Precise determination of proton magnetic radius from electron scattering data

C. Weiss (JLab), IWHSS2020 Hadron Structure and Spectroscopy, Trieste, 16-18 Nov 2020
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Radius extraction using theory-based method: Dispersively improved chiral EFT

- Combines dispersion theory (analyticity, sum rules) and χEFT (dynamics, controled accuracy)
- Correlates values of radii with FF behavior at finite $Q^2 \lesssim 1$ GeV^2
- Enables reliable determination of magnetic radius

Method: J. M. Alarcon, C. Weiss, PLB 784 (2018) 373; PRC **97,** 055203 (2018);
J. M. Alarcon, A. N. Hiller Blin, M. Vicente Vacas, C. Weiss, NPA **964**, 18 (2017)

Radius extraction: J. M. Alarcon, D. Higinbotham, C. Weiss, PRC 102 (2020) 035203 ←
See also: J. M. Alarcon, D. Higinbotham, C. Weiss, Z. Ye, PRC 99 (2019) 044303

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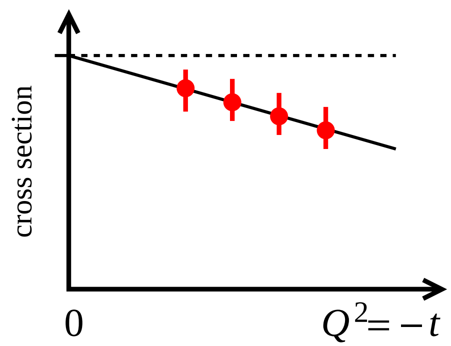
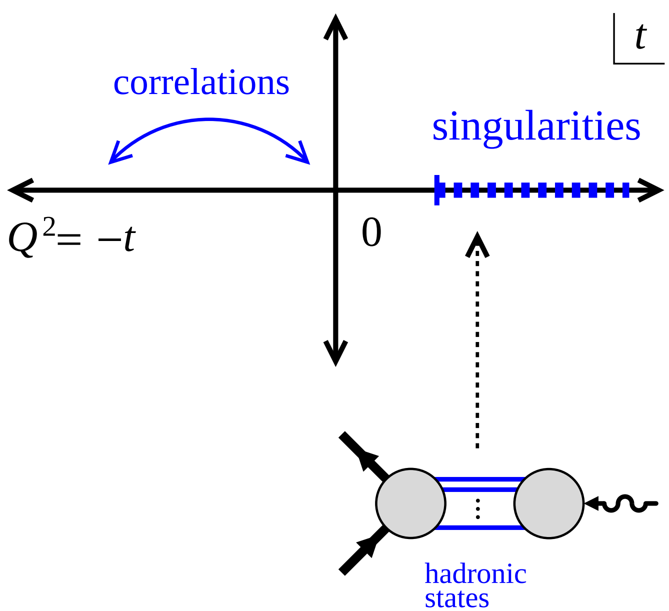
Motivation: Analyticity in radius extraction





- Challenges in proton radius extraction

  Derivative at $Q^2 = 0$ from data at finite $Q^2 > 0$

  Extrapolation $Q^2 \to 0$: Stability, functional bias?
  see e.g. Barcus, Higinbotham, McClellan 2020

  Magnetic radius: Contribution of G_M^p to elastic ep cross section $\propto \tau/\epsilon$, vanishes for $Q^2 \to 0$

- Analyticity

  FFs analytic functions of $t = -Q^2$

  Singularities at $t > 0$: Hadronic exchanges

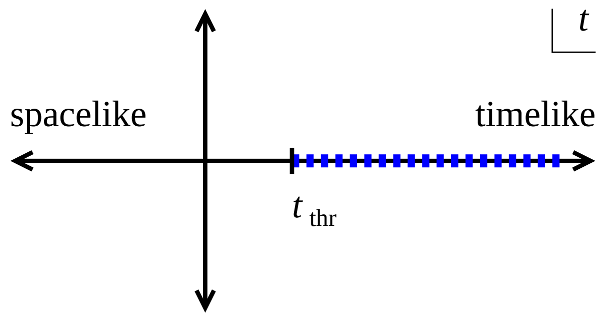
  Correlates functional behavior of FF at $Q^2 > 0$ with derivative at $Q^2 = 0$

  Predicts size of higher derivatives

  Global properties: Sum rules

  Use in radius extraction!

