Ultraviolet divergences and evolution in weighted transverse moments

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Based on work with Jianwei Qiu and Bowen Wang:

Phys.Rev.D 101 (2020) 11, 116017, 2004.13193
and 2008.05351

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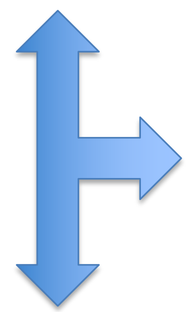
Transverse momentum in correlation functions and in cross sections

- Correlation functions
  - Parton densities (pdfs), fragmentation functions, others…

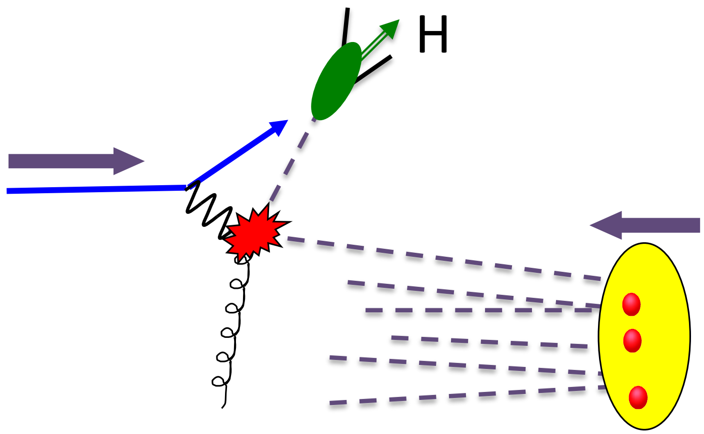


Factorization

- Cross sections
  - Semi-inclusive deep inelastic scattering, Drell-Yan, etc…

Semi-inclusive deep inelastic scattering

$$\frac{d\sigma^{\mathrm{SIDIS}}}{dx\,dy\,dz\,d^2\mathbf{P}_{hT}}$$

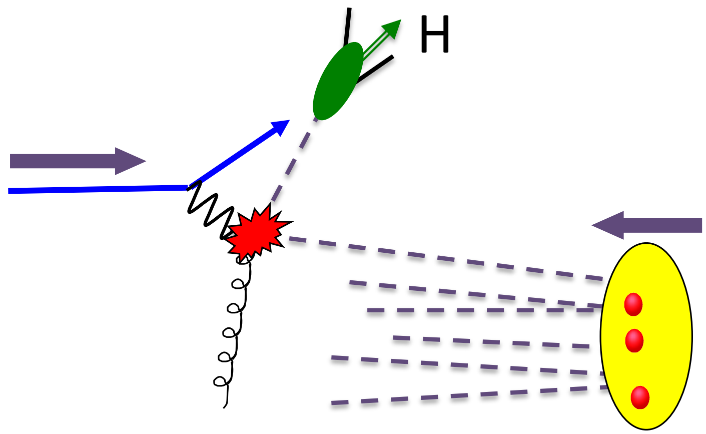


- *Large P_T, insensitive to intrinsic parton transverse momentum.*

➡ *Collinear factorization and evolution (DGLAP, etc)*

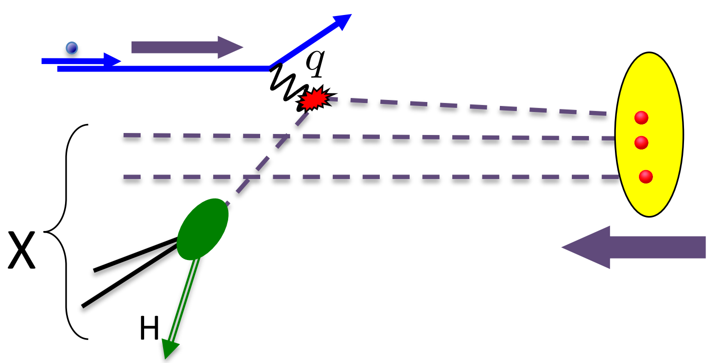
Semi-inclusive deep inelastic scattering

$$\frac{\mathrm{d}\sigma^{\mathrm{SIDIS}}}{\mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}^2\mathbf{P}_{hT}}$$



- *Large P_T, insensitive to intrinsic parton transverse momentum.*

➡ *Collinear factorization and evolution (DGLAP, etc)*



- *Small P_T, access to intrinsic parton transverse momentum*

➡ *TMD factorization, TMD evolution, Sudakov, etc*

Transverse momentum dependence and factorization

$$\frac{d\sigma}{dx\,dz\,dQ\,d\boldsymbol{q}_T} = \underbrace{H(Q) f(x, \boldsymbol{k}_{1T}) \otimes d(z, z\boldsymbol{k}_{2T})}_{\substack{\text{Small } q_T/\text{Q} \\ \text{TMD factorization}}} + \underbrace{Y(x, z, \boldsymbol{q}_T, Q)}_{\substack{q_T \sim Q \\ \text{collinear factorization} \\ (\text{Small } m/q_T)}} + O(m/Q)$$

Transverse momentum dependence and factorization

Extra scales for TMD evolution

Collinear / DGLAP evolution scale, μ

$$\frac{d\sigma}{dx\ dz\ dQ\ d\boldsymbol{q}_T} = \underbrace{H(Q) f(x, \boldsymbol{k}_{1T}) \otimes d(z, z\boldsymbol{k}_{2T})}_{\text{Small } q_T/Q \text{ TMD factorization}} + \underbrace{Y(x, z, \boldsymbol{q}_T, Q)}_{q_T \sim Q \text{ collinear factorization (Small } m/q_T)} + O(m/Q)$$

Small q_T/Q
TMD factorization

$q_T \sim Q$
collinear factorization
(Small m/q_T)

Transverse momentum dependence and factorization

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$$\frac{d\sigma}{dx\,dz\,dQ\,d\boldsymbol{q}_T} = H(Q) f(x, \boldsymbol{k}_{1T}) \otimes d(z, z\boldsymbol{k}_{2T}) + Y(x, z, \boldsymbol{q}_T, Q) + O(m/Q)$$

Small q_T/Q
TMD factorization

$q_T \sim Q$
collinear factorization
(Small m/q_T)

- *There is an overlapping collinear/TMD description for $m \ll q_T \ll Q$*

For single-spin asymmetries:

X. Ji, J.-W. Qiu, W. Vogelsang, and F. Yuan, Phys. Rev. Lett. 97, 082002 (2006)
X. Ji, J.-W. