

IWHSS, 16-18 NOVEMBER 2020, TRIESTE

# JETS AND TMD IN SIDIS AND E+E- EXPERIMENTS

IGNAZIO SCIMEMI

UNIVERSIDAD COMPLUTENSE DE MADRID  
(UCM AND IPARCOS)

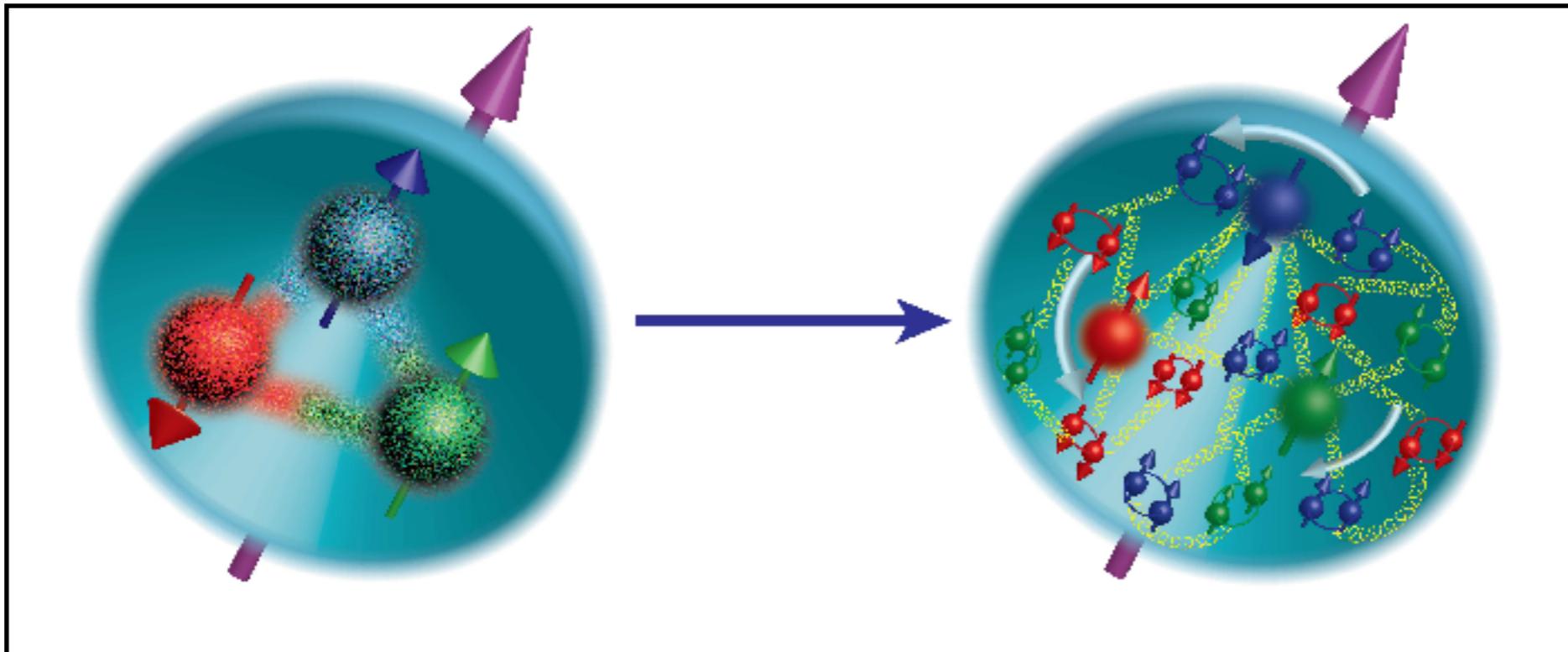


Based on :

D. Gutierrez-Reyes, I.S., W. Waalewijn, L. Zoppi PRL 121,  
162001(2018)

D. Gutierrez-Reyes, I.S., W. Waalewijn, L. Zoppi JHEP 1910 (2019) 031





The 3D mapping of hadrons is our challenge...



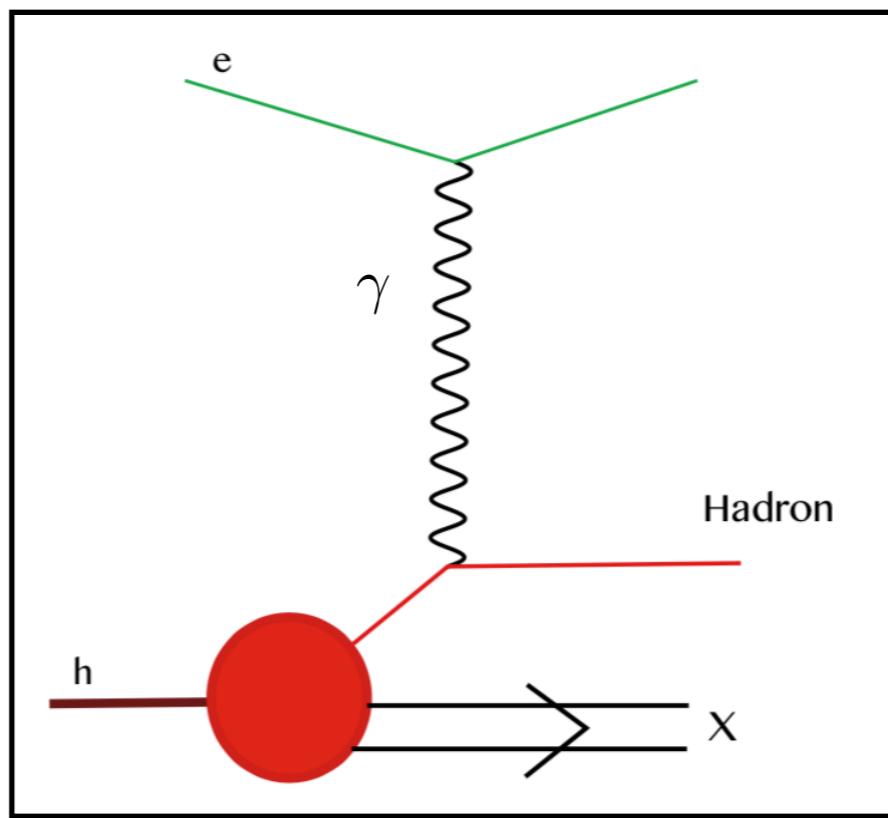
Let us use **jets** to investigate hadron structure!

Expectations from jet (to be checked!!):

Better control of hadronization effects with respect to  
fragmentation functions

An alternative study of TMD, using different combinations of jets  
and TMD

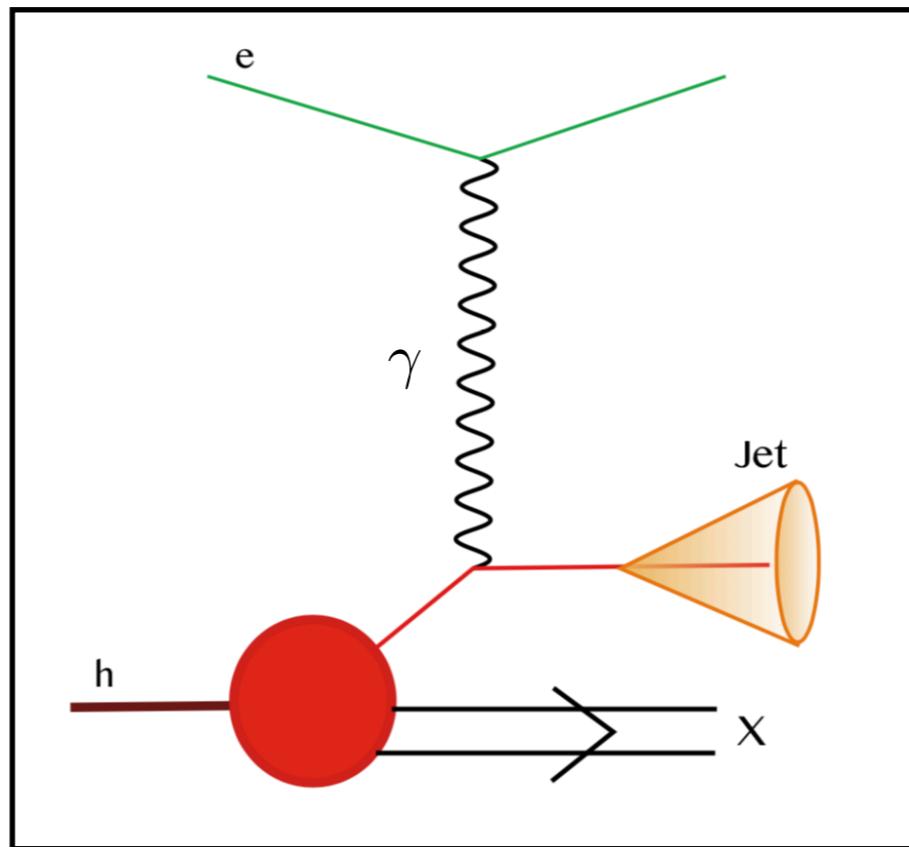
# TMDS WITHOUT JETS



Factorization theorem for **SIDIS**

$$\frac{d\sigma_{eN \rightarrow eN'X}}{dQ^2 dx dz d\mathbf{q}} = \sum_a \mathcal{H}_a(Q^2, \mu) \int \frac{d\mathbf{b}}{(2\pi)^2} e^{-i\mathbf{b}\mathbf{q}} \underbrace{f_{a/N}(x, \mathbf{b}, \mu, \zeta)}_{\text{TMDPDF}} \underbrace{d_{a/N'}(z, \mathbf{b}, \mu, \zeta)}_{\text{TMDFF}}$$