DIχEFT: Dispersively improved chiral EFT

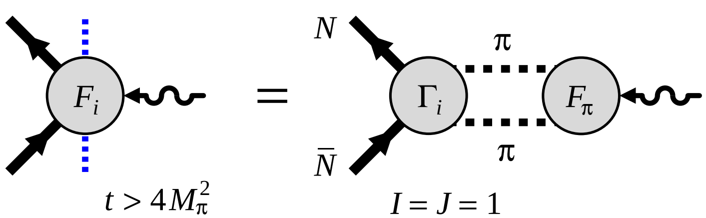


- Dispersive representation

$$F_i(t) \; = \int\limits_{t_{\mathrm{thr}}}^{\infty} \frac{dt'}{\pi} \, \frac{\mathrm{Im}\, F_i(t')}{t' - t - i0}$$

Expresses analytic structure

$\mathrm{Im}\, F_i$ spectral function, constructed theoretically



$$\mathrm{Im} F_i(t) = \frac{k_{\mathrm{cm}}^3}{\sqrt{t}} \, \Gamma_i(t) \;\; F_\pi^*(t)$$

$$= \frac{k_{\mathrm{cm}}^3}{\sqrt{t}} \, \underbrace{\frac{\Gamma_i(t)}{F_\pi(t)}}_{\chi\text{EFT}} \;\; \underbrace{|F_\pi(t)|^2}_{\text{Data}}$$

- Spectral function in $\pi\pi$ region

Elastic unitarity relation
Frazer, Fulco 1960; Höhler et al 1975+

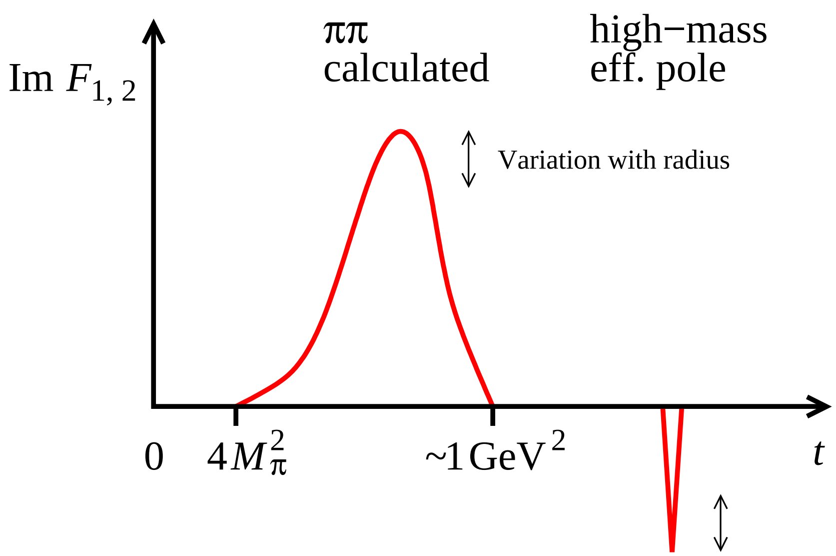
Factorize $\pi\pi$ rescattering using N/D method

Γ_i / F_π: $\pi\pi$–NN coupling, calculated in χEFT good convergence

$|F_\pi|^2$: $\pi\pi$ rescattering, taken from e^+e^- data

Presently implemented LO + NLO + partial N2LO
Alarcon, Weiss, PLB 784 (2018) 373; PRC 97 (2018) 055203

DIχEFT: Sum rules and parameters



$$\frac{1}{\pi}\int_{t_{\mathrm{thr}}}^{\infty} dt\, \frac{\mathrm{Im}F_1(t)}{t} = Q$$

$$\frac{1}{\pi}\int_{t_{\mathrm{thr}}}^{\infty} dt\, \frac{\mathrm{Im}F_1(t)}{t^2} = \frac{1}{6}\langle r^2\rangle_1$$

$$\frac{1}{\pi}\int_{t_{\mathrm{thr}}}^{\infty} dt\, \frac{\mathrm{Im}F_2(t)}{t} = \kappa$$

$$\frac{1}{\pi}\int_{t_{\mathrm{thr}}}^{\infty} dt\, \frac{\mathrm{Im}F_2(t)}{t^2} = \frac{1}{6}\kappa\langle r^2\rangle_2$$

