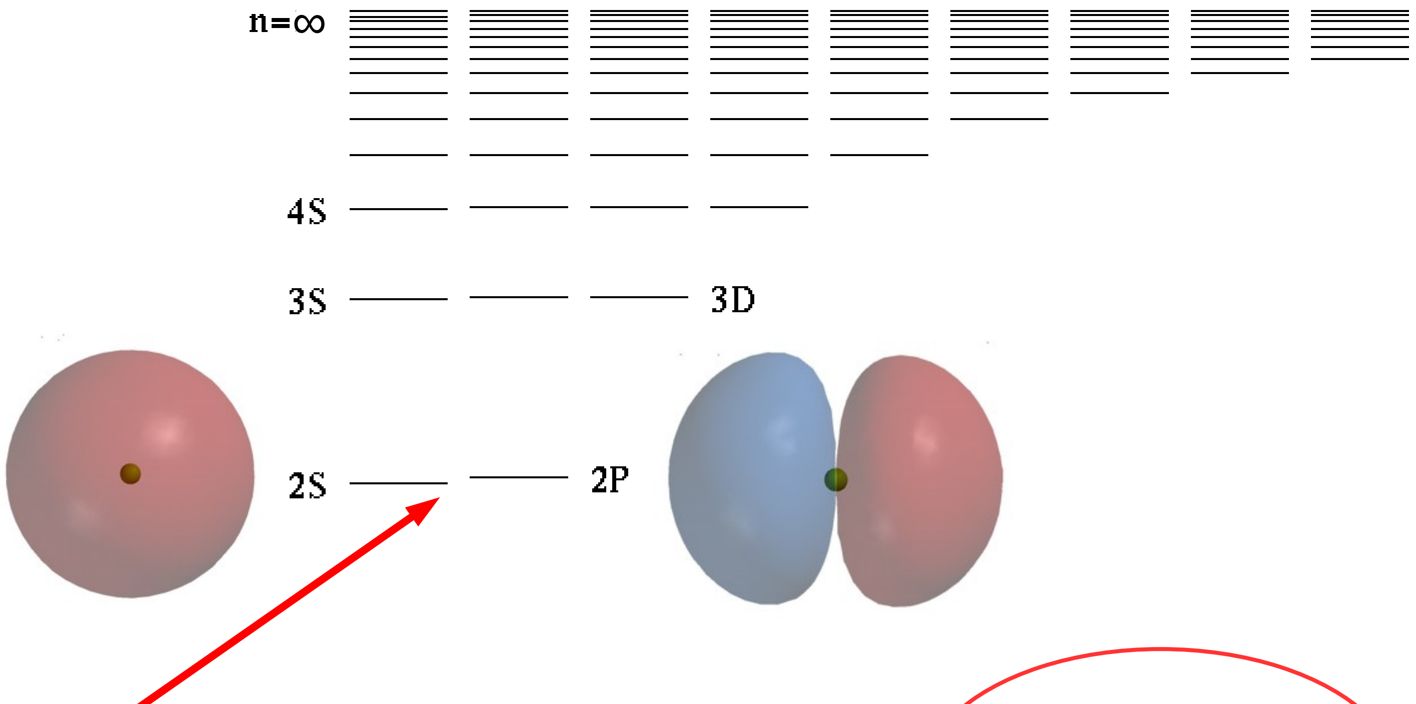
Rydberg constant

finite size effect

RP et al., Metrologia 54, L1 (2017)

Energy levels of hydrogen



2S-2P Lamb shift

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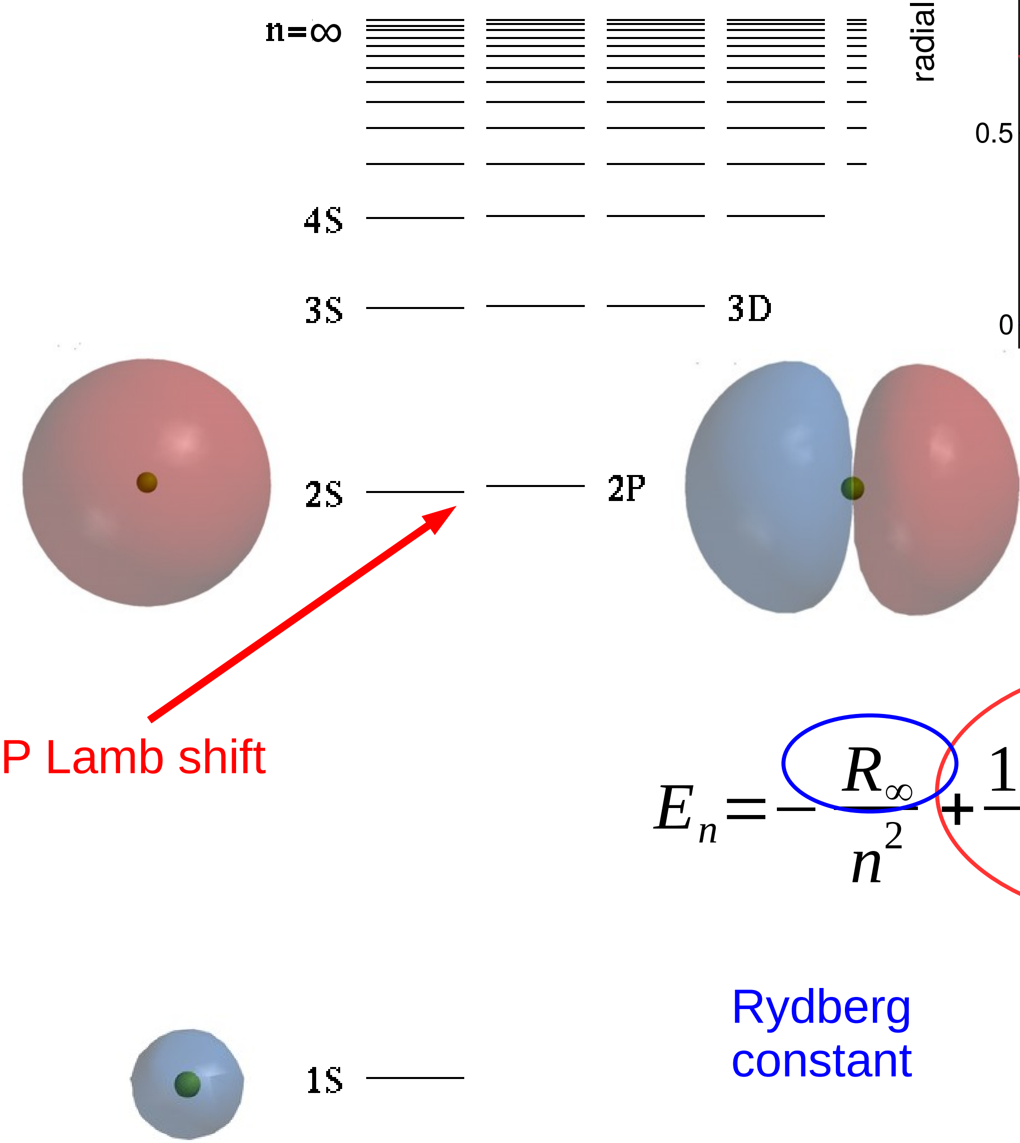

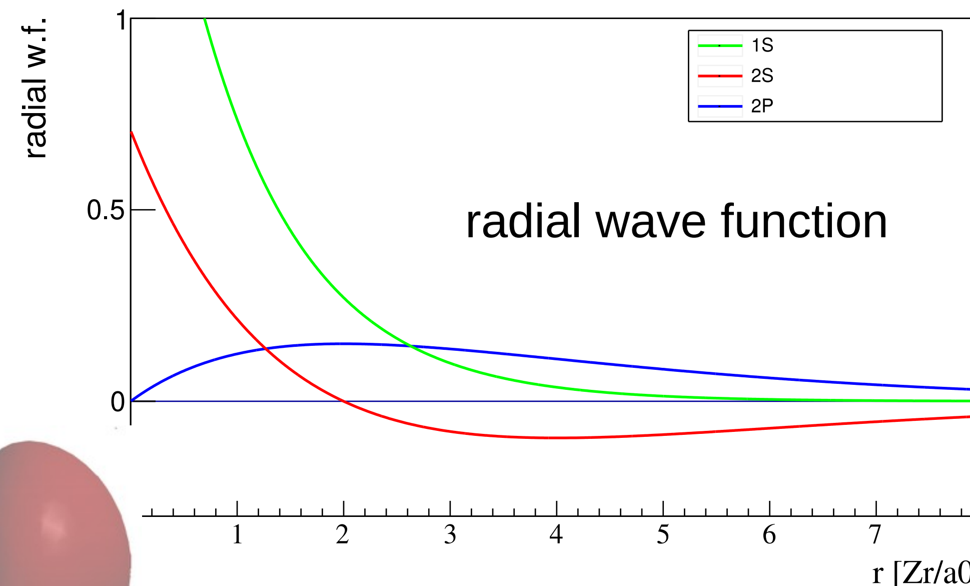

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Rydberg constant

finite size effect

RP et al., Metrologia 54, L1 (2017)

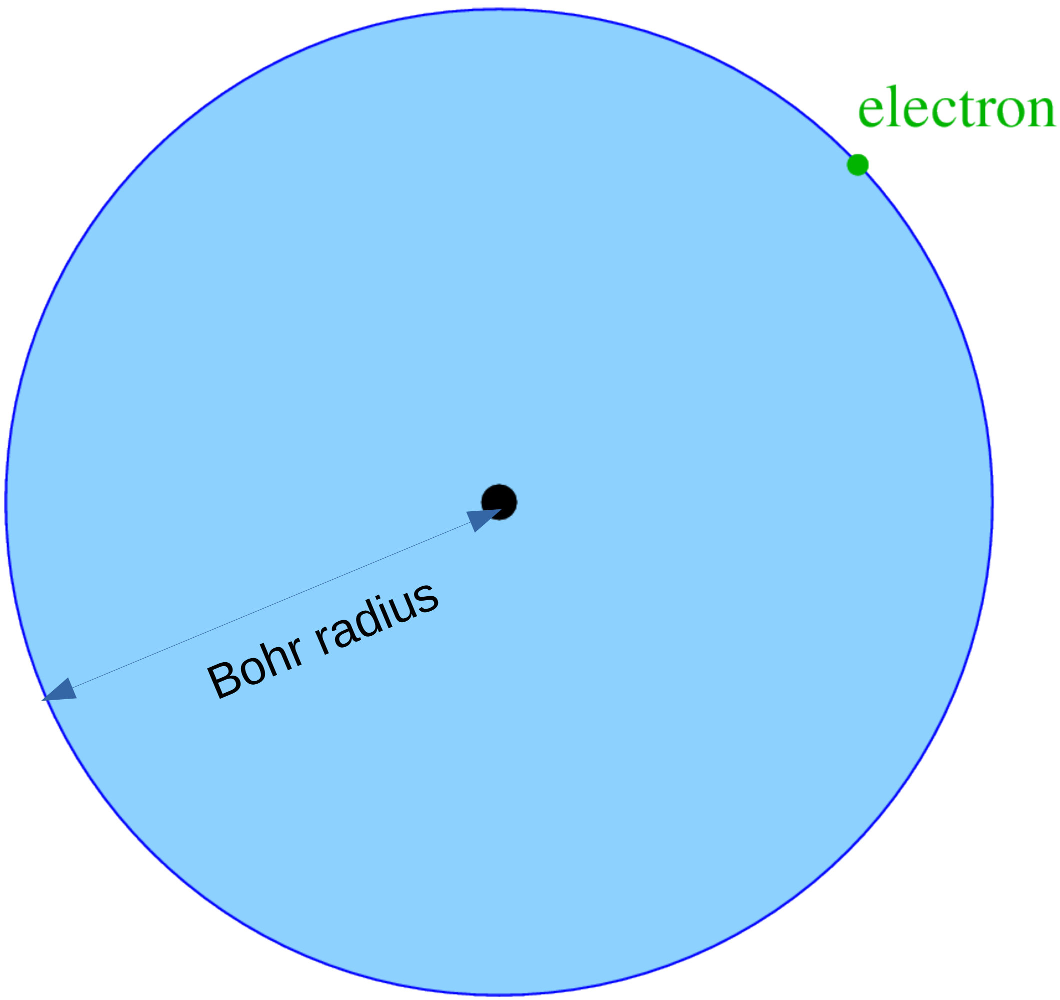
Muonic Hydrogen

A proton, orbited by a **negative muon.**

Electronic and muonic atoms

Regular hydrogen:

Proton + Electron



Muonic hydrogen:

Proton + Muon

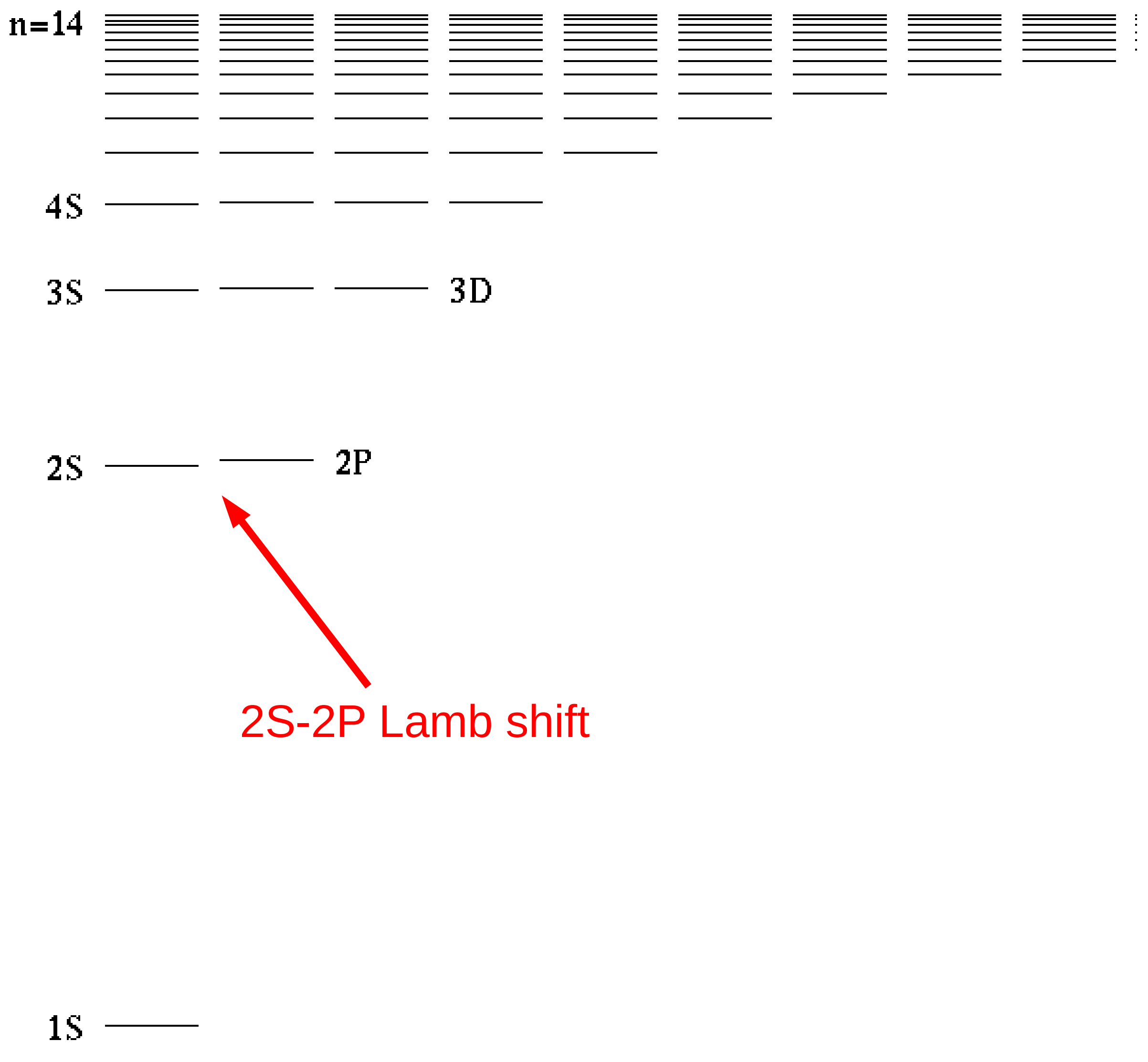
Muon **mass** = **200** * electron mass

Bohr **radius** = **1/200** of H

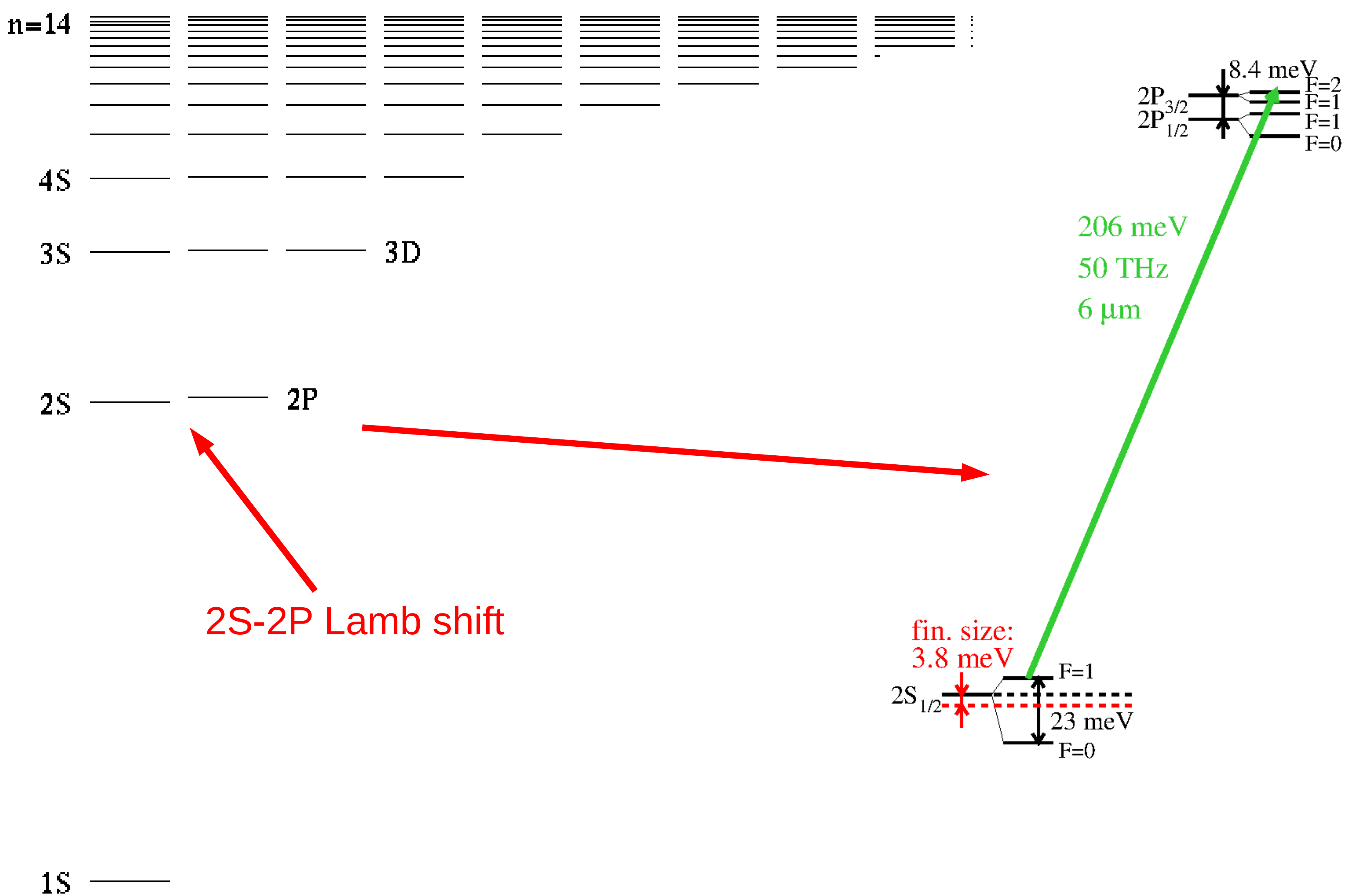
$\mathbf{200^3}$ = a **few million times** more sensitive to proton size

Vastly not to scale!!

Muonic Hydrogen



Muonic Hydrogen



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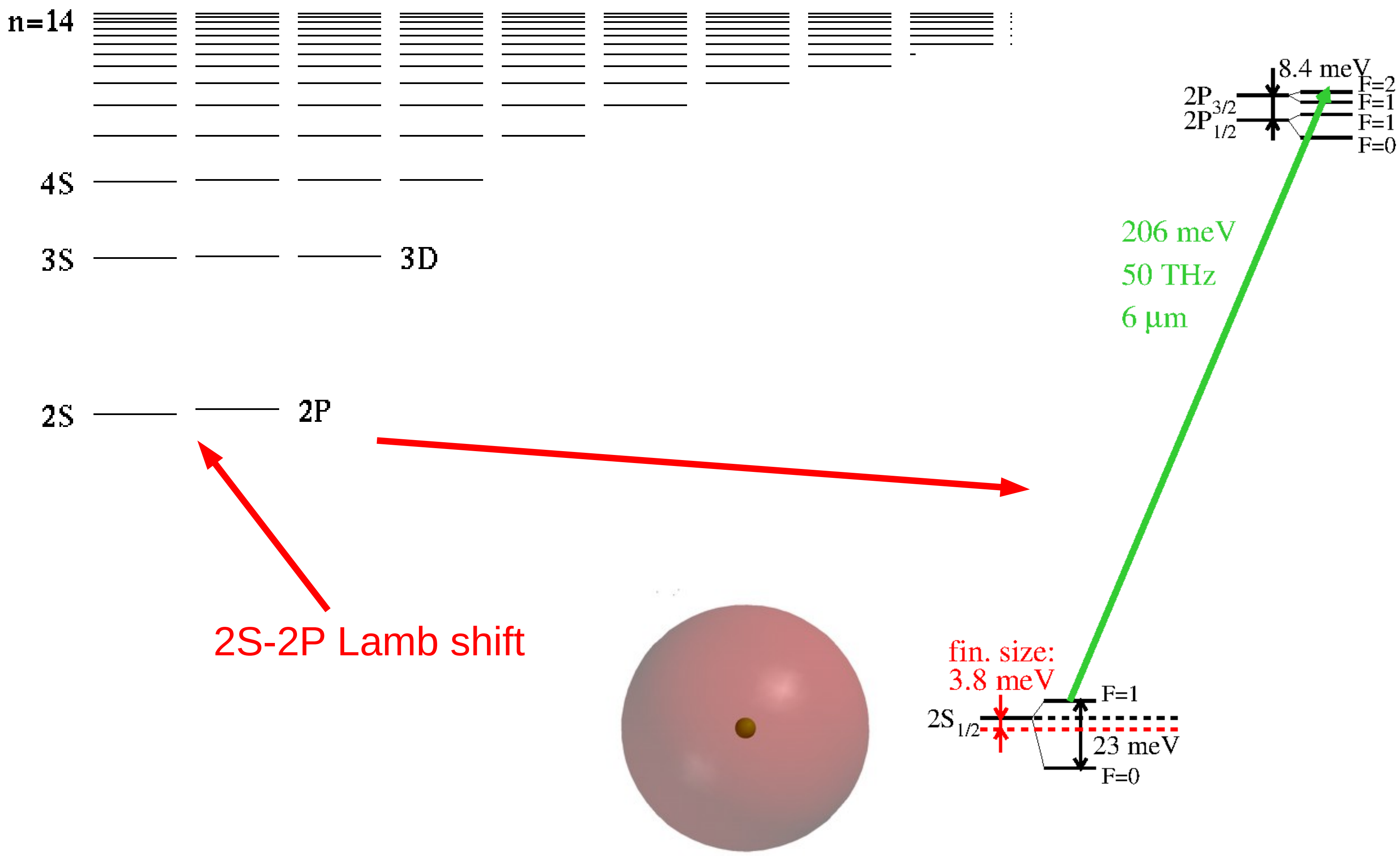
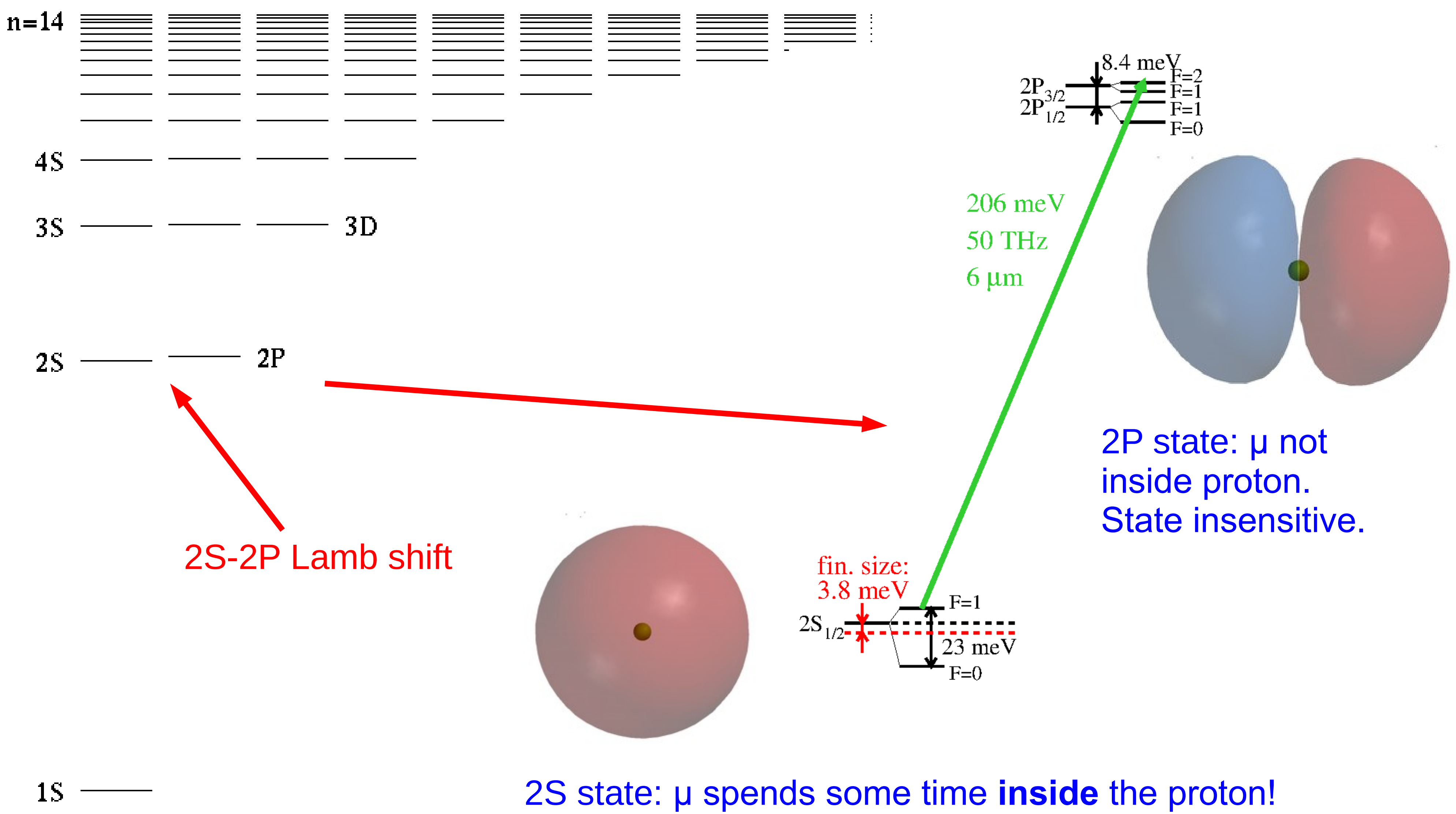


2S state: μ spends some time **inside** the proton!
State is sensitive to the proton size.

Muonic Hydrogen

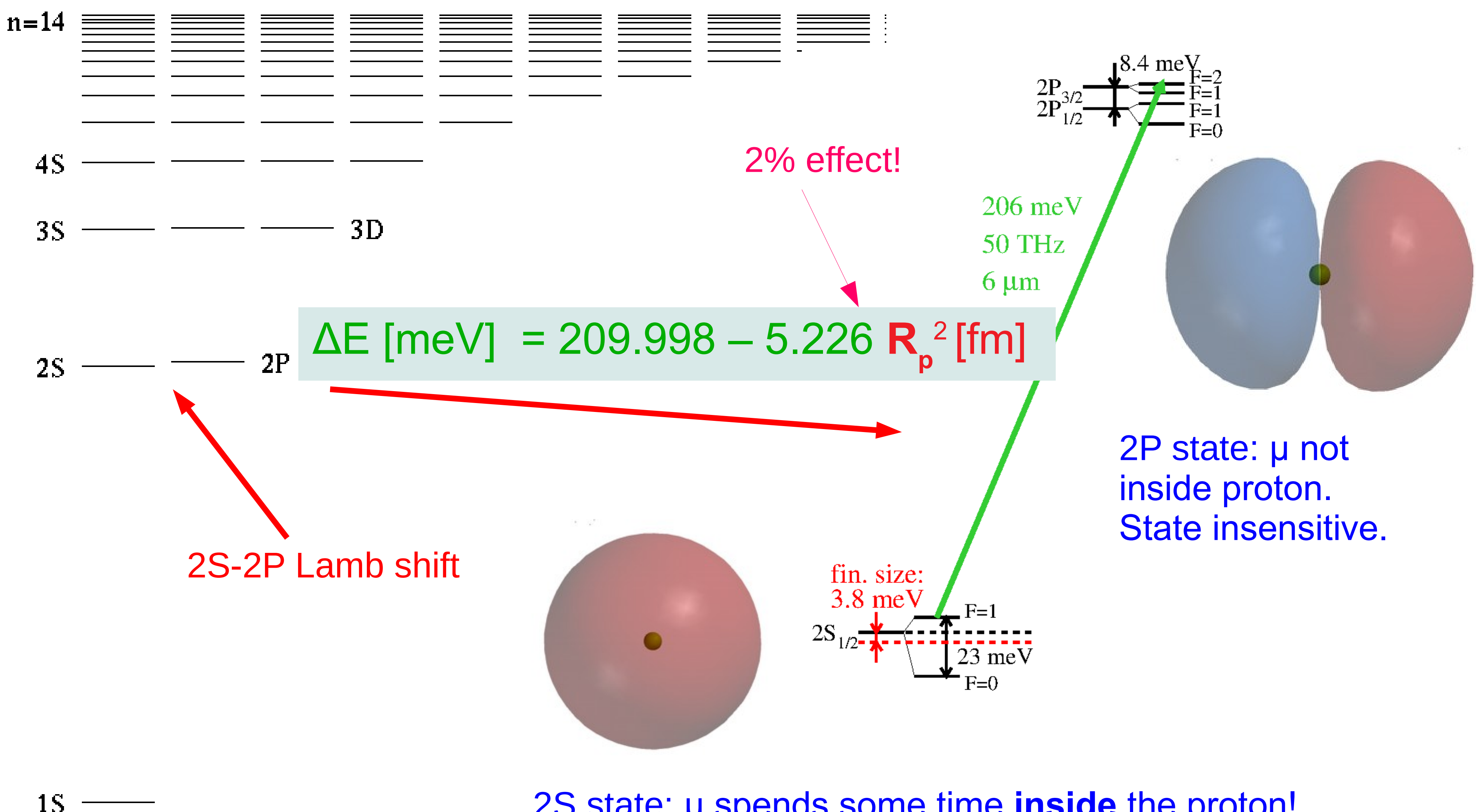


2S-2P Lamb shift

2P state: μ not inside proton. State insensitive.

2S state: μ spends some time **inside** the proton! State is sensitive to the proton size.

Muonic Hydrogen



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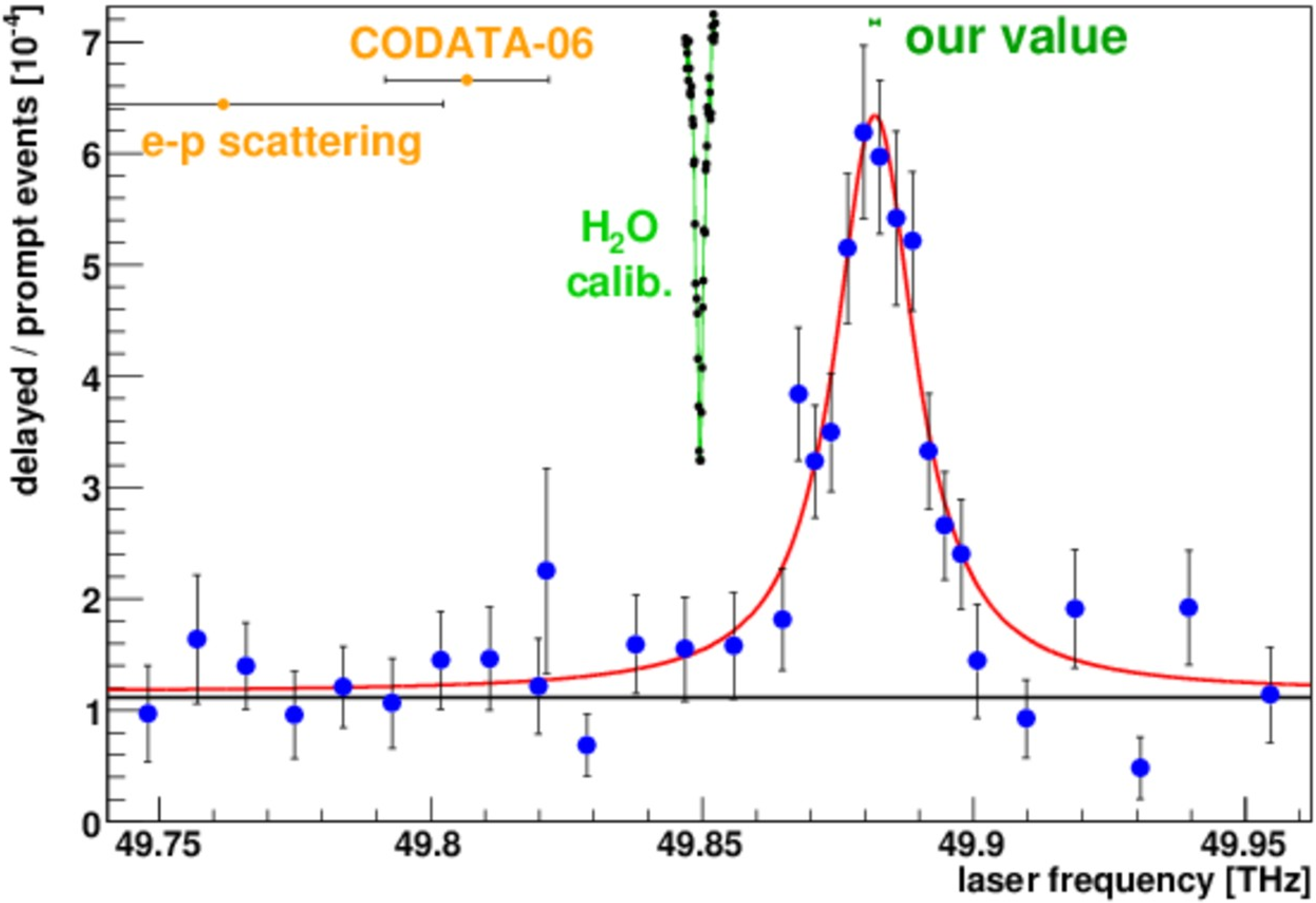
The resonance line



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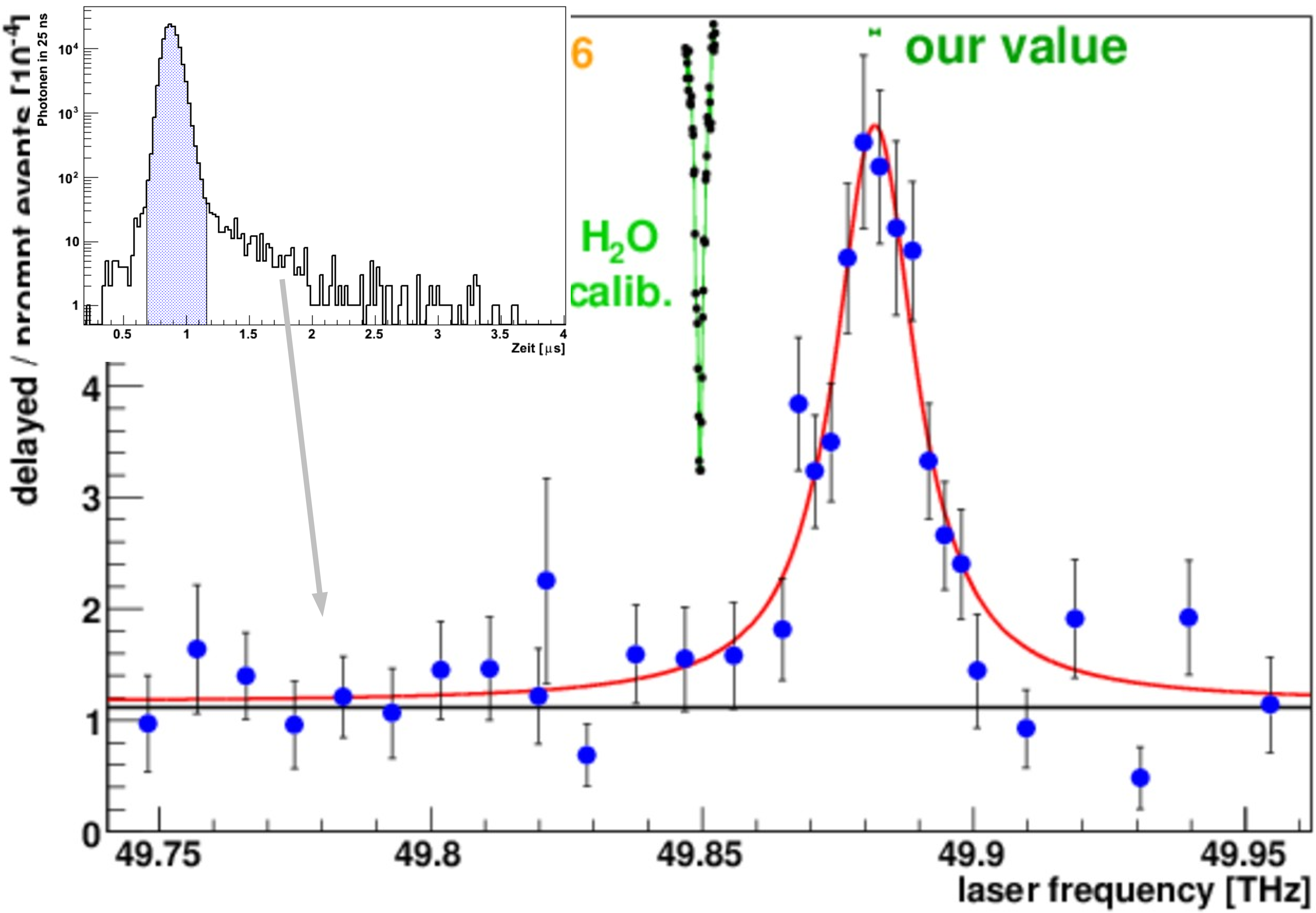

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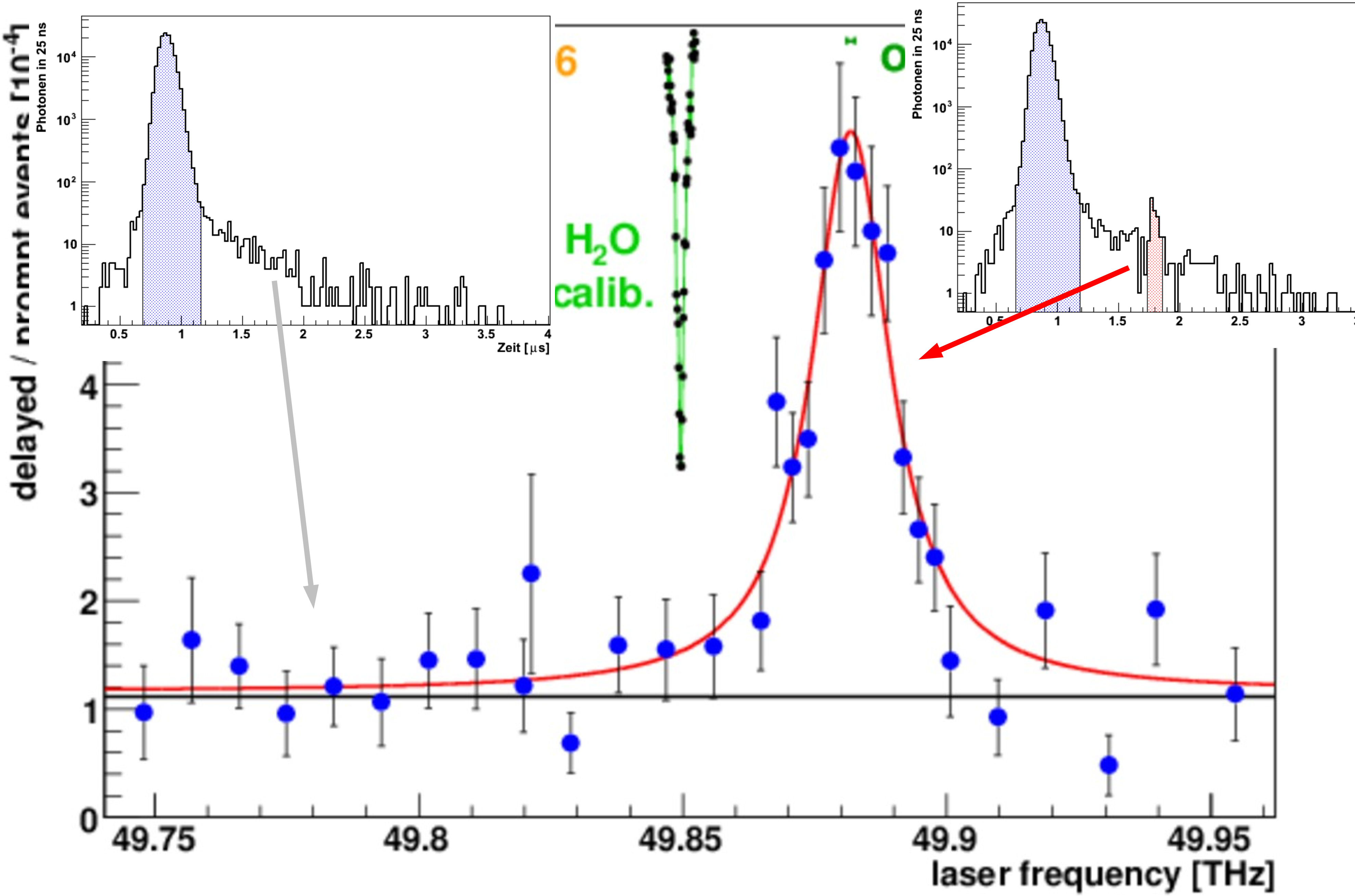

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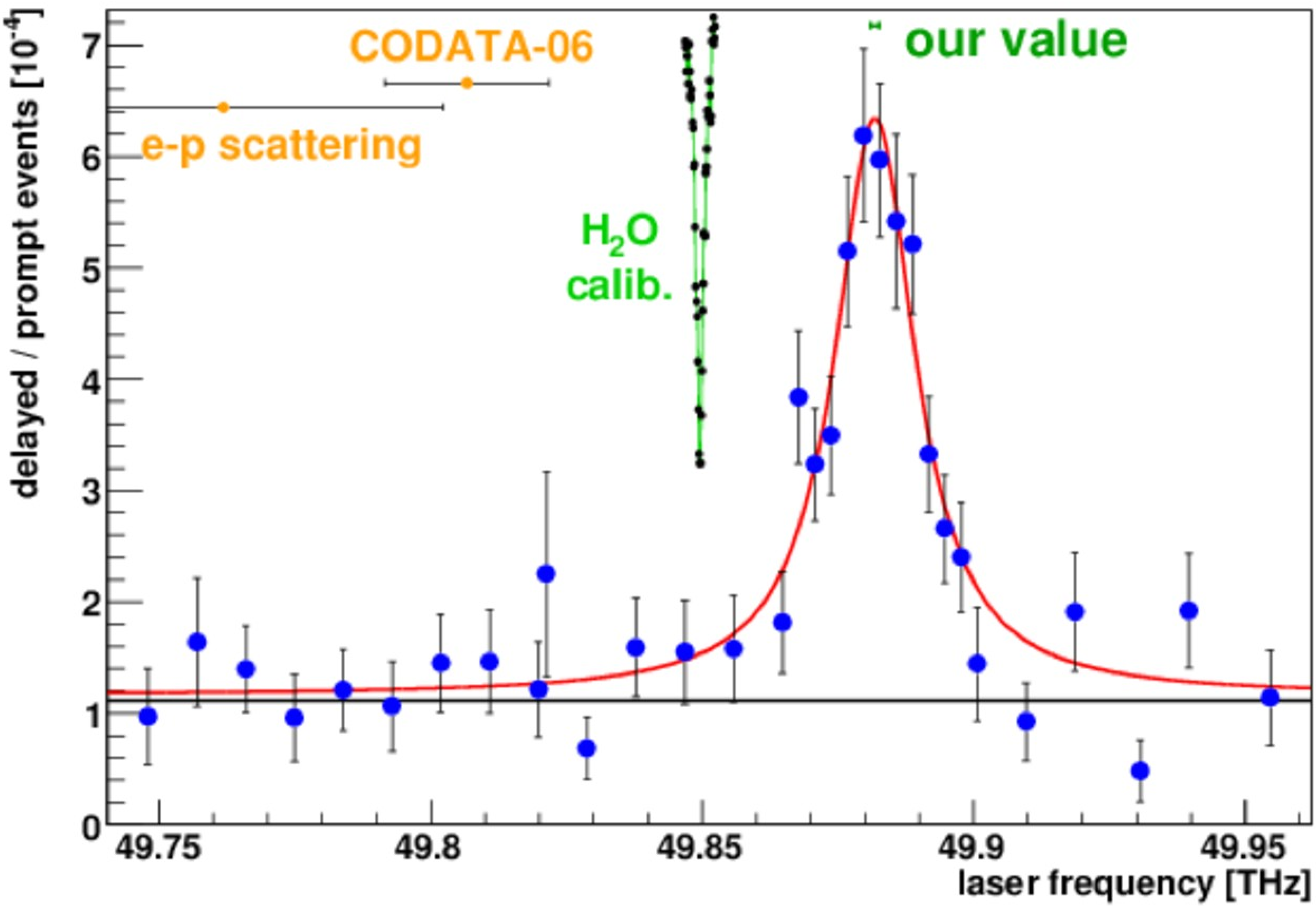
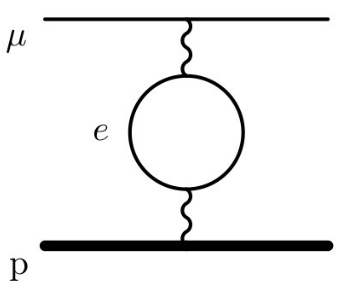
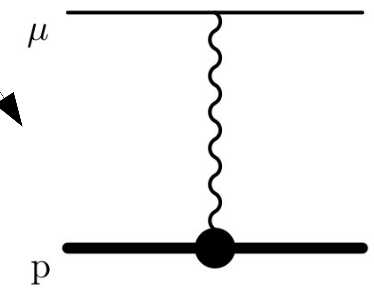


Theory in muonic H

ΔE_{Lamb} = 206.0336 (15) meV_{QED} + 0.0332 (20) meV_{TPE} – 5.2275 (10) meV/fm^2 * $\mathbf{R_p^2}$

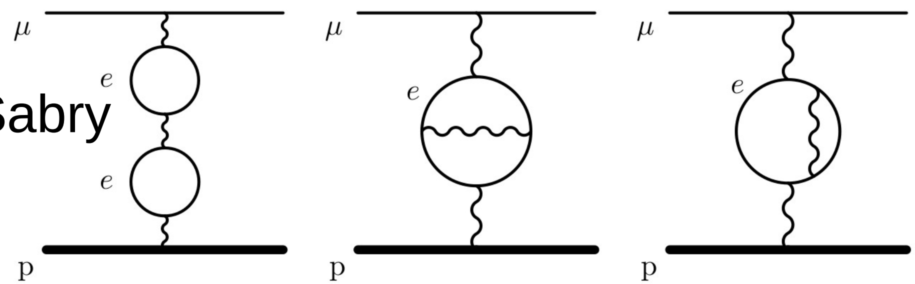
Uehling



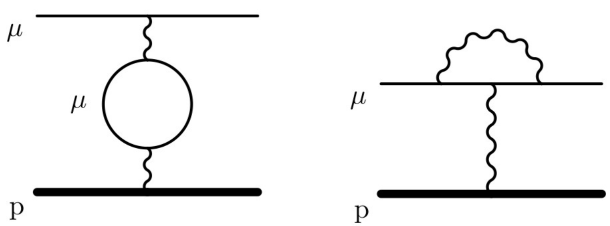


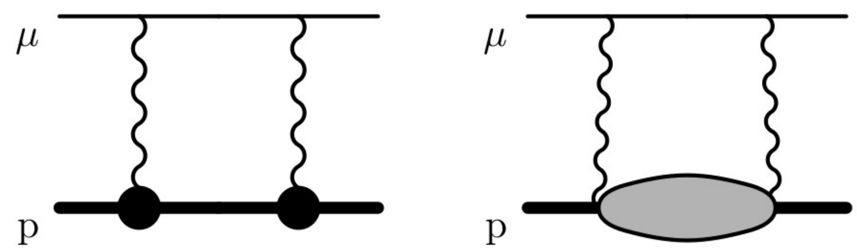
Proton form factor

Källen-Sabry



Muon SE+VP





elastic and inelastic two-photon exchange
(Friar moment and polarizability)

and 20+ more….

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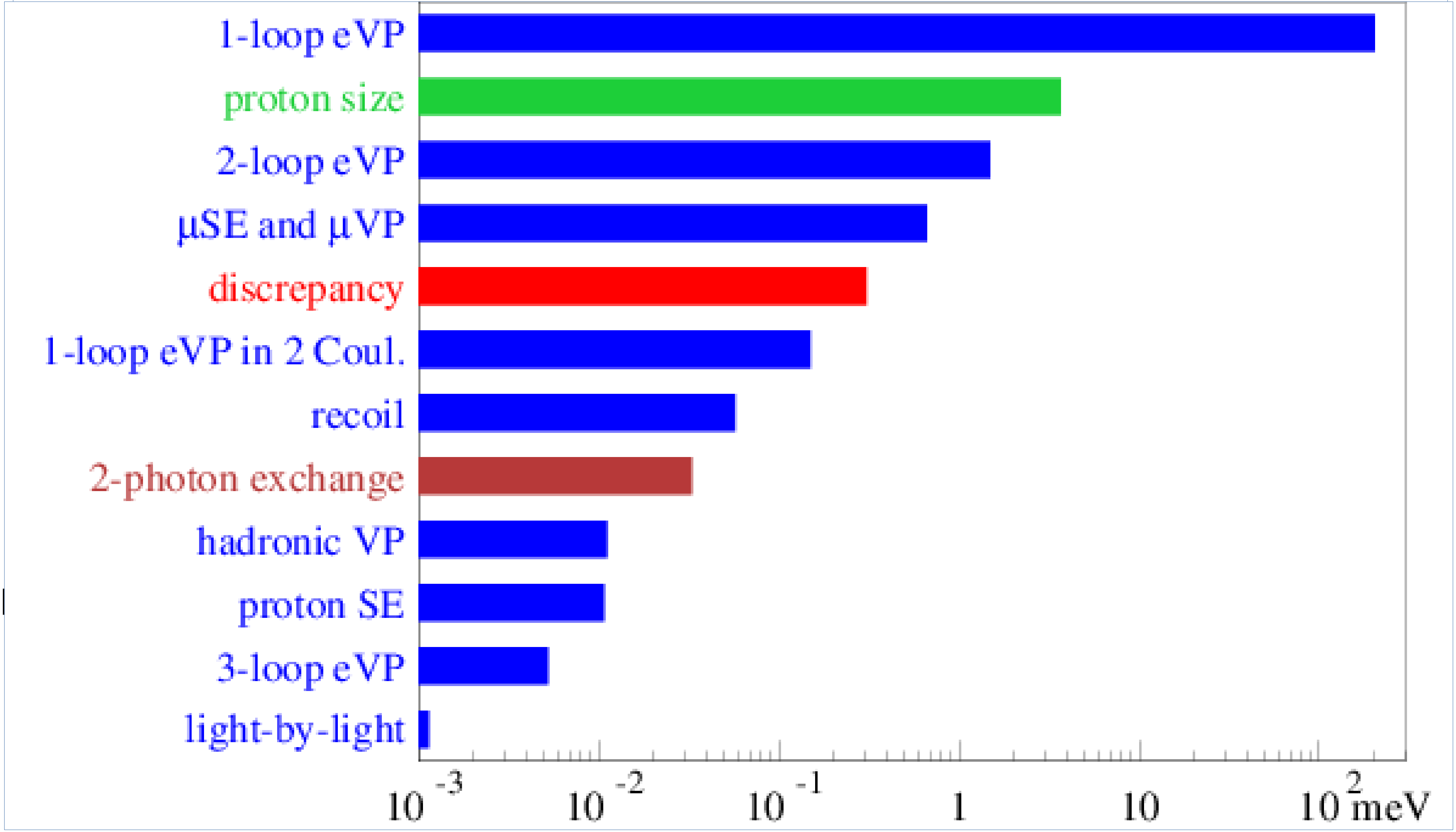

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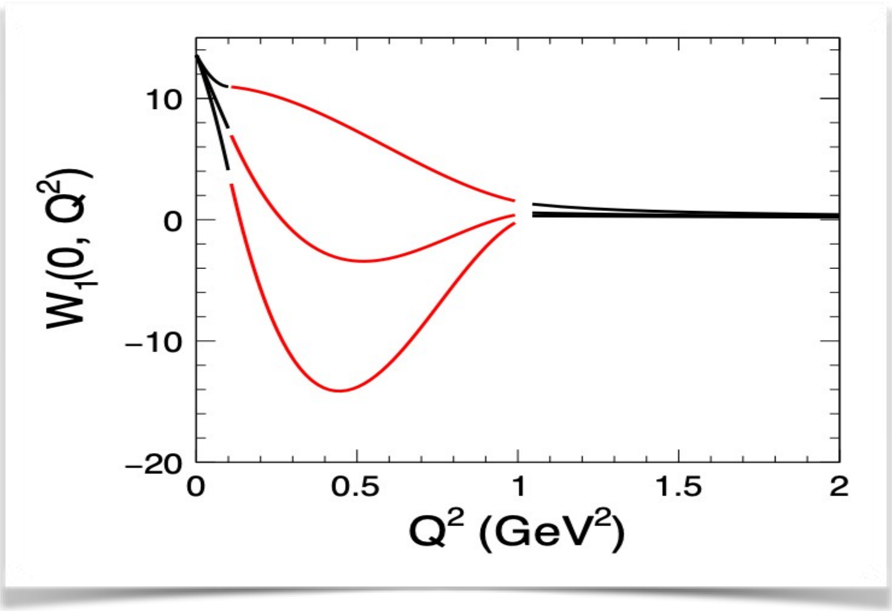
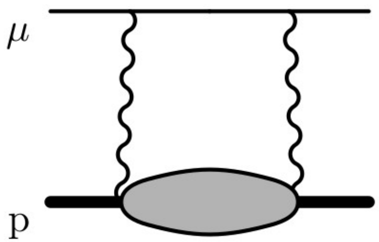


Pachucki, Carlson, Birse, McGovern, Pineda, Peset, Gorchtein, Pascalutsa, Vanderhaeghen, Tomalak, Martynenko, Alarcon, Miller, Paz, Hill, Hagelstein…

(25) Pascalutsa et al.
(100) Hill & Paz



inelastic two-photon exchange
(polarizability)

Theory in muonic H

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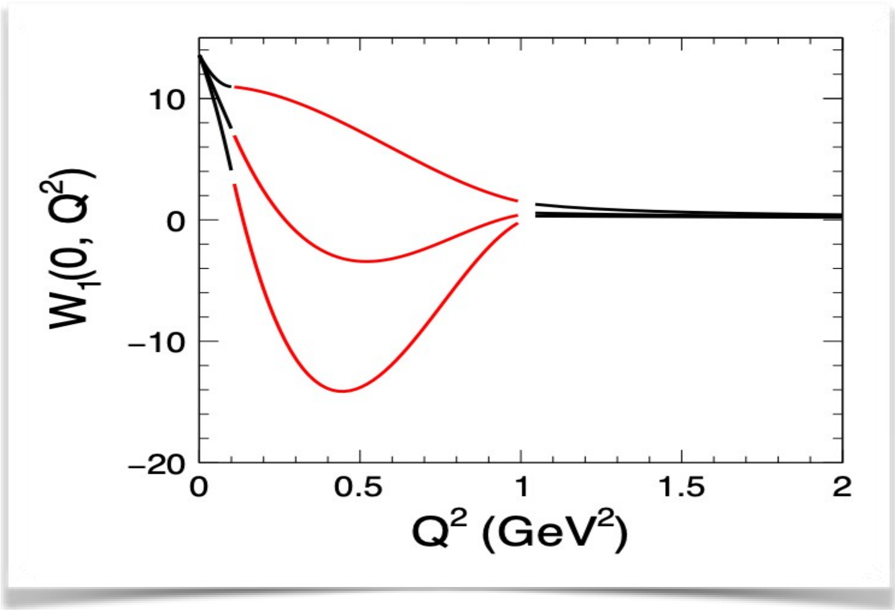

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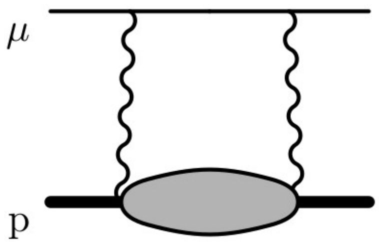

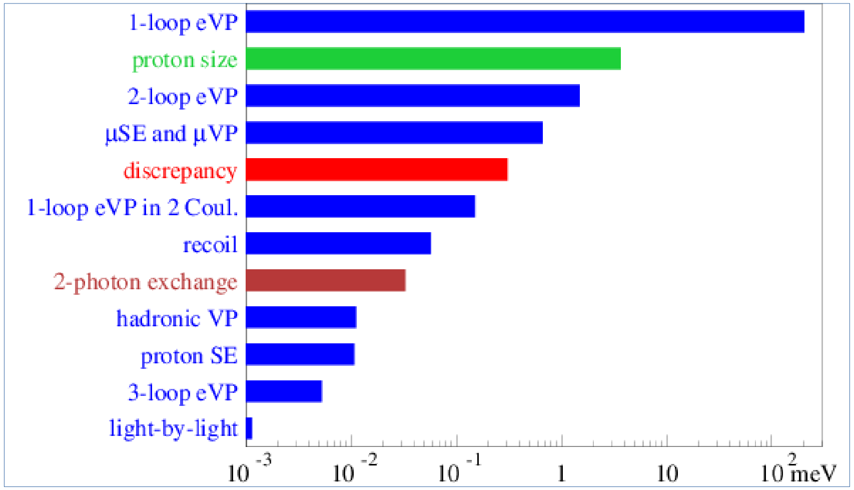

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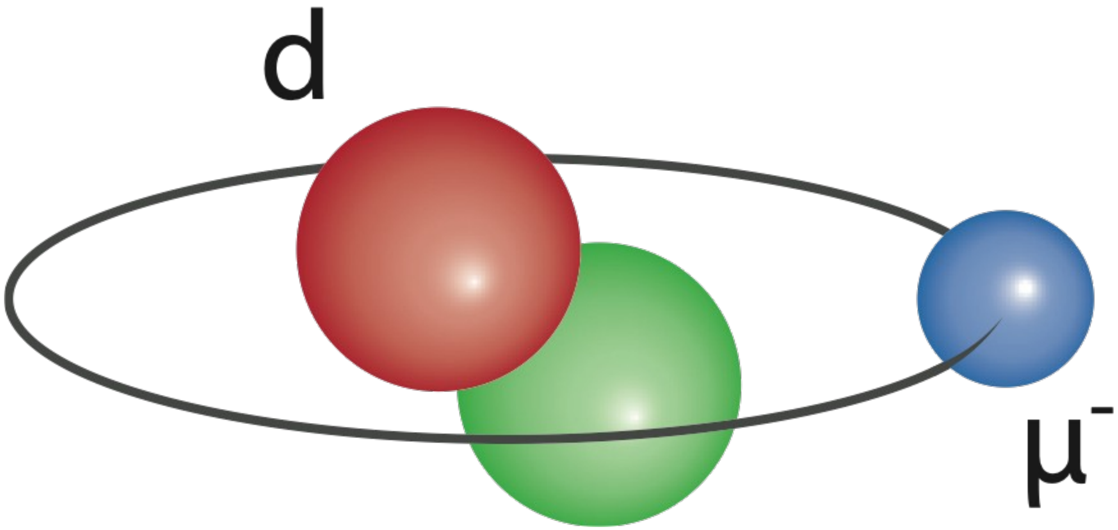
Muonic Deuterium



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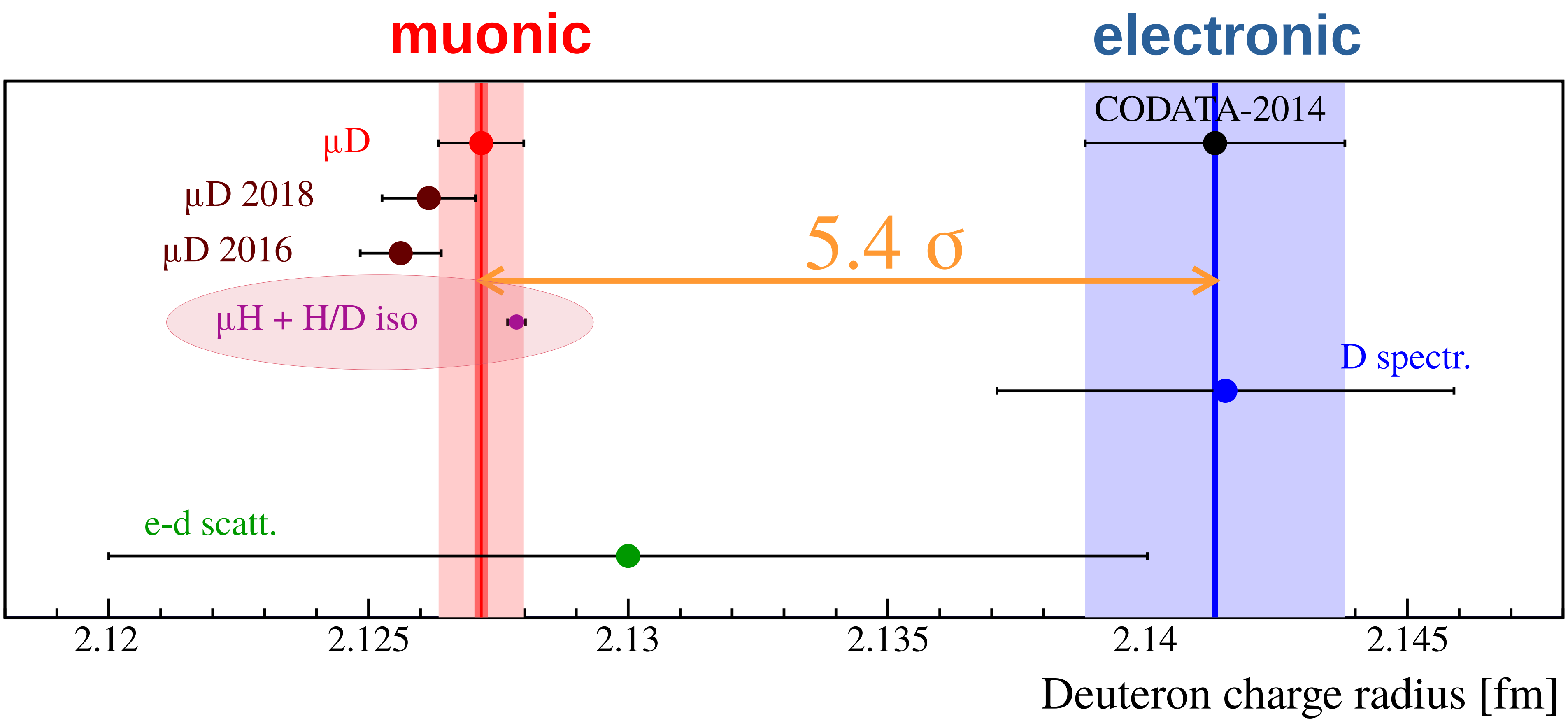


μD: 2.12717 $(13)_{exp}$ $(82)_{theo}$ fm (theo = nucl. polarizability)

μH + H/D(1S-2S): 2.12785 (17) fm

CODATA-2014: 2.1**4**130 (250) fm

H/D 1S-2S isotope shift:
$r_d^2 - r_p^2 = 3.82070(31)\ fm^2$

Pachucki et al., PRA 97, 062511 (2018)

H/D 1S-2S. Parthey, RP et al. (MPQ Garching), PRL 104, 233001 (2010)
PRL 107, 203001 (2011)

Theory in muonic D

$\Delta E^{\mu D}_{Lamb}$ = 228.7854 (13) meV_{QED} + 1.7500 (**210**) meV_{TPE} – 6.1103 (3) meV/fm^2 * $\mathbf{R_d^2}$

ΔE_{TPE} (theo) = 1.7500 +- 0.0210 meV (Kalinowski, 2018)

vs. **+- 0.0034 meV experimental uncertainty**

(1) **charge radius**, using **calculated TPE**

r_d (μD) = 2.12717 (13) $_{exp}$ (82) $_{theo}$ fm vs.

r_d (CODATA-14) = 2.1**4**130 (250) fm

(2) **polarizability**, using **charge radius from isotope shift**

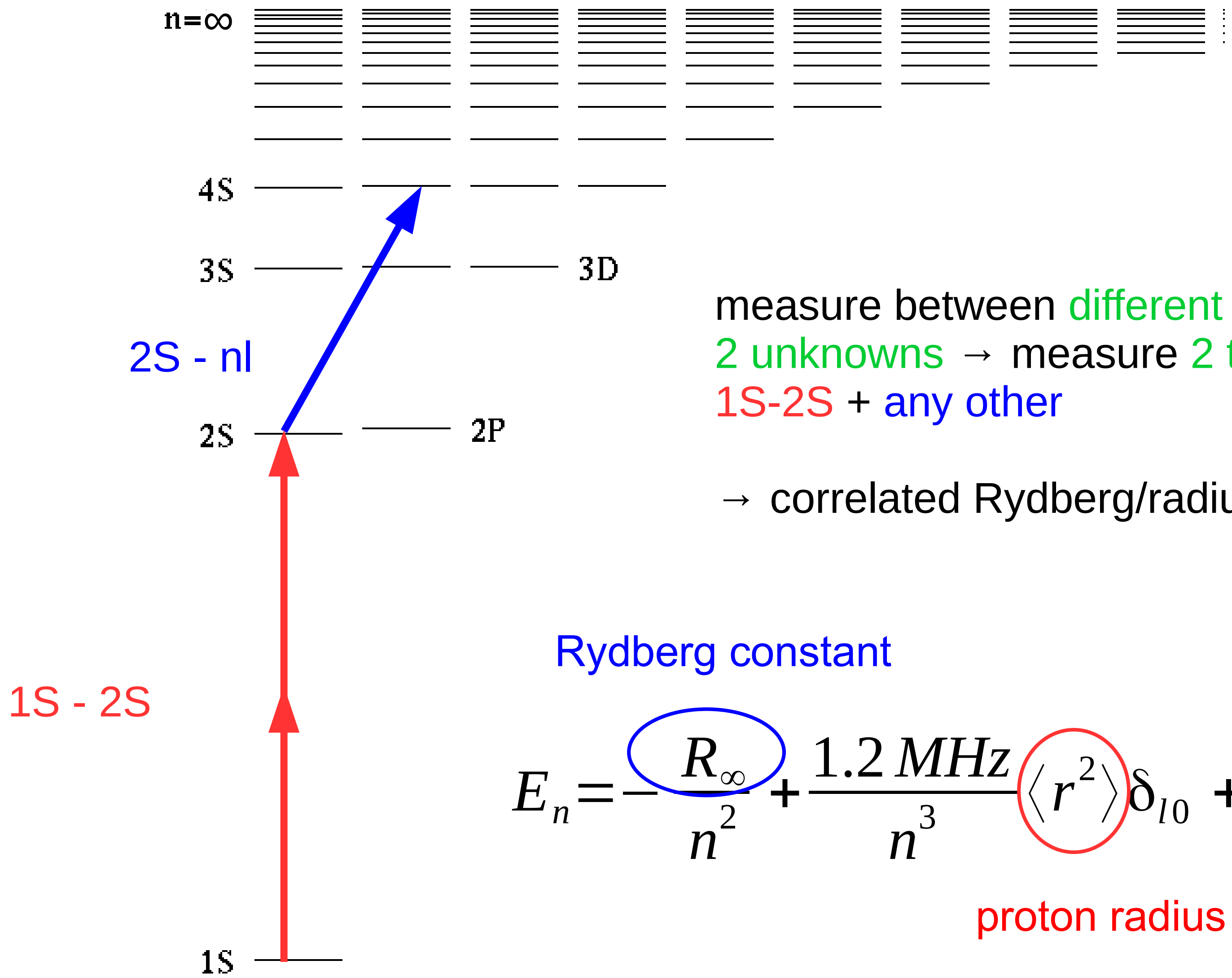
ΔE_{TPE} (theo) = 1.7500 (210) meV vs.

ΔE_{TPE} (exp) = 1.7591 (59) meV 3.5x more accurate

Krauth et al. (2016) + Pachucki et al. (2018) + Hernandez et al. (2018) + Kalinowski (2018)

Hydrogen

Energy levels of hydrogen



measure between different **n**
2 unknowns → measure 2 transitions:
1S-2S + any other

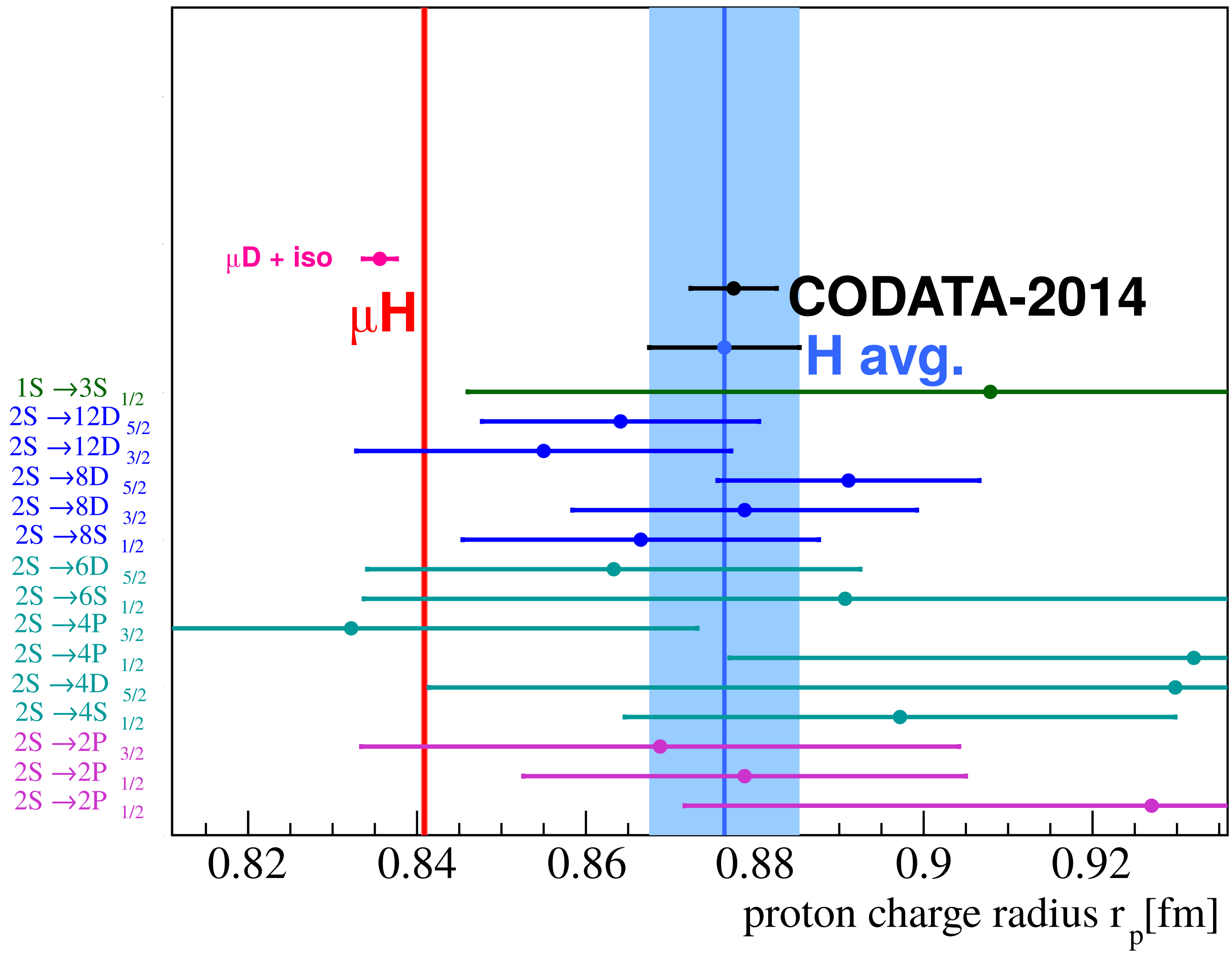
→ correlated Rydberg/radius pairs

Rydberg constant

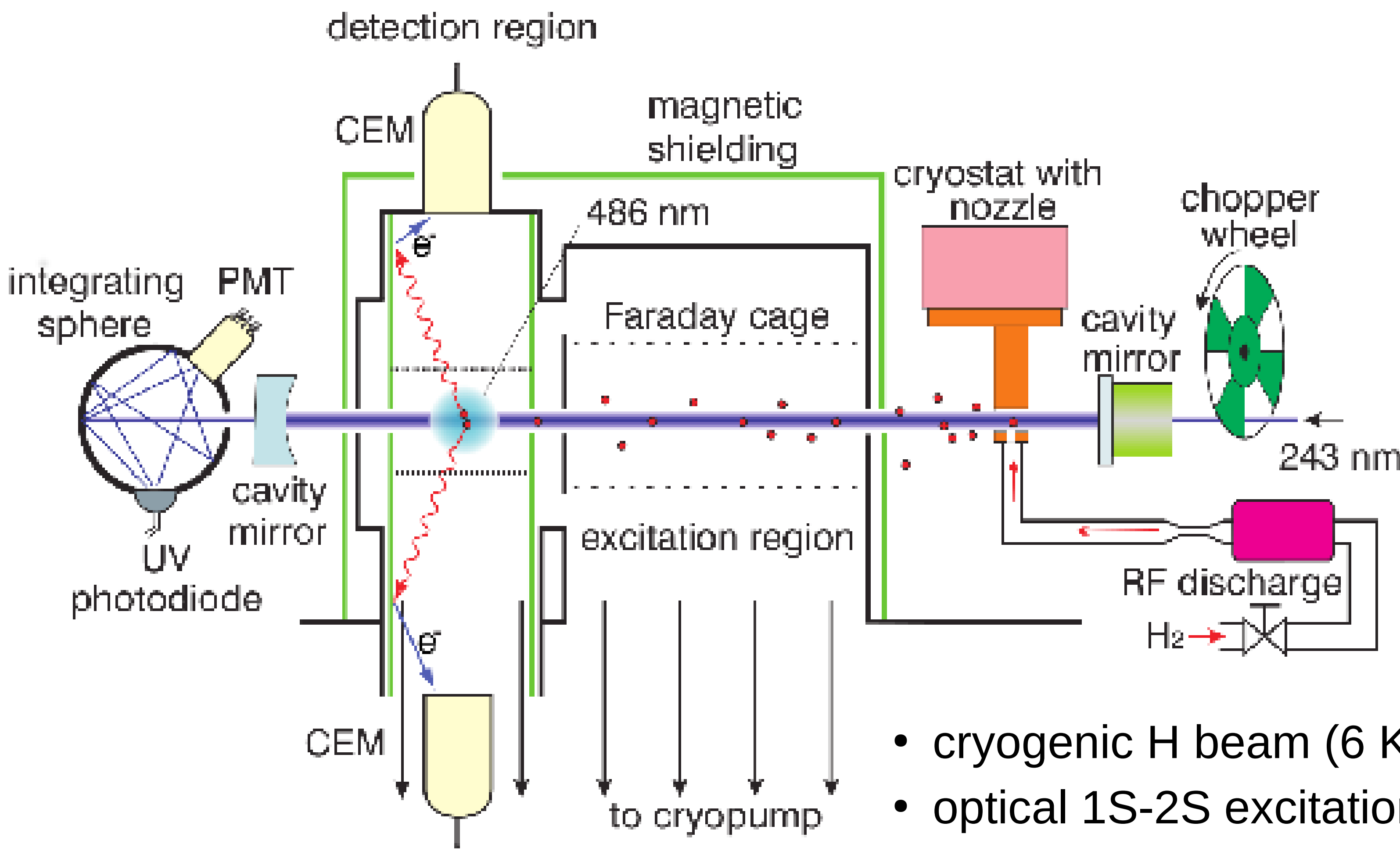
$$E_n = -\frac{R_\infty}{n^2} + \frac{1.2\,MHz}{n^3}\langle r^2\rangle \delta_{l0} + \Delta(n,l,j)$$

proton radius

Rp from H spectroscopy



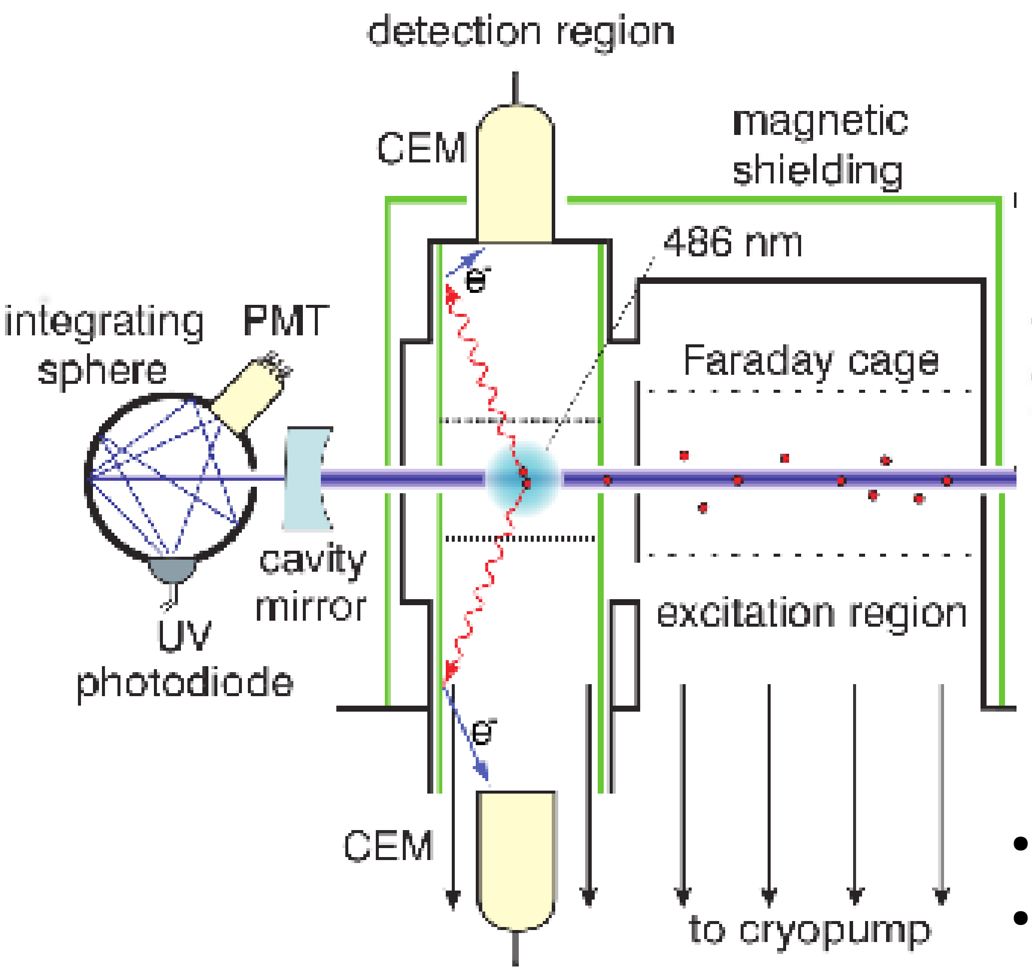
Garching H(2S-4P)



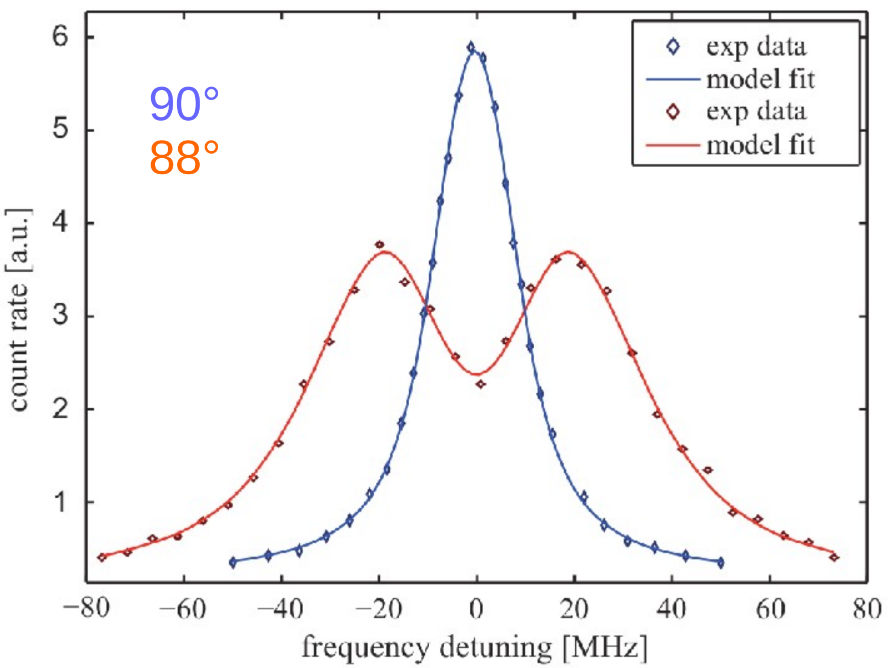
- cryogenic H beam (6 K)
- optical 1S-2S excitation (2S, F=0)
- 2S-4P transition is 1-photon: retroreflector
- split line to 10^{-4} !!!
- 2.3 kHz vs. 9 kHz PRP
- large systematics

Beyer, Maisenbacher, RP et al, Science 358, 79 (2017)

Garching H(2S-4P)



1st order Doppler cancellation



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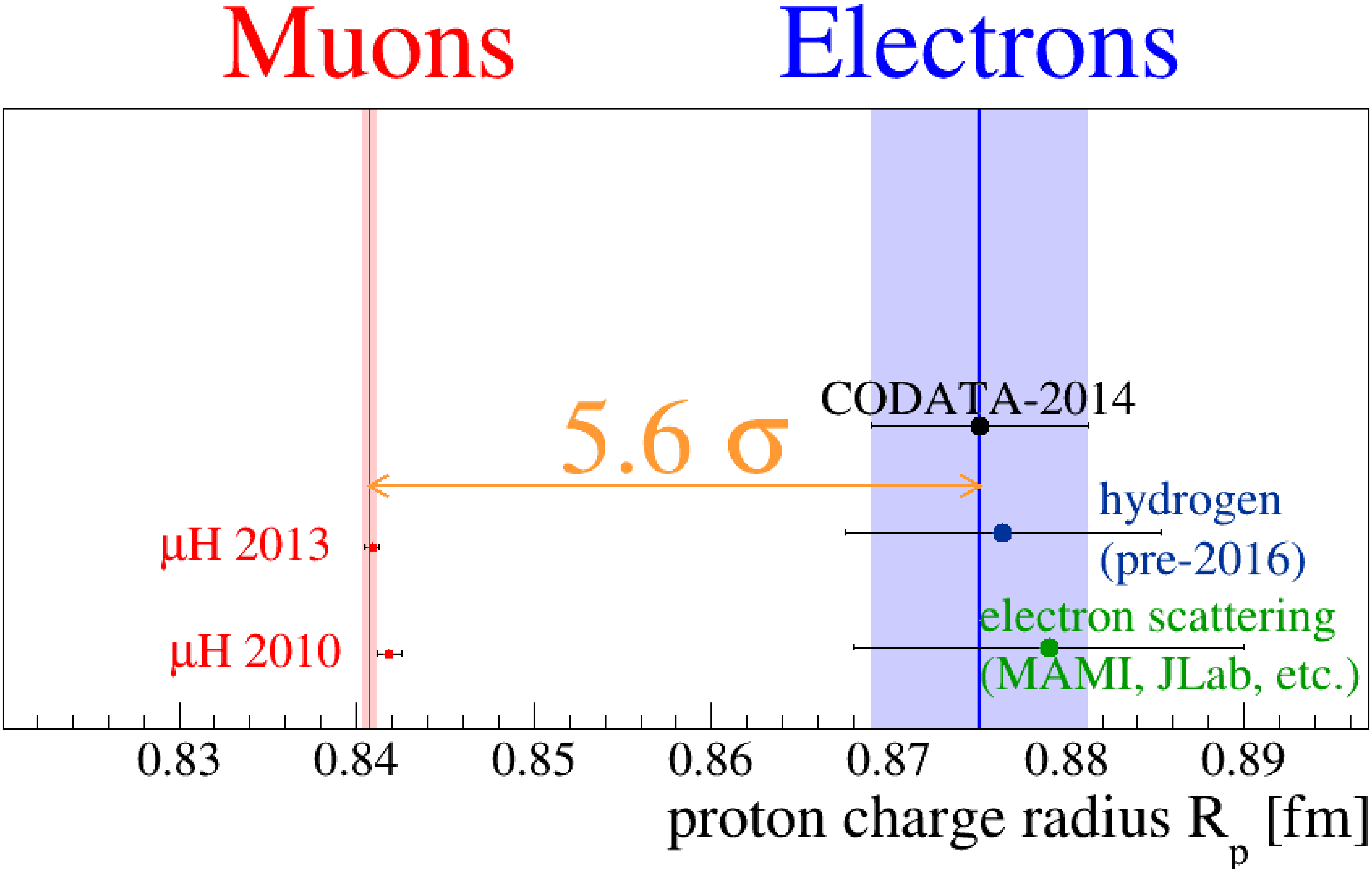
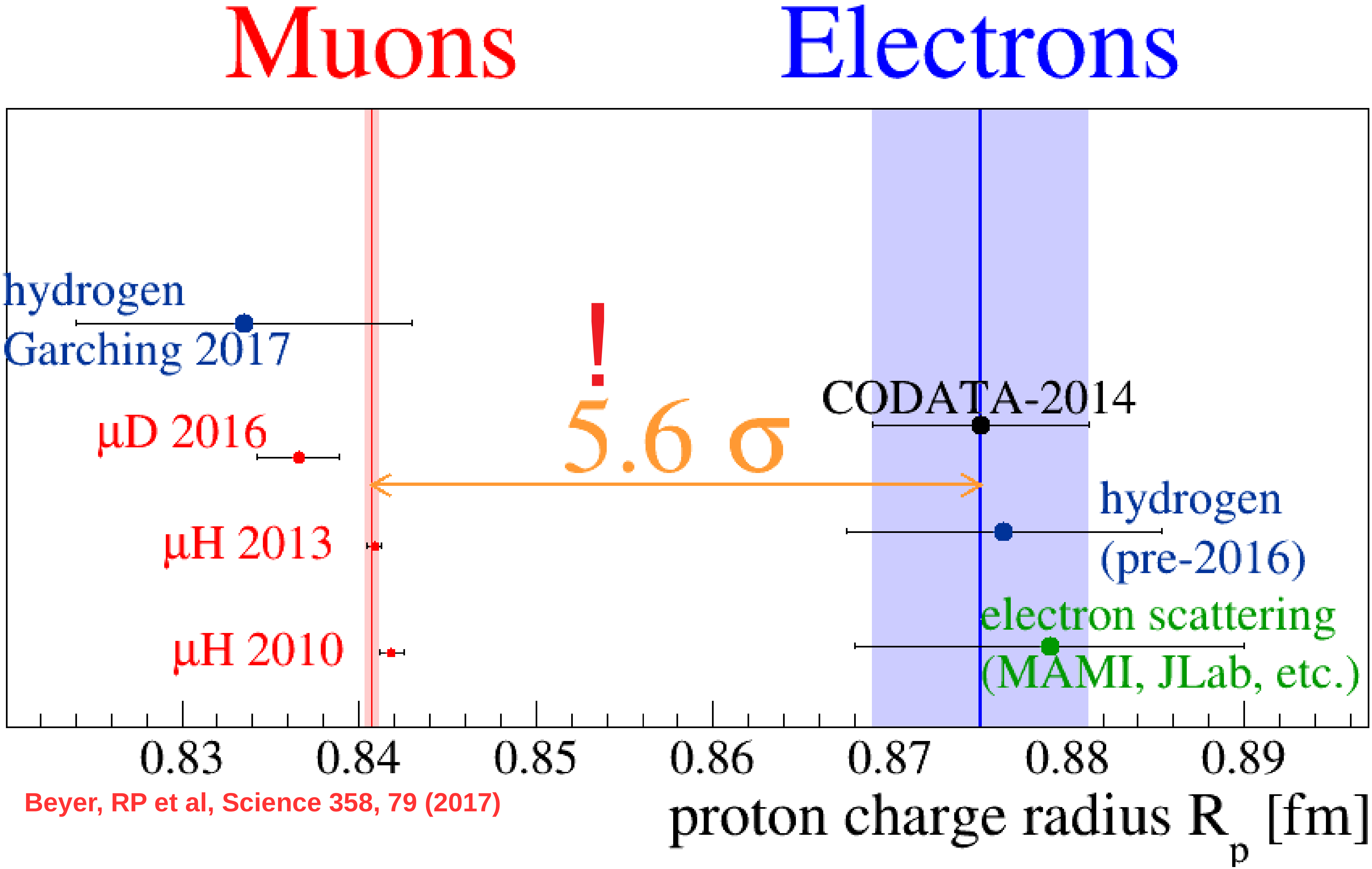
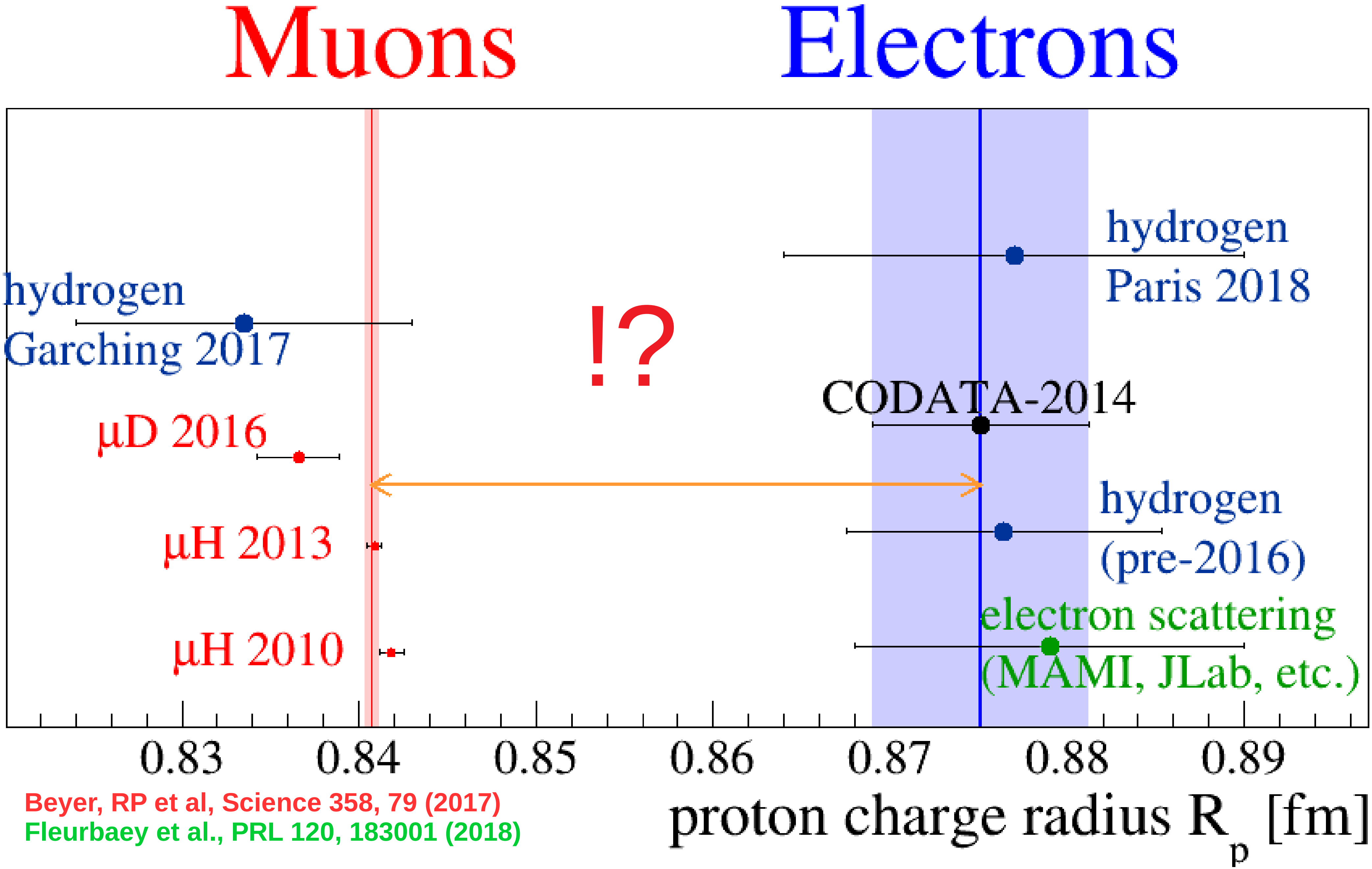


New Measurements: Garching 2S-4P



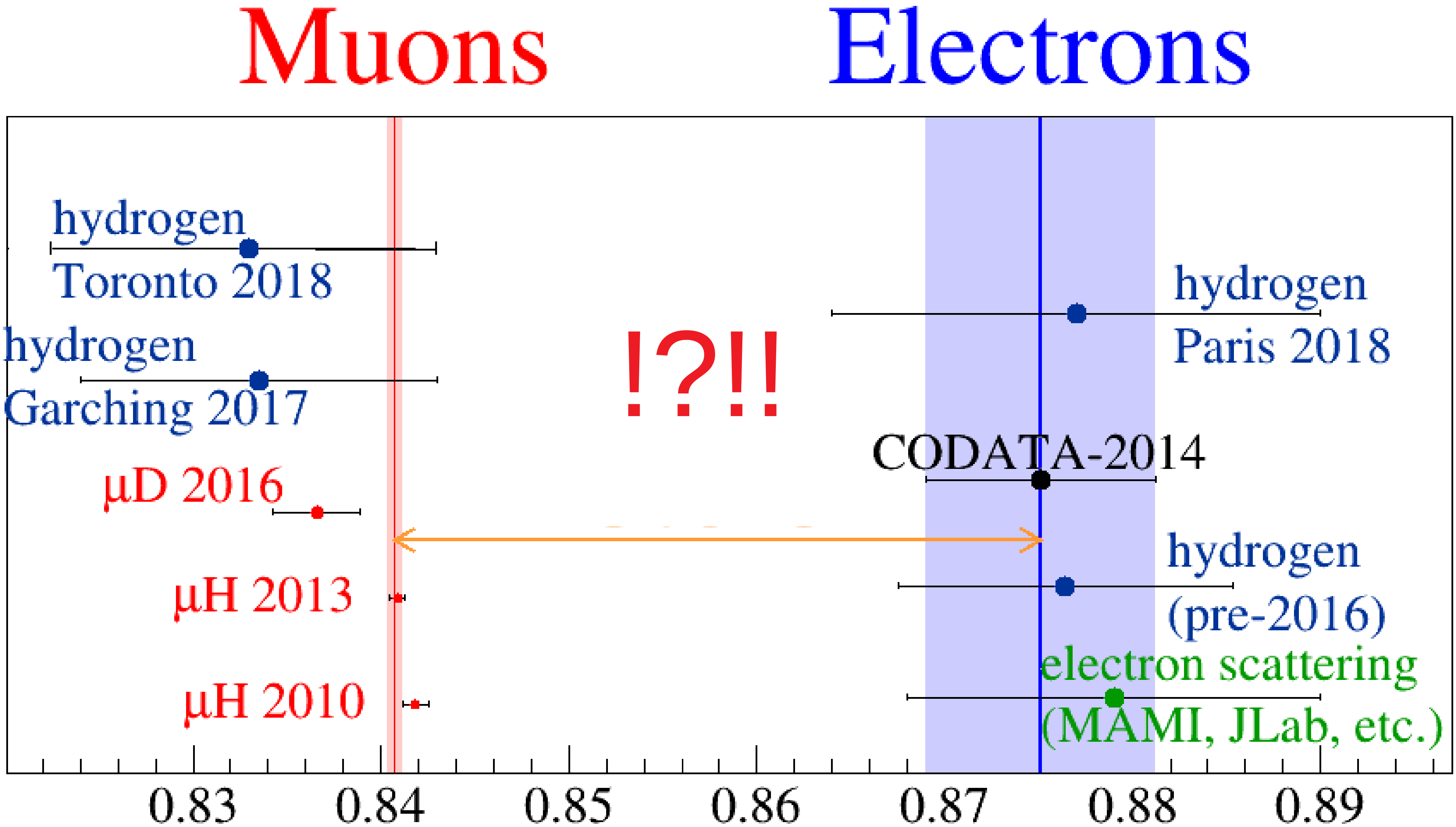
Beyer, RP et al, Science 358, 79 (2017)

New Measurements: Paris 1S-3S



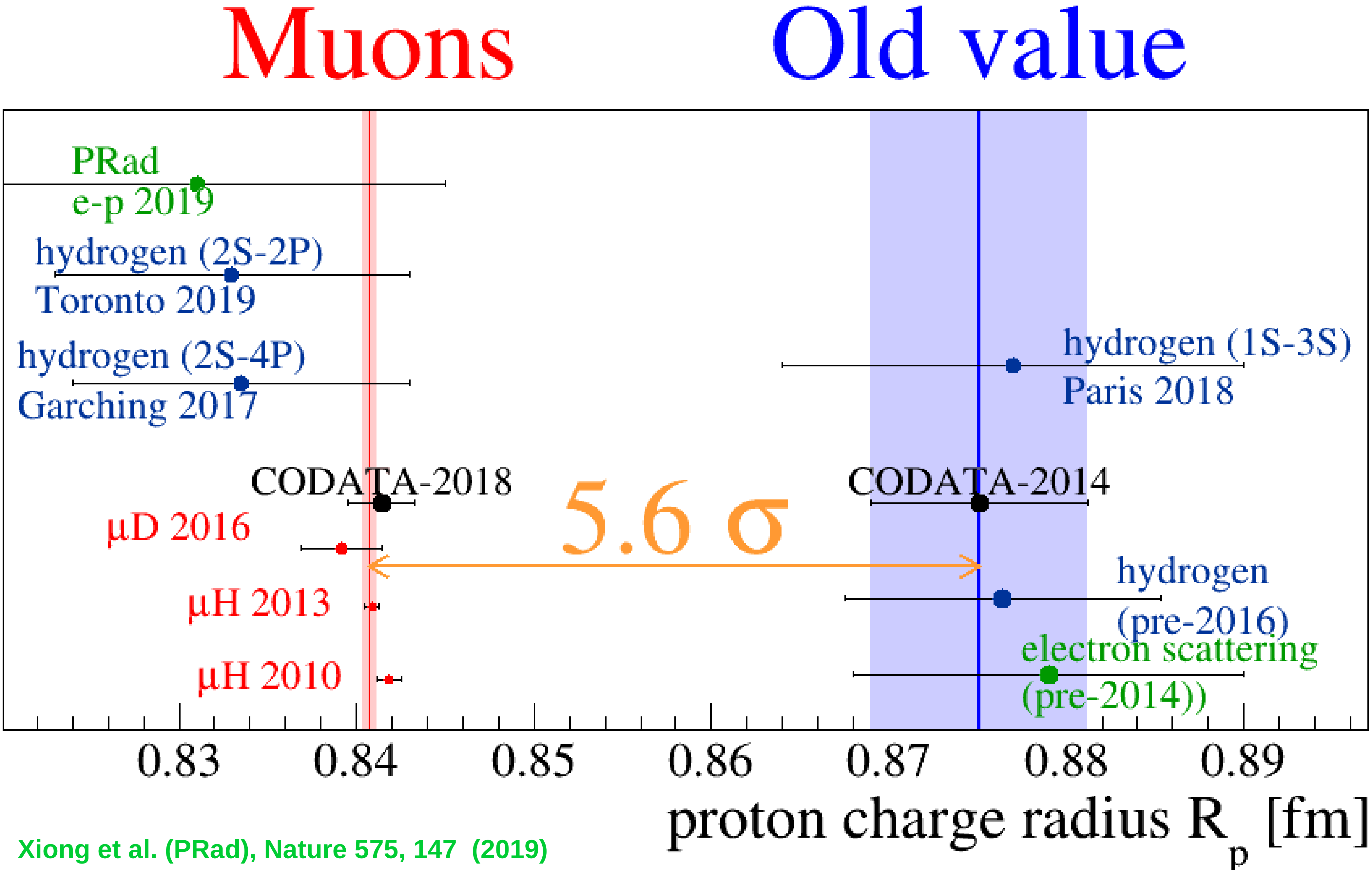
Beyer, RP et al, Science 358, 79 (2017)
Fleurbaey et al., PRL 120, 183001 (2018)

New Measurements: Toronto 2S-2P



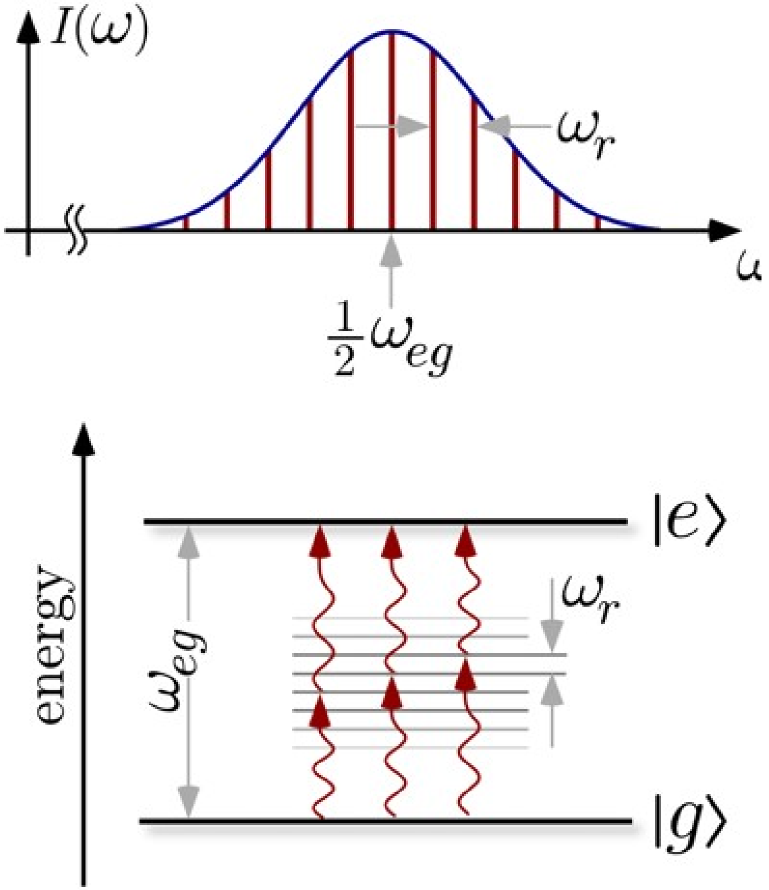
Beyer, RP et al, Science 358, 79 (2017)
Fleurbaey et al., PRL 120, 183001 (2018)
Bezginov et al., Science 365, 1007 (2019)

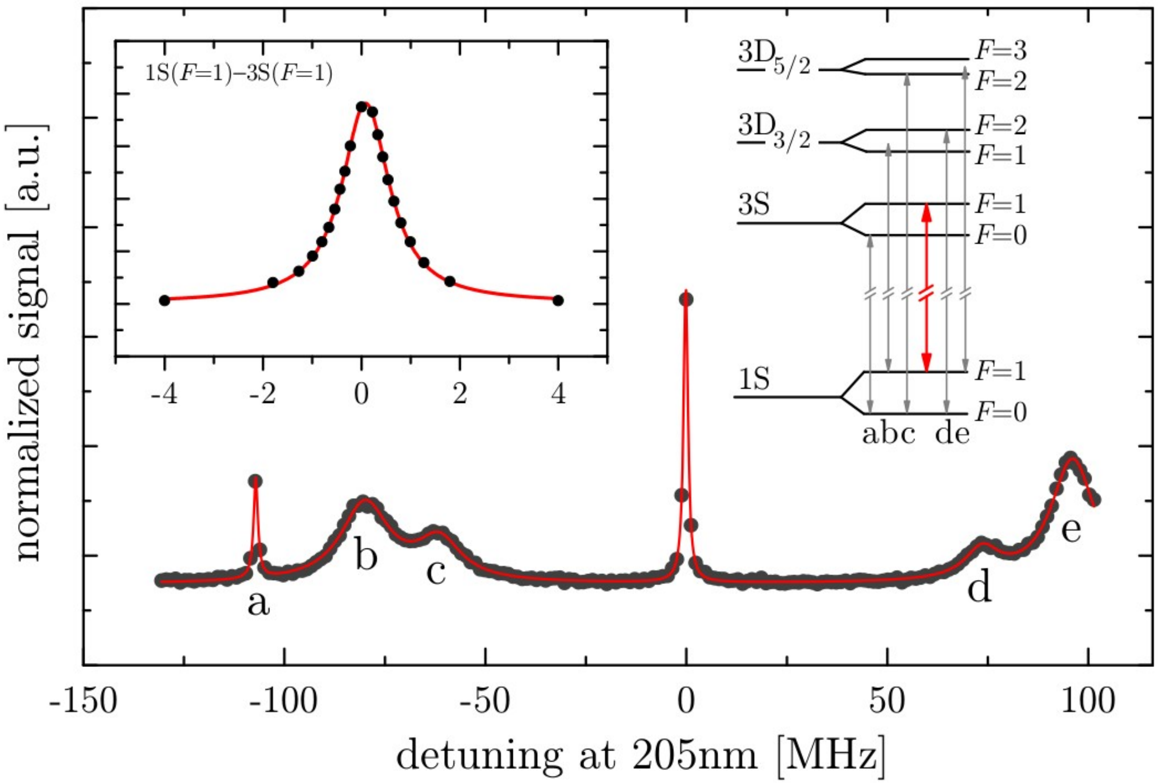
New Measurements: PRad



Xiong et al. (PRad), Nature 575, 147 (2019)

Garching H(1S-3S)

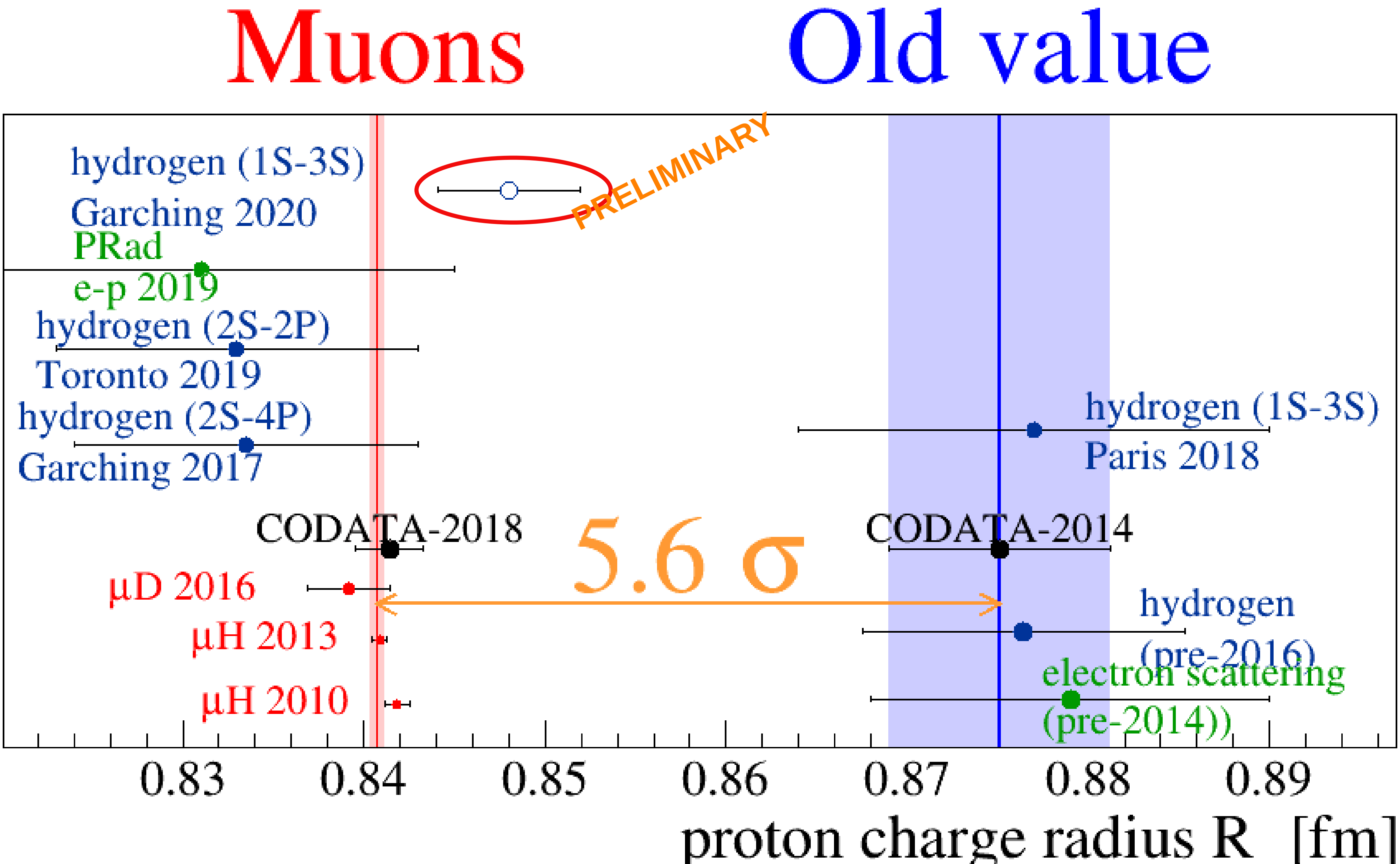




- Direct Frequency Comb Spectroscopy
- cryogenic H beam (6 K)

Grinin, Matveev,.. RP et al, Science, next week

H(1S-3S) Garching 2020



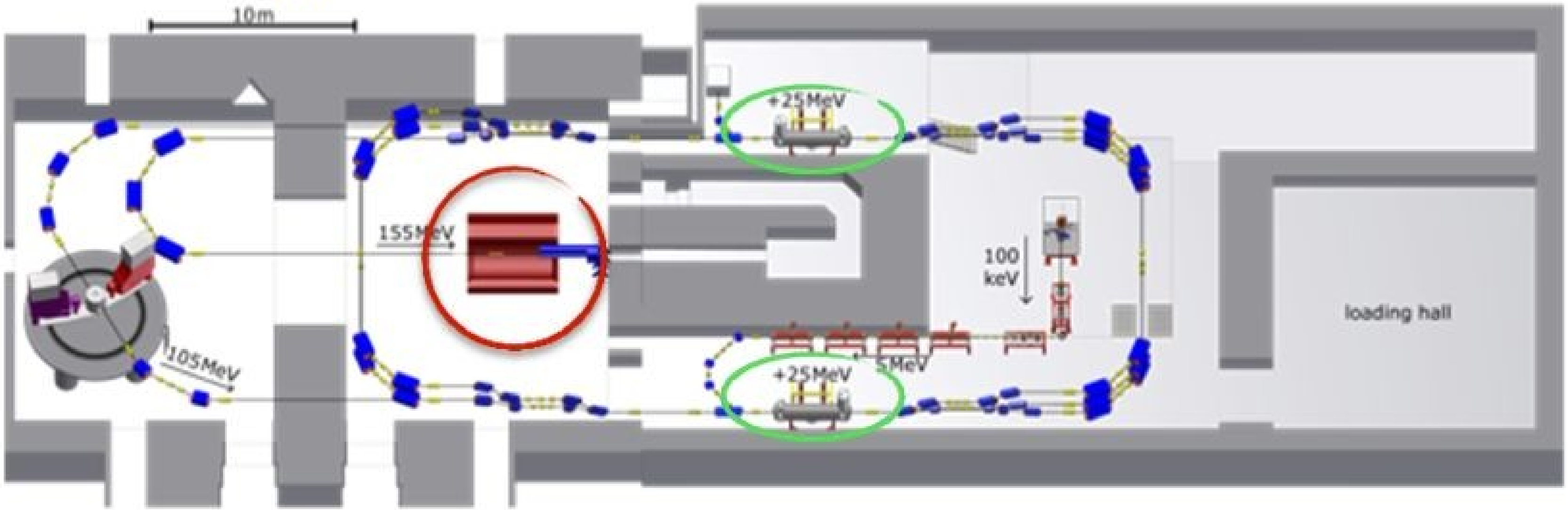
Grinin, Matveev,.. RP et al, Science, next week

New Mainz electron accelerator **MESA**

Kurt Aulenbacher

MESA — "**M**ainz **E**nergy-Recovering **S**uperconducting **A**ccelerator

Beam energy: 105 MeV / 155 MeV Current: 1–2 mA

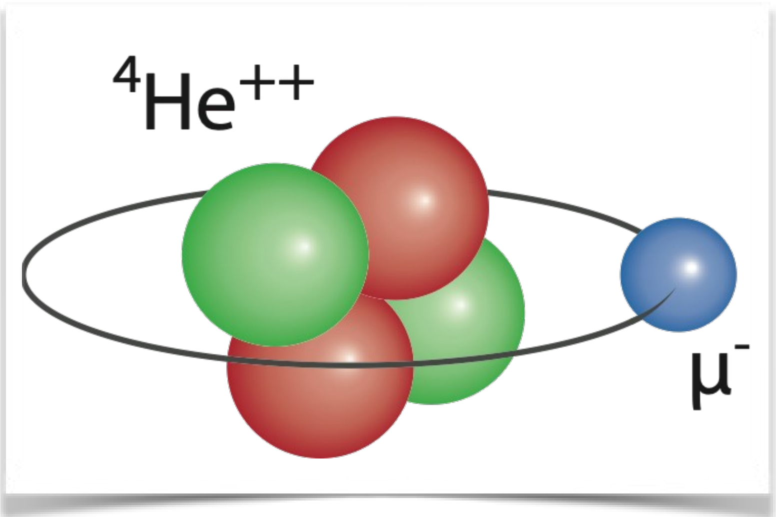


Being built on **Campus of JGU Mainz**

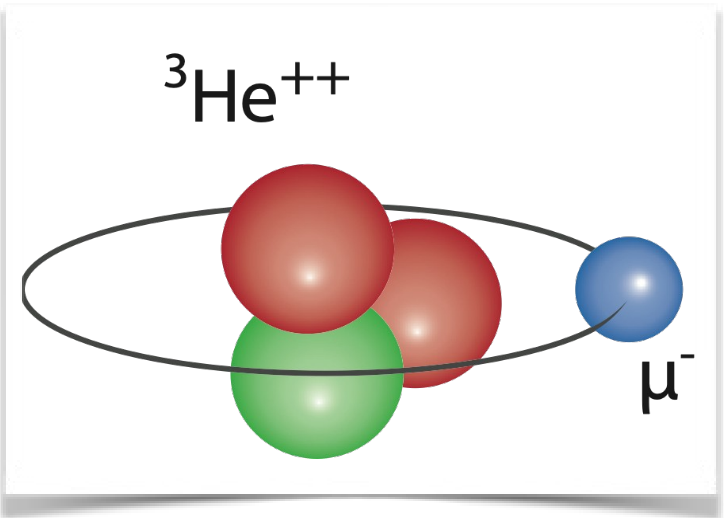
MAGIX: windowless (gas-jet) target, lowest Q^2

Cluster of Excellence
PRiSMA+

Muonic Helium

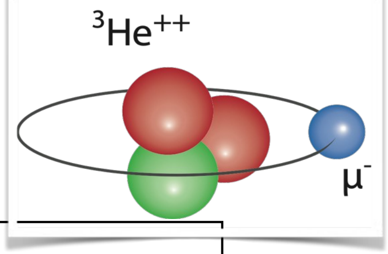


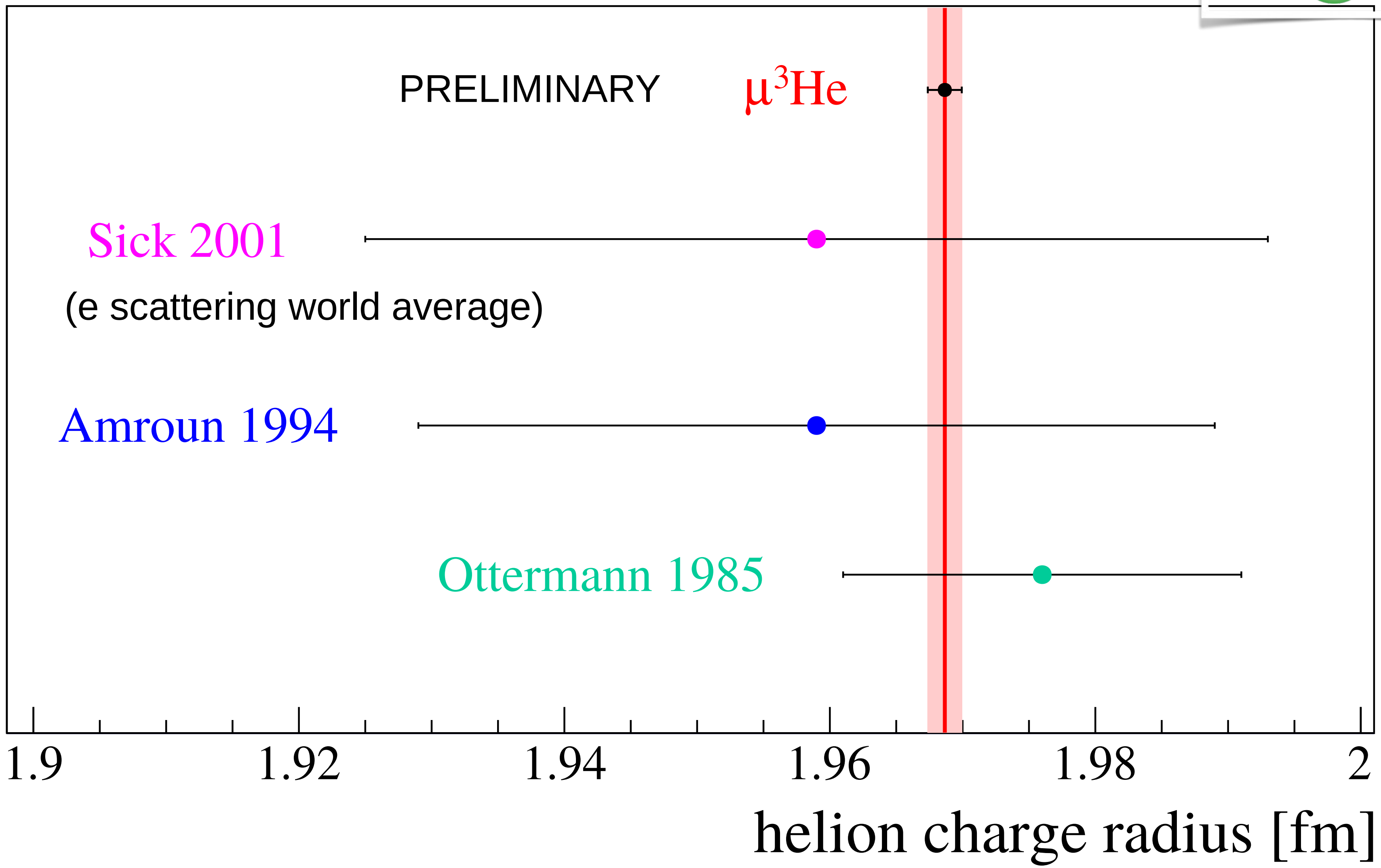
Accepted (Nature 2020)



Measured

Muonic Helium-3

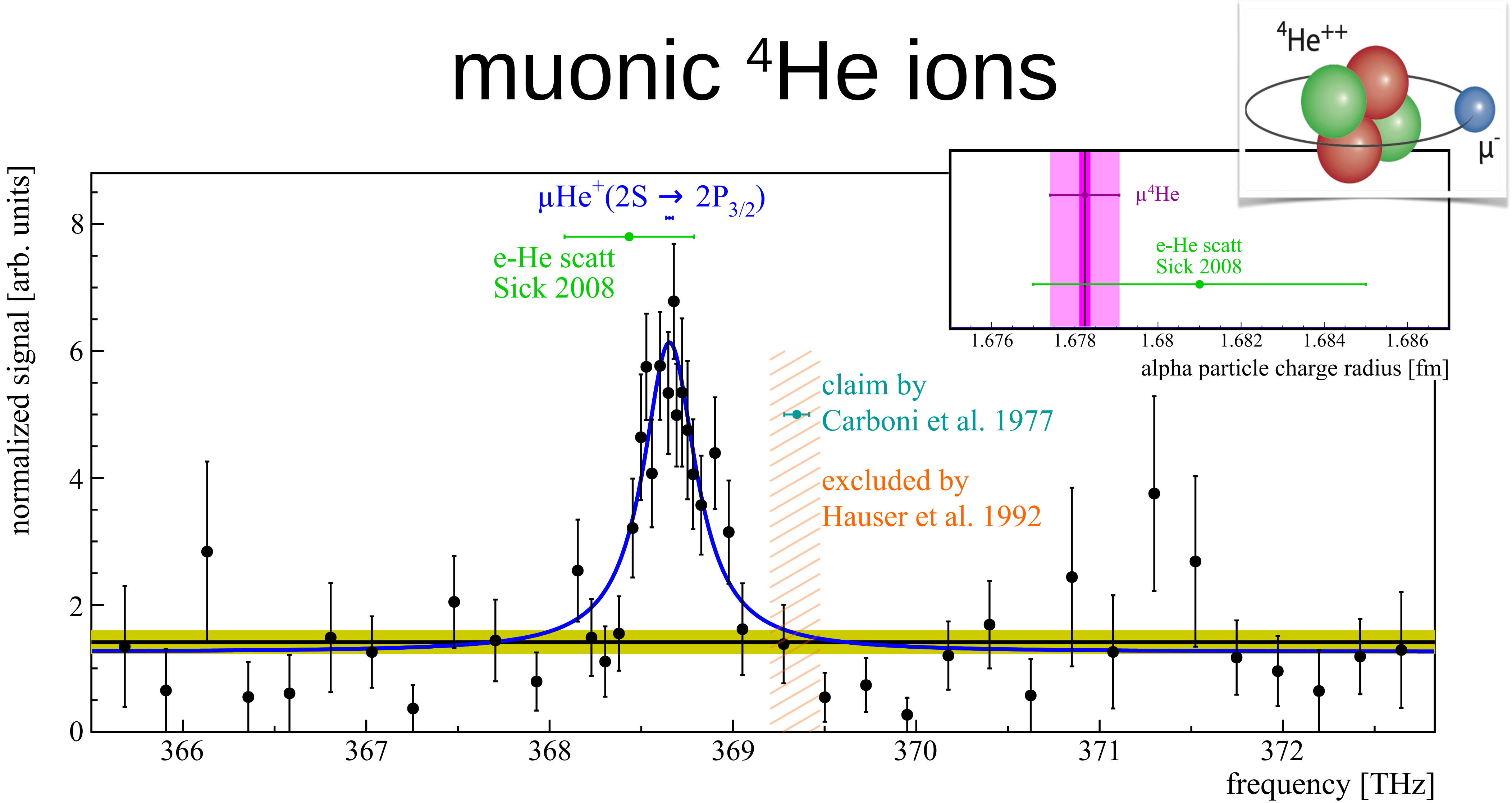




prel. accuracy: exp **+- 0.00012** fm, theo **+- 0.00128** fm (nucl. polarizability)

Theory: see Franke et al. EPJ D 71, 341 (2017) [1705.00352]

muonic ^{4}He ions



$R(^4He) = 1.67824\ (13)_{exp}\ (82)_{theo}$ fm

Krauth, RP et al. (CREMA Coll.)
Nature, accepted (2020)

Theory: Diepold et al., Ann. Phys. (2018)
incl. 3-photon nuclear polarizability (Pachucki, 2018)

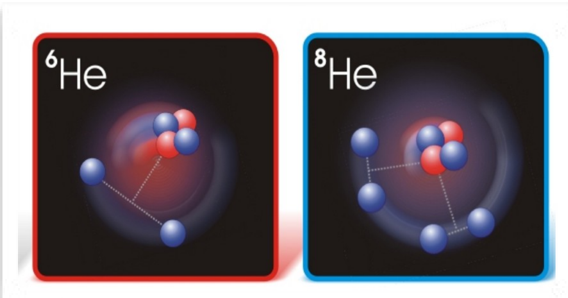
Impact of μ⁴He⁺ measurements

Few-nucleon theories

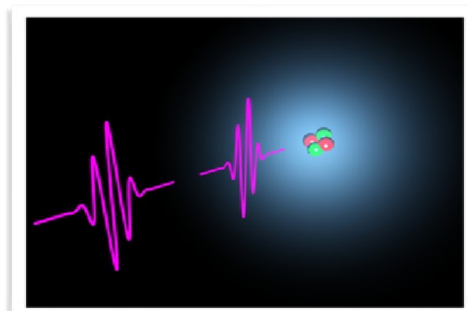
- r_α represents a benchmark for few-nucleon theories.
- r_α can be used also to fix a low-energy constant of nuclear potential.
- r_α improves ^{6}He and ^{8}He radii

BSM physics

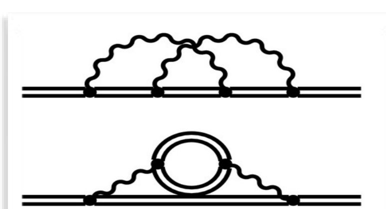
- Agreement constrains BSM models suggested to explain the R_p puzzle



Müller, Lu



Udem, MPQ
Eikema, LaserLab



Combined with upcoming He⁺ (He) exp.

- bound-state QED test He⁺(1S-2S): 60 kHz, $u_r = 6\text{x}10^{-12}$
- Rydberg constant: 24 kHz
- **2PE+3PE in μHe with 0.1 meV uncertainty**

Conclusions

Muonic atoms / ions provide:

- **~10x more accurate charge radii**, when combined with

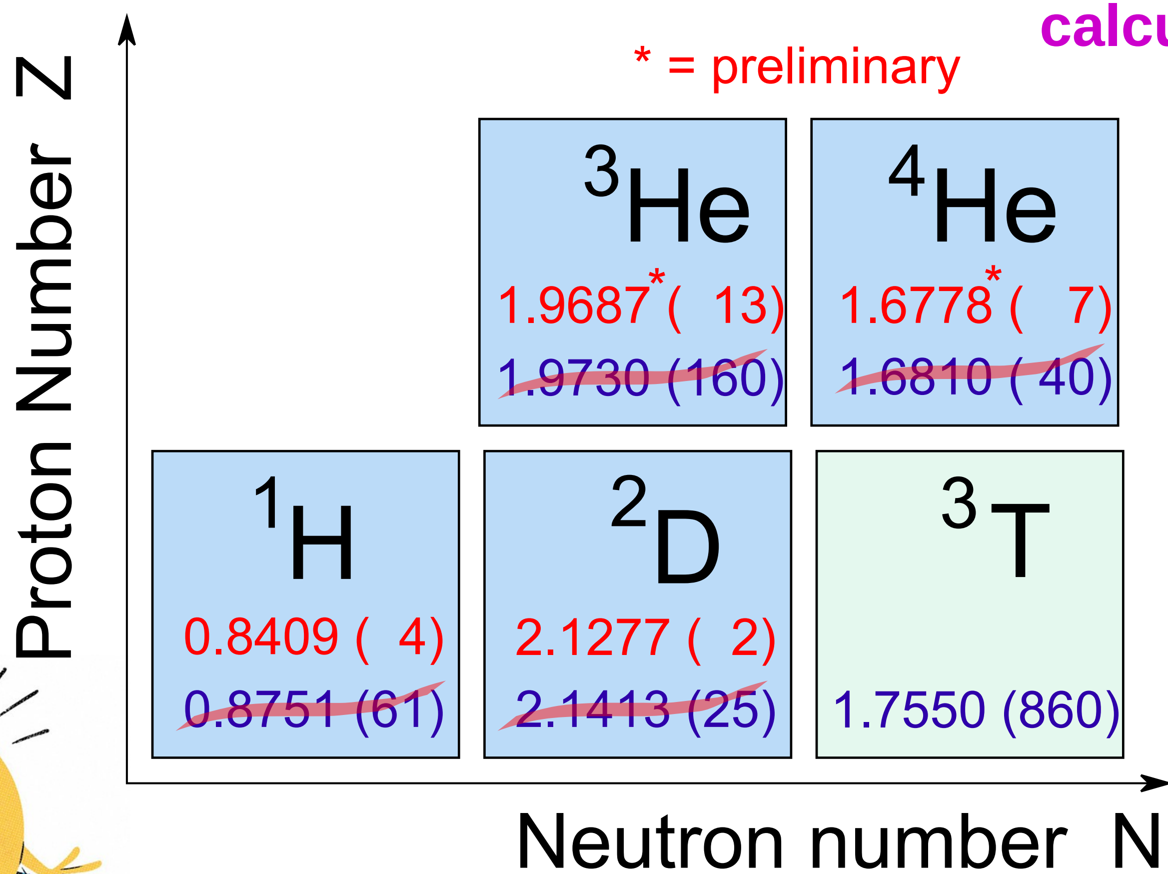
  calculated polarizability

Conclusions

Muonic atoms / ions provide:

- **~10x more accurate charge radii**, when combined with

calculated polarizability



Intermediate conclusions

Muonic atoms / ions provide:

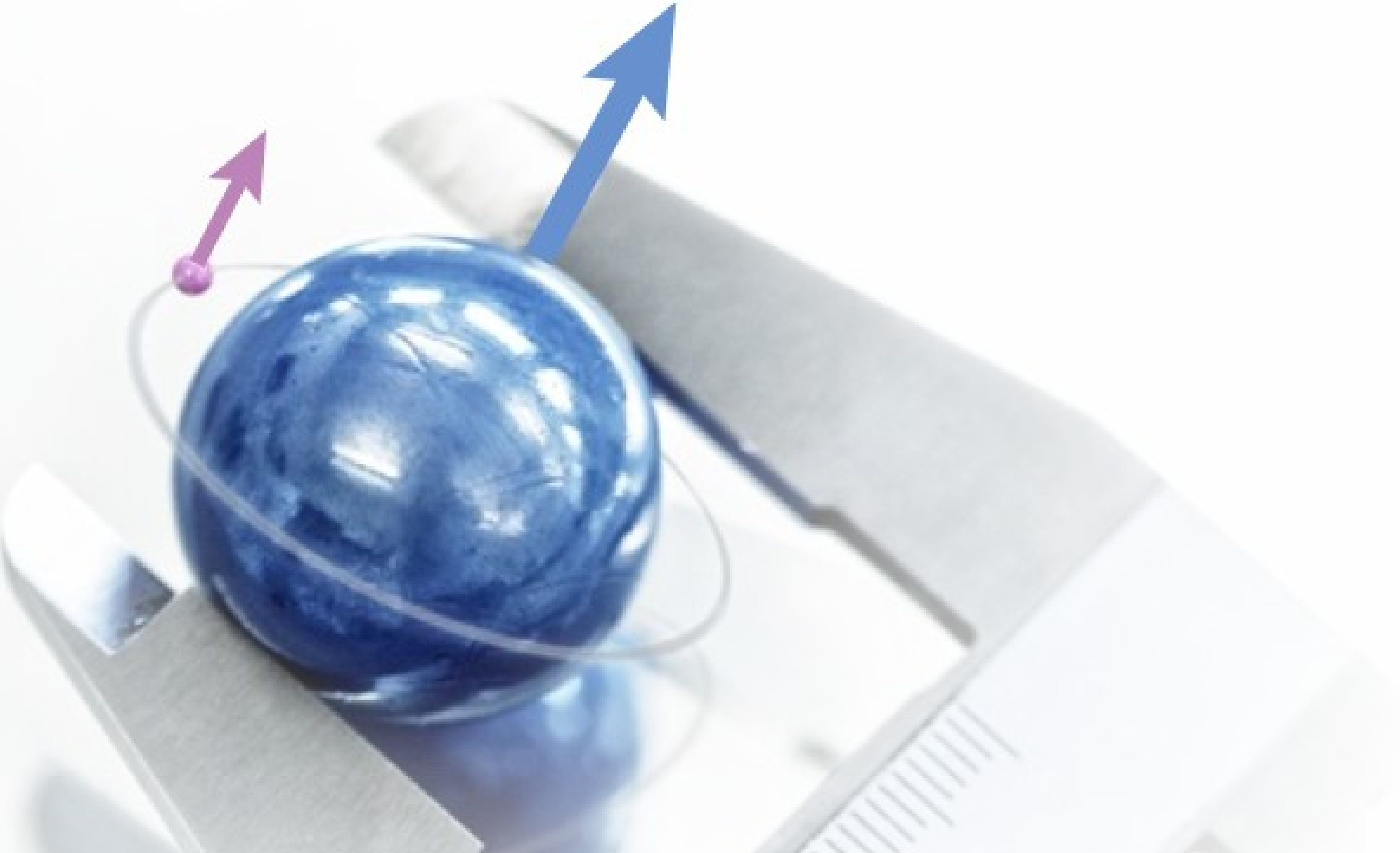
- **~10x more accurate charge radii**, when combined with **calculated polarizability**
- few times more accurate **nuclear polarizability**, when combined with **charge radius from regular atoms**

Muonic atoms are a novel tool for proton and new-nucleon properties!

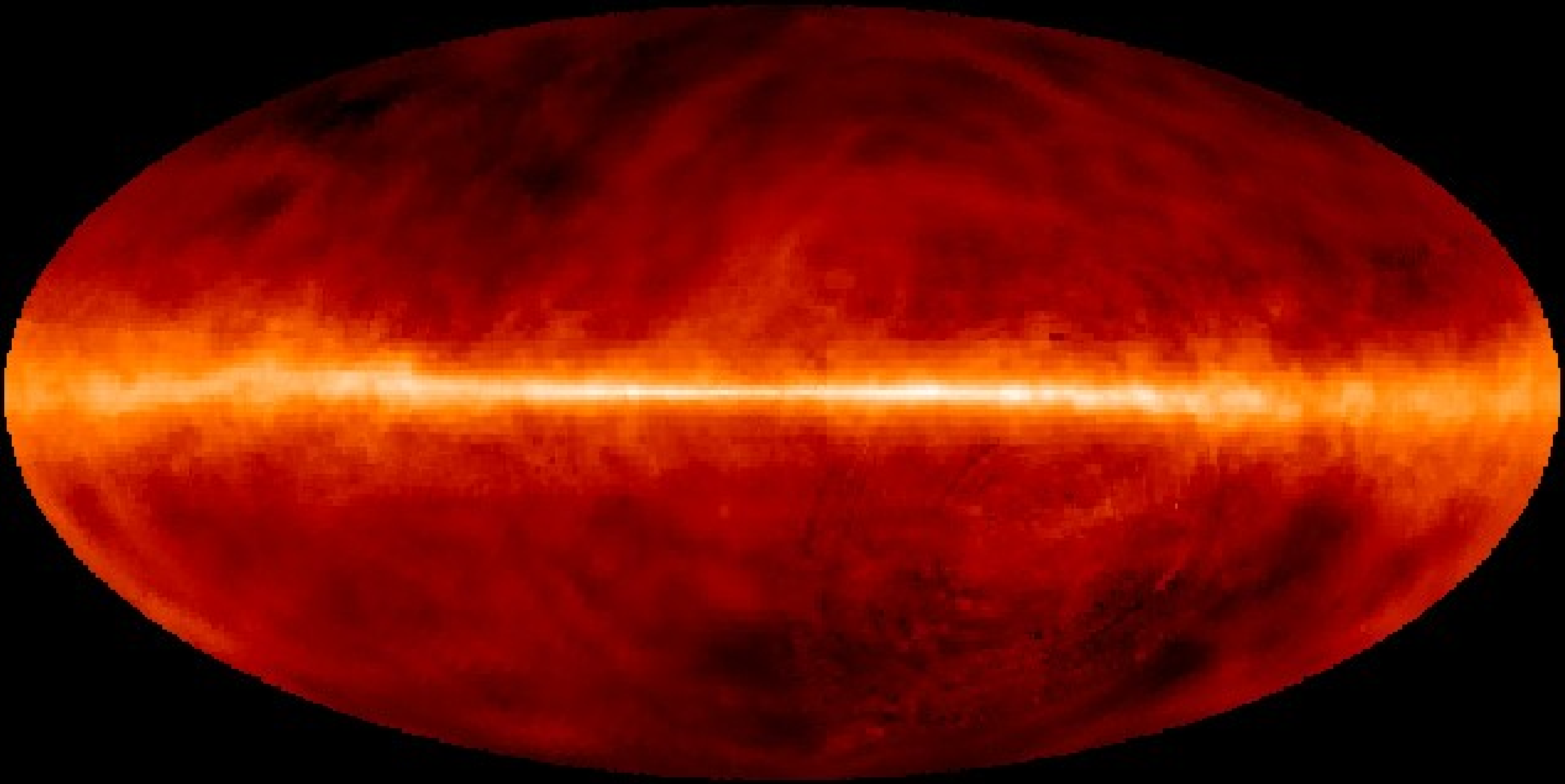
The Present

Hyperfine structure in muonic H

CREMA-3 / HyperMu at PSI
(R16.02)



The sky in hydrogen



Hyperfine structure in H / μp

The 21 cm line in hydrogen (1S hyperfine splitting) has been **measured** to 12 digits (0.001 Hz) in 1971:

$$\nu_{exp} = 1\,420\,405.\,751\,766\,7 \pm 0.000\,001 \text{ kHz}$$

Essen et al., Nature 229, 110 (1971)

Hyperfine structure in H / μp

The 21 cm line in hydrogen (1S hyperfine splitting) has been **measured** to 12 digits (0.001 Hz) in 1971:

$$\nu_{exp} = \mathbf{1\ 420\ 405.\ 751\ 766\ 7\ \ \pm 0.000\ 001\ kHz}$$

Essen et al., Nature 229, 110 (1971)

QED test is limited to 6 digits (800 Hz) because of proton structure effects:

$$\nu_{theo} = \mathbf{1\ 420\ 403.\ 1\ \ \pm 0.6_{proton\ size}\ \ \pm 0.4_{polarizability}\ kHz}$$

Eides et al., Springer Tracts 222, 217 (2007)

Proton Zemach radius

HFS depends on “Zemach” radius:

$$\Delta E = -2(Z\alpha) m \langle r \rangle_{(2)} E_F$$

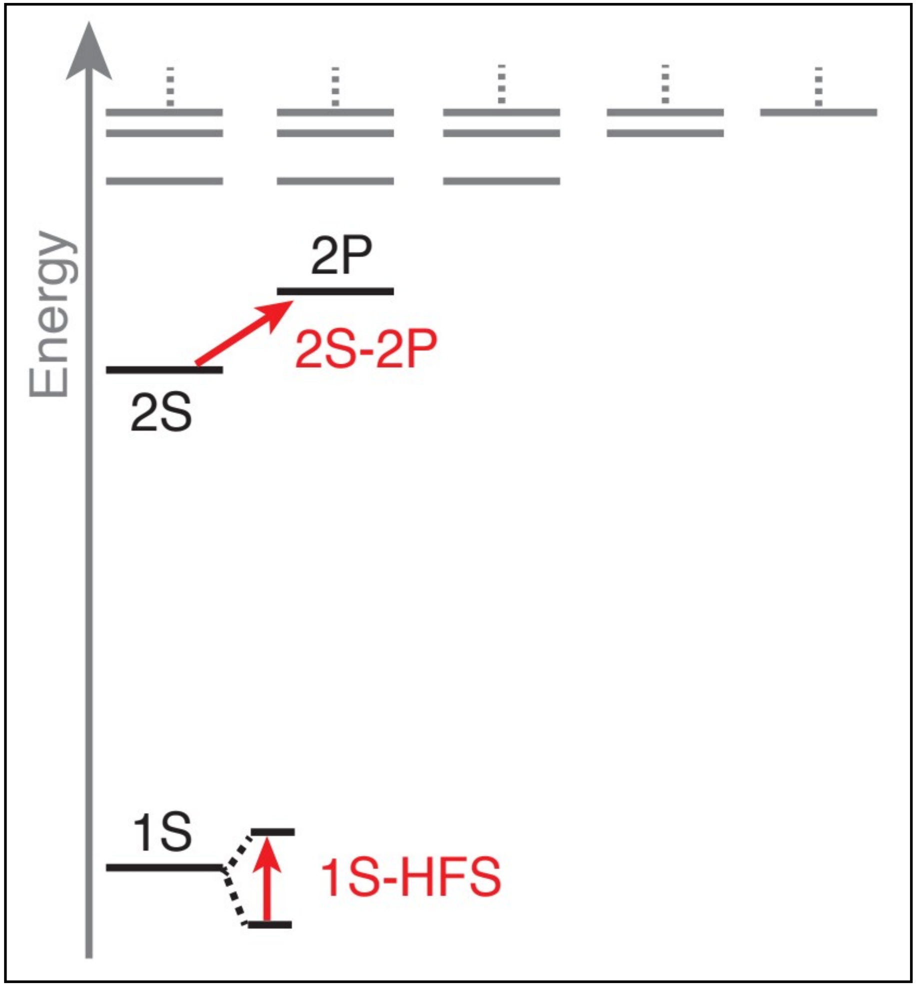
$$\langle r \rangle_{(2)} = \int d^3r\, d^3r' \rho_E(r) \rho_M(r') |r - r'|$$

Zemach, Phys. Rev. 104, 1771 (1956)

Form factors and momentum space

$$\Delta E = \frac{8(Z\alpha) m}{\pi n^3} E_F \int_0^{\infty} \frac{dk}{k^2} \left[\frac{G_E(-k^2) G_M(-k^2)}{1+\kappa} \right]$$

From charge to magnetic properties



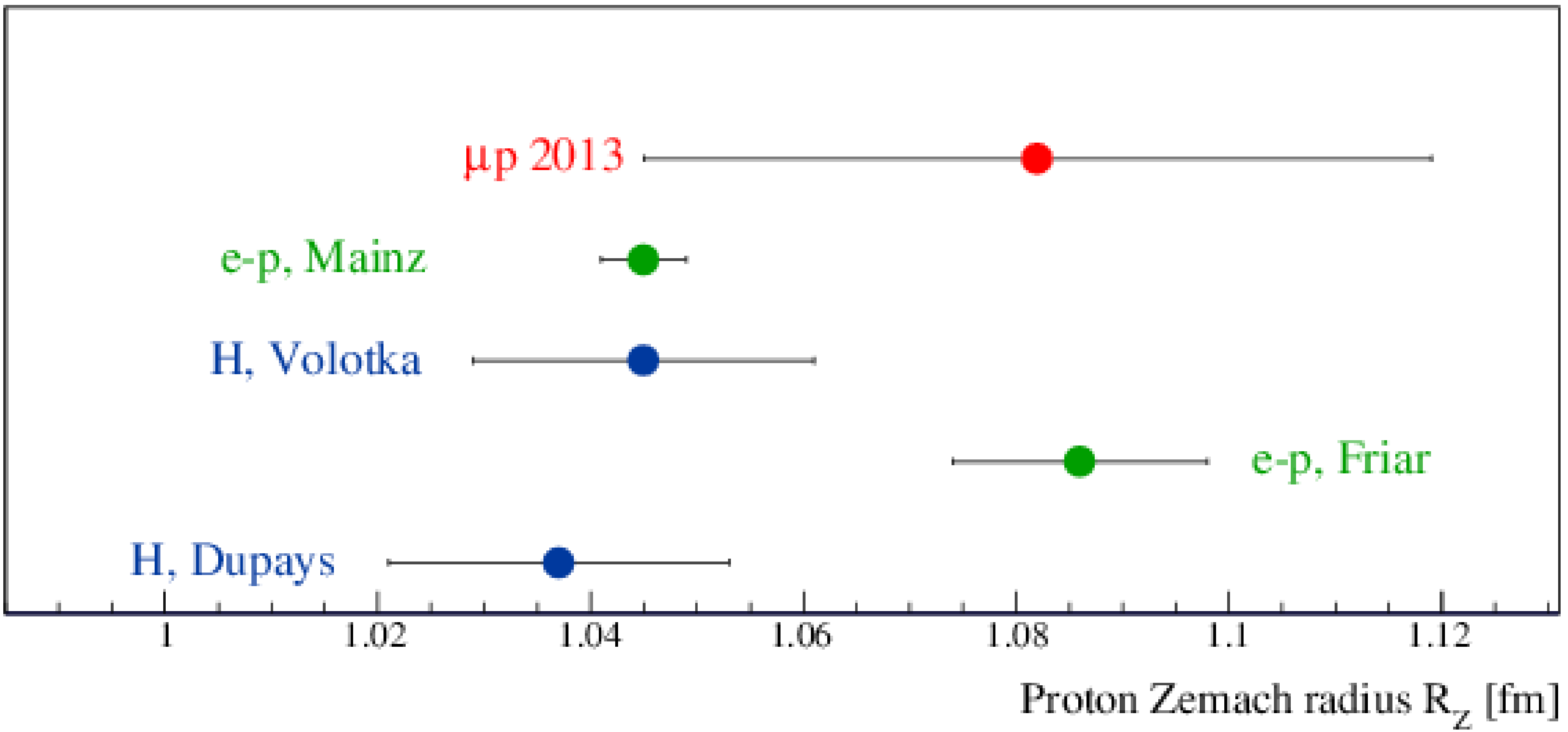
2S-2P = Lamb shift

is sensitive to CHARGE radius

1S-HFS = Hyperfine splitting

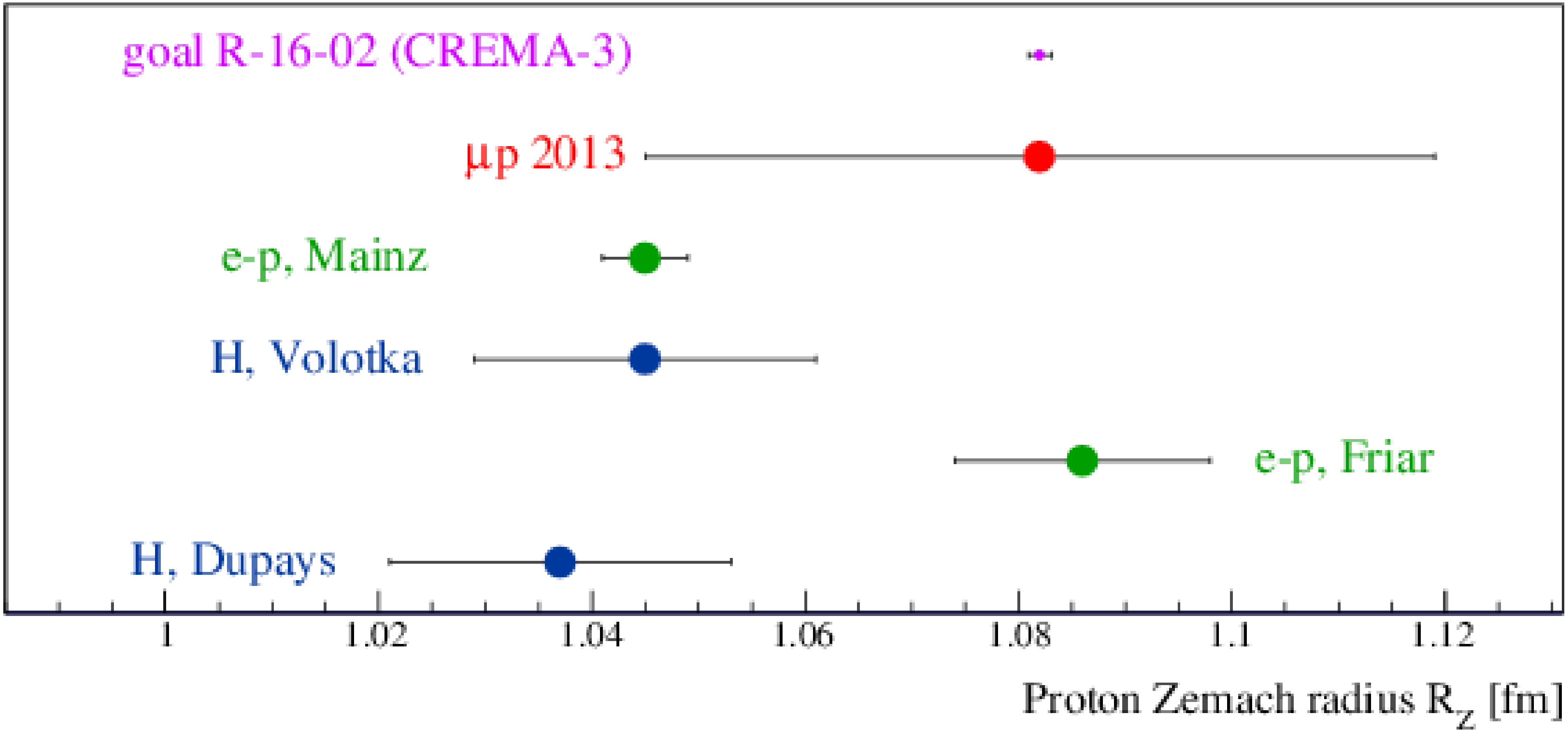
is sensitive to ZEMACH radius

Proton Zemach radius from μp



μp 2013: Antognini et al. (CREMA Coll.), Science 339, 417 (2013)

Proton Zemach radius from μp

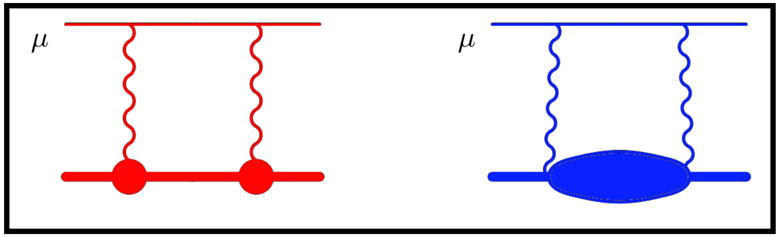


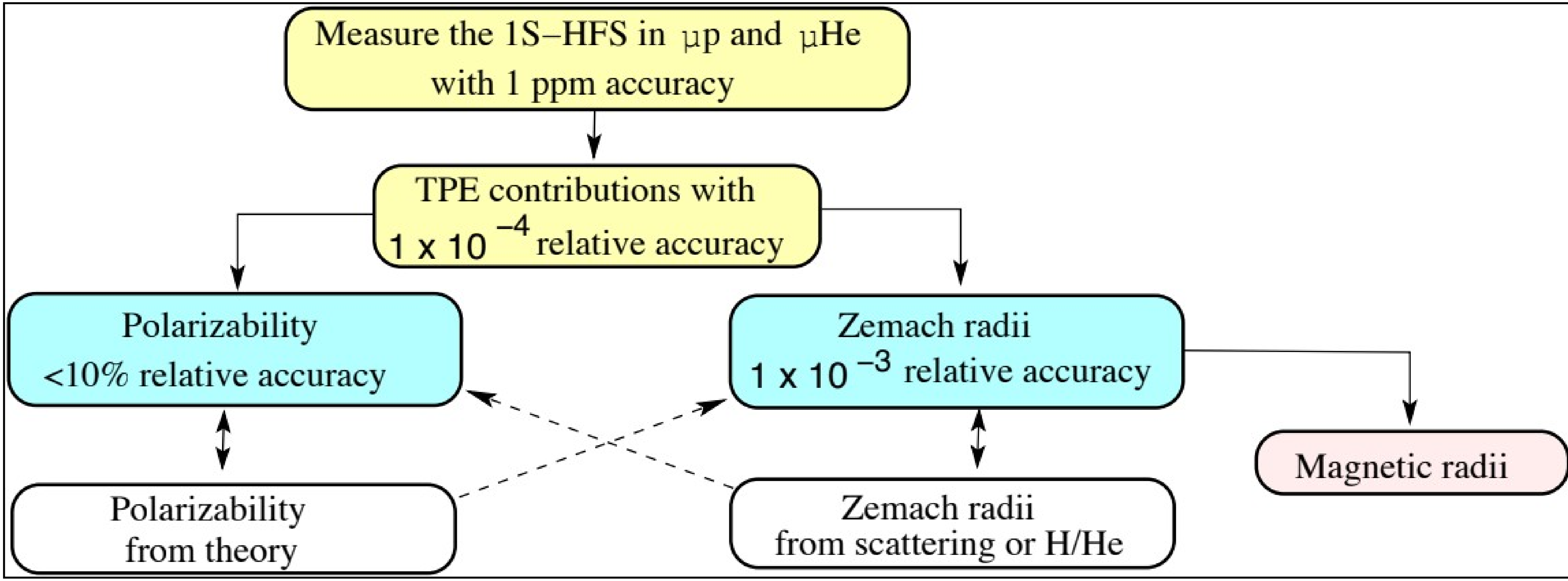
PSI Exp. R-16-02: Antognini, RP et al. (CREMA-3 / HyperMu)

see e.g. Schmidt, RP et al., J. Phys. Conf. Ser 1138, 012010 (2018); arXiv 1808.07240

HFS in μp

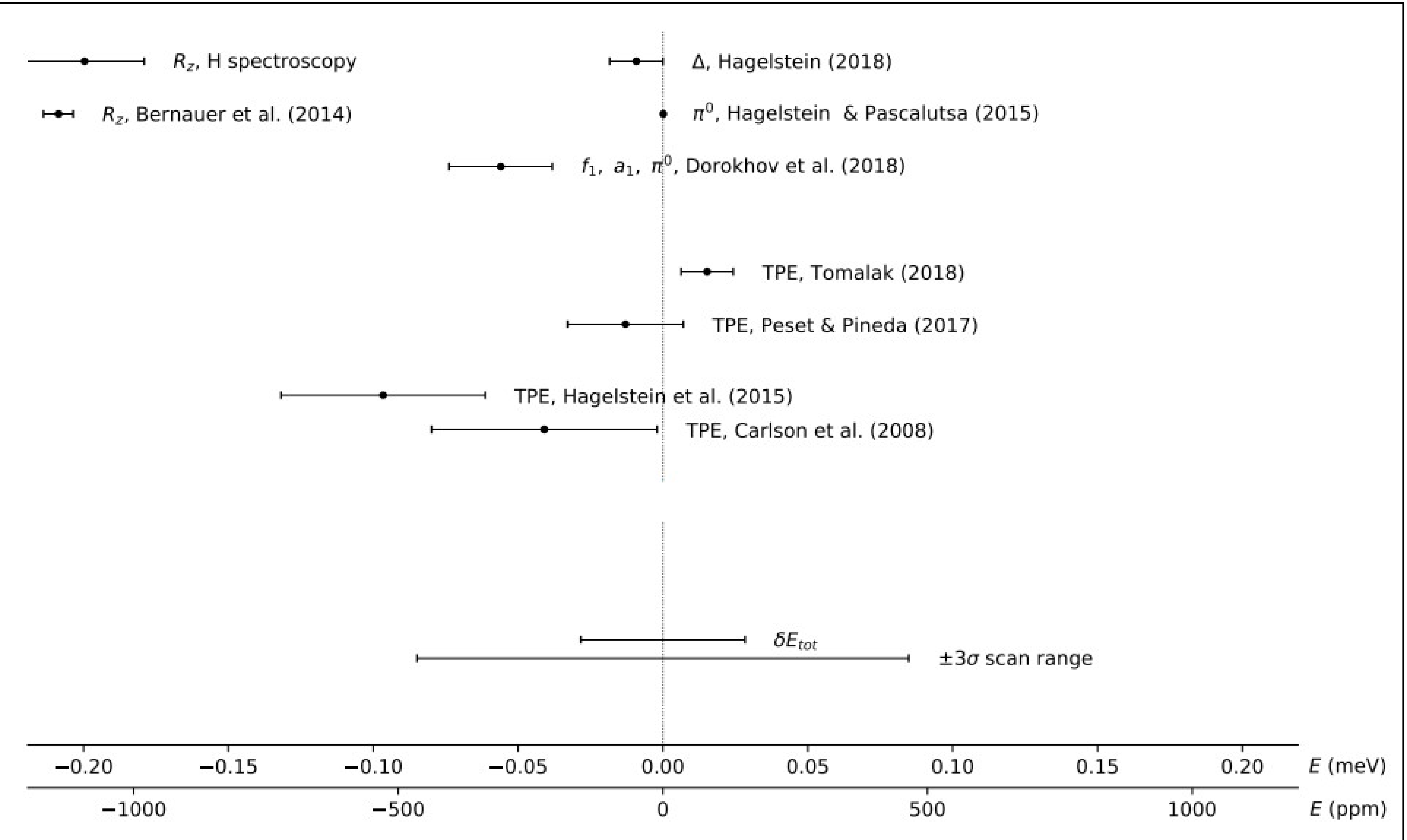
$$\Delta E^{\rm th}_{\rm HFS} = 182.819(10) - \underbrace{1.301 R_Z + 0.064(21)}_{\rm TPE} + \cdots \quad {\rm meV}$$



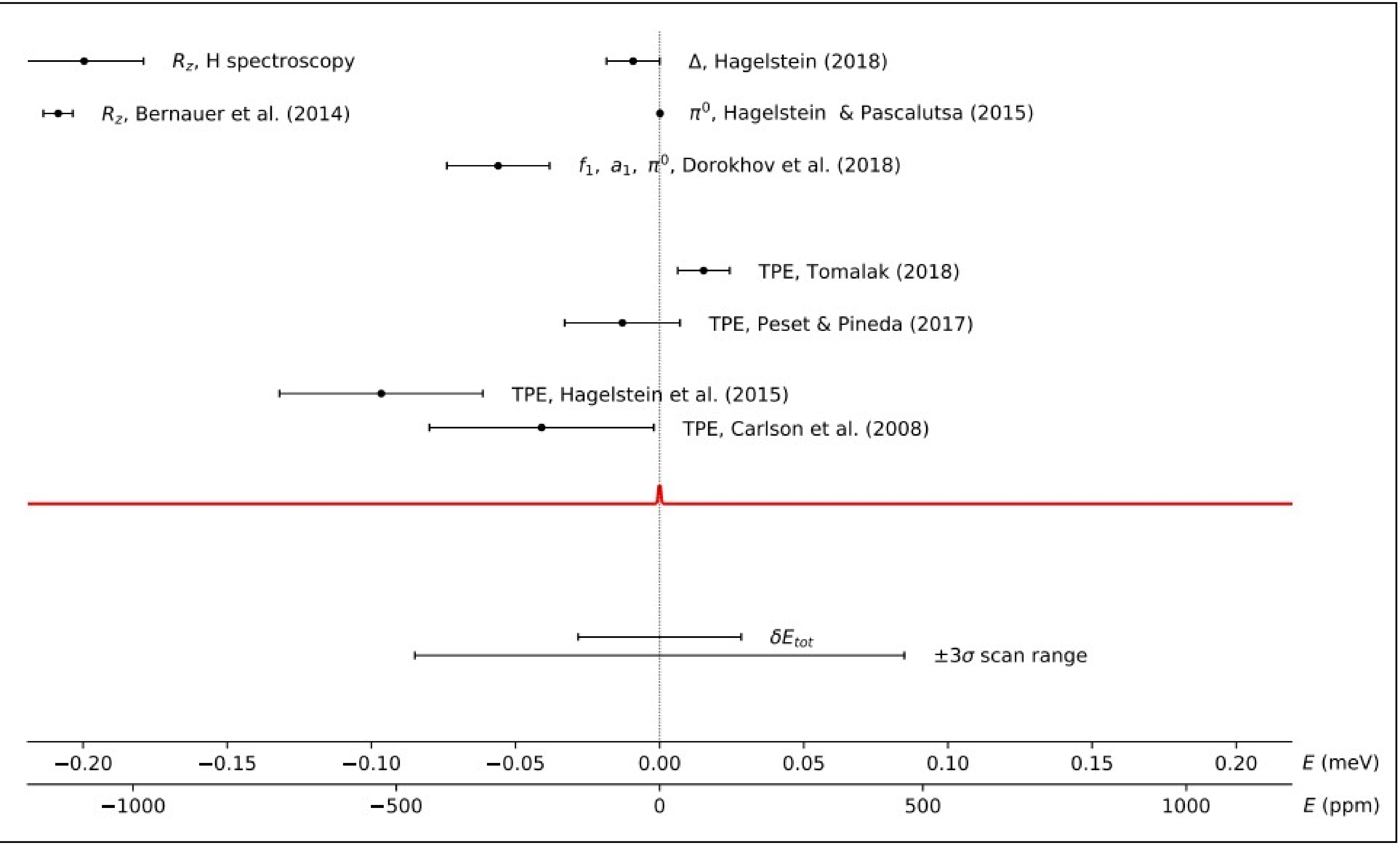


related propsals: FAMU at RIKEN/RAL, muonic H at J-PARC

Predicting the resonance position

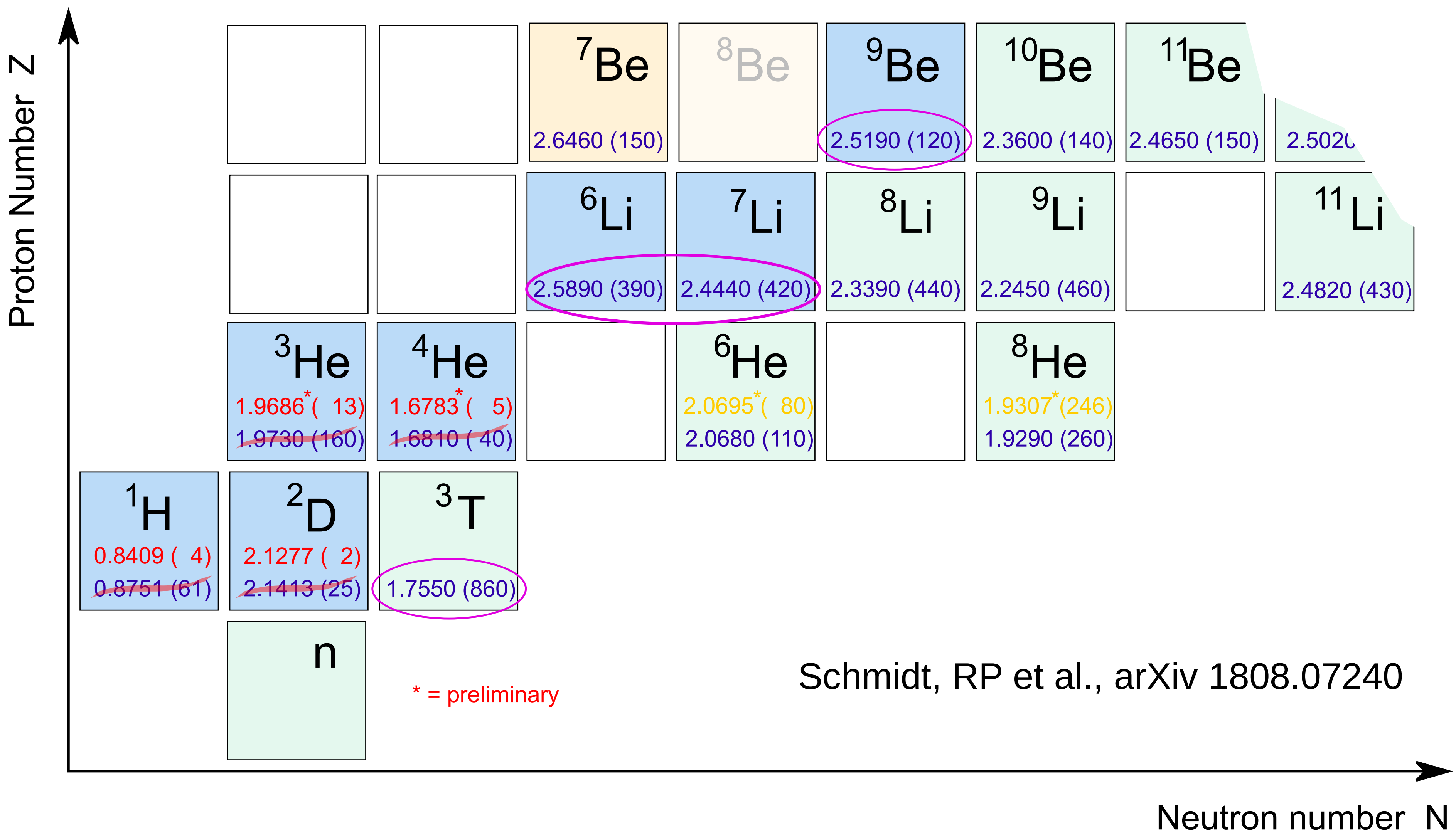


The resonance position

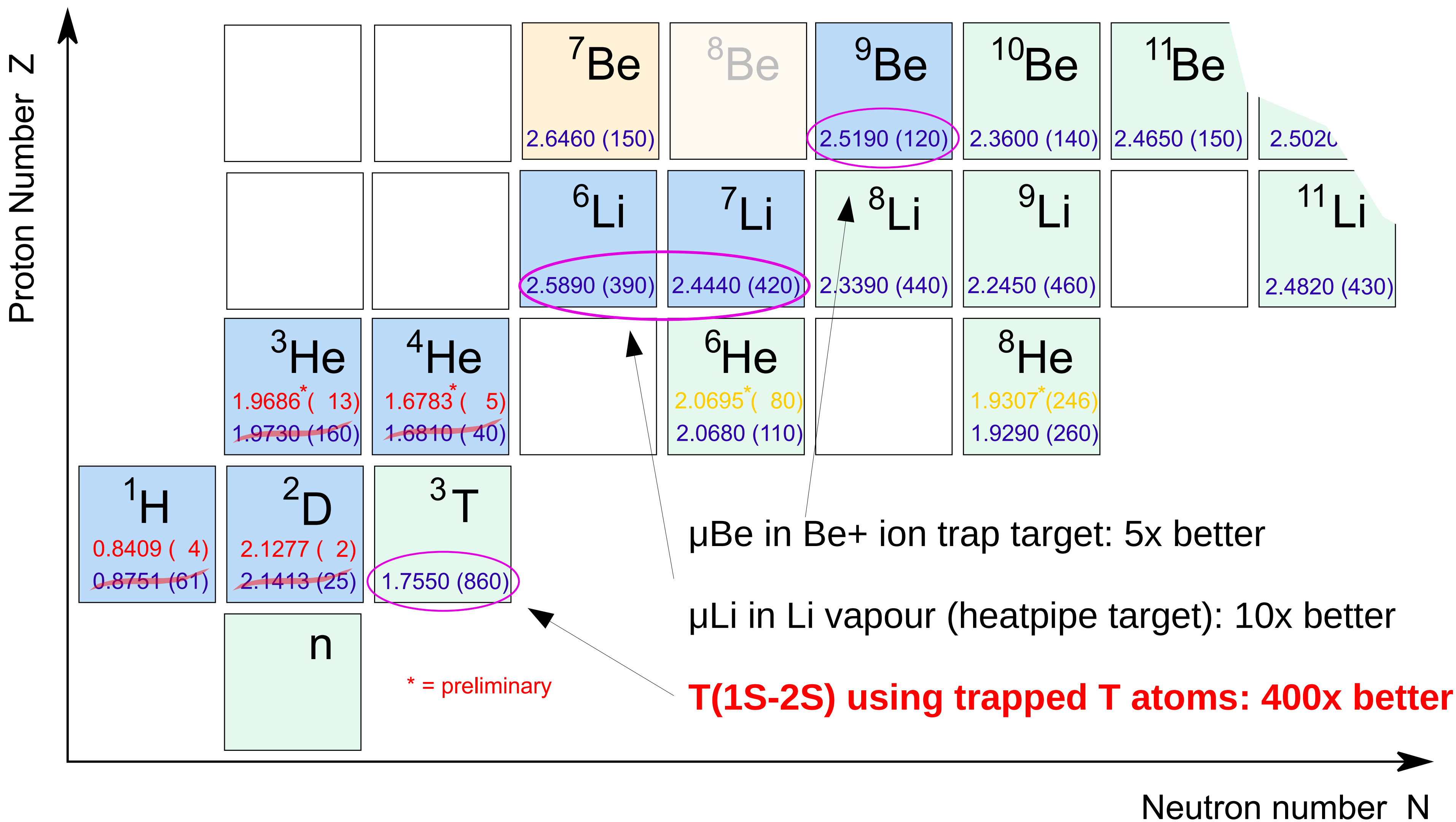


The Future

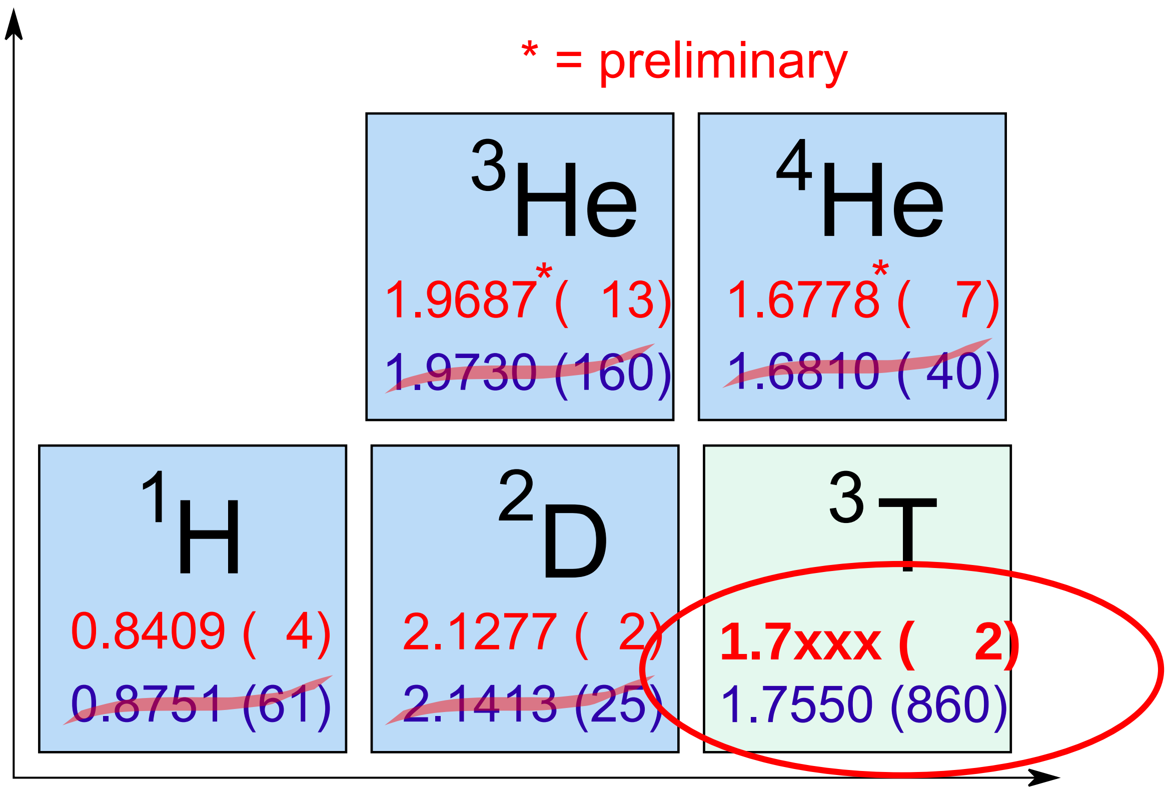
Charge radii: The future



Charge radii: The future

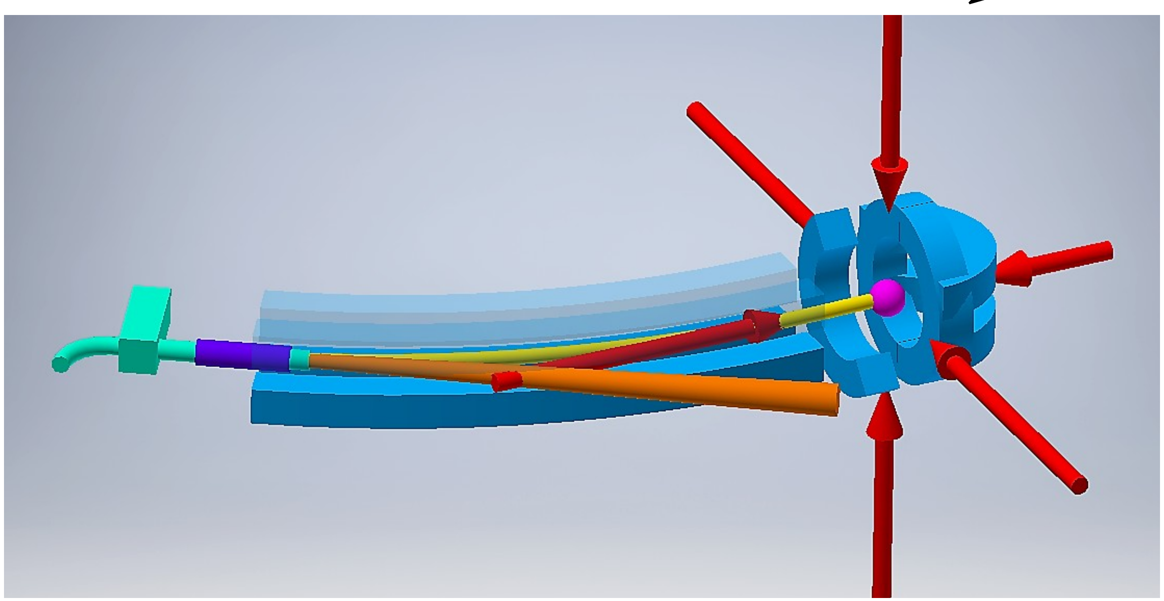


Tritium 1S-2S in a trap



Triton-Radius EXperiment
Mainz

400x better radius with 1 kHz measurement
(vs. 0.01 kHz for H, D)



- cryogenic H nozzle (4.2K)
- magnetic quadrupole guide
- Li MOT -> cold buffer gas
- magnetic trapping of H/D/T

Thanks a lot for your attention

My new Mainz group:

Konrad Franz, Lukas Görner, Jan Haack, Merten Heppener, Rishi Horn, Jonas Klingelhöfer, Ahmed Ouf, Gregor Schwendler, Lukas Schumacher, Hendrik Schürg, Benedikt Tscharn, Marcel Willig

The Garching Hydrogen Team:

Axel Beyer, Lothar Maisenbacher, Arthur Matveev, RP, Ksenia Khabarova, Alexey Grinin, Tobias Lamour, Dylan C. Yost, Theodor W. Hänsch, Nikolai Kolachevsky, Thomas Udem

The CREMA Collaboration:

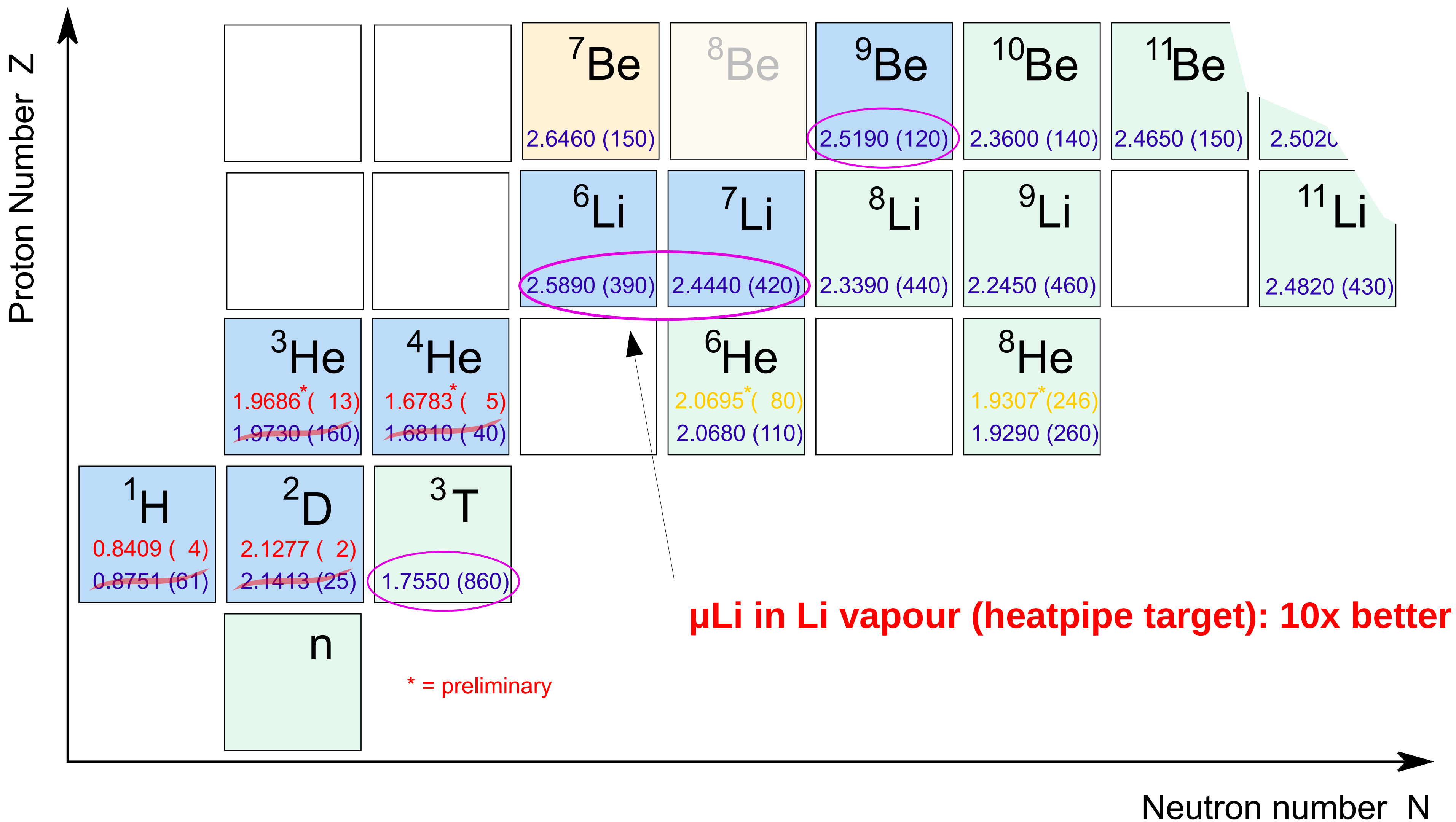
Aldo Antognini, Fernando D. Amaro, François Biraben, João M. R. Cardoso, Daniel S. Covita, Andreas Dax, Satish Dhawan, Marc Diepold, Luis M. P. Fernandes, Adolf Giesen, Andrea L. Gouvea, Thomas Graf, Theodor W. Hänsch, Paul Indelicato, Lucile Julien, Paul Knowles,Franz Kottmann, Juilian J. Krauth, Eric-Olivier Le Bigot, Yi-Wei Liu, José A. M. Lopes, Livia Ludhova, Cristina M. B. Monteiro, Françoise Mulhauser, Tobias Nebel, François Nez, Paul Rabinowitz, Joaquim M. F. dos Santos, Lukas A. Schaller, Karsten Schuhmann, Catherine Schwob, David Taqqu, João F. C. A. Veloso, RP

Group at JGU Mainz

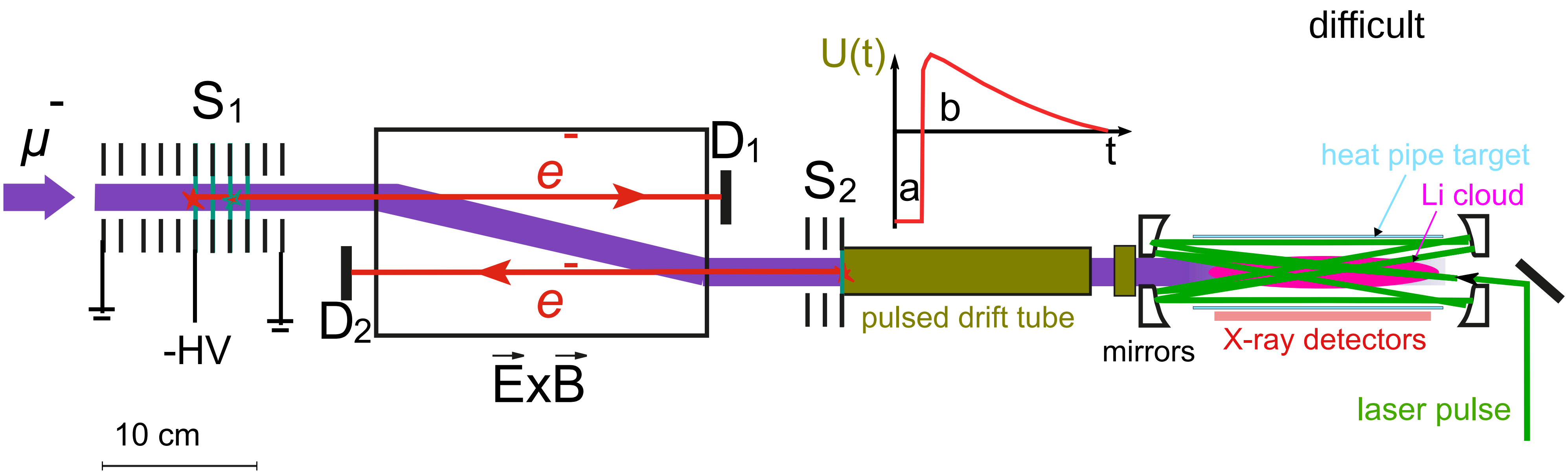
Group at JGU Mainz

...

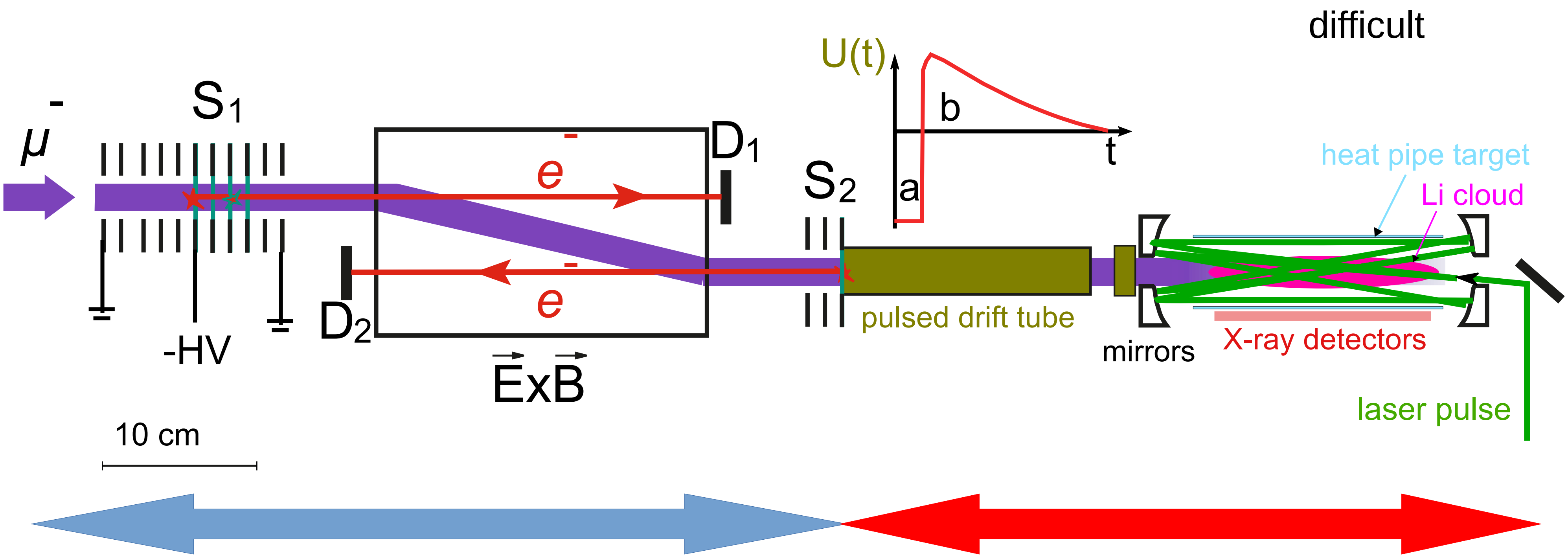
Charge radii: The future



muonic Li with heat pipe target



muonic Li with heat pipe target

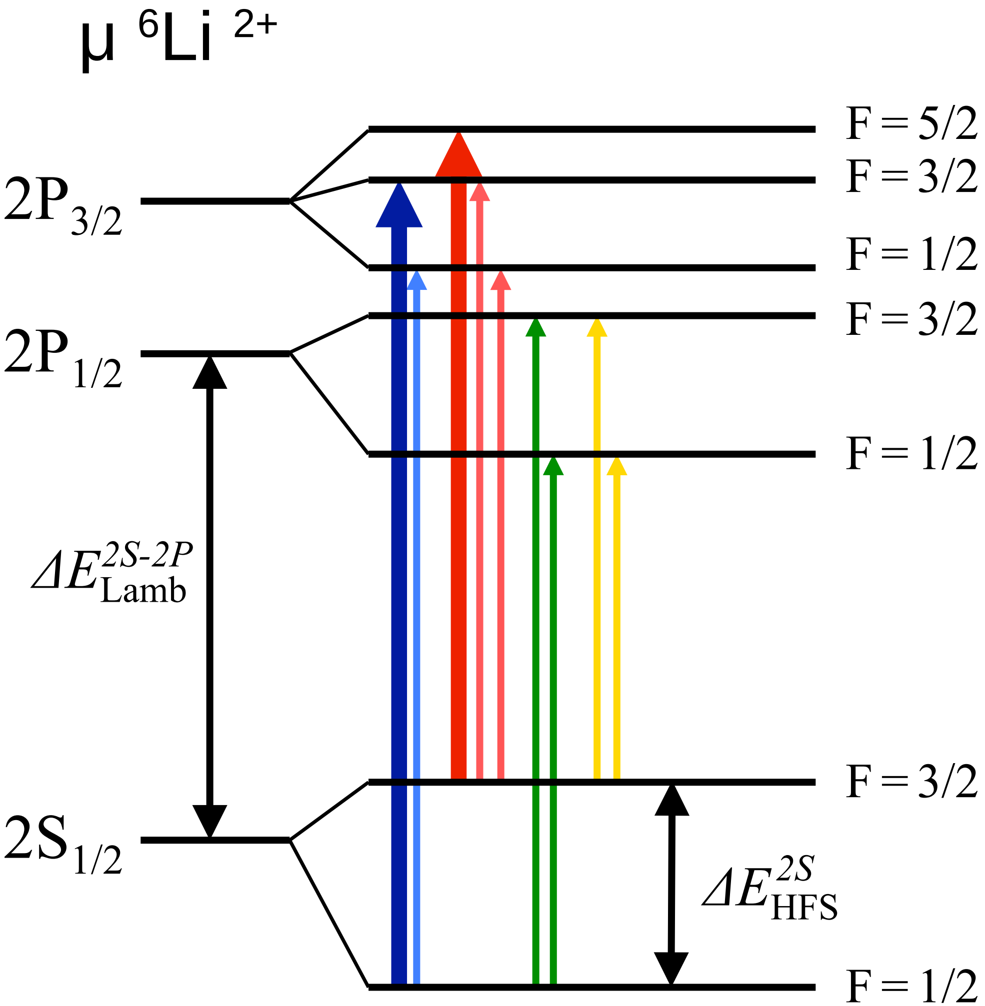


existing beam line:
1000/s at ~1 keV

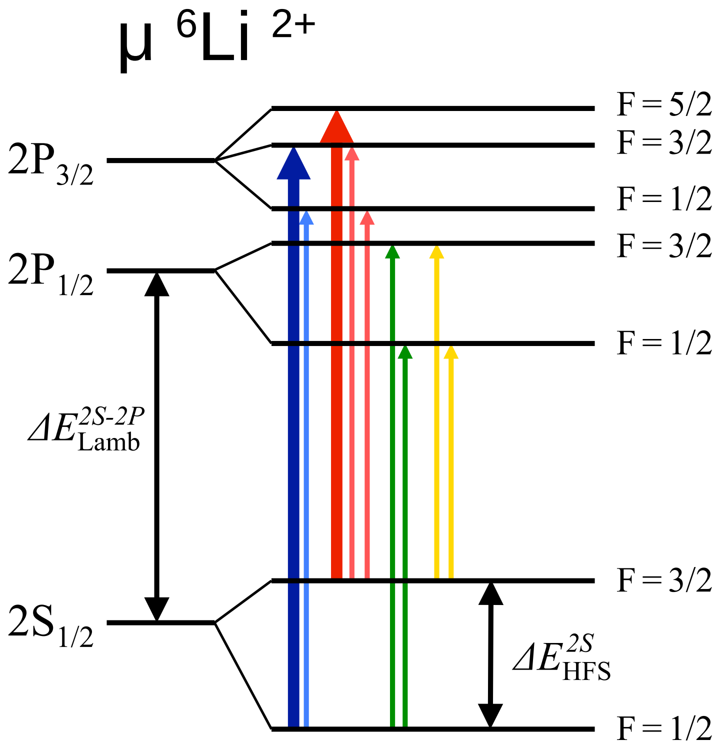
to be built:

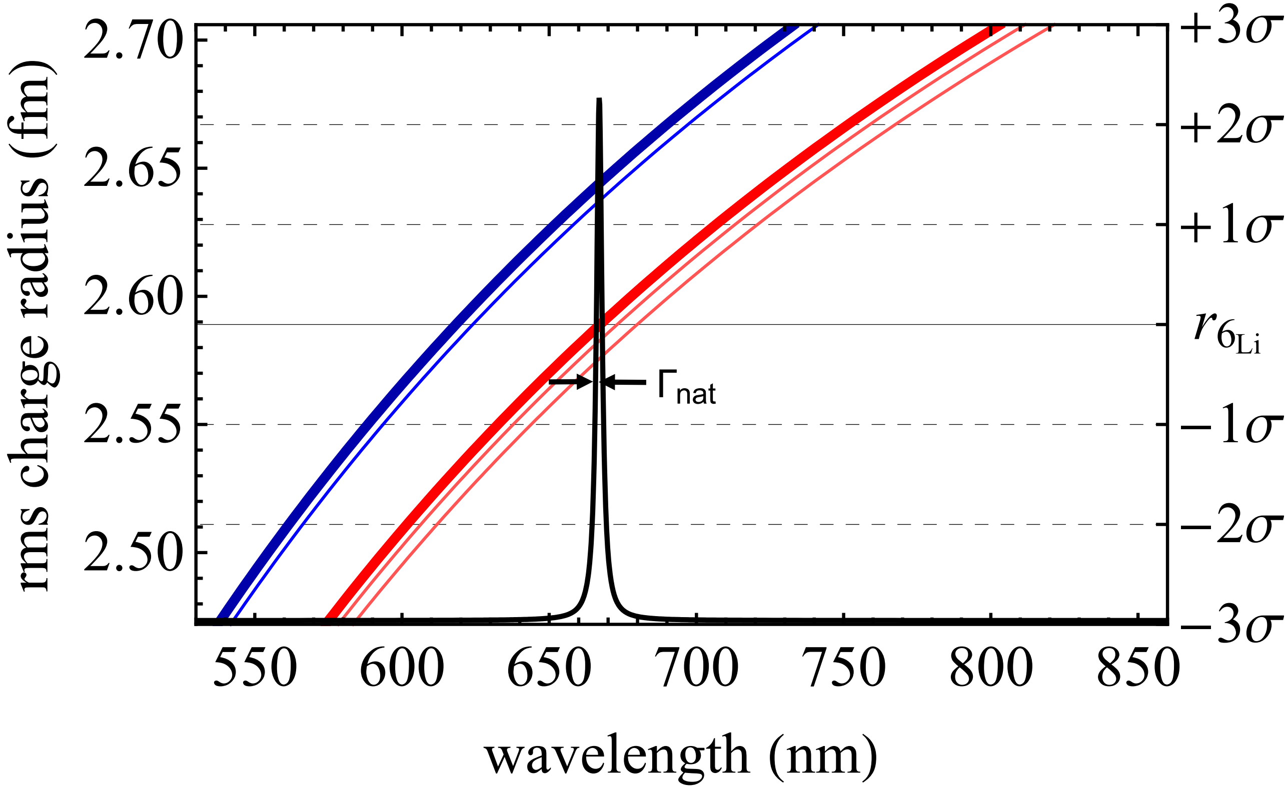
drift tube to 100 .. 10 eV
lasers
detectors (easy)

muonic Li with heat pipe target



muonic Li with heat pipe target





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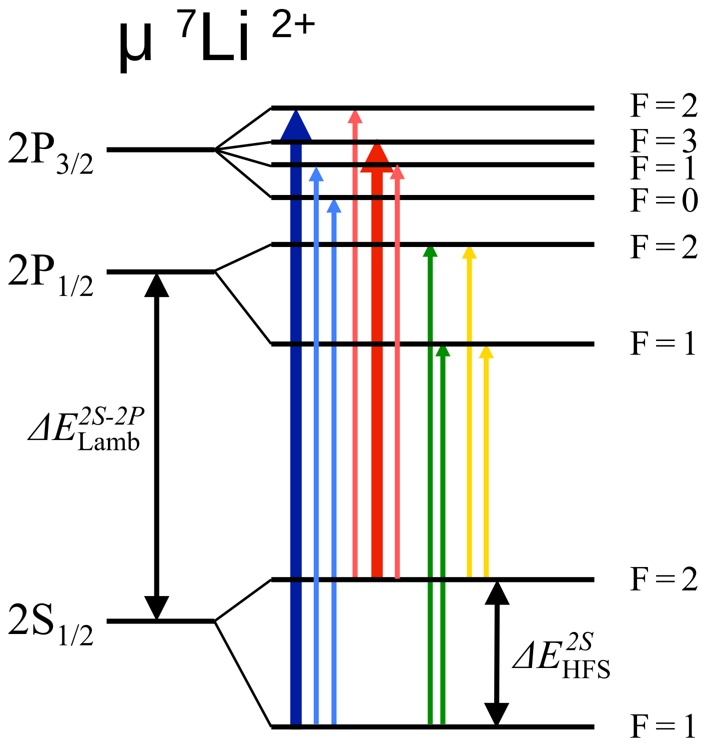
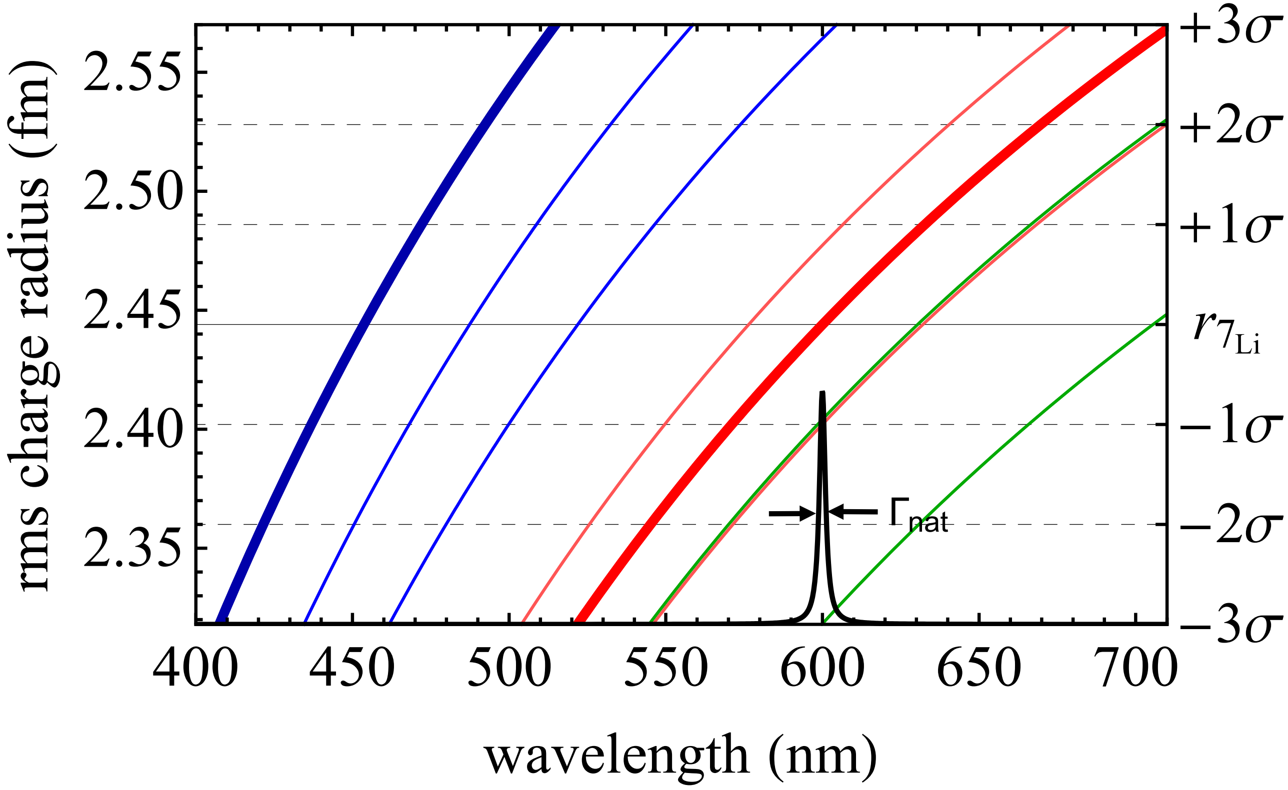




muonic Li: theory and accuracy

Item	$(\mu\,^6\mathrm{Li})^{2+}$	$(\mu\,^7\mathrm{Li})^{2+}$
QED Lamb shift [meV]	4654.4(0.1)	4671.4(0.1)
finite size [meV]	-3712(112)	-3335(117)
nucl. shape (Friar moment) [meV]	223(9)	191(9)
nucl. polarizability [meV]	15(4)	21(4)
total Lamb shift [meV]	1162(112)(10)	1532(117)(10)
experimental accuracy goal ($\Gamma/10$) [meV]	0.7	0.7
wavelengths ($\pm 3\sigma$ in charge radius)	575-800 nm	520-710 nm

line width Γ (nm/meV)	2.3 nm $\equiv 6.8\,$meV
K_α energy	18.7 keV
2S lifetime $\tau(2S)$	830 ns

$$\boldsymbol{r}(^6\mathbf{Li}) = 2.58900(\mathbf{3900})\ \mathrm{fm}\ [31] \quad \rightarrow \quad 2.58xxx(\ \mathbf{40})^{\mathrm{exp}}(\mathbf{400})^{\mathrm{th}}\ \mathrm{fm} \qquad (\mu\,^6\mathrm{Li})^{2+}$$

$$\boldsymbol{r}(^7\mathbf{Li}) = 2.44400(\mathbf{4200})\ \mathrm{fm}\ [31] \quad \rightarrow \quad 2.44xxx(\ \mathbf{40})^{\mathrm{exp}}(\mathbf{400})^{\mathrm{th}}\ \mathrm{fm} \qquad (\mu\,^7\mathrm{Li})^{2+}$$

exp: 100x better radius, but polarizability -> “only” 10x better

muonic Li and Li^+

Item	$(\mu\,^6\mathrm{Li})^{2+}$	$(\mu\,^7\mathrm{Li})^{2+}$
QED Lamb shift [meV]	4654.4(0.1)	4671.4(0.1)
finite size [meV]	-3712(112)	-3335(117)
nucl. shape (Friar moment) [meV]	223(9)	191(9)
nucl. polarizability [meV]	15(4)	21(4)
total Lamb shift [meV]	1162(112)(10)	1532(117)(10)
experimental accuracy goal ($\Gamma/10$) [meV]	0.7	0.7