# TMDS WITH JETS



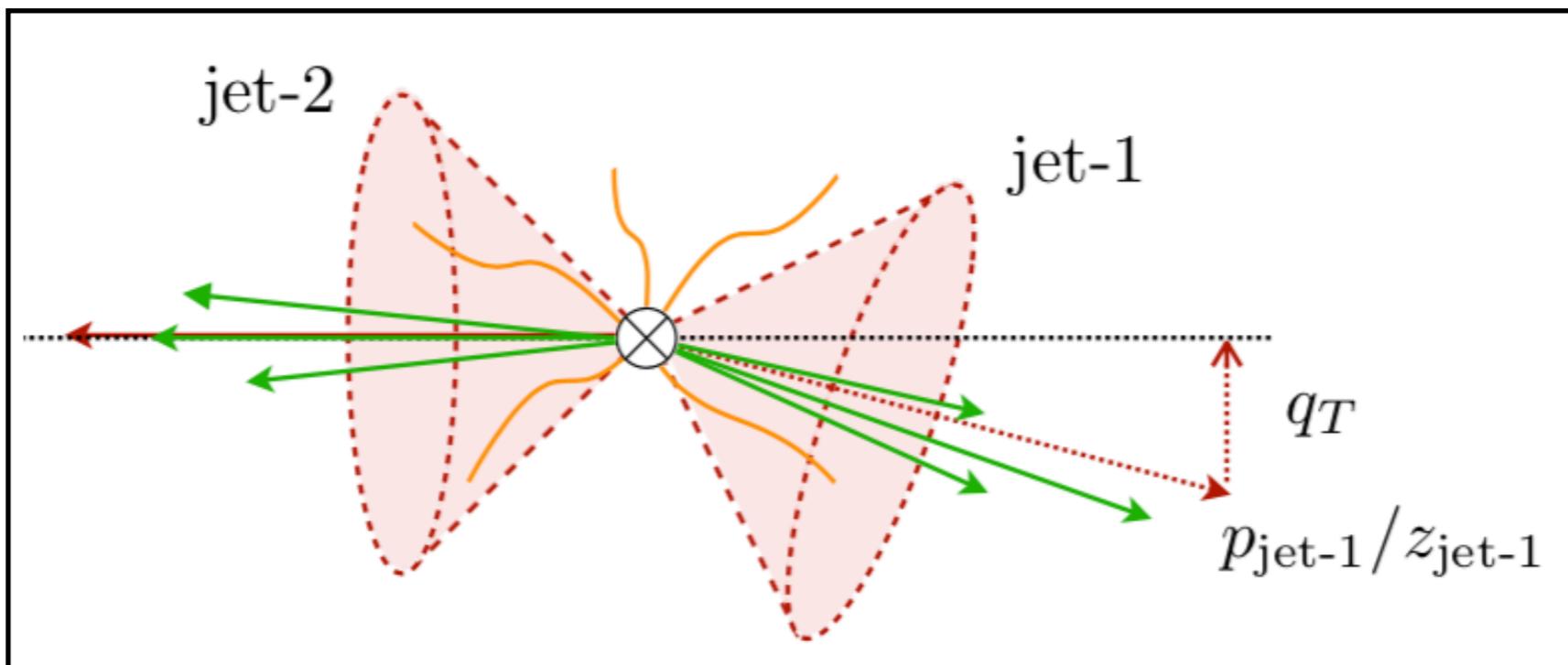
Factorization theorem for SIDIS

$$\frac{d\sigma_{eN \rightarrow eJX}}{dQ^2 dx dz d\mathbf{q}} = \sum_a \mathcal{H}_a(Q^2, \mu) \int \frac{d\mathbf{b}}{(2\pi)^2} e^{-i\mathbf{b}\mathbf{q}} f_{a/N}(x, \mathbf{b}, \mu, \zeta) J_q^{\text{axis}}(z, \mathbf{b}, Q R, \mu, \zeta)$$

—————      —————

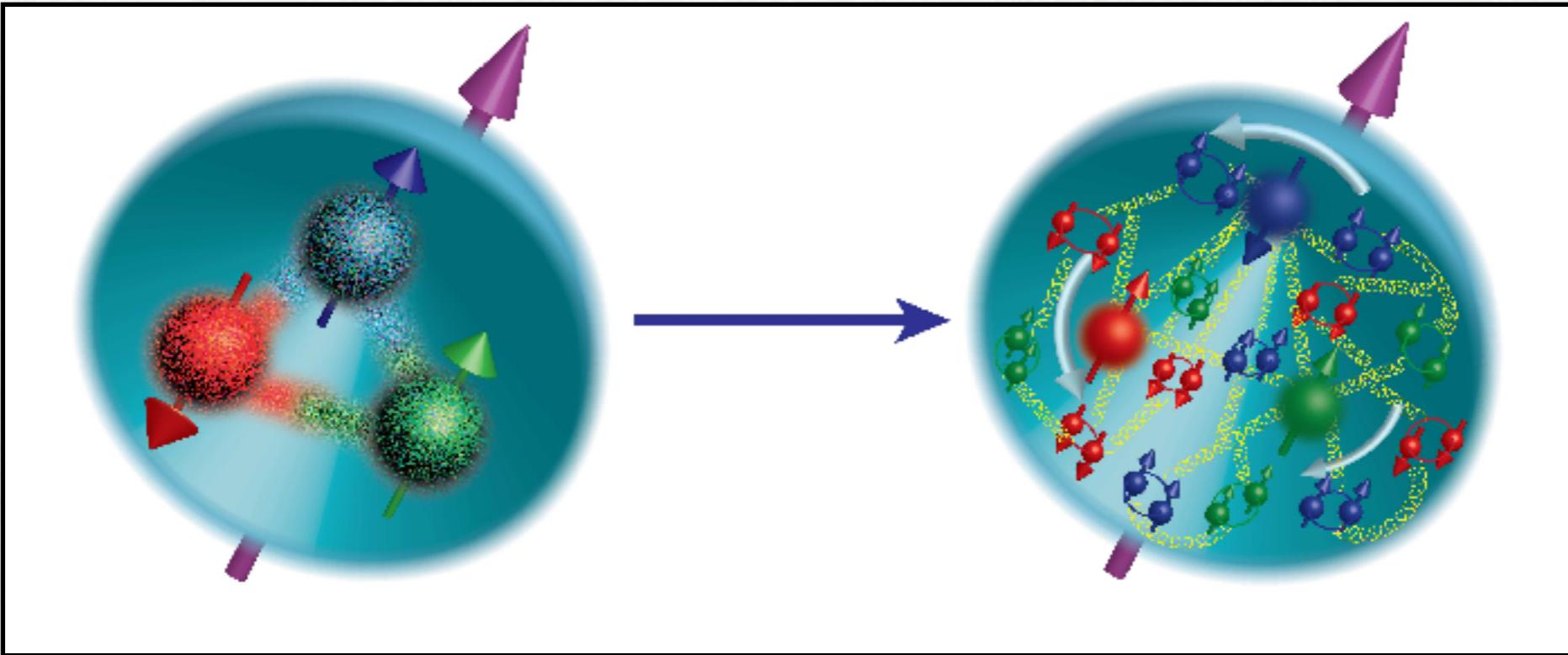
TMDPDF      TMD jet function

# THE TMD EVOLUTION KERNEL



$$\frac{d\sigma_{e^+e^- \rightarrow J J X}}{dz_1 dz_2 d\mathbf{q}} = \sigma_0^{e^+e^-}(s) H_{e^+e^-}(s, \mu) \int \frac{d\mathbf{b}}{(2\pi)^2} e^{-i\mathbf{b} \cdot \mathbf{q}} J_q(z_1, \mathbf{b}, \frac{\sqrt{s}}{2}\mathcal{R}, \mu, \zeta) J_{\bar{q}}(z_2, \mathbf{b}, \frac{\sqrt{s}}{2}\mathcal{R}, \mu, \zeta)$$

When only jets are present in the final state we can study the evolution kernel by itself!



We want to study the non-perturbative part of the evolution kernel:  
 We consider the possibility to reduce non-perturbative effects in final states using jets.

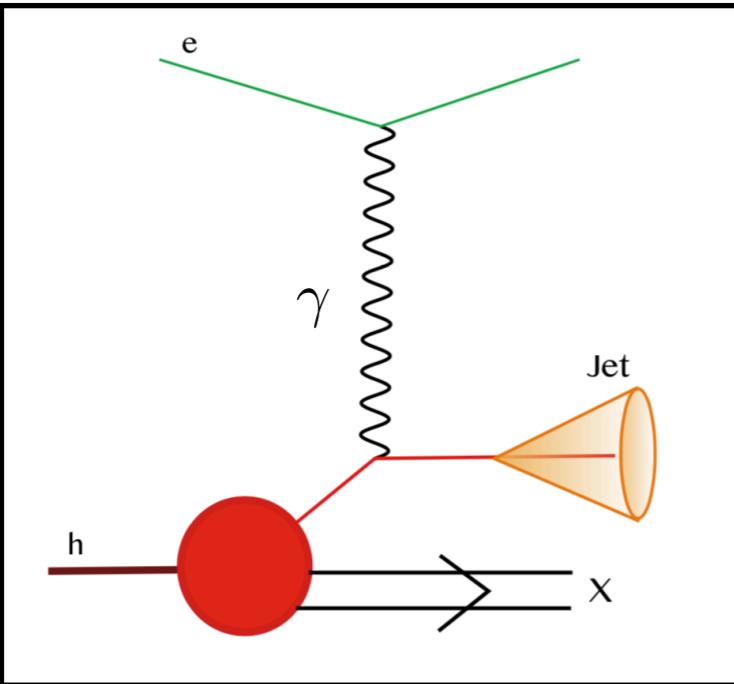
In SIDIS  
 With a jet:

$$\frac{d\sigma}{dQ^2 dx dz dq_T^2} = \sigma_0 \sum_{f_1, f_2} \int \frac{d^2 \mathbf{b}}{4\pi} e^{i \mathbf{b} \mathbf{q}_T} \{R[\mathbf{b}; (Q, Q^2)]\}^2 F_{f_1 \leftarrow h}(x, \mathbf{b}) J_{f_2 \rightarrow J}(z, \mathbf{b})$$

In e+e-:

$$\frac{d\sigma}{dz_1 dz_2 dq_T^2} = \sigma_0 \sum_{f_1, f_2} \int \frac{d^2 \mathbf{b}}{4\pi} e^{i \mathbf{b} \mathbf{q}_T} \{R[\mathbf{b}; (Q, Q^2)]\}^2 J_{f_1 \rightarrow J_1}(x, \mathbf{b}) J_{f_2 \rightarrow J_2}(z, \mathbf{b})$$

# TMDS WITH JETS



## Questions:

Can we write TMD factorization theorems for **processes with jets?**



Can we write factorization theorems for jets **regardless of the size of the jet?**



(With the same evolution kernel as in the standard TMD case)

Experimental set-ups are not always ideal, so compatibility with factorization theorem should be checked

Hadronization effects can be considered (**ungroomed/groomed jets**)

# Dijet decorrelation

In all cases  
 $\theta \ll 1$

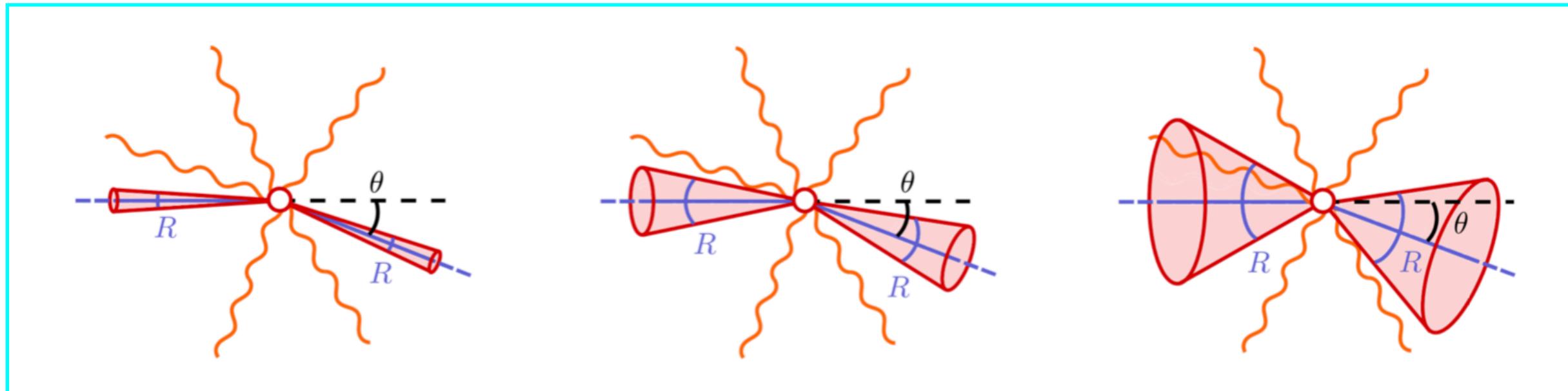
$$e^+ e^- \rightarrow \text{dijet} + X$$

$$\begin{aligned} q &= \frac{\mathbf{p}_1}{z_1} + \frac{\mathbf{p}_2}{z_2} \\ \tan \theta &= 2q_T/\sqrt{s} \quad z_i = 2E_i/\sqrt{s} \end{aligned}$$

$$e^- + H \rightarrow e^- + \text{jet} + X$$

$$\begin{aligned} \mathbf{q} &= \frac{\mathbf{P}_J}{z} + \mathbf{q}_{in} \\ \tan \theta &= 2q_T/Q \quad z_i = 2E_J/Q \end{aligned}$$

$\theta$  competes with  $R$



$\theta \gg R$

$\theta \sim R$

$\theta \ll R$

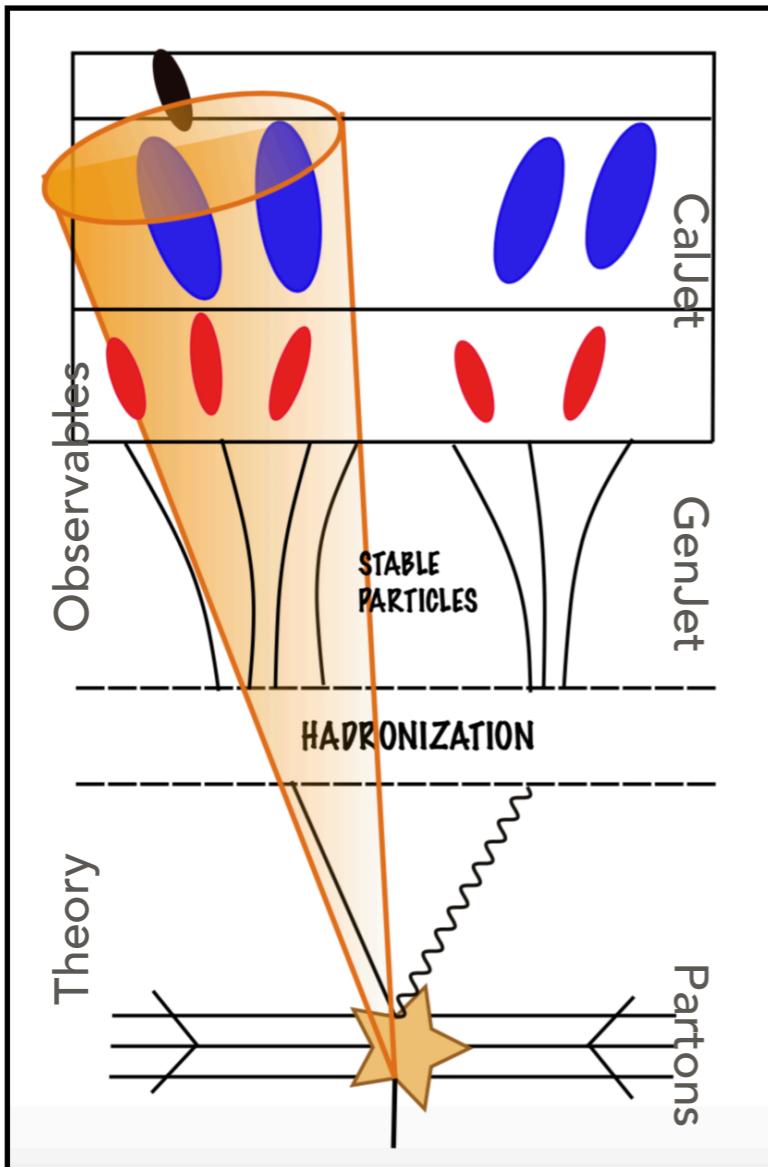
For  $R \rightarrow 0$  one recovers the (hadronic) TMD limit

Most interesting case for  
Belle and EIC!