- Spectral function in high-mass region

  Parameterized by effective pole

  Sufficient for low-Q^2 form factors, uncertainty quantified
  Alarcon, Weiss PLB 784 (2018) 373

- Sum rules and parameters

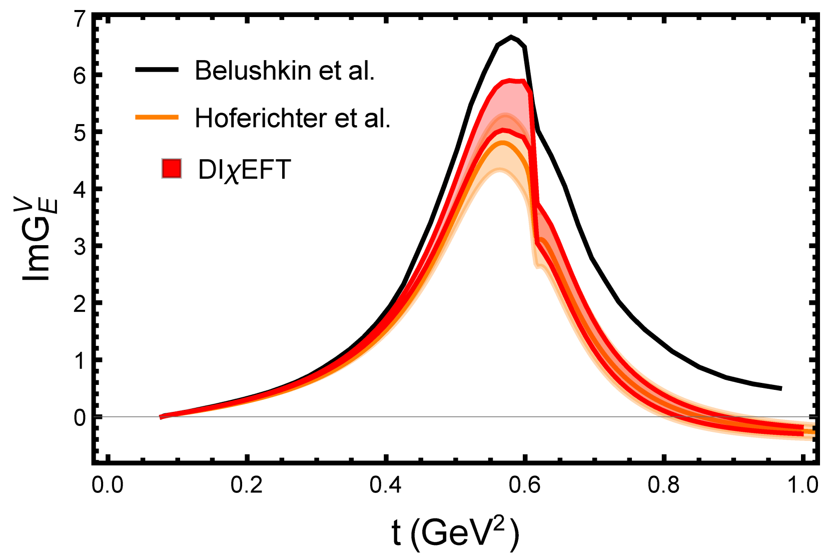
  Sum rules for $F(0)$, $F'(0)$ = charges, radii

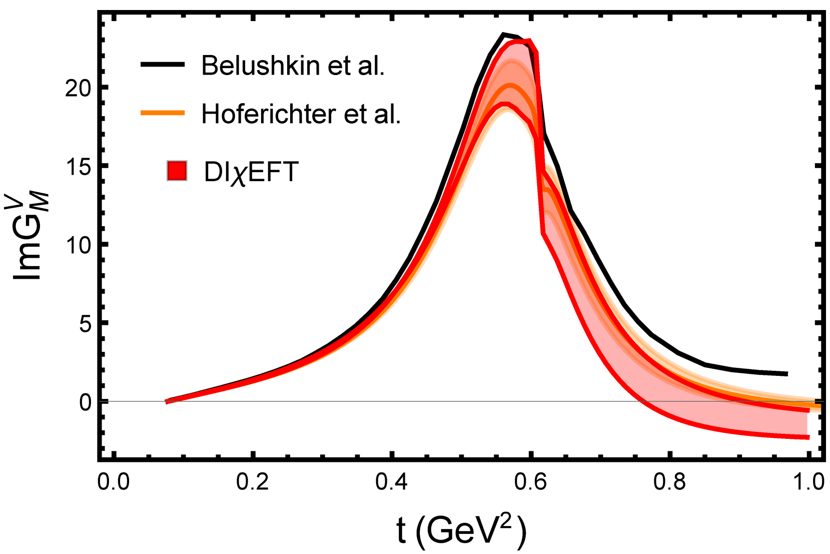
  Express χEFT LEC in terms of radii

  Radii appear directly as parameters of spectral functions, control behavior

  [Different from traditional dispersion analysis, where $\pi\pi$ spectral function taken as fixed input
  Höhler et al 1975; Belushkin, Hammer Meissner 2006; Lorenz, Hammer, Meissner 2012]