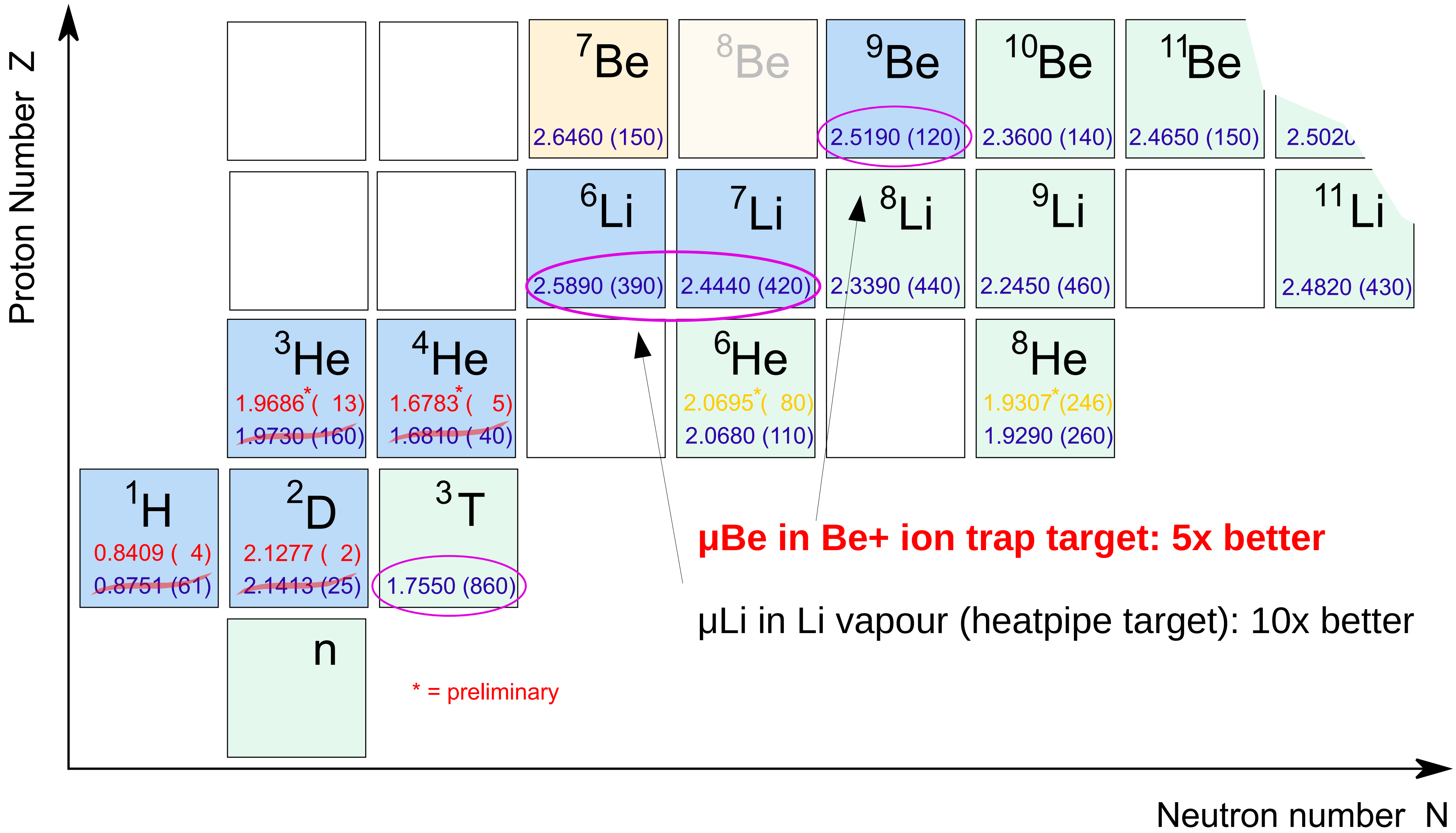
$$\boldsymbol{r(^6\mathrm{Li})} = 2.58900(\mathbf{3900})\ \mathrm{fm}\ [31] \quad \rightarrow \quad 2.58xxx(\ \mathbf{40})^{\mathrm{exp}}(\mathbf{400})^{\mathrm{th}}\ \mathrm{fm} \qquad (\mu\,^6\mathrm{Li})^{2+}$$

$$\boldsymbol{r(^7\mathrm{Li})} = 2.44400(\mathbf{4200})\ \mathrm{fm}\ [31] \quad \rightarrow \quad 2.44xxx(\ \mathbf{40})^{\mathrm{exp}}(\mathbf{400})^{\mathrm{th}}\ \mathrm{fm} \qquad (\mu\,^7\mathrm{Li})^{2+}$$

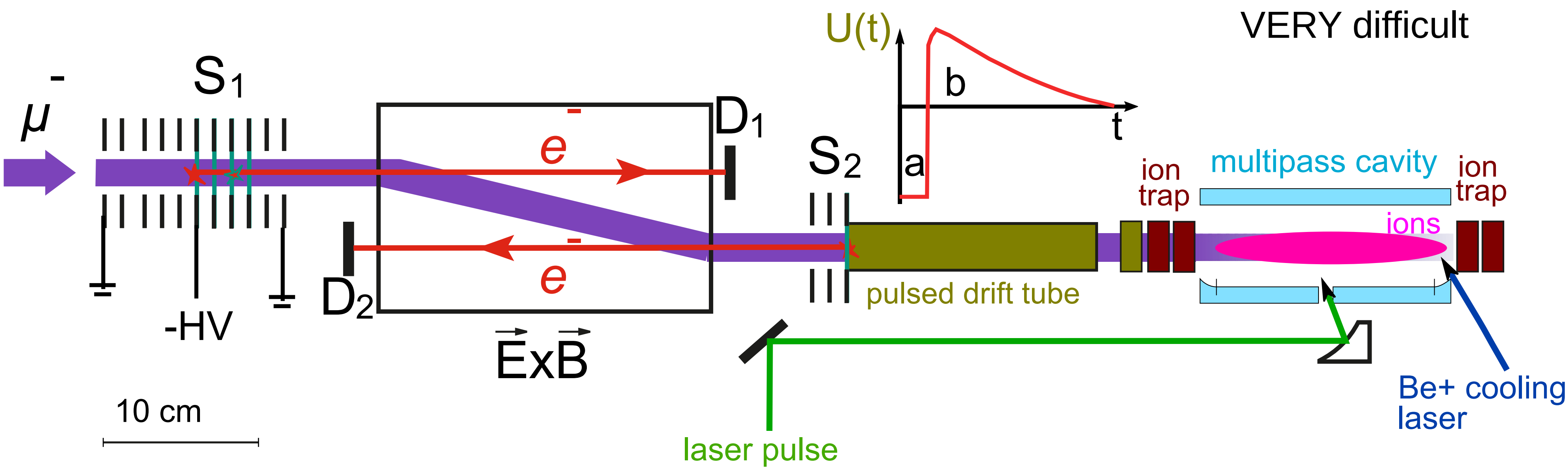
when combined with normal Li^+ (Th. Udem)

-> QED test: He/μHe vs. Li^+/μLi and
H/μH vs. He^+/μHe
Rydberg constant H, He, Li, ...
100x better radius AND 10x better polariability,

Charge radii: The future

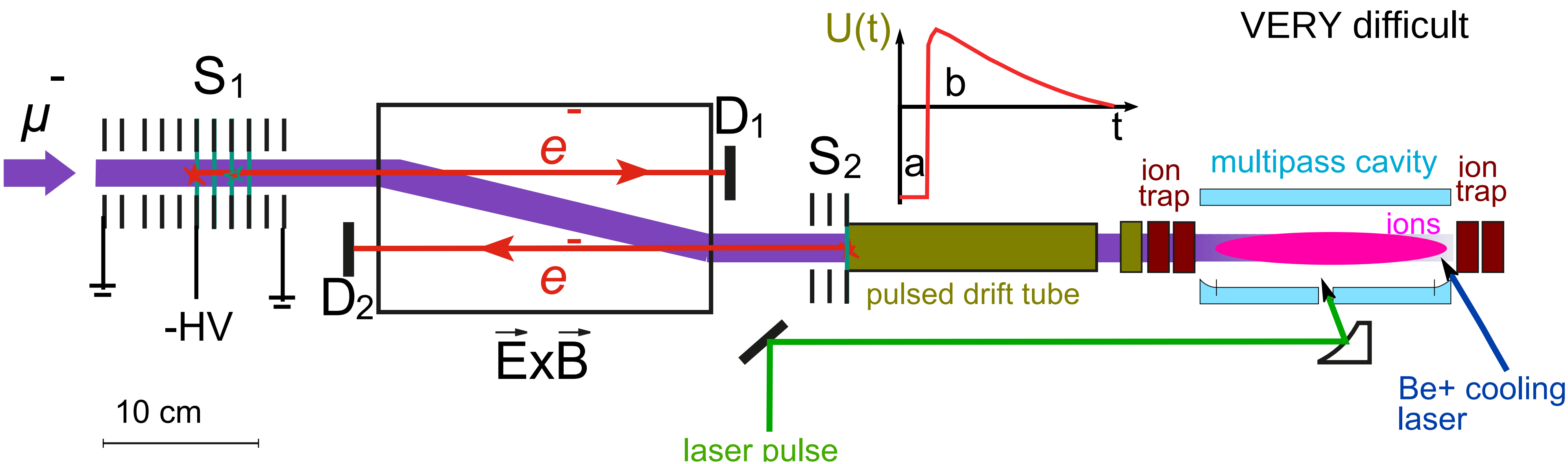


muonic Be with Penning trap target



simulations: μBe in Penning trap could maybe work, if muon energy <1eV

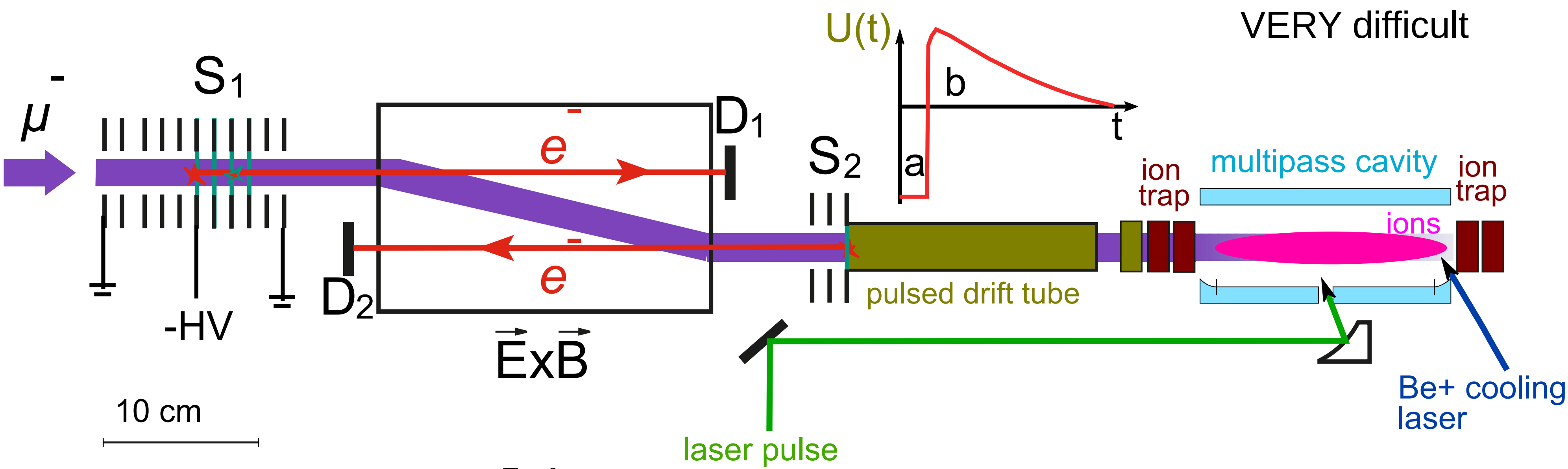
muonic Be with Penning trap target

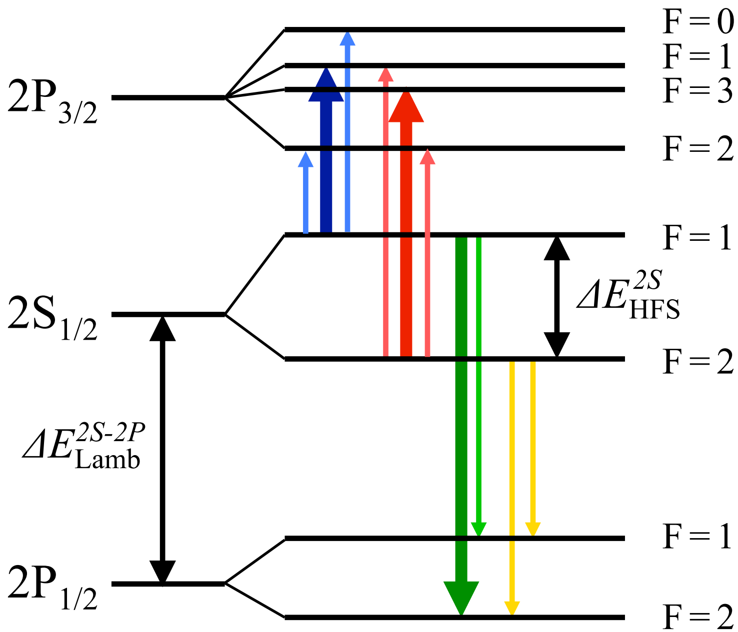


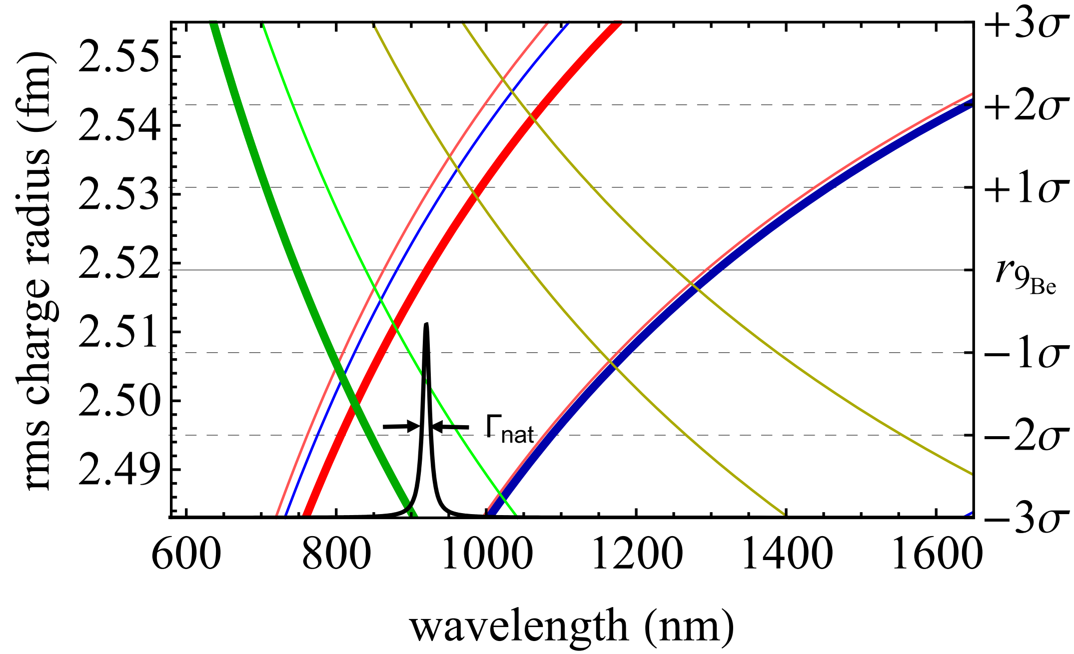
Item	$(\mu\,^6\mathrm{Li})^{2+}$	$(\mu\,^7\mathrm{Li})^{2+}$	$(\mu\,^9\mathrm{Be})^{3+}$
QED Lamb shift [meV]	4654.4(0.1)	4671.4(0.1)	9192.2(0.1)
finite size [meV]	-3712(112)	-3335(117)	-11329(107)
nucl. shape (Friar moment) [meV]	223(9)	191(9)	891(12)
nucl. polarizability [meV]	15(4)	21(4)	82(16)
total Lamb shift [meV]	1162(112)(10)	1532(117)(10)	-1252(107)(20)
experimental accuracy goal (Γ/10) [meV]	0.7	0.7	2
wavelengths ($\pm 3\sigma$ in charge radius)	575-800 nm	520-710 nm	755-1000 nm or 640-910 nm
line width Γ (nm/meV)	2.3 nm ≡ 6.8 meV		11 nm ≡ 21.5 meV
K_α energy	18.7 keV		33.4 keV
2S lifetime $\tau(2S)$	830 ns		150 ns

muonic Be with Penning trap target

VERY difficult





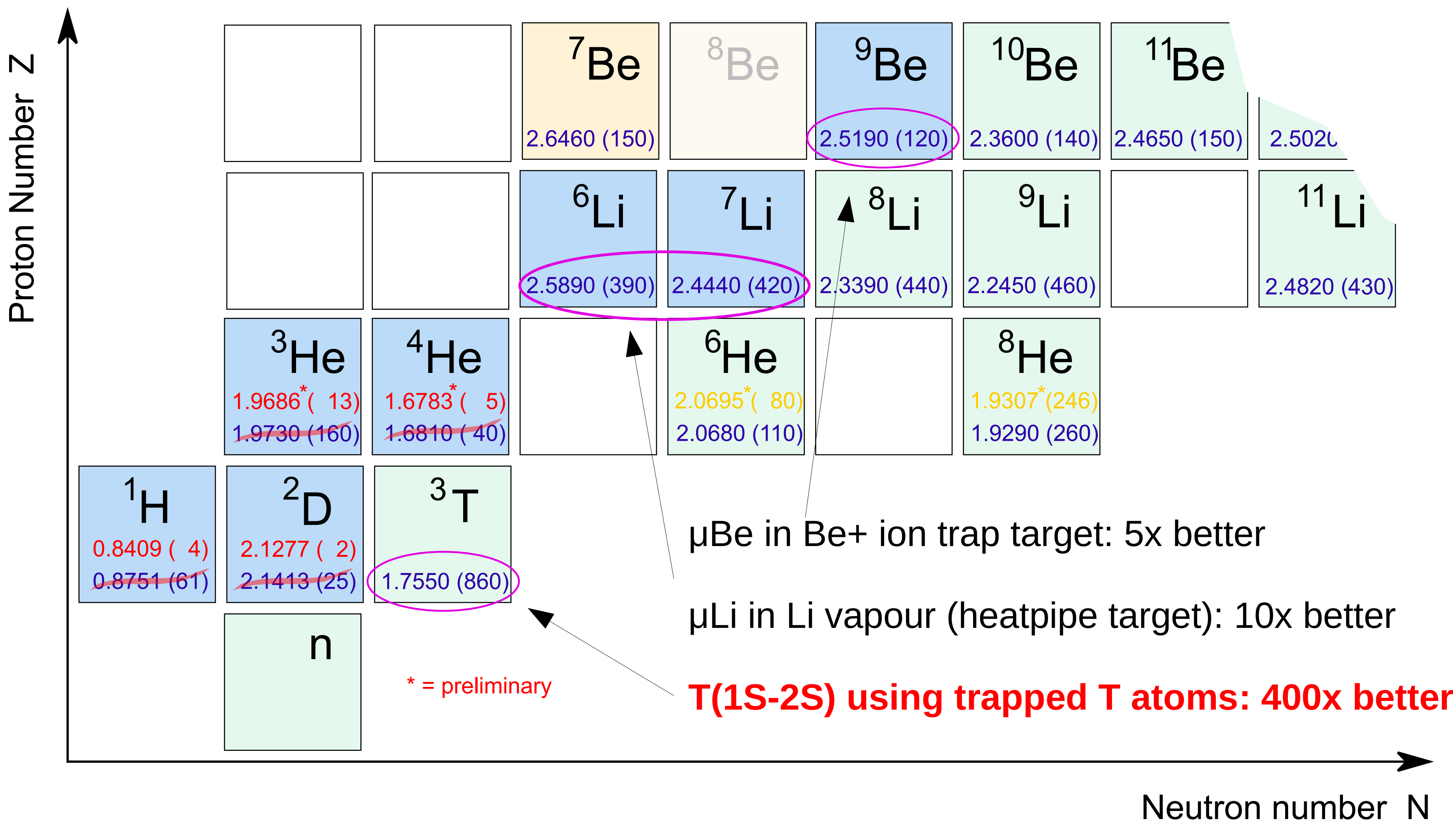


muonic Be with Penning trap target

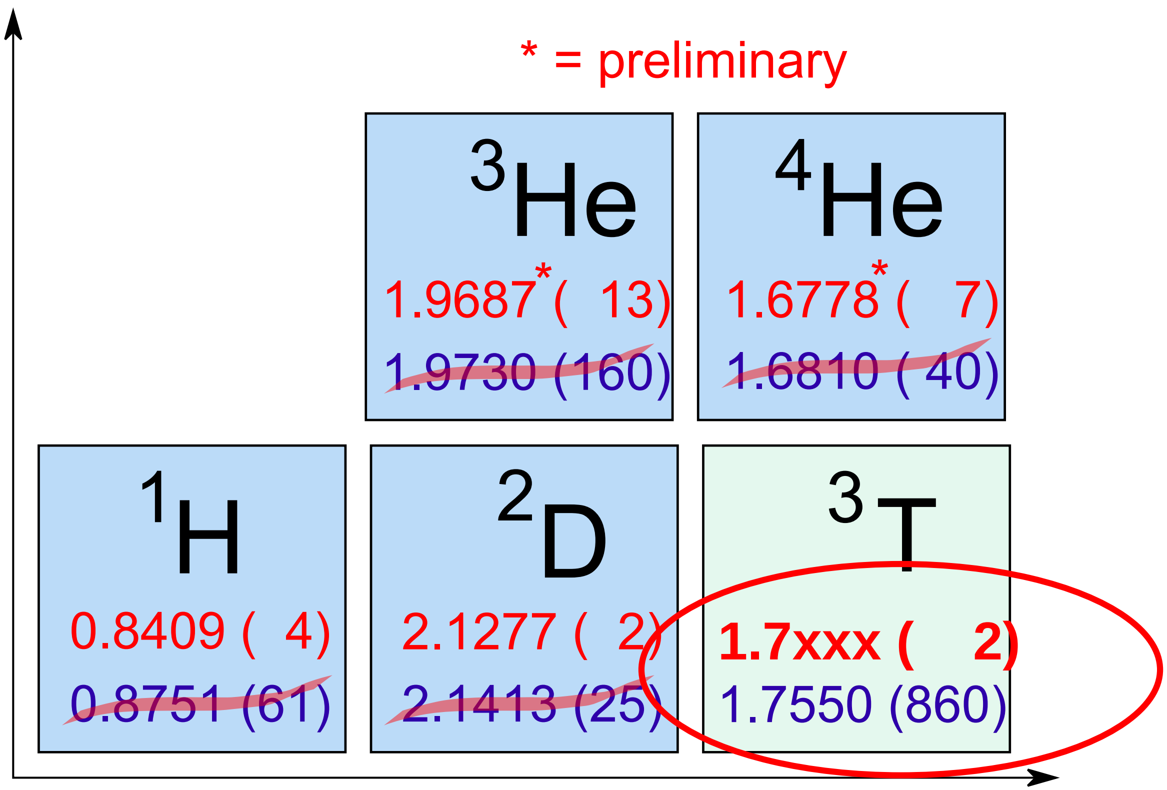
VERY difficult

$\boldsymbol{r(^6\mathrm{Li})} = 2.58900(\mathbf{3900})$ fm [31]	$\rightarrow$	$2.58xxx(\,\mathbf{40})^{\mathrm{exp}}(\mathbf{400})^{\mathrm{th}}$ fm	$(\mu\,^6\mathrm{Li})^{2+}$	
$\boldsymbol{r(^7\mathrm{Li})} = 2.44400(\mathbf{4200})$ fm [31]	$\rightarrow$	$2.44xxx(\,\mathbf{40})^{\mathrm{exp}}(\mathbf{400})^{\mathrm{th}}$ fm	$(\mu\,^7\mathrm{Li})^{2+}$	
$\boldsymbol{r(^9\mathrm{Be})} = 2.51900(\mathbf{1200})$ fm [31]	$\rightarrow$	$2.51xxx(\,\mathbf{25})^{\mathrm{exp}}(\mathbf{230})^{\mathrm{th}}$ fm	$(\mu\,^9\mathrm{Be})^{3+}$	
$r(p) = 0.87510(\;610)$ fm [65]	$\rightarrow$	$0.84087(\;26)^{\mathrm{exp}}(\;29)^{\mathrm{th}}$ fm	μp	[3]
$r(d) = 2.14130(\;250)$ fm [65]	$\rightarrow$	$2.12562(\;13)^{\mathrm{exp}}(\;77\;)^{\mathrm{th}}$ fm	μd	[4]
	$\rightarrow$	$2.12771(\;22)$ fm	μp + H/D iso	[3,
$r(^3\mathrm{He}) = 1.97300(1400)$ fm [68]	$\rightarrow$	$1.96866(\;12)^{\mathrm{exp}}(128)^{\mathrm{th}}$ fm	$(\mu\,^3\mathrm{He})^+$ prel.	[66,
$r(^4\mathrm{He}) = 1.68100(\;400)$ fm [69]	$\rightarrow$	$1.67779(\;13)^{\mathrm{exp}}(\;71)^{\mathrm{th}}$ fm	$(\mu\,^4\mathrm{He})^+$ prel.	[67,

Charge radii: The future



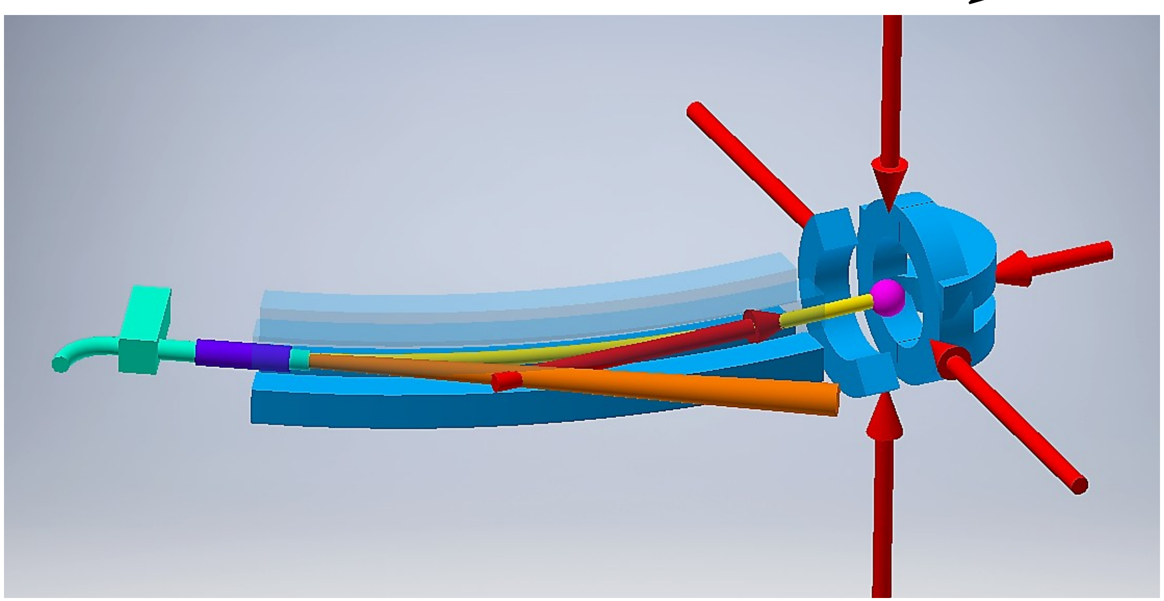
Tritium 1S-2S in a trap



Triton-Radius EXperiment
Mainz

400x better radius with 1 kHz measurement
(vs. 0.01 kHz for H, D)

- cryogenic H nozzle (4.2K)
- magnetic quadrupole guide
- Li MOT -> cold buffer gas
- magnetic trapping of H/D/T



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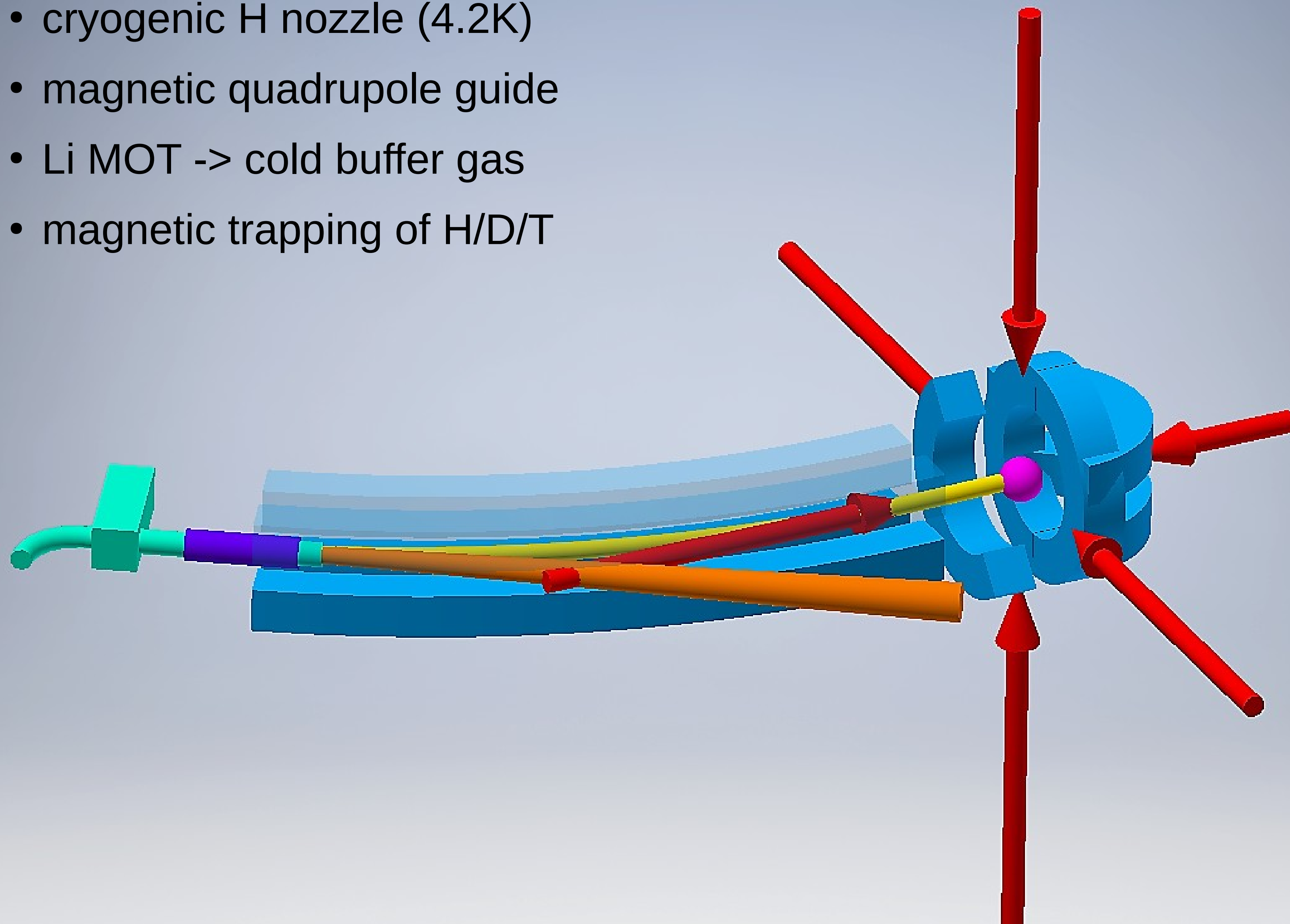
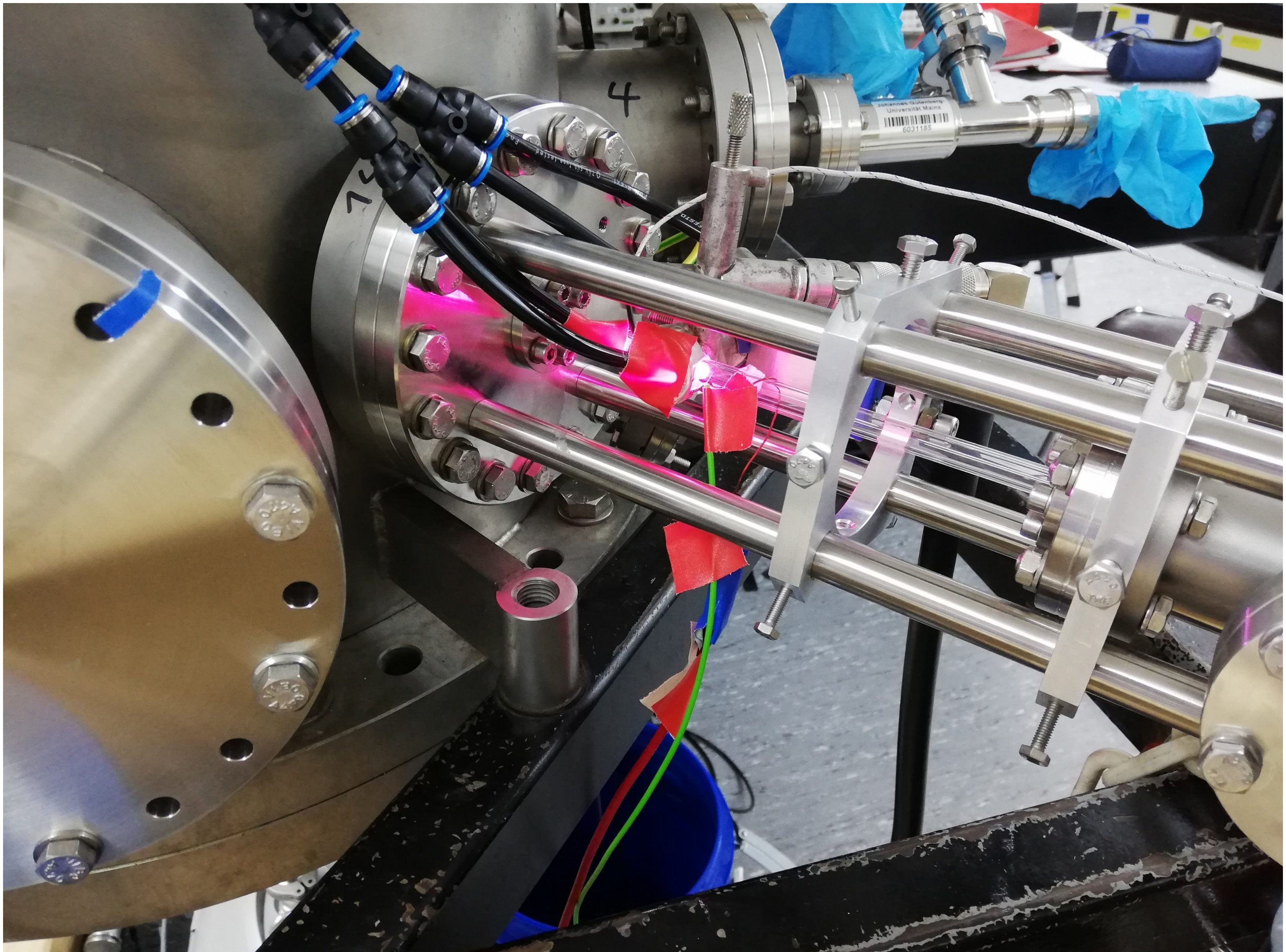
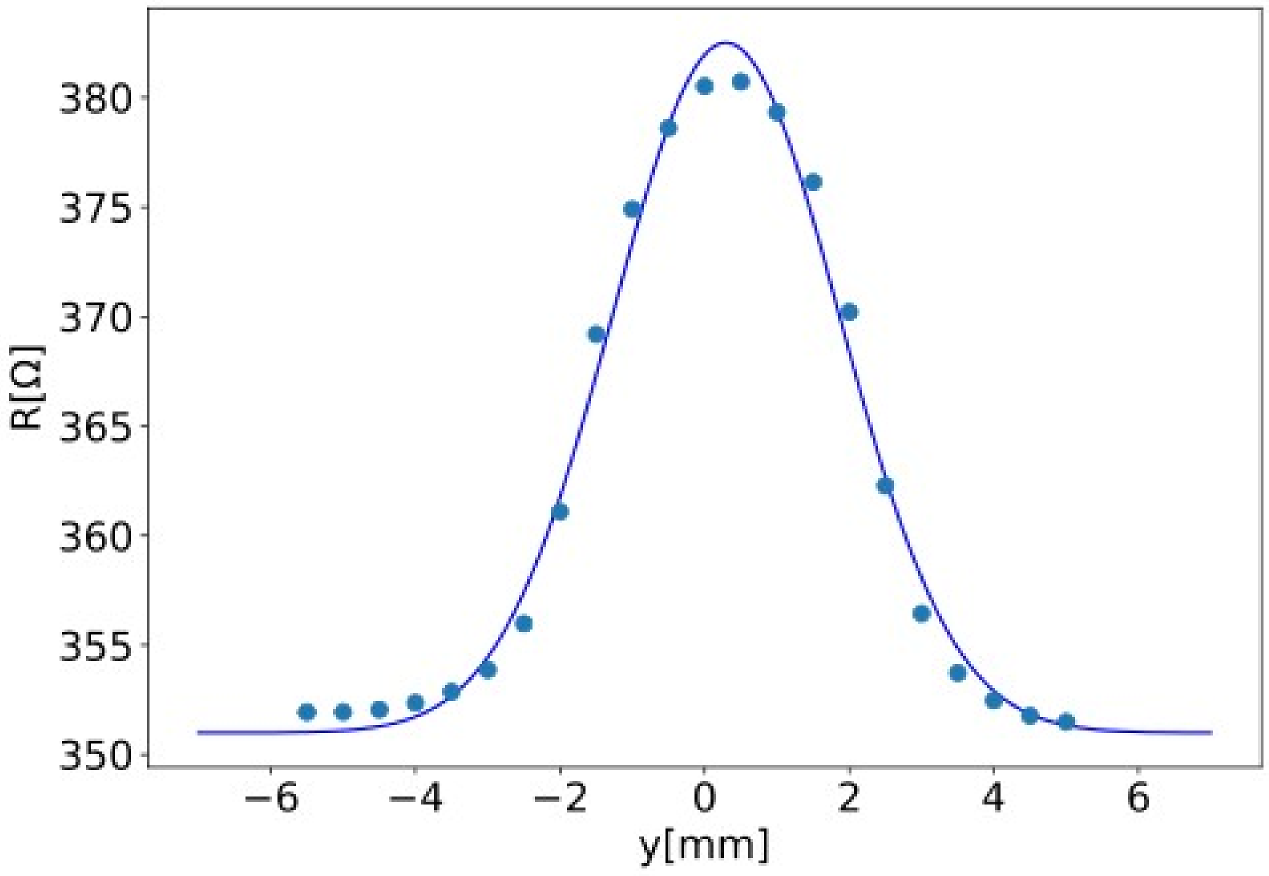