# RADIUS AND JET ALGORITHMS

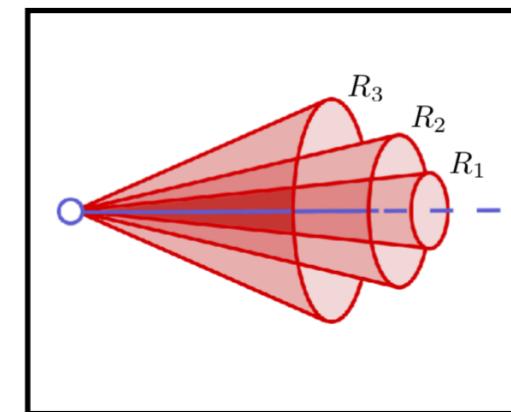
Standard kt-type algorithms

$$d_{ij} = \min(k_{T,1}^{2w}, k_{T,2}^{2w}) \frac{\Delta R_{ij}}{R}$$
$$w \in \{-1, 0, 1\}$$

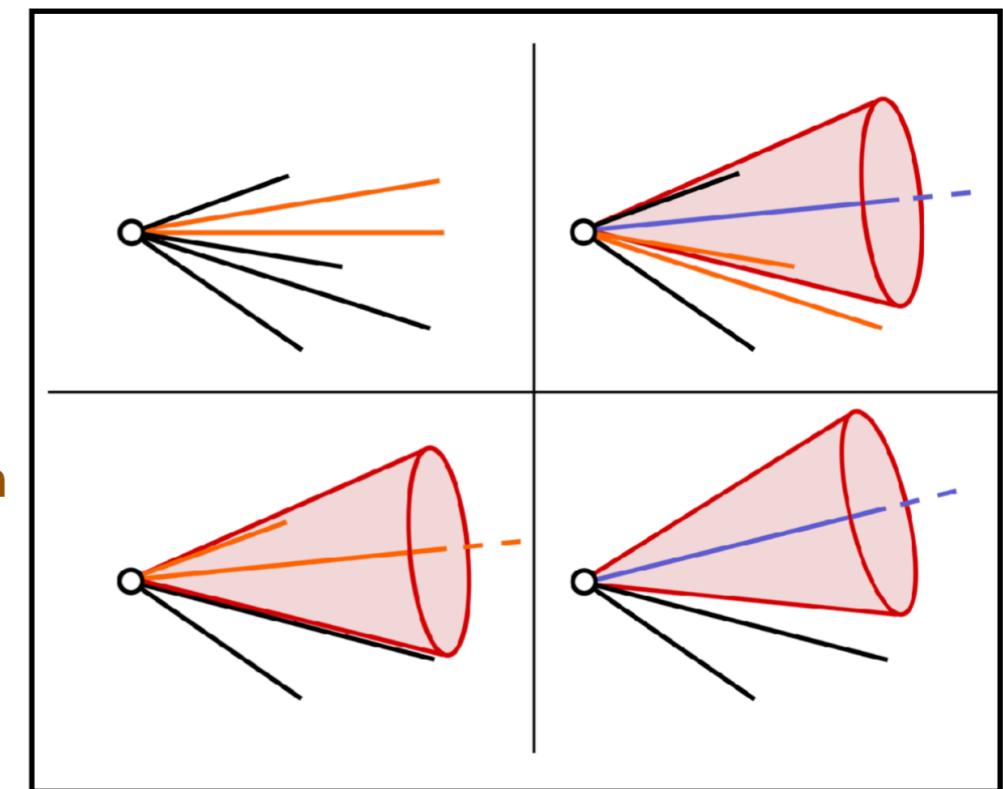


Building a jet...

Step 1:  
Set a size  
(Radius)



Step 2:  
Run a jet  
algorithm



# JET FUNCTIONS AND MODES

$$J_q(z, \mathbf{b}, E\mathcal{R}) = \frac{z}{2N_c} \text{Tr} \left[ \frac{\vec{\eta}}{2} \langle 0 | [\delta(2E/z - \bar{n} \cdot P) e^{i\mathbf{b} \cdot \mathbf{P}} \chi_n(0)] \sum_X |J_{\text{alg}, R} X\rangle \langle J_{\text{alg}, R} X | \bar{\chi}_n | 0 \rangle \right]$$

$$\mathcal{R} \equiv 2 \tan \frac{R}{2}$$

Mode	$R \ll \theta \ll 1$	$\theta \sim R \ll 1$	$\theta \ll R$ (WTA)	$\theta \ll R \ll 1$ (SJA)	$\theta \ll R \sim 1$ (SJA)
hard	(1,1,1)	(1,1,1)	(1,1,1)	(1,1,1)	(1,1,1)
$n$ -coll.	$(1, \sqrt{\theta}, \theta)$	$(1, \sqrt{\theta}, \theta)$	$(1, \sqrt{\theta}, \theta)$		
$\bar{n}$ -coll.	$(\sqrt{\theta}, 1, \theta)$	$(\sqrt{\theta}, 1, \theta)$	$(\sqrt{\theta}, 1, \theta)$		
$n$ -coll <sub>2</sub>	$(1, \sqrt{R}, R)$			$(1, \sqrt{R}, R)$	
$\bar{n}$ -coll <sub>2</sub>	$(\sqrt{R}, 1, R)$			$(\sqrt{R}, 1, R)$	
$n$ -csoft				$\theta/R(1, \sqrt{R}, R)$	
$\bar{n}$ -csoft				$\theta/R(\sqrt{R}, 1, R)$	
soft	$(\theta, \theta, \theta)$	$(\theta, \theta, \theta)$	$(\theta, \theta, \theta)$	$(\theta, \theta, \theta)$	$(\theta, \theta, \theta)$

# JET AXES

Larkoski, Neill, Thaler `14  
arXiv: 1401.2158

## Standard jet axis (SJA)

The sum of the momentum of collinear and soft particles is zero

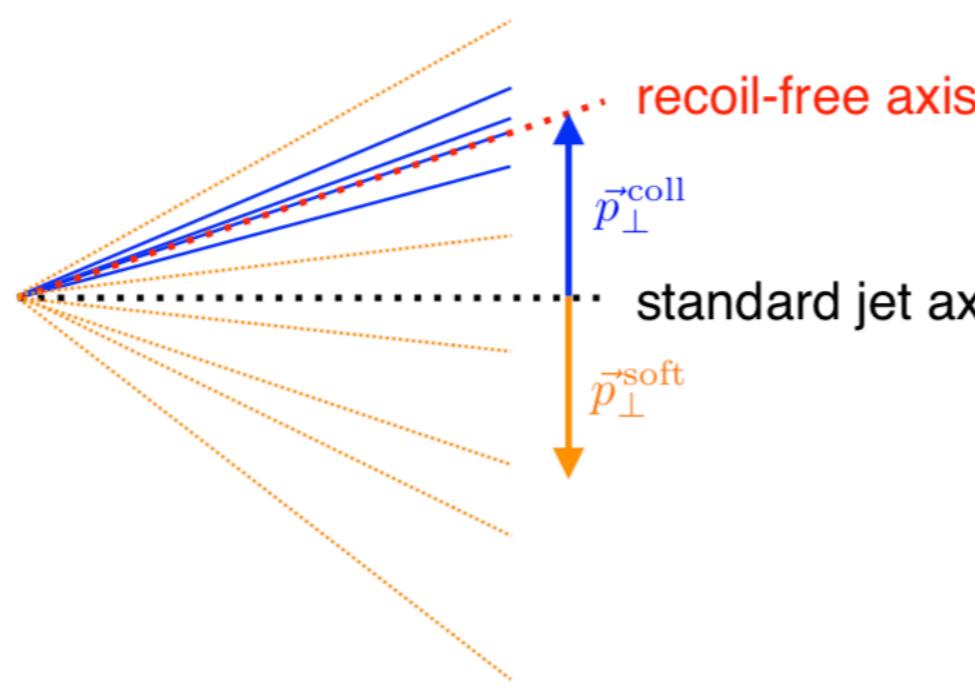
Introduces soft-sensitivity to the axis definition. Important with unintegrated transverse momentum

$$E_{(12)} = E_1 + E_2, \quad \vec{p}_{(12)} = \vec{p}_1 + \vec{p}_2,$$

Large R



$$E_{(12)} = E_1 + E_2, \quad \vec{p}_{(12)} = E_{(12)} \left[ \frac{\vec{p}_1}{|\vec{p}_1|} \theta(E_1 - E_2) + \frac{\vec{p}_2}{|\vec{p}_2|} \theta(E_2 - E_1) \right]$$