DIχEFT: Spectral functions





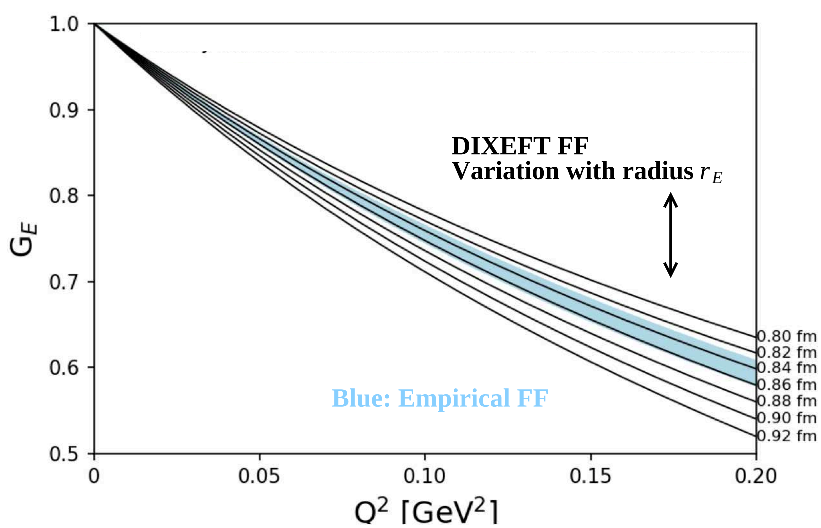
- Spectral functions in $\pi\pi$ region

  Band shows variation with radii (PDG range)

  Good agreement with Roy-Steiner results
  Hoferichter et al 2017

Alarcon, Weiss, PLB 784 (2018) 373
Bands: Variation with nucleon radii (PDG range)

DIχEFT: Form factors



G_M similar, dependence on r_M

Alarcon, Higinbotham, Weiss, Ye PRC 99 (2019) 044303
Empirical FF: Global fit Ye et al 2017

- Form factors from dispersion integral

$$G_{E,M}(t) = \int_{4M_\pi^2}^{\infty} \frac{dt'}{\pi} \frac{\mathrm{Im}\, G_{E,M}(t')}{t' - t - i0}$$

- Family of FFs depending on radii

  Each member respects analyticity, sum rules

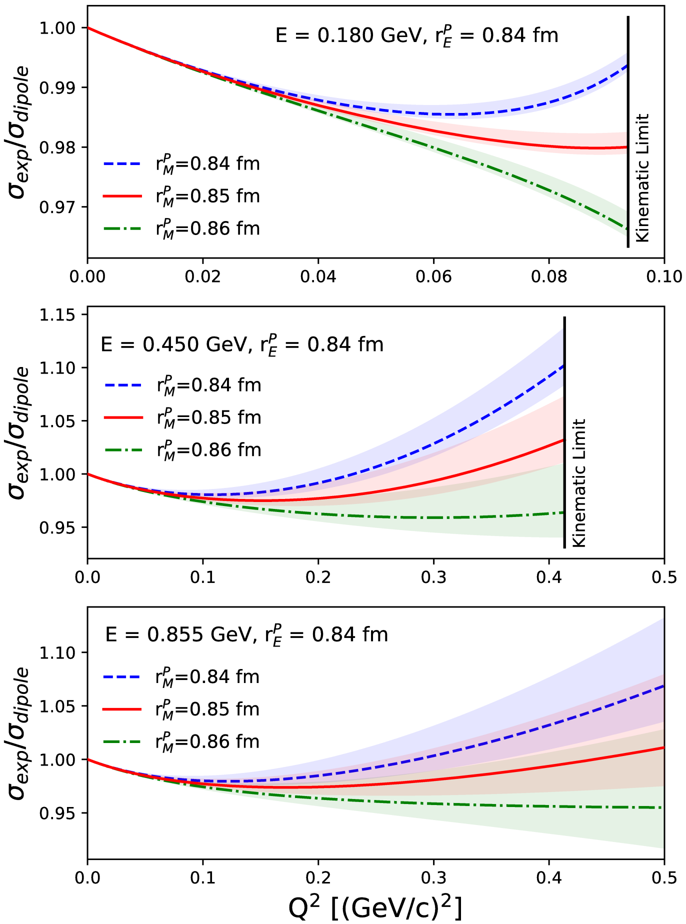
  Each has intrinsic theoretical uncertainty

- Radius correlated with finite-Q^2 behavior

  Provided by analyticity

  Use for radius extraction!