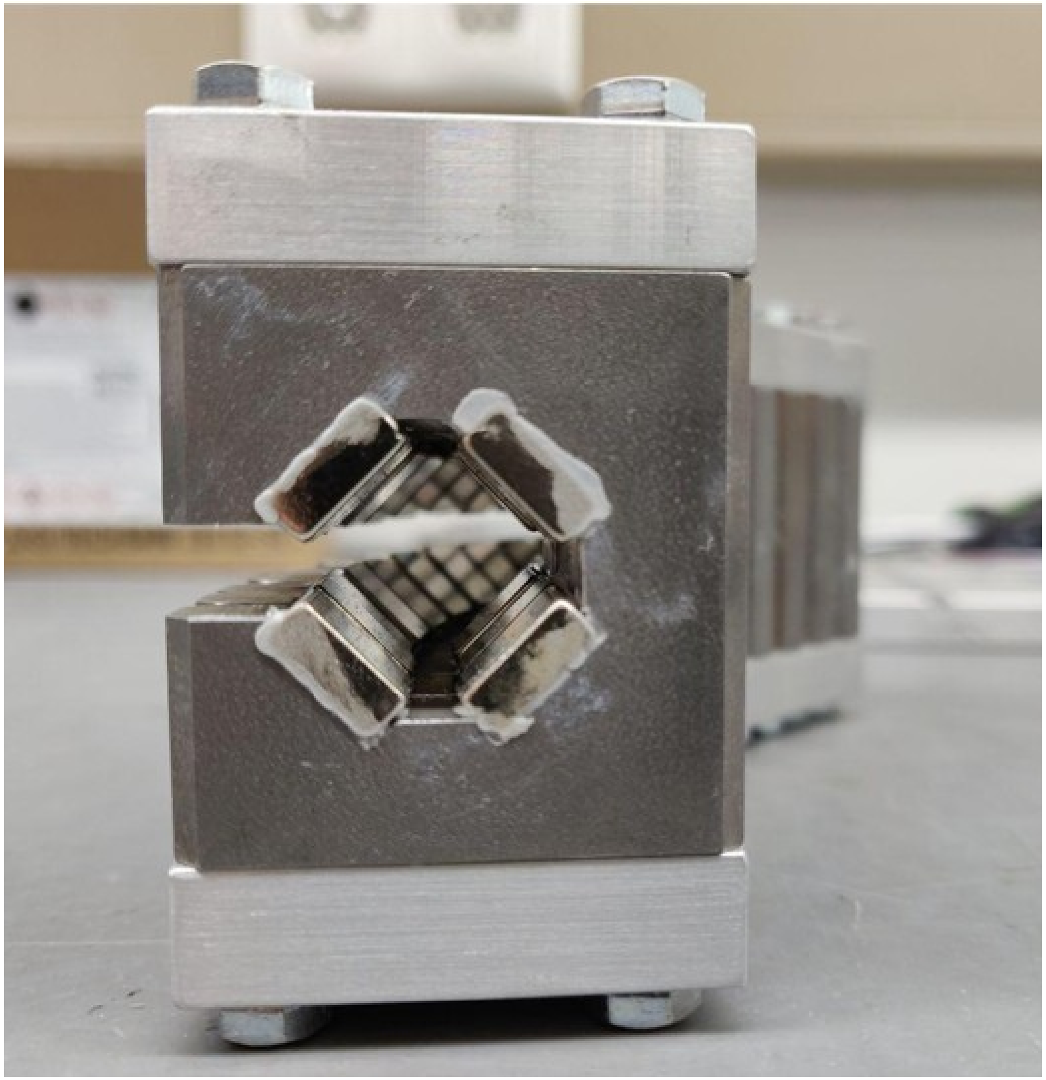
Atomic H source



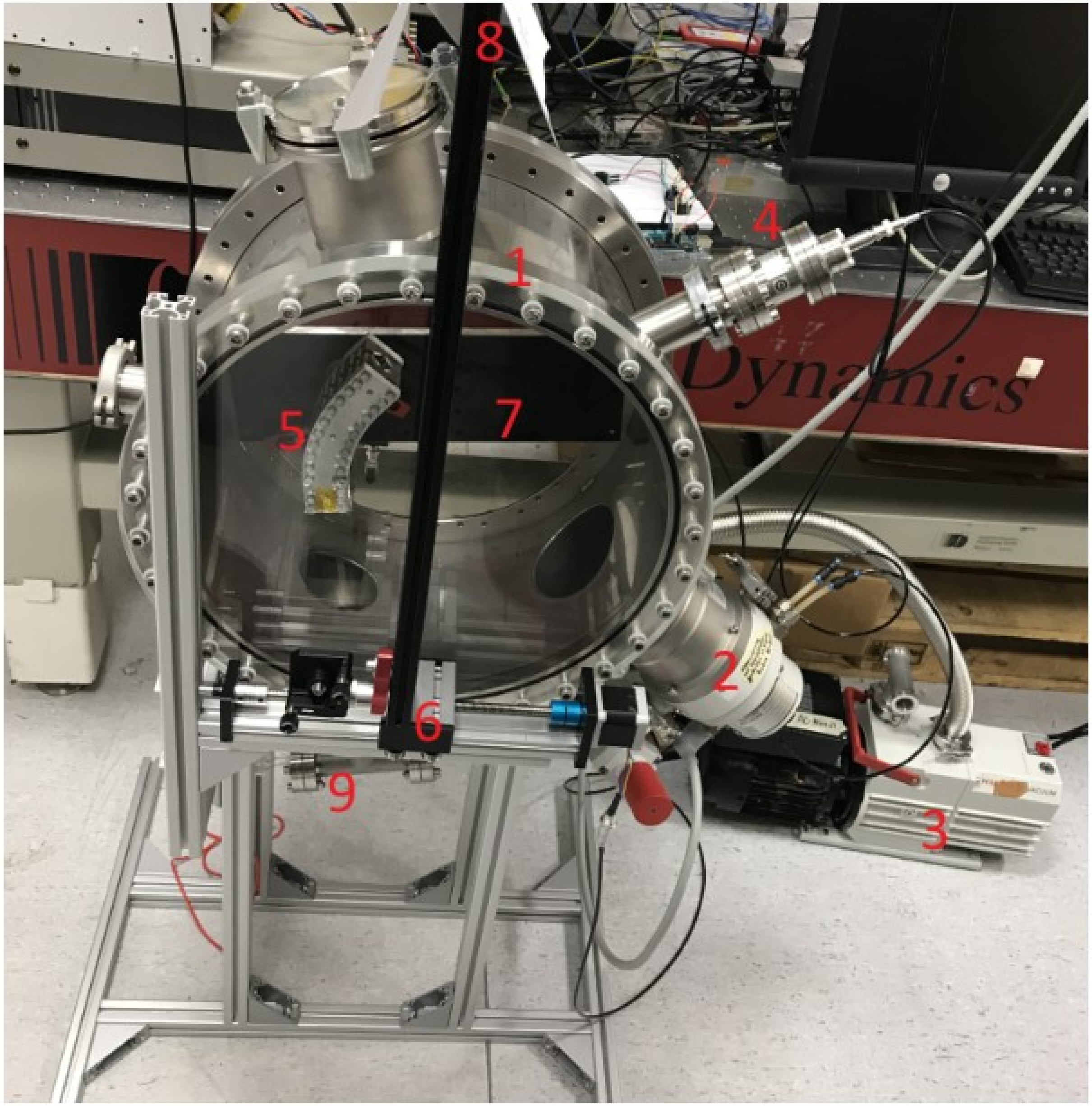
H beam profile



Magnetic quadrupole guide



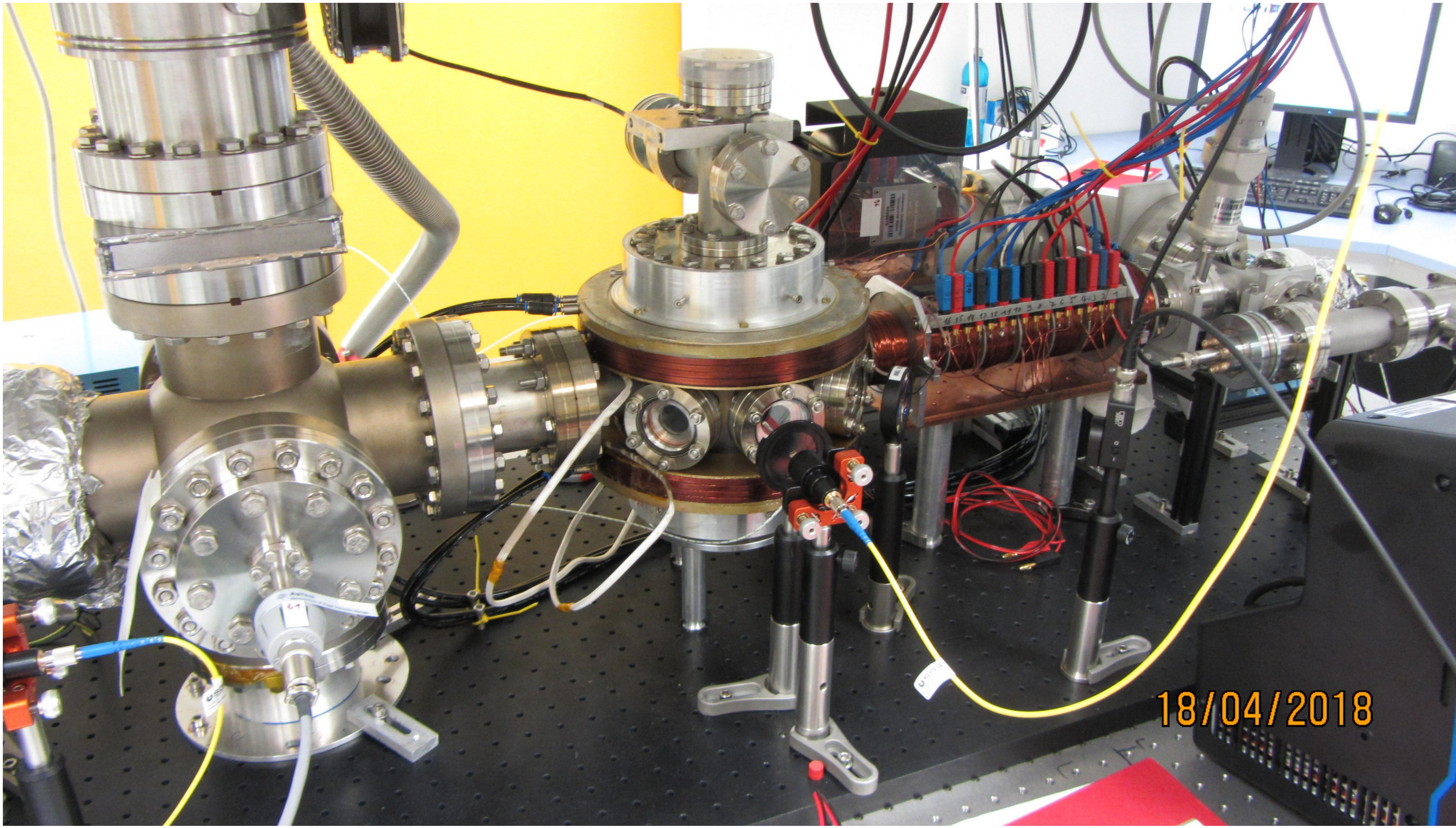
Towards progress in guiding



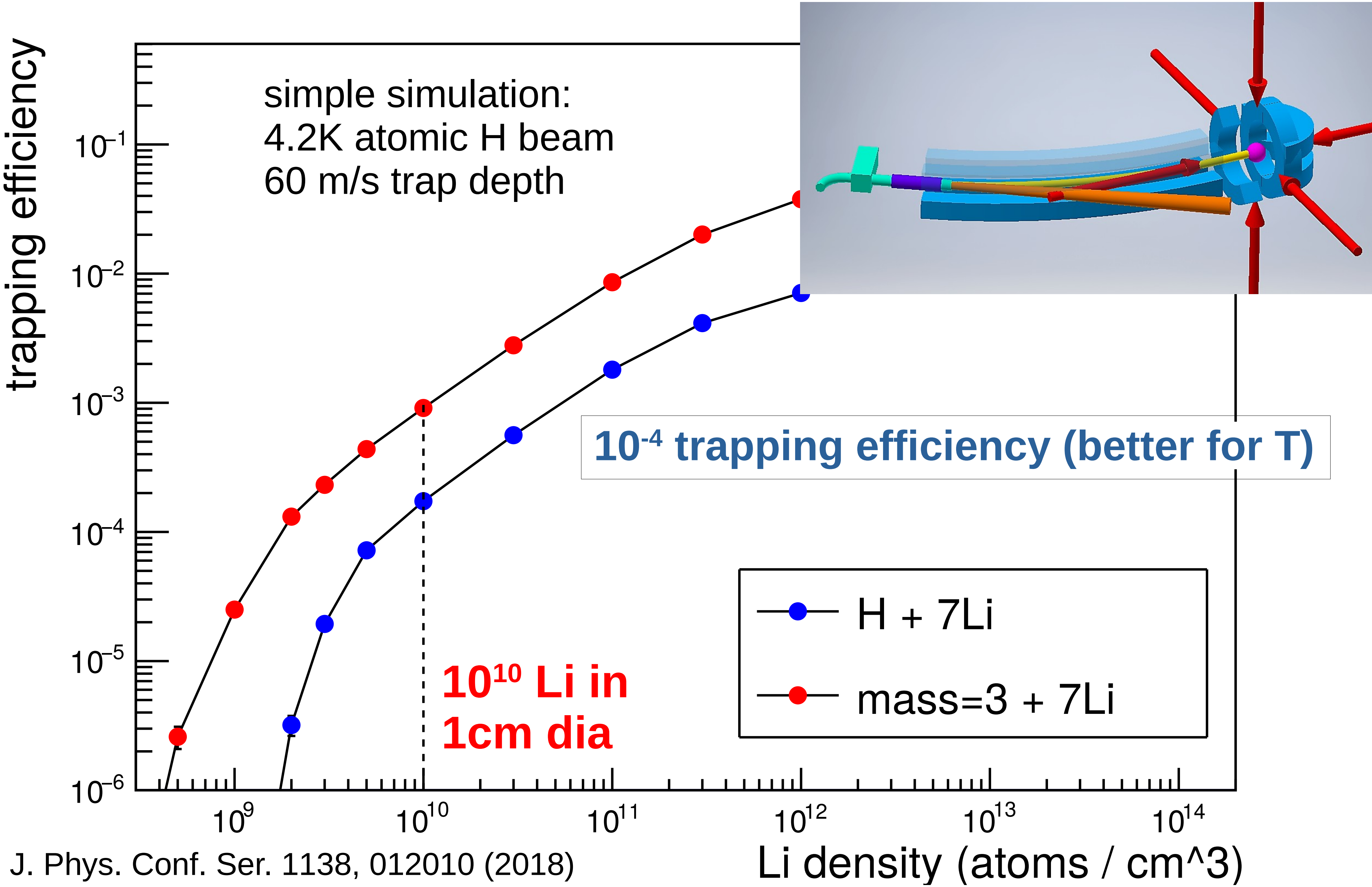
Rb source,

no guiding yet.

Li MOT apparatus

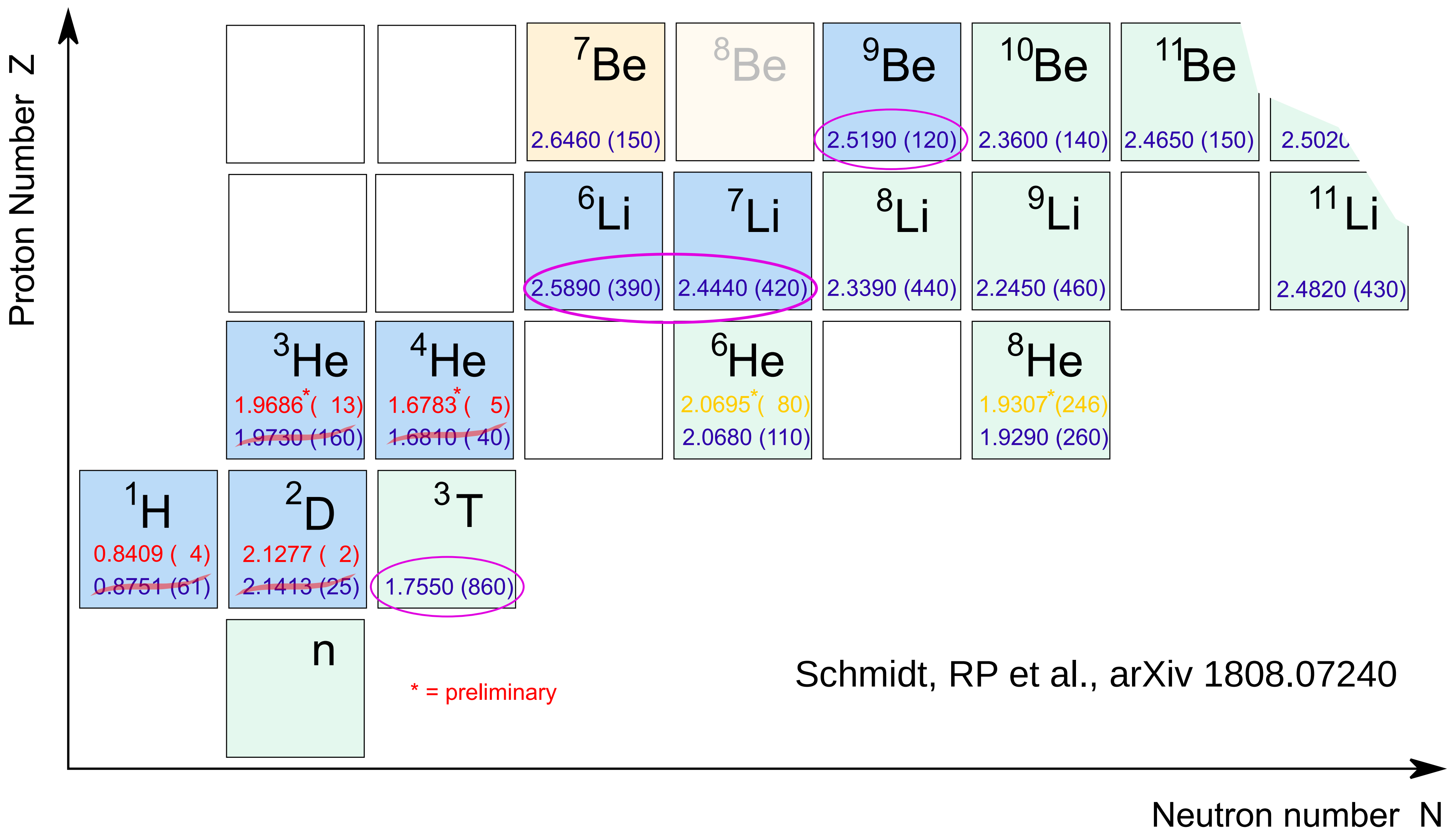


Simulated trapping efficiency



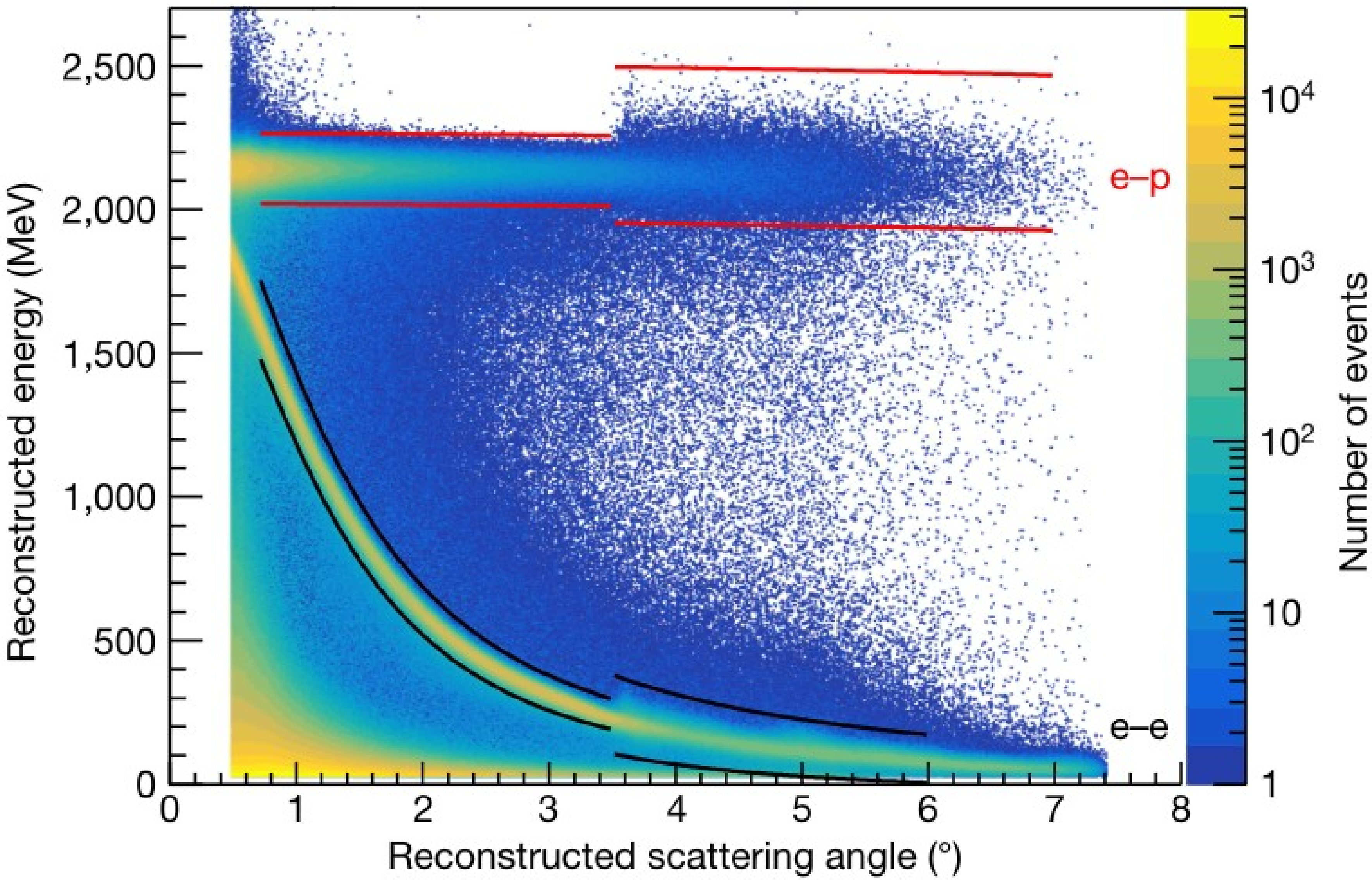
J. Phys. Conf. Ser. 1138, 012010 (2018)

Charge radii: The future



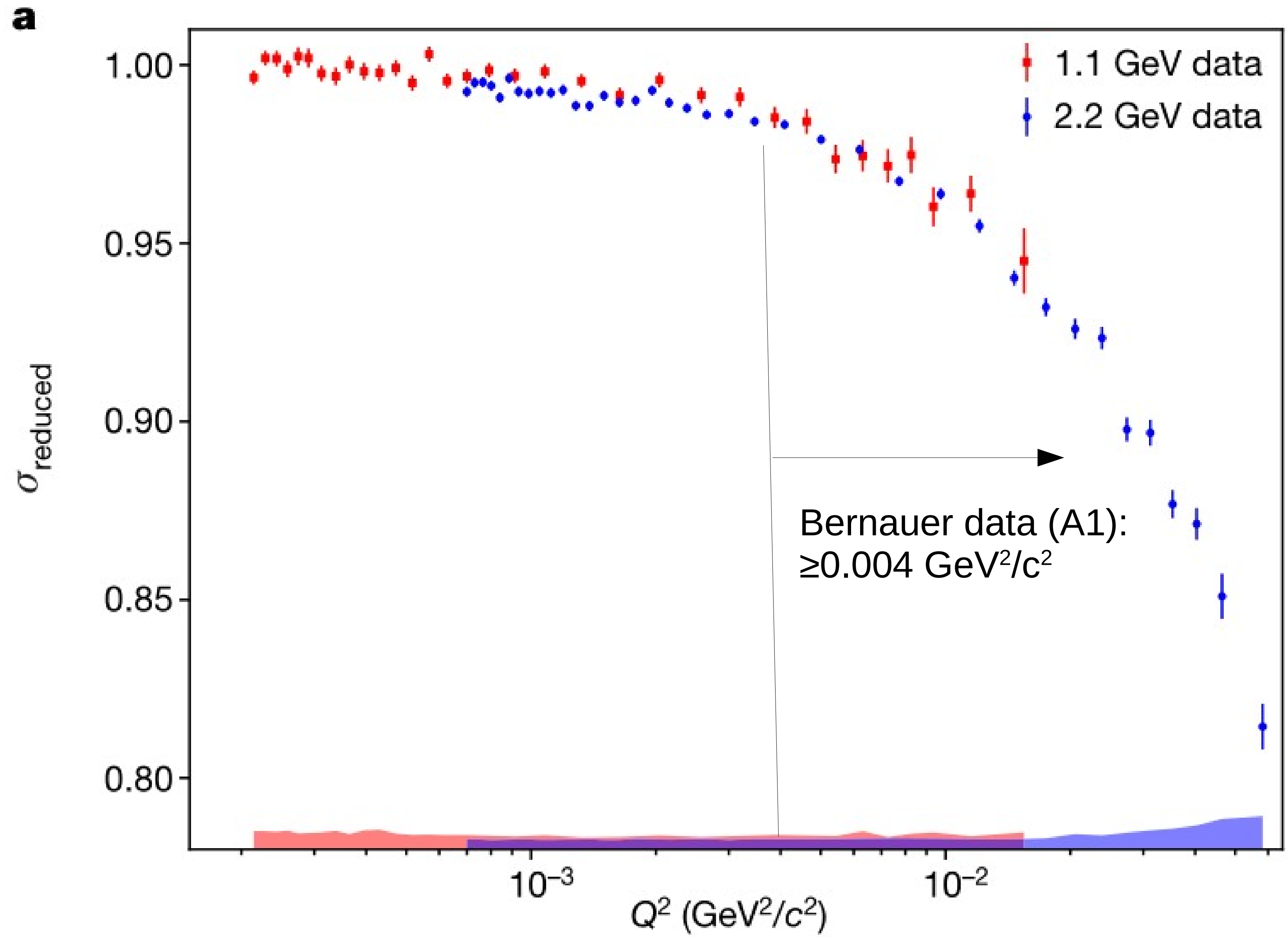
Schmidt, RP et al., arXiv 1808.07240

e-p and e-e



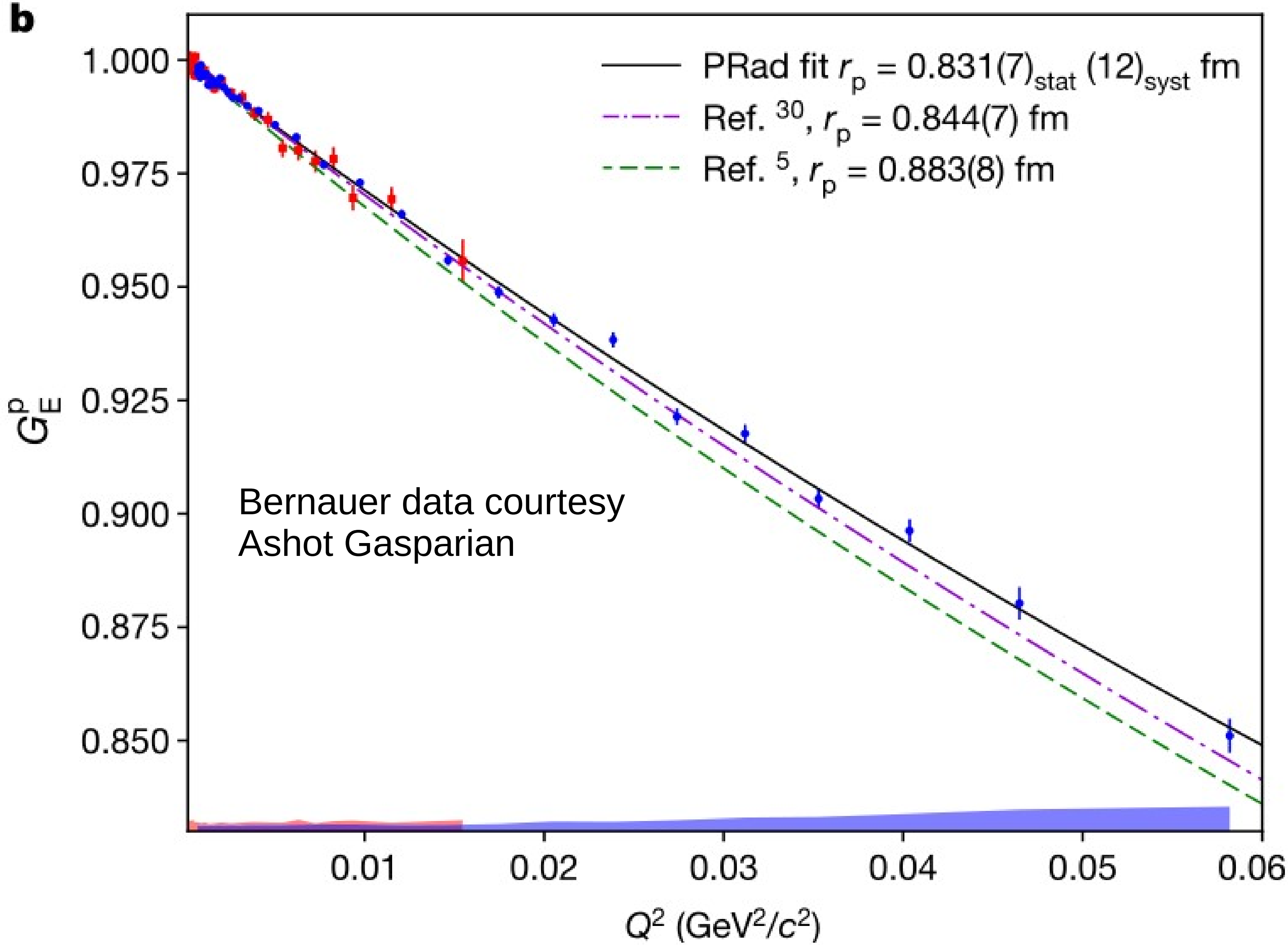
Xiong et al., Nature 575, 147 (2019)

$$\sigma_{\text{reduced}} = \left(\frac{d\sigma}{d\Omega}\right)_{e\text{-}p} \Big/ \left[\left(\frac{d\sigma}{d\Omega}\right)_{\text{point-like}} ((4M_p^2 E'/E)/(4M_p^2 + Q^2))\right]$$



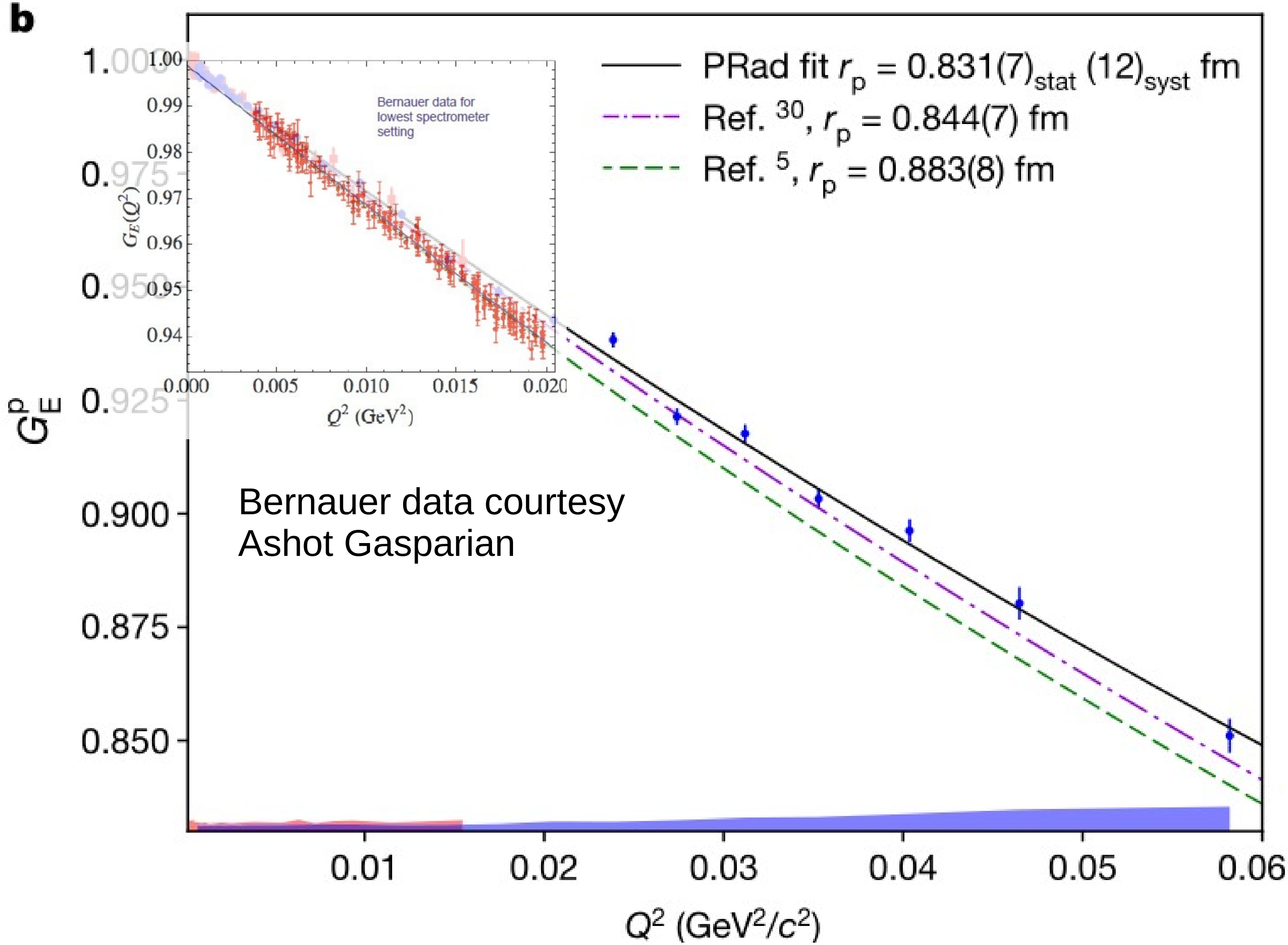
Xiong et al., Nature 575, 147 (2019)

Cross sections and fits



Xiong et al., Nature 575, 147 (2019)

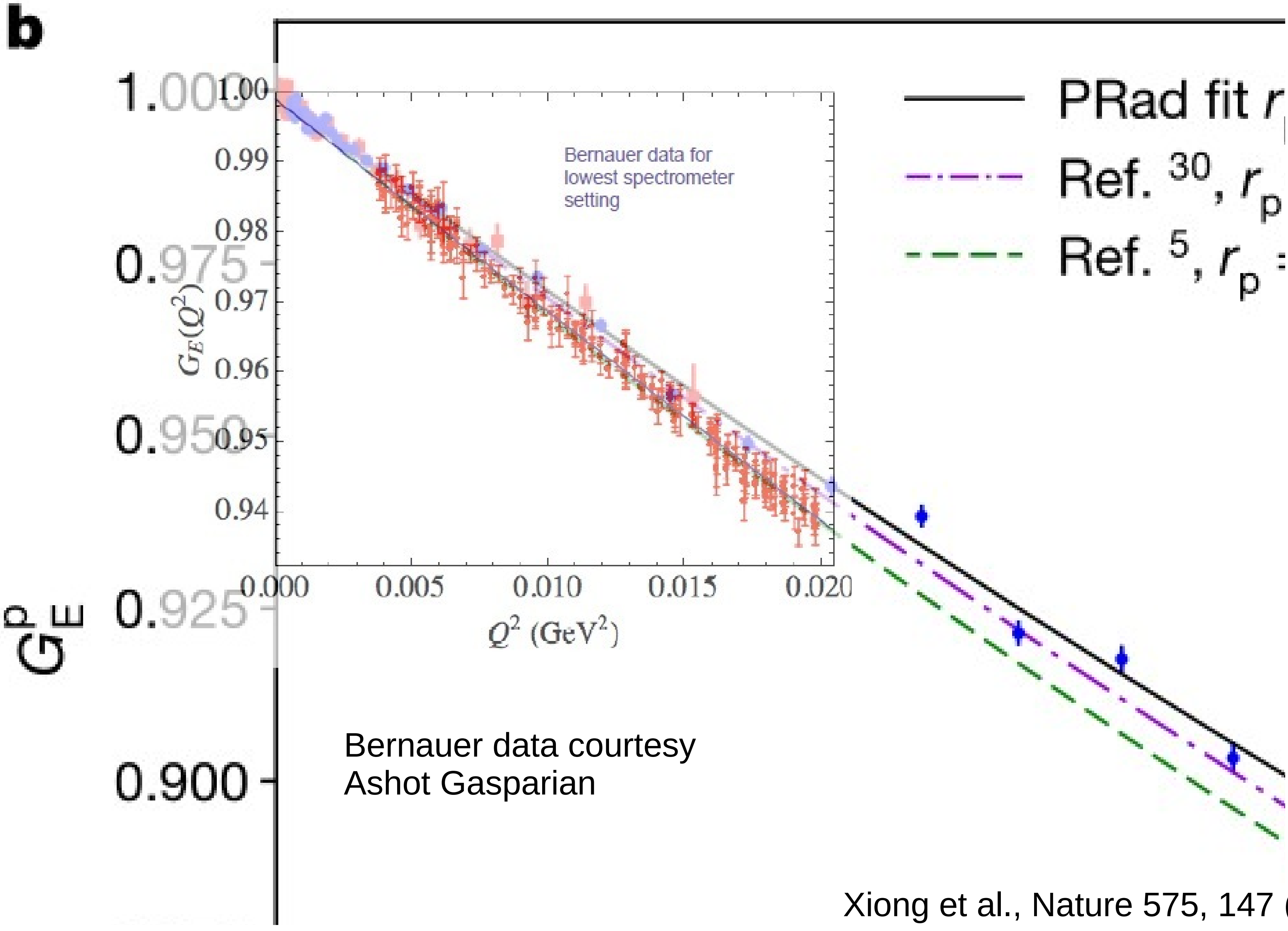
Cross sections and fits



Bernauer data courtesy
Ashot Gasparian

Xiong et al., Nature 575, 147 (2019)

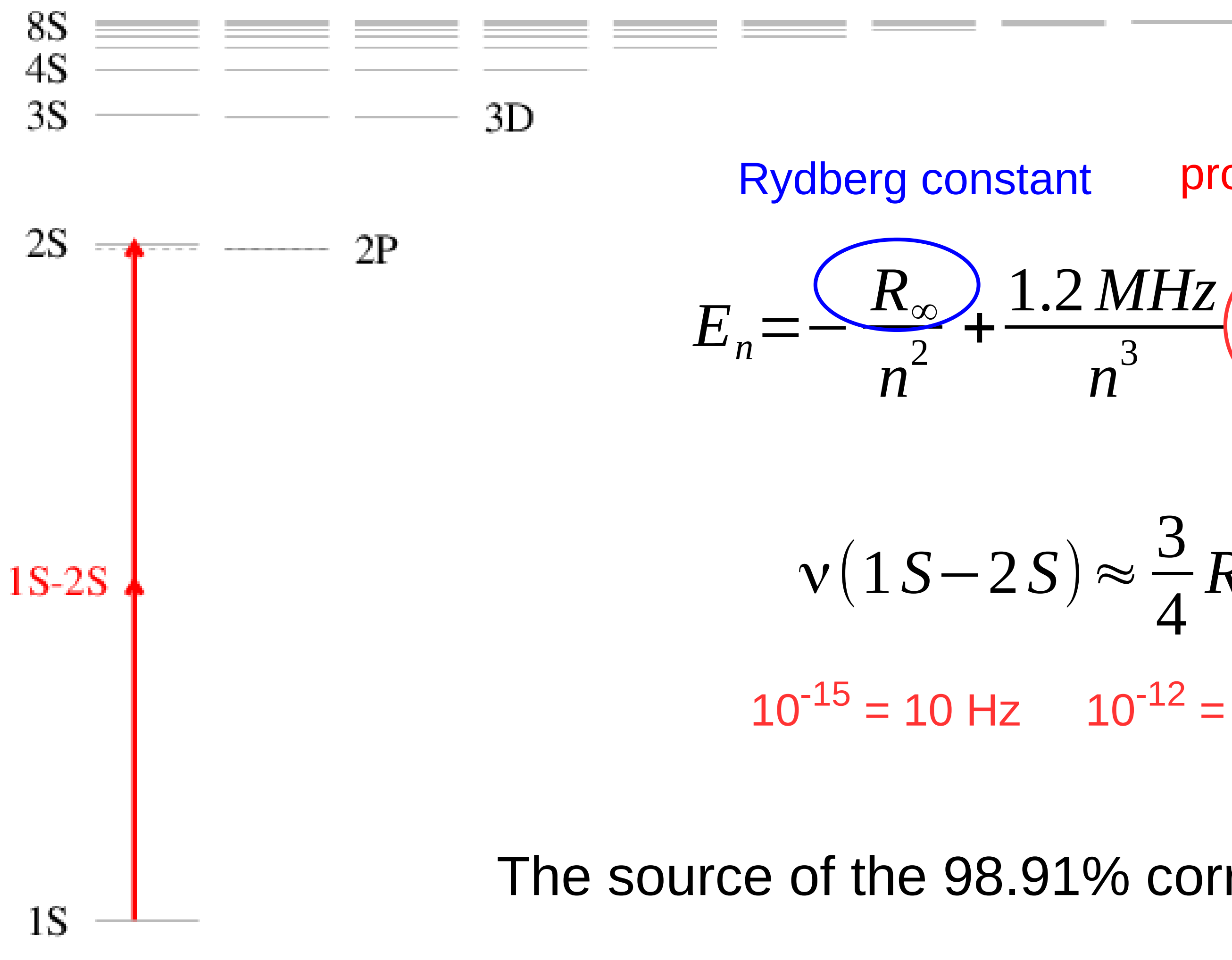
Low Q^2 comparison



Bernauer data courtesy
Ashot Gasparian

Xiong et al., Nature 575, 147 (2019)

Correlation between R_∞ and R_p / R_d



Rydberg constant   proton radius

$$E_n = -\frac{R_\infty}{n^2} + \frac{1.2\,MHz}{n^3}\langle r^2\rangle \delta_{l0} + \Delta(n, l, j)$$

$$\nu(1S-2S) \approx \frac{3}{4}R_\infty - \frac{7}{8}E_{NS}$$

10^{-15} = 10 Hz 10^{-12} = 20 kHz

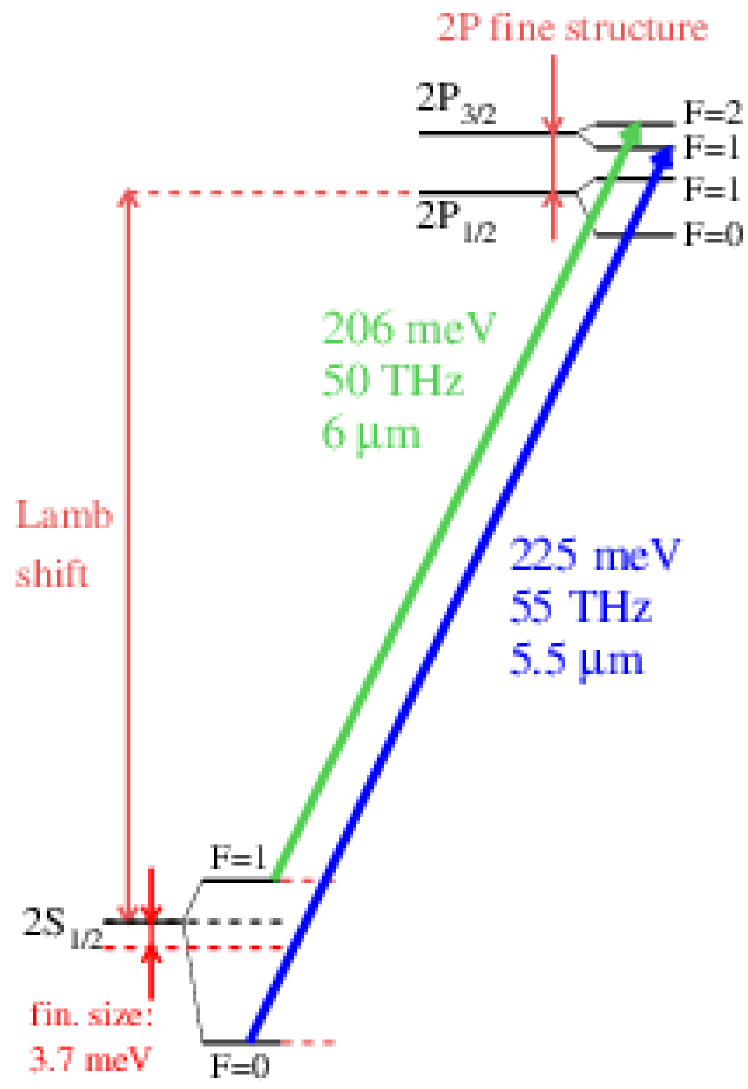
The source of the 98.91% correlation of R_∞ and R_p

1S-2S: Parthey, RP et al., PRL 107, 203001 (2011)

Theory in muonic H

$\Delta E_{Lamb} = 206.0336\,(15)\,\mathrm{meV}_{QED} + 0.0332\,(20)\,\mathrm{meV}_{TPE} - 5.2275\,(10)\,\mathrm{meV/fm}^2 * \mathbf{R_p}^2$

Simple-looking formula
based on decades of work by
E. Borie, M.C. Birse, P. Blunden, C.E. Carlson,
M.I. Eides, R. Faustov, J.L. Friar, G. Paz,
A. Pineda, J. McGovern, K. Griffioen, H. Grotch,
F. Hagelstein, H.-W. Hammer, R.J Hill, P.Indelicato,
U.D. Jentschura, S.G. Karshenboim, E.Y. Korzinin,
V.G. Ivanov, I.T. Lorenz, A.P. Martynenko,
G.A. Miller, U.-G. Meissner, P.J. Mohr,
K. Pachucki, V. Pascalutsa, J. Rafelski,
V.A. Shelyuto, I. Sick, A.W. Thomas,
M. Vanderhaeghen, V. Yerokhin,
…..
(shout if I missed your name!)



Antognini, RP at al., Ann. Phys. (N.Y.) 331, 127 (2013)

Theory in muonic H

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Annals of Physics 331 (2013) 127–145

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Theory of the 2S–2P Lamb shift and 2S hyperfine splitting in muonic hydrogen

CrossMark

Aldo Antognini [a,*], Franz Kottmann [a], François Biraben [b], Paul Indelicato [b], François Nez [b], Randolf Pohl [c]

[a] Institute for Particle Physics, ETH Zurich, 8093 Zurich, Switzerland
[b] Laboratoire Kastler Brossel, École Normale Supérieure, CNRS and Université P. et M. Curie, 75252 Paris, CEDEX 05, France
[c] Max-Planck-Institut für Quantenoptik, 85748 Garching, Germany

Our attempt to summarize all the original work by many theorists….

Theory I: “pure” QED

Table 1
All known radius-*independent* contributions to the Lamb shift in μp from different authors, and the one we selected. Values are in meV. The entry # in the first column refers to Table 1 in Ref. [13]. The “finite-size to relativistic recoil correction” (entry #18 in [13]), which depends on the proton structure, has been shifted to Table 2, together with the small terms #26 and #27, and the proton polarizability term #25. SE: self-energy, VP: vacuum polarization, LBL: light-by-light scattering, Rel: relativistic, NR: non-relativistic, RC: recoil correction.

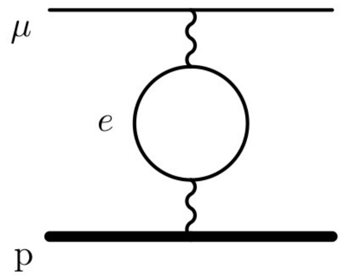
#	Contribution	Pachucki [10,11]	Nature [13]	Borie-v6 [79]	Indelicato [80]	Our choice	Ref.
1	NR one-loop electron VP (eVP)	205.0074					
2	Rel. corr. (Breit–Pauli)	0.0169[a]					
3	Rel. one-loop eVP		205.0282	205.0282	205.02821	205.02821	[80] Eq. (54)
19	Rel. RC to eVP, $\alpha(Z\alpha)^4$	(incl. in #2)[b]	−0.0041	−0.0041		−0.00208[c]	[77,78]
4	Two-loop eVP (Källén–Sabry)	1.5079	1.5081	1.5081	1.50810	1.50810	[80] Eq. (57)
5	One-loop eVP in 2-Coulomb lines $\alpha^2(Z\alpha)^5$	0.1509	0.1509	0.1507	0.15102	0.15102	[80] Eq. (60)
7	eVP corr. to Källén–Sabry	0.0023	0.00223	0.00223	0.00215	0.00215	[80] Eq. (62), [87]
6	NR three-loop eVP	0.0053	0.00529	0.00529		0.00529	[87,88]
9	Wichmann–Kroll, “1:3” LBL		−0.00103	−0.00102	−0.00102	−0.00102	[80] Eq. (64), [89]
10	Virtual Delbrück, “2:2” LBL		0.00135	0.00115		0.00115	[74,89]
New	“3:1” LBL			−0.00102		−0.00102	[89]
20	μSE and μVP	−0.6677	−0.66770	−0.66788	−0.66761	−0.66761	[80] Eqs. (72) + (76)
11	Muon SE corr. to eVP $\alpha^2(Z\alpha)^4$	−0.005(1)	−0.00500	−0.004924[d]		−0.00254	[85] Eq. (29a)[e]
12	eVP loop in self-energy $\alpha^2(Z\alpha)^4$	−0.001	−0.00150			[f]	[74,90–92]
21	Higher order corr. to μSE and μVP		−0.00169	−0.00171[g]		−0.00171	[86] Eq. (177)
13	Mixed eVP + μVP		0.00007	0.00007		0.00007	[74]
New	eVP and μVP in two Coulomb lines				0.00005	0.00005	[80] Eq. (78)
14	Hadronic VP $\alpha(Z\alpha)^4 m_r$	0.0113(3)	0.01077(38)	0.011(1)		0.01121(44)	[93–95]
15	Hadronic VP $\alpha(Z\alpha)^5 m_r$		0.000047			0.000047	[94,95]
16	Rad corr. to hadronic VP		−0.000015			−0.000015	[94,95]
17	Recoil corr.	0.0575	0.05750	0.0575	0.05747	0.05747	[80] Eq. (88)
22	Rel. RC $(Z\alpha)^5$	−0.045	−0.04497	−0.04497	−0.04497	−0.04497	[80] Eq. (88), [74]
23	Rel. RC $(Z\alpha)^6$	0.0003	0.00030		0.0002475	0.0002475	[80] Eq. (86)+Tab.II
New	Rad. (only eVP) RC $\alpha(Z\alpha)^5$					0.000136	[85] Eq. (64a)
24	Rad. RC $\alpha(Z\alpha)^n$ (proton SE)	−0.0099	−0.00960	−0.0100		−0.01080(100)	[43][h] [74]
	Sum	206.0312	206.02915	206.02862		206.03339(109)	

Antognini, RP at al., Ann. Phys. (N.Y.) 331, 127 (2013)

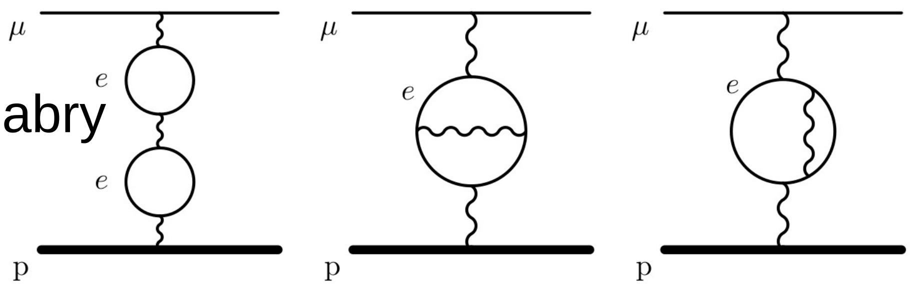
Theory in muonic H

$$\Delta E_{\text{Lamb}} = 206.0336\ (15)\ \text{meV}_{\text{QED}} + 0.0332\ (20)\ \text{meV}_{\text{TPE}} - 5.2275\ (10)\ \text{meV/fm}^2 * \mathbf{R_p}^{\mathbf{2}}$$

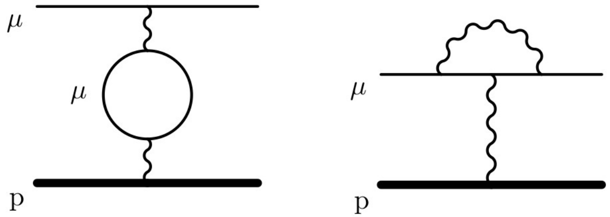
Uehling



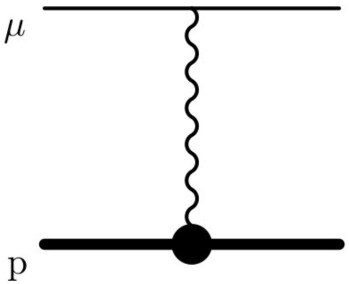
Källen-Sabry



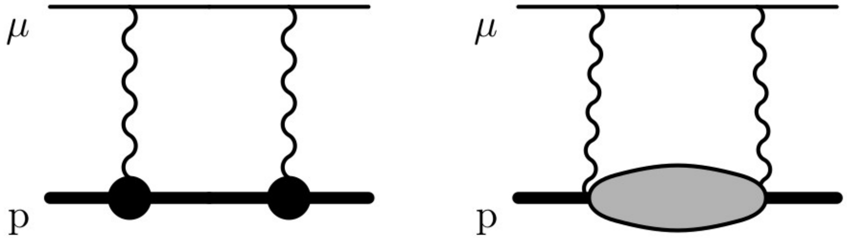
Muon SE+VP



and 20+ more….



Proton form factor



elastic and inelastic two-photon exchange
(Friar moment and polarizability)

Theory in muonic D

$$\Delta E^{\mu D}_{Lamb} = 228.7854\,(13)\ \text{meV}_{QED} + 1.7150\,(\mathbf{230})\ \text{meV}_{TPE} - 6.1103\,(3)\ \text{meV/fm}^2 * \mathbf{R_d}^{\mathbf{2}}$$

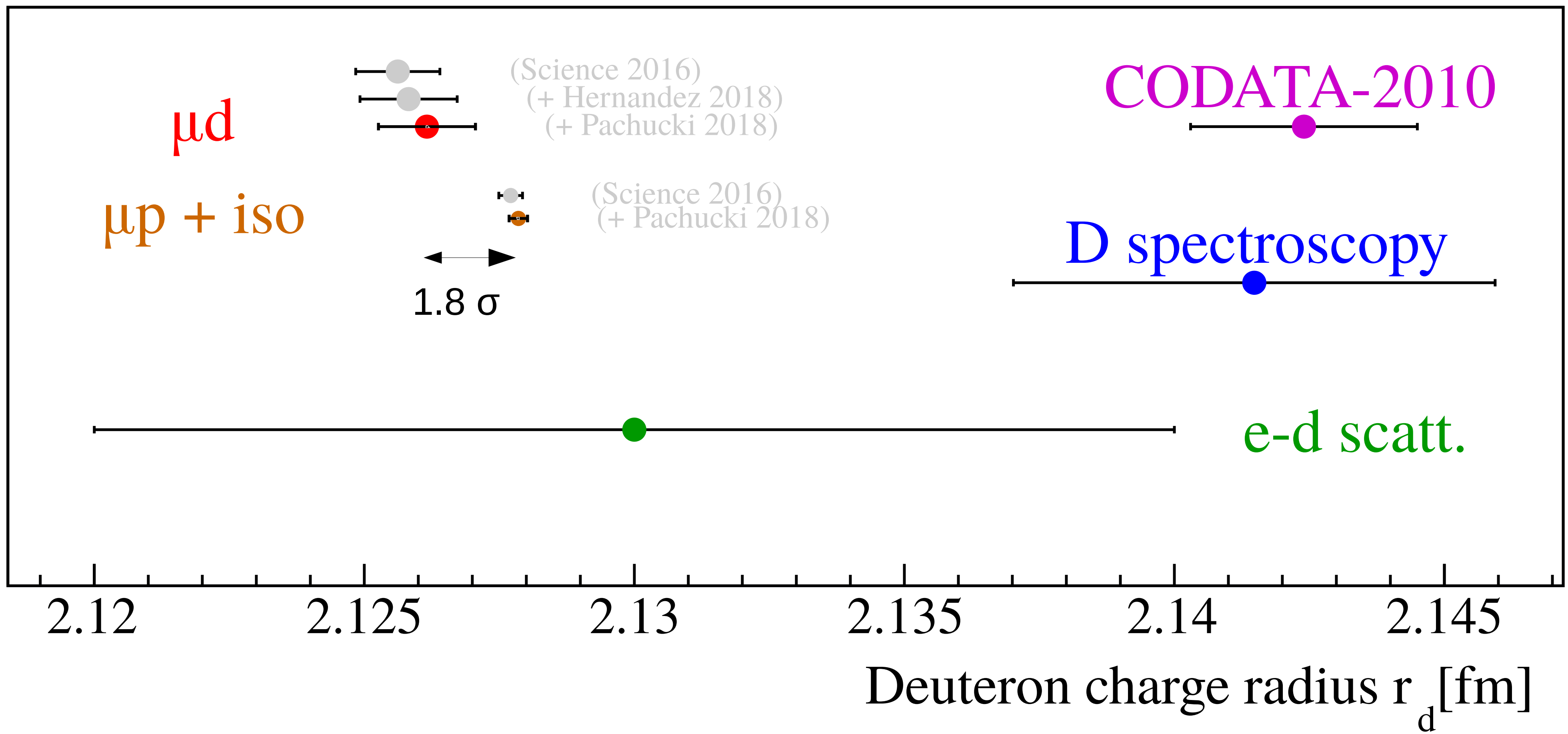
Nuclear structure contributions to the Lamb shift in muonic deuterium.

Item	Contribution	Pachucki [55]		Friar [60]		Hernandez *et al.* [58]			Pach.& Wienczek [65]		Carlson *et al.* [64]	Our choice		
		AV18		ZRA		AV18	N^3LO †		AV18		data	value		source
	Source	1		2		3	4		5		6			
p1	Dipole	1.910	$\delta_0 E$	1.925	Leading C1	1.907	1.926	$\delta^{(0)}_{D1}$	1.910	$\delta_0 E$		1.9165	± 0.0095	3-5
p2	Rel. corr. to p1, longitudinal part	−0.035	$\delta_R E$	−0.037	Subleading C1	−0.029	−0.030	$\delta^{(0)}_L$	−0.026	$\delta_R E$				
p3	Rel. corr. to p1, transverse part					0.012	0.013	$\delta^{(0)}_T$						
p4	Rel. corr. to p1, higher-order								0.004	$\delta_{HO} E$				
sum	Total rel. corr., p2+p3+p4	−0.035		−0.037		−0.017	−0.017		−0.022			−0.0195	± 0.0025	3-5
p5	Coulomb distortion, leading	−0.255	$\delta_{C1} E$						−0.255	$\delta_{C1} E$				
p6	Coul. distortion, next order	−0.006	$\delta_{C2} E$						−0.006	$\delta_{C2} E$				
sum	Total Coulomb distortion, p5+p6	−0.261				−0.262	−0.264	$\delta^{(0)}_C$	−0.261			−0.2625	± 0.0015	3-5
p7	El. monopole excitation	−0.045	$\delta_{Q0} E$	−0.042	C0	−0.042	−0.041	$\delta^{(2)}_{R2}$	−0.042	$\delta_{Q0} E$				
p8	El. dipole excitation	0.151	$\delta_{Q1} E$	0.137	Retarded C1	0.139	0.140	$\delta^{(2)}_{D1D3}$	0.139	$\delta_{Q1} E$				
p9	El. quadrupole excitation	−0.066	$\delta_{Q2} E$	−0.061	C2	−0.061	−0.061	$\delta^{(2)}_Q$	−0.061	$\delta_{Q2} E$				
sum	Tot. nuclear excitation, p7+p8+p9	0.040		0.034	C0 + ret-C1 + C2	0.036	0.038		0.036			0.0360	± 0.0020	2-5
p10	Magnetic	−0.008 ◇ [a]	$\delta_M E$	−0.011	M1	−0.008	−0.007	$\delta^{(0)}_M$	−0.008	$\delta_M E$		−0.0090	± 0.0020	2-5
SUM_1	Total nuclear (corrected)	1.646		1.648 [b]		1.656	1.676		1.655			1.6615	± 0.0103	
p11	Finite nucleon size			0.021	Retarded C1 f.s.	0.020 ◇ [c]	0.021 ◇ [c]	$\delta^{(2)}_{NS}$	0.020	$\delta_{FS} E$				
p12	n p charge correlation			−0.023	pn correl. f.s.	−0.017	−0.017	$\delta^{(1)}_{np}$	−0.018	$\delta_{FZ} E$				
sum	p11+p12			−0.002		0.003	0.004		0.002			0.0010	± 0.0030	2-5
p13	Proton elastic 3rd Zemach moment	} 0.043(3)	$\delta_P E$	0.030	$\langle r^3 \rangle^{pp}_{(2)}$				} 0.043(3)	$\delta_P E$		0.0289	± 0.0015	Eq.(13) [d]
p14	Proton inelastic polarizab.					} 0.027(2)		δ^N_{pol} [64]			} 0.028(2) ΔE^{hadr}	} 0.0280	± 0.0020	6
p15	Neutron inelastic polarizab.								0.016(8)	$\delta_N E$				
p16	Proton & neutron subtraction term											−0.0098	± 0.0098	Eq.(15) [e]
sum	Nucleon TPE, p13+p14+p15+p16	0.043(3)		0.030		0.027(2)			0.059(9)			0.0471	± 0.0101	[f]
SUM_2	Total nucleon contrib.	0.043(3)		0.028		0.030(2)			0.061(9)			0.0476	± 0.0105	
	Sum, published	1.680(16)		1.941(19)		1.690(20)			1.717(20)		2.011(740)			
	Sum, corrected			1.697(19) [g]		1.714(20) [h]			1.707(20) [i]		1.748(740) [j]	**1.7096**	**± 0.0147**	

Krauth, RP at al., Ann. Phys. (N.Y.) 366, 168 (2016)

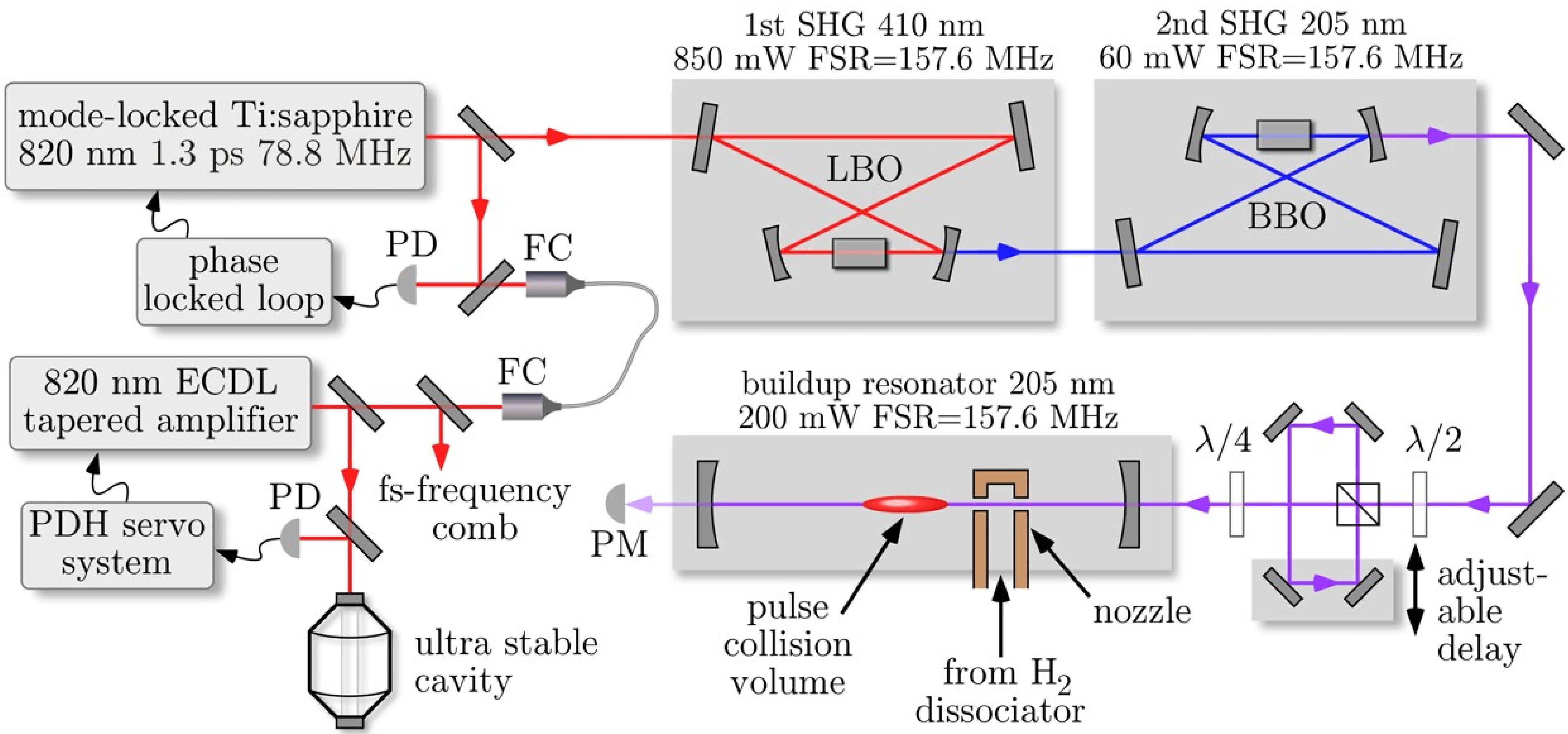
+ Pachucki et al., PRA 97, 062511 (2018)
+ Hernandez et al., PLB 778, 377 (2018)

Deuteron radius



Hernandez et al, Phys. Lett. B 778, 377 (2018)
Pachucki et al., PRA 97, 062511 (2018)

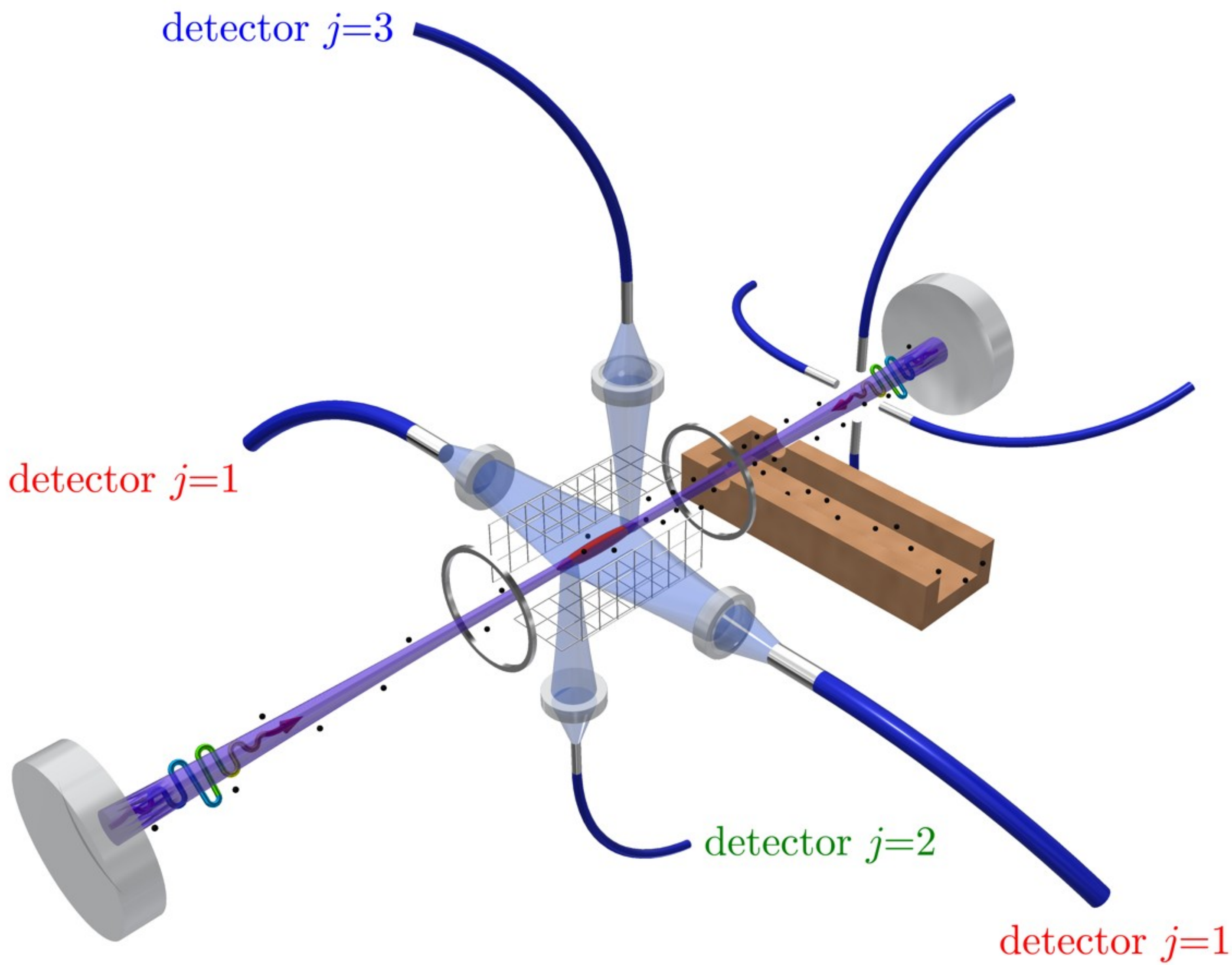
Garching H(1S-3S)



- Direct Frequency Comb Spectroscopy
- cryogenic H beam (6 K)
- 740 Hz (total), 110 Hz (stat.)

Grinin, Matveev,.. RP et al, submitted (2020)

Garching H(1S-3S)



- Direct Frequency Comb Spectroscopy
- cryogenic H beam (6 K)

Grinin, Matveev,.. RP et al, submitted (2020)

Garching H(1S-3S)

contribution	average effect	correction	uncertainty
statistics	——	——	0.11
CIFODS	+0.79	——	0.08
SOD	−3.19	——	0.26
AC–Stark	+4.59	——	0.30
pressure shift	+0.87	——	0.35
residual Doppler	——	——	0.48
DC–Stark	+0.031	−0.031	0.015
Zeeman shift	−0.002	+0.002	0.002
line pulling	−0.30	+0.30	0.050
MP CIFODS	——	——	0.10
maser	−0.30	+0.30	0.030
total		+0.57	0.74

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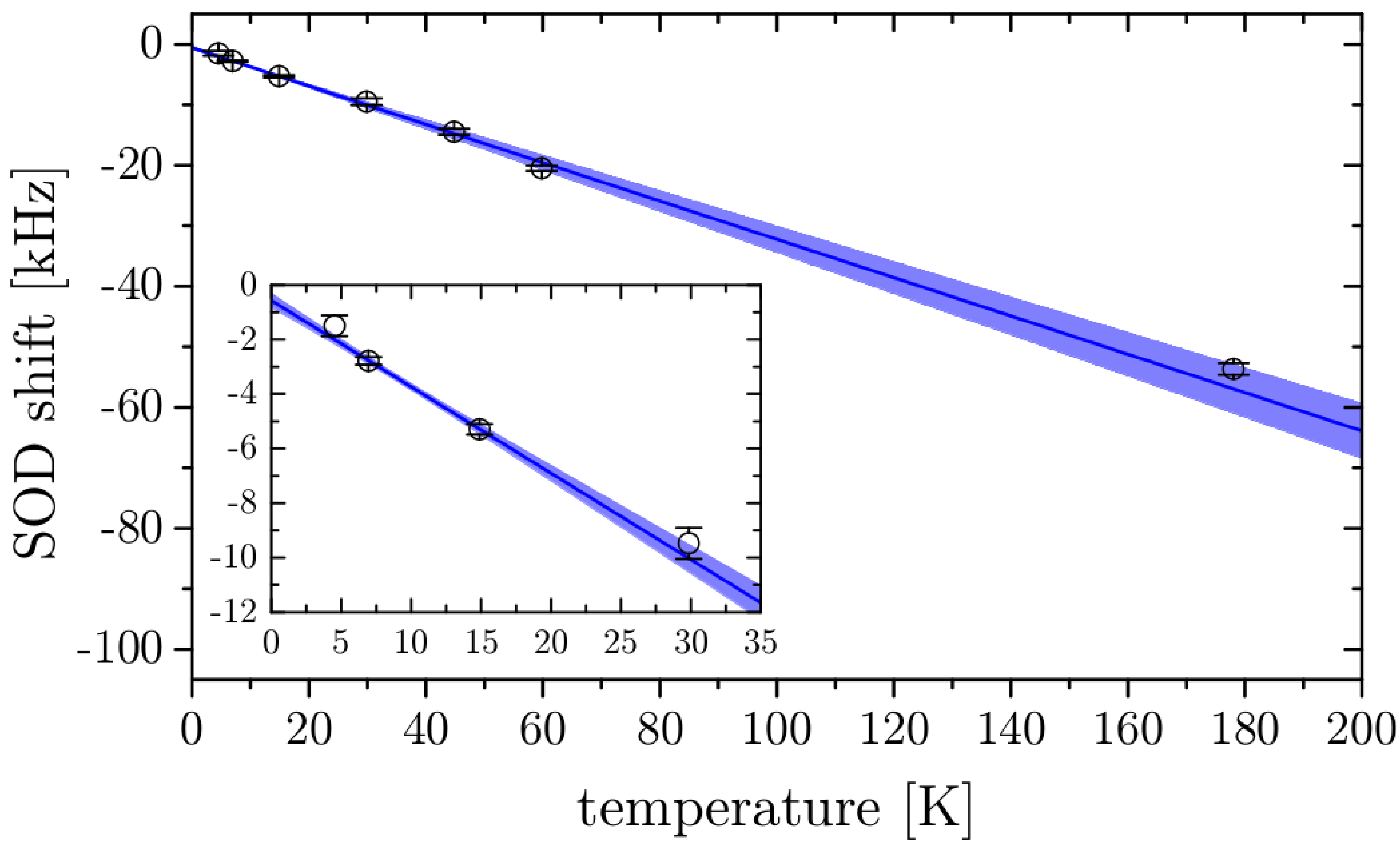
Grinin, Matveev,.. RP et al, submitted (2020)

Garching H(1S-3S)



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