## Winner-take-all (WTA)

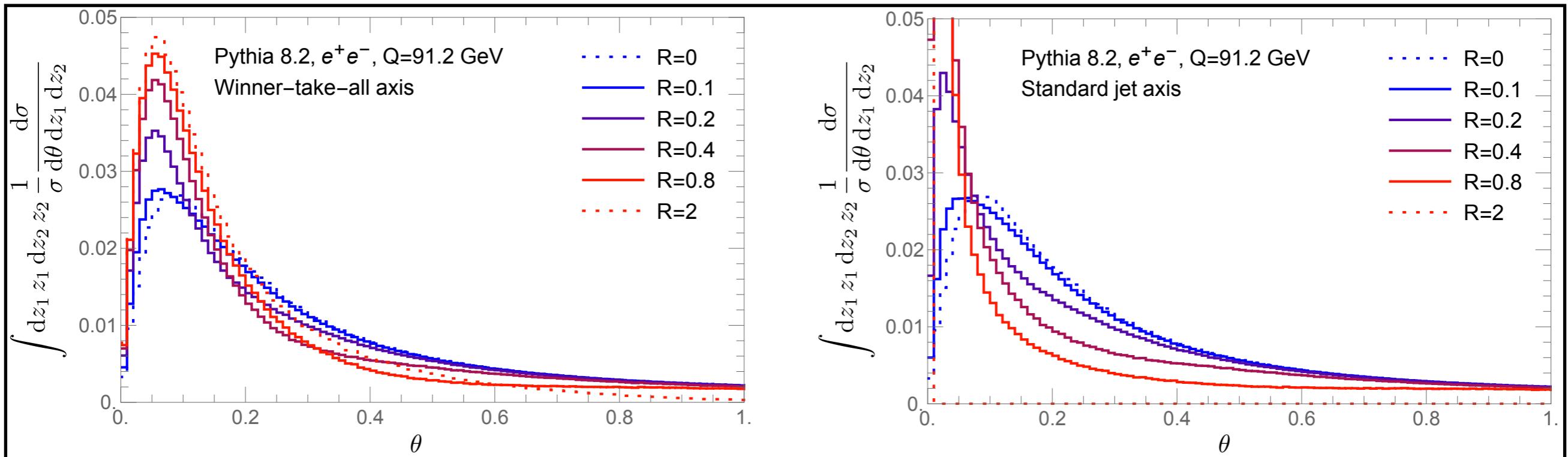
It always follows the direction of the most energetic particle

Recoil invariant. It is not sensitive to soft radiation

Large R



# CHECKING WITH PYTHIA 8.2.



Cross-section of angular decorrelation for different values of the radii of the jets

For small values of R the cross-section for both axis elections agrees!

For big values of R the cross section in **SJA** is inconsistent!

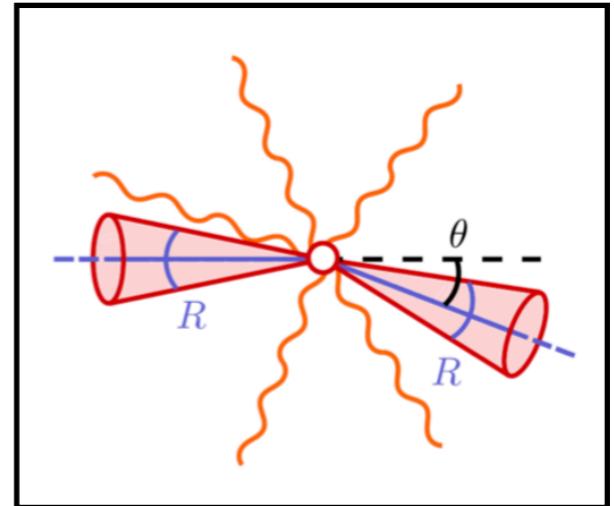
Factorization is broken

WTA axis **solves the problems!**

# FACTORIZATION

$$\theta \sim R \ll 1$$

The wide angle soft radiation does not resolve individual collinear emissions in the jet



$$\frac{d\sigma_{ee \rightarrow JJJX}}{dz_1 dz_2 d\mathbf{q}} = H(s, \mu) \int \frac{d\mathbf{b}}{(2\pi)^2} e^{-i\mathbf{b}\mathbf{q}} J_q^{\text{axis}} \left( z_1, \mathbf{b}, \frac{\sqrt{s}}{2}R, \mu, \zeta_1 \right) J_q^{\text{axis}} \left( z_2, \mathbf{b}, \frac{\sqrt{s}}{2}R, \mu, \zeta_2 \right) \left[ 1 + \mathcal{O} \left( \frac{\mathbf{q}^2}{s} \right) \right]$$

The soft function is the same as in (hadronic) TMD fragmentation

$$R \ll \theta \ll 1$$

Refactorization as in standard TMD case

$$J_i^{\text{axis}} = \sum_j \int \frac{dz'}{z'} \left[ (z')^2 \mathbb{C}(z', \mathbf{b}, \mu, \zeta) \right] \mathcal{J}_i \left( \frac{z}{z'}, \frac{\sqrt{s}}{2}R, \mu \right) \left[ 1 + \mathcal{O} \left( b^2 s^2 R^2 / 4 \right) \right]$$

NNLO TMDFFs Coefficients  
Echevarría, S., Vladimirov  
arXiv: 1604.07869



Coll. jet Function  
Kang, Ringer, Vitev, arXiv:1606.06732  
Dal, Kim, Leibovich, arXiv:1606.07411

# TMD SEMI-INCLUSIVE JET FUNCTION AT NLO

$$\begin{aligned}\theta &\ll 1 \\ \theta &\ll R\end{aligned}$$

Collinear radiation of typical angle  $\theta$   
sees the jet boundary infinitely far away



Independence of the radius of the jet!

The collinear radiation is mostly  
inside the jet



The dependence on  $z$  is power suppressed!

$$J_i^{\text{WTA}}(z, \mathbf{b}, ER, \mu, \zeta) = \delta(1 - z) \mathcal{J}_i^{\text{WTA}}(\mathbf{b}, \mu, \zeta) \left[ 1 + \mathcal{O} \left( \frac{1}{\mathbf{b}^2 E^2 R^2} \right) \right]$$

where

$$\mathcal{J}_i^{\text{WTA}}(\mathbf{b}, \mu, \zeta) = \frac{1}{2N_c(\bar{n} \cdot p_J)} \text{Tr} \left\{ \frac{\not{p}}{2} \langle 0 | e^{-i\mathbf{b}\mathbf{P}_\perp} \chi_n(0) | J_{\text{alg}}^{\text{WTA}} \rangle \langle J_{\text{alg}}^{\text{WTA}} | \bar{\chi}_n(0) | 0 \rangle \right\}$$

$$\mathcal{J}_i^{[0]\text{WTA}}(\mathbf{b}, \mu, \zeta) = 1$$

$$\mathcal{J}_i^{[1]\text{WTA}}(\mathbf{b}, \mu, \zeta) = 2 \left\{ N_i + L_\mu \left[ \mathcal{C}'_i + \mathcal{C}_i \left( \mathbf{l}_\zeta - \frac{1}{2} L_\mu \right) \right] \right\}$$

$$N_q = C_F \left( \frac{7}{2} - \frac{5\pi^2}{12} - 3 \ln 2 \right)$$

$$N_g = C_A \left( \frac{131}{36} - \frac{5\pi^2}{12} \right) - \frac{17}{18} n_f T_R - \beta_0 \ln 2$$

# TMD SEMI-INCLUSIVE JET FUNCTION AT NNLO

We know the evolution of the TMD jet function and in this limit it does not depend on radius or z

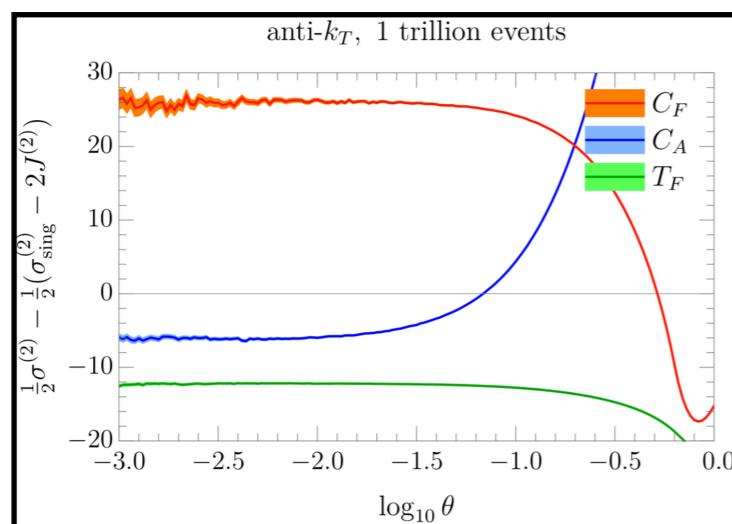


We can predict the **log behavior** of the two-loop jet function solving renormalization group equations and the **constant** can be numerically calculated (EVENT2)

$$\boxed{\begin{array}{l} \theta \ll 1 \\ \theta \ll R \end{array}}$$

Predicted by RG equations    
 Predicted by EVENT2

$$\mathcal{J}^{[2]\text{WTA}}(\mathbf{b}, \mu, \zeta) = \sum_{k=1}^4 \sum_{l=0}^k C_{kl} L_\mu^k \mathbf{l}_\zeta^l + C_0$$



$$C_0 = j_{C_F} + j_{C_A} + n_f j_{T_F}$$

# SUMMING UP ON WTA AXIS

Measuring jet decorrelation using WTA axes we have that  
factorization theorem holds for all jet radii.