Magnetic radius extraction: Procedure



$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} \frac{\epsilon\,[G_E^p]^2 + \tau [G_M^p]^2}{\epsilon(1+\tau)}$$

- Use DIχEFT $G^p_{E,M}(Q^2)$ with params r^p_E, r^p_M

- Fit Mainz A1 cross section data

  E = 0.18–0.855 GeV, Q^2 = 0.003–1.0 GeV^2

  Fit original cross secns with floating normalizations

  Alt: Fit reanalyzed cross secns of Lee Arrington Hill 2015 with recalc uncertainties: Same radii, lower χ^2

- Impact on magnetic radius

  Sensitivity of cross section to G^p_M

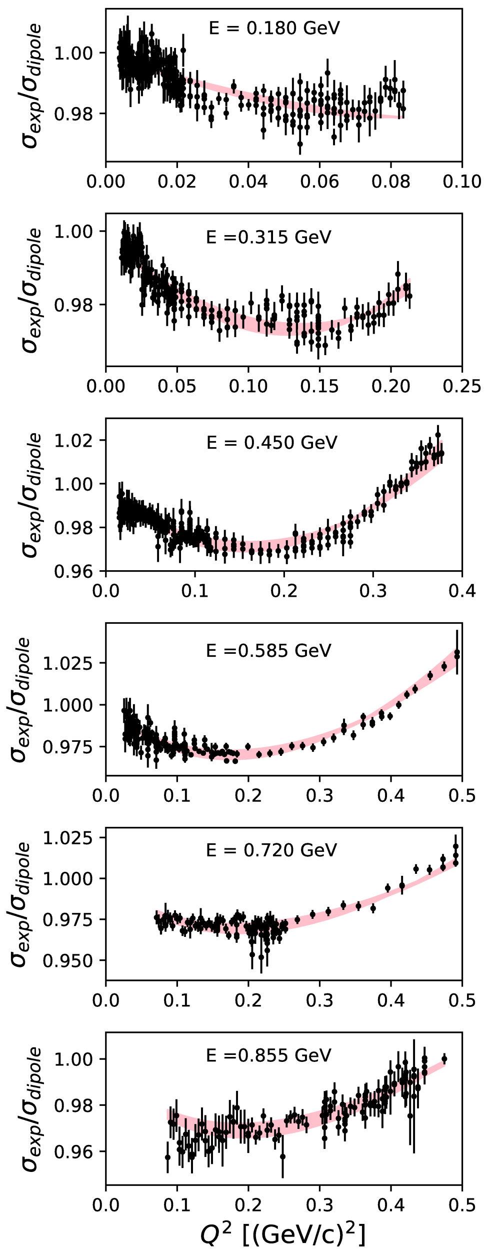
  Dependence of DIχEFT G^p_M on r^p_M

  Theoretical uncertainty from high-mass pole

  Use data up to $Q^2 \approx$ 0.5 GeV^2

Alarcon, Higinbotham, Weiss, PRC 102 (2020) 035203

Magnetic radius extraction: Results



- Extracted radii

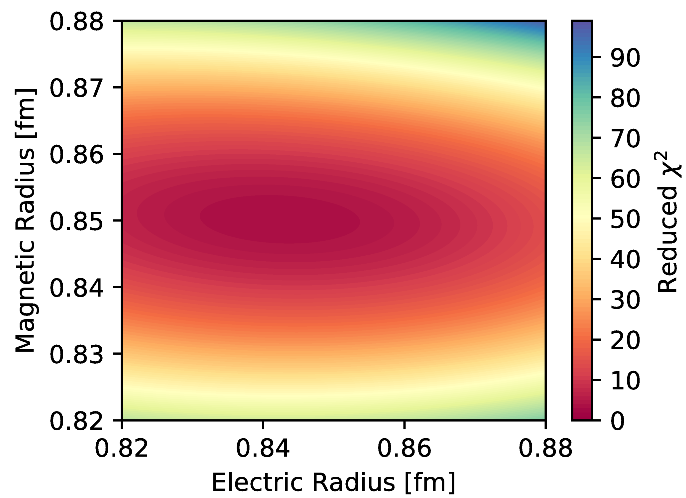
$r_E^p = 0.842 \pm 0.002$ (fit 1σ) $^{+0.005}_{-0.002}$ (theory full-range) fm

$r_M^p = 0.850 \pm 0.001$ (fit 1σ) $^{+0.009}_{-0.004}$ (theory full-range) fm

Magnetic radius has smaller fit uncertainty, larger theory unc

Magnetic radius needs theory-based extraction method

Consistent with results of empirical dispersive fits
Lorenz, Hammer, Meissner 2012