$$R \ll \theta \ll 1$$
$$\theta \sim R \ll 1$$

$$\frac{d\sigma_{ee \rightarrow JJX}}{dq} = H(Q^2, \mu) \int \frac{d\mathbf{b}}{(2\pi)^2} e^{-i\mathbf{b}\mathbf{q}} J_f^{\text{WTA}} \left( \mathbf{b}, z, \frac{\sqrt{s}}{2}R; \mu_i, \zeta_i \right) J_f^{\text{WTA}} \left( \mathbf{b}, z \frac{\sqrt{s}}{2}R; \mu_i, \zeta_i \right) R^2[\mathbf{b}; (\mu_i, \zeta_i) \rightarrow (\mu, \zeta)]$$

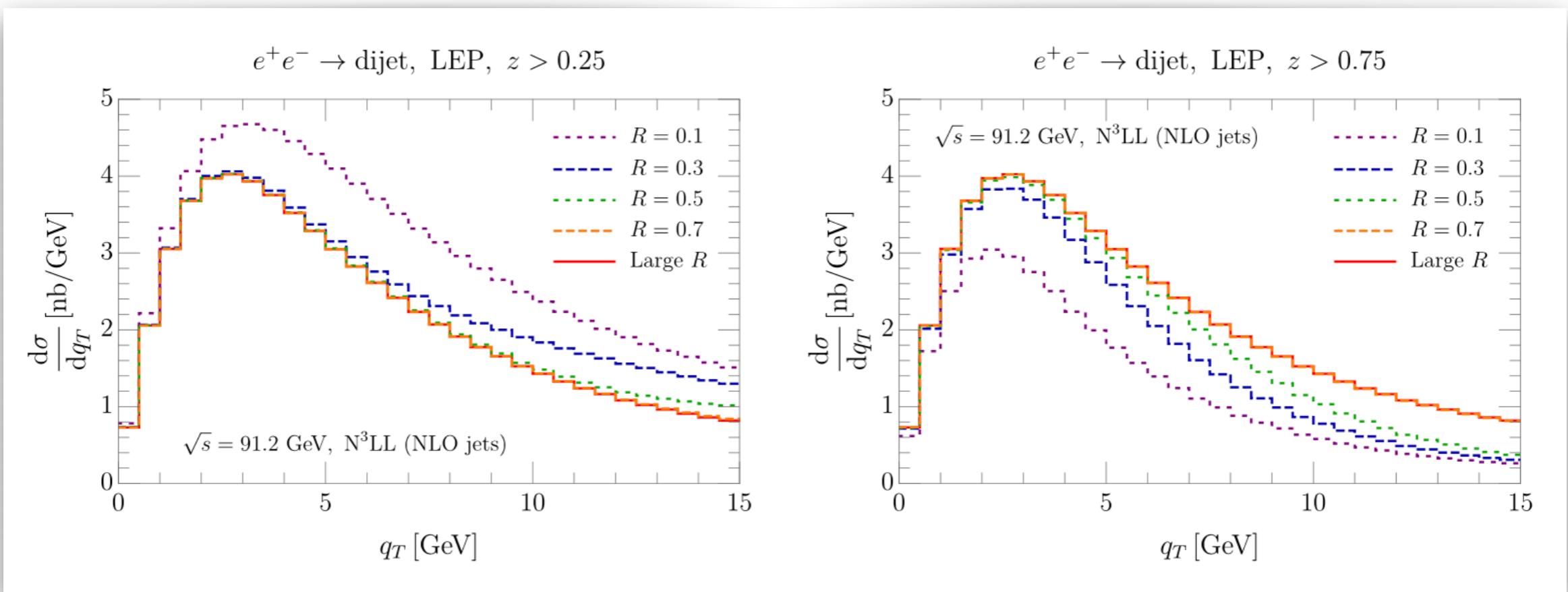
$$\theta \ll 1$$

$$\theta \ll R$$

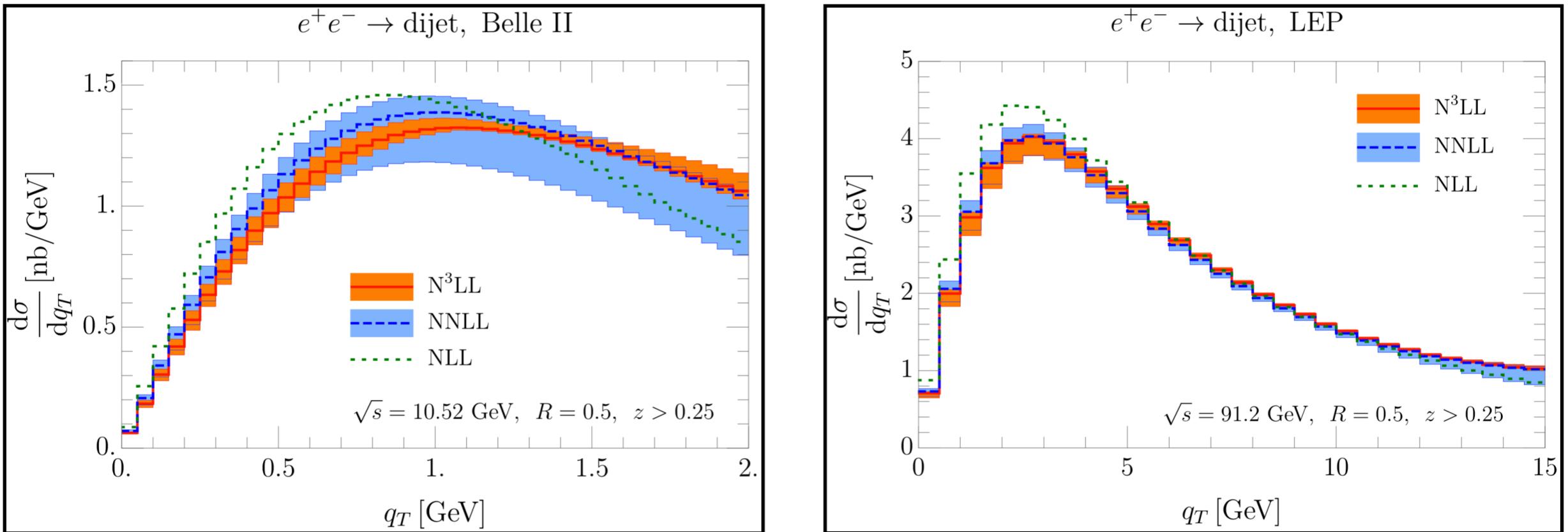
**TMD evolution kernel**

$$\frac{d\sigma_{ee \rightarrow JJX}}{dq} = H(Q^2, \mu) \int \frac{db}{(2\pi)^2} e^{-i\mathbf{b}\mathbf{q}} \mathcal{J}_f^{\text{WTA}}(\mathbf{b}; \mu_i, \zeta_i) \mathcal{J}_f^{\text{WTA}}(\mathbf{b}; \mu_i, \zeta_i) R^2[\mathbf{b}; (\mu_i, \zeta_i) \rightarrow (\mu, \zeta)]$$

# RADIUS DEPENDENCE AT LEP



# PERTURBATIVE CONVERGENCE: LARGE RADIUS

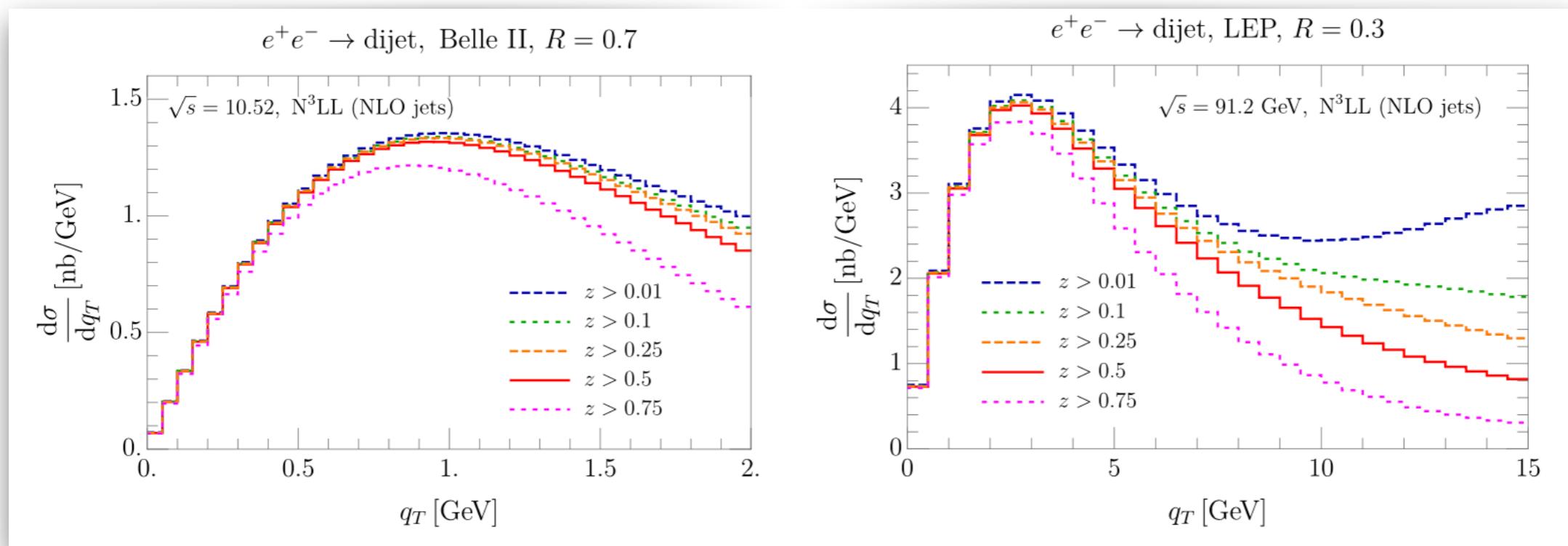


As the large-R jet function does not depend on radius or  $z$ , we can predict the two loop jet function by RG + Numerical constant (Event2)

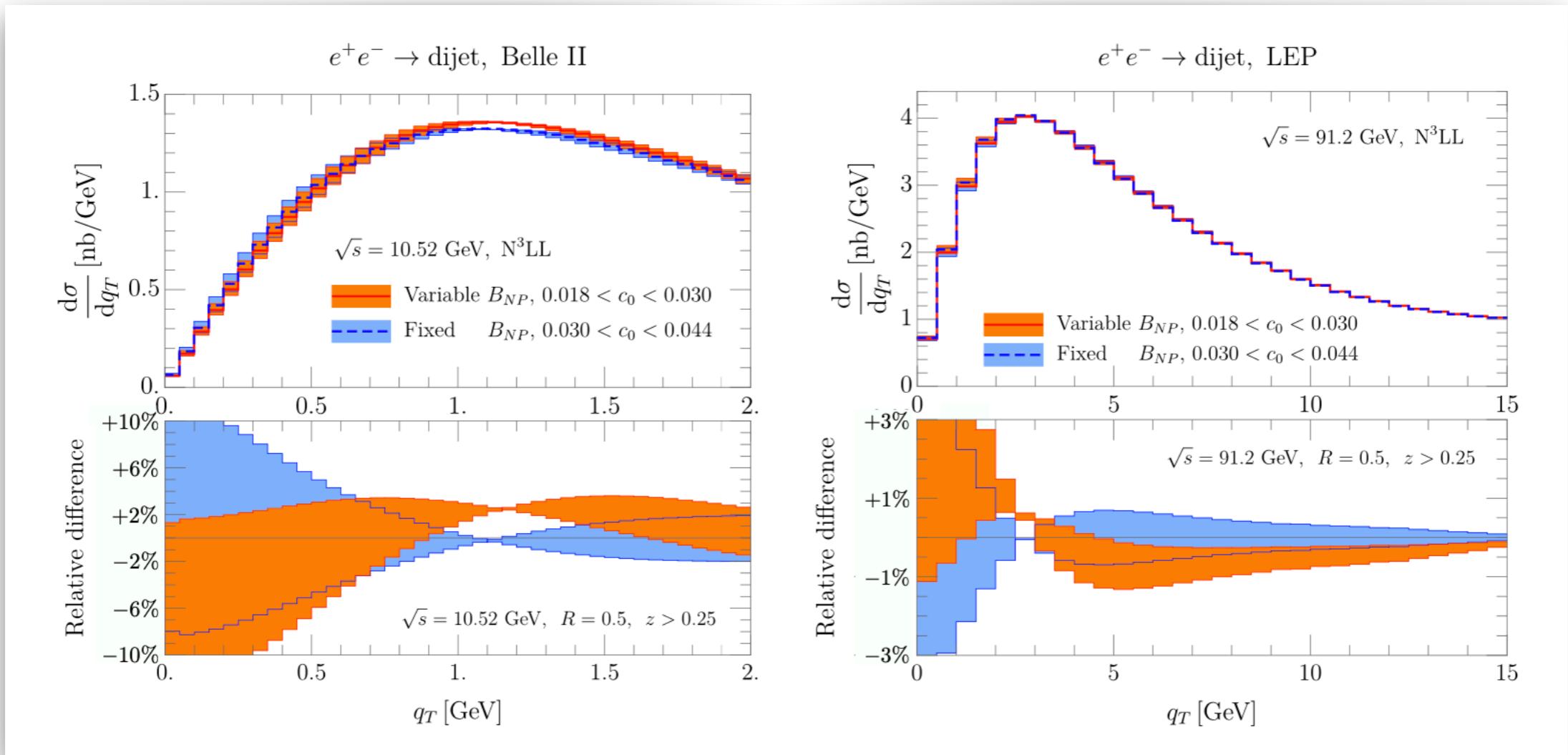
We use an improved NLO + NNLO large-R jet functions  
Resummation up to N3LL!

Theoretical errors are reduced when the perturbative order is increased!

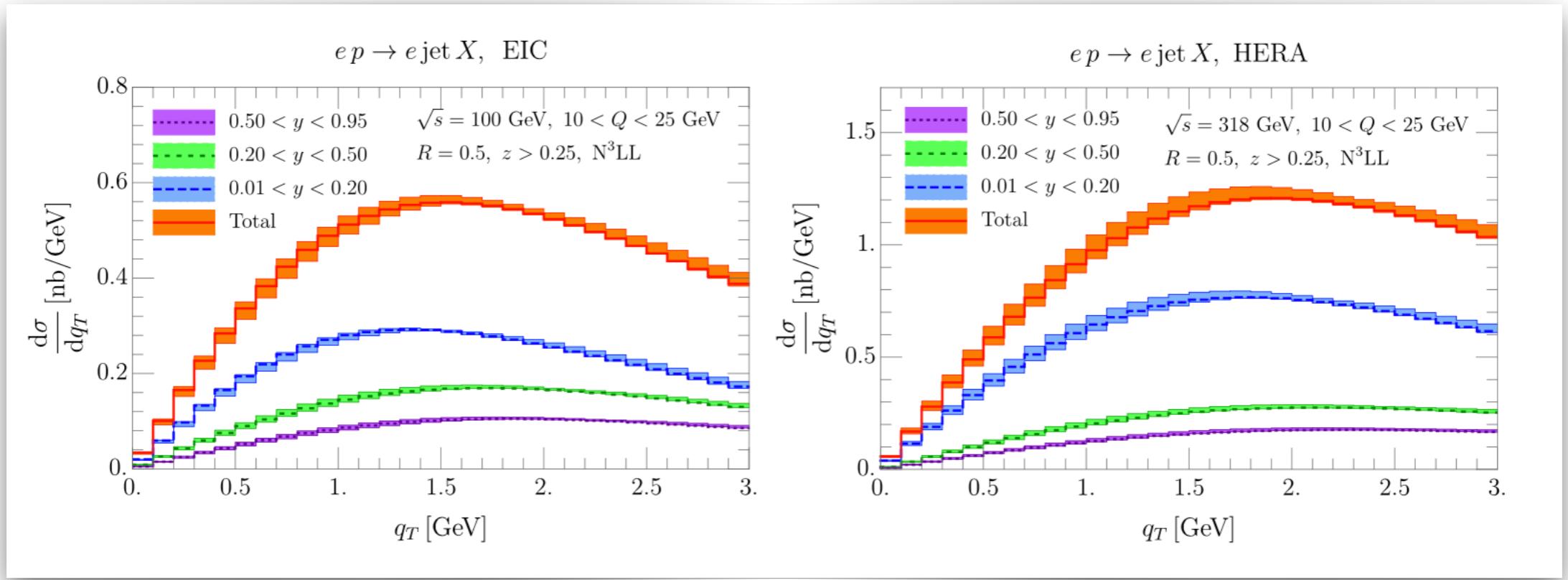
# Z-DEPENDENCE (BELLE, LEP)