Alarcon, Higinbotham, Weiss, PRC 102 (2020) 035203

Summary

- DIχEFT describes nucleon FFs combining dispersion theory and χEFT

  Includes $\pi\pi$ rescattering and ρ resonance through unitarity

  Enables predictive calculations, controlled theoretical accuracy

  Excellent agreement with empirical FFs up to $Q^2 \sim 1$ GeV^2 and beyond

- DIχEFT enables theory-based radius extraction

  Correlates $Q^2 = 0$ derivatives with finite-Q^2 behavior through analyticity $+$ sum rules

  Employs radii directly as parameters $\leftrightarrow$ LECs

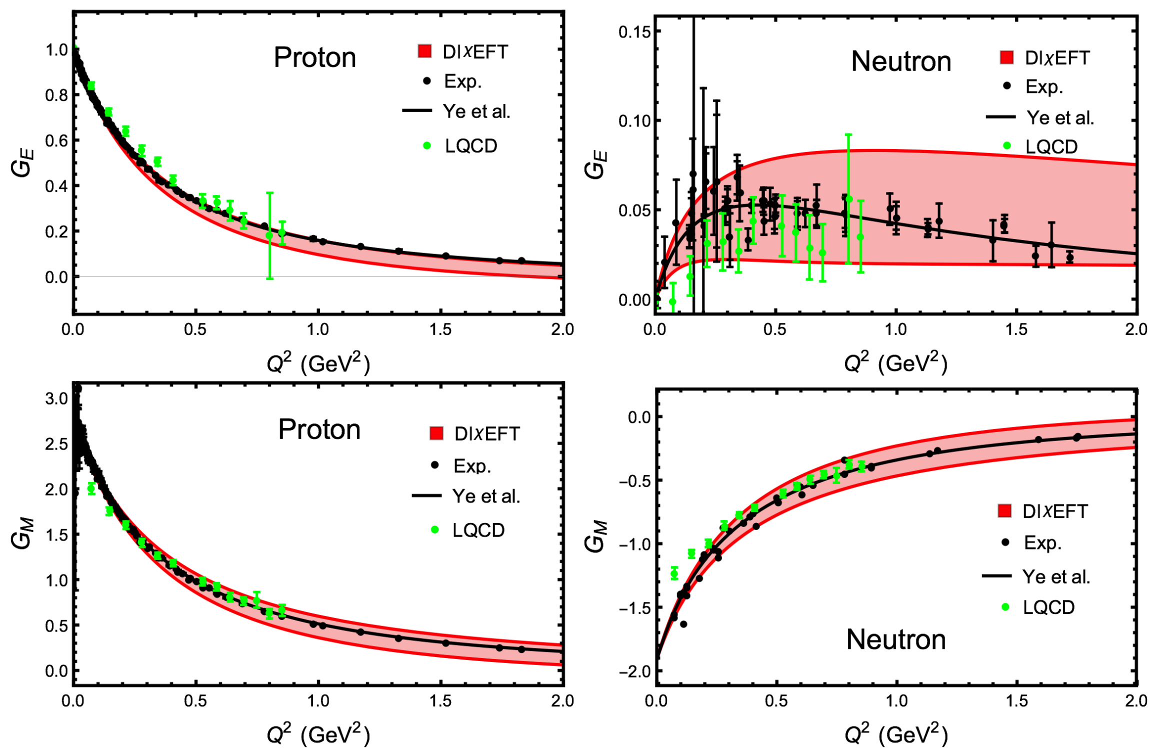
  Enables reliable determination of magnetic radius from finite-Q^2 data

- Other DIχEFT applications

  Nucleon transverse charge/magnetization densities $\leftrightarrow$ GPDs
  Alarcon, Weiss, in progress

  Nucleon scalar FF $\leftrightarrow$ σ term, nucleon mass decomposition
  Alarcon, Weiss, PRC **96,** 055206 (2017)

Supplement: DIχEFT form factors



$$G_{E,M}(t) = \int_{4M_\pi^2}^{\infty} \frac{dt'}{\pi} \frac{\operatorname{Im} G_{E,M}(t')}{t' - t - i0}$$

Alarcon, Weiss, PLB 784 (2018) 373
Bands: Variation with nucleon radii (PDG range)

- DIχEFT form factors

  Evaluated using dispersion integral with spectral functions

  Band shows variation with radii (PDG range).
  Also quantified uncertainty from high-mass states

  Excellent agreement with data. Not fit, but prediction based on dynamics