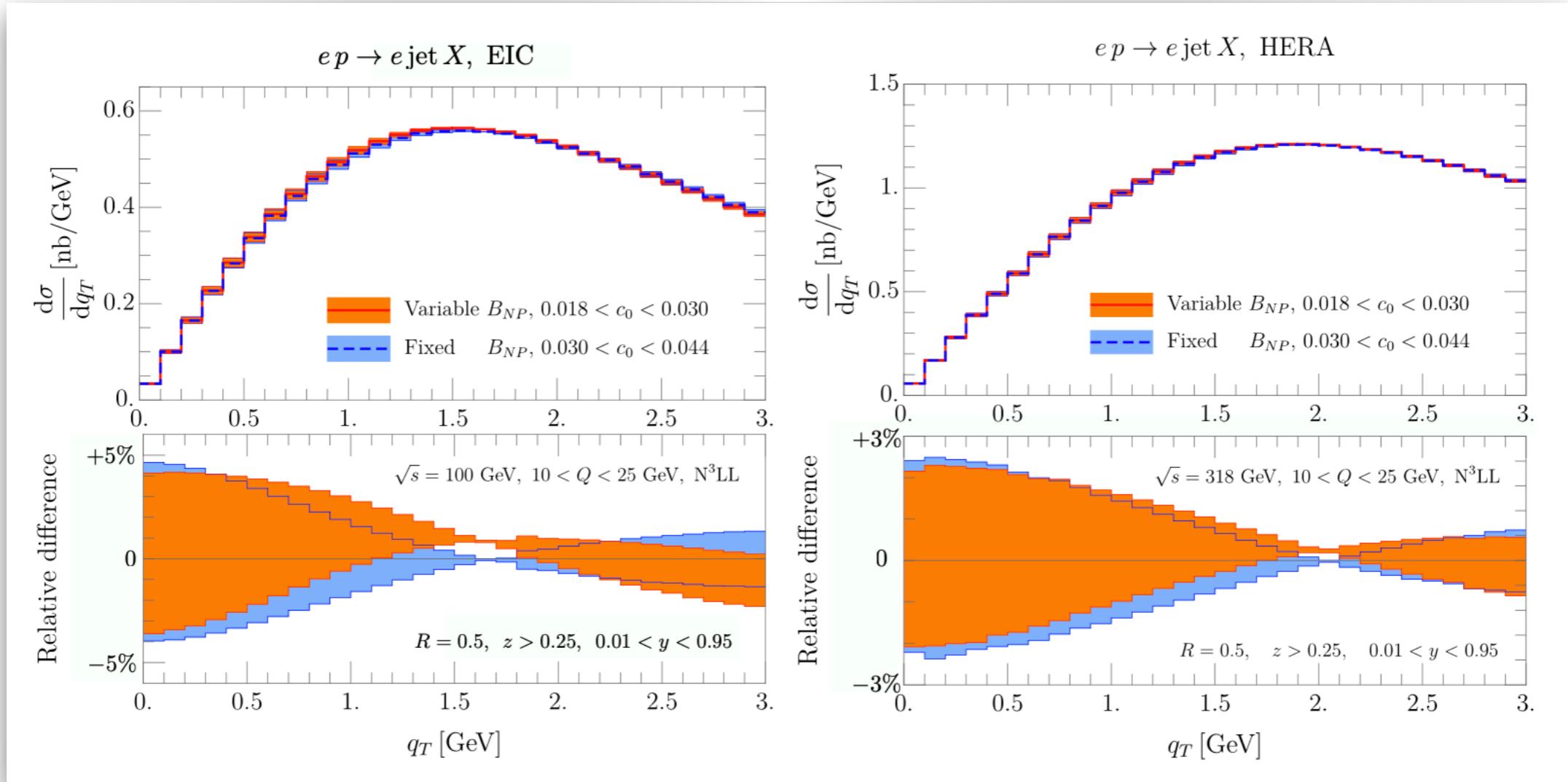
# TMD KERNEL (BELLE, LEP)



# SIDIS (EIC, HERA)



# SIDIS (EIC, HERA)



TODO: there can be a stronger dependence on the PDF sets

# CONCLUSIONS

- ✓ Jets offer an opportunity to study TMD and hadronization
- ✓ *There exist observables for which the factorization theorems have the same soft function for TMD and are independent of jet details like radius, algorithms*
- ✓ We expect that perturbation theory is more controllable with jets than with hadrons
- ✓ For the e+e- case we can achieve an incredible precision for the TMD evolution kernel

# BACKUP SLIDES

# DOUBLE-SCALE RENORMALIZATION GROUP EVOLUTION

Same RG evolution for **hadronic TMDs** and for **TMD jet functions!**

TMDs

$$\mu^2 \frac{d}{d\mu^2} D_i(z, \mathbf{b}, \mu, \zeta) = \frac{1}{2} \gamma_F^i(\mu, \zeta) D_i(z, \mathbf{b}, \mu, \zeta)$$

$$\zeta \frac{d}{d\zeta} D_i(z, \mathbf{b}, \mu, \zeta) = -\mathcal{D}^i(\mu, \mathbf{b}) D_i(z, \mathbf{b}, \mu, \zeta)$$

Jets

$$\mu^2 \frac{d}{d\mu^2} J_i^{\text{axis}}(z, \mathbf{b}, QR, \mu, \zeta) = \frac{1}{2} \gamma_F^i(\mu, \zeta) J_i^{\text{axis}}(z, \mathbf{b}, QR, \mu, \zeta)$$

$$\zeta \frac{d}{d\zeta} J_i^{\text{axis}}(z, \mathbf{b}, QR, \mu, \zeta) = -\mathcal{D}^i(\mu, \mathbf{b}) J_i^{\text{axis}}(z, \mathbf{b}, QR, \mu, \zeta)$$

# DOUBLE-SCALE RENORMALIZATION GROUP EVOLUTION

They have a common **evolution factor**

TMDs

$$D_i(z, \mathbf{b}, \mu_f, \zeta_f) = \exp \left[ \int_{(\mu_i, \zeta_i)}^{(\mu_f, \zeta_f)} \left( \gamma_F^i(\mu, \zeta) \frac{d\mu}{\mu} - \mathcal{D}^i(\mu, \mathbf{b}) \frac{d\zeta}{\zeta} \right) \right] D_i(z, \mathbf{b}, \mu_i, \zeta_i)$$

Jets

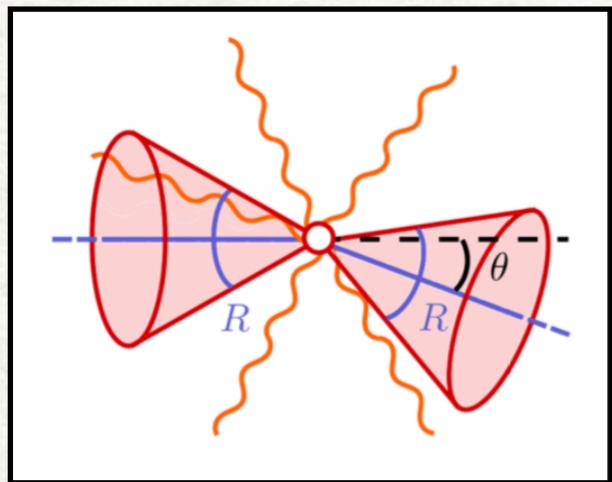
$$J_i^{\text{axis}}(z, \mathbf{b}, QR, \mu_f, \zeta_f) = \exp \left[ \int_{(\mu_i, \zeta_i)}^{(\mu_f, \zeta_f)} \left( \gamma_F^i(\mu, \zeta) \frac{d\mu}{\mu} - \mathcal{D}^i(\mu, \mathbf{b}) \frac{d\zeta}{\zeta} \right) \right] J_i^{\text{axis}}(z, \mathbf{b}, QR, \mu_i, \zeta_i)$$

This fact makes phenomenological analysis simpler!

# FACTORIZATION: UNGROOMED JETS

$$\theta \ll 1 \quad \theta \ll R$$

## Standard Jet Axis



The SJA is aligned with the total momentum of the jet



Hard splittings with typical angle  $R$  are allowed inside the jet, generating additional soft radiation



Factorization broken!

$$\frac{d\sigma_{(ee \rightarrow JJX)}^{\text{SJA}}}{d\mathbf{q}} = \sum_{m=2}^{\infty} \text{Tr}_c [\mathcal{H}_m(\{n_i\}) \otimes \mathcal{S}_m(\mathbf{q}, \{n_i\})]$$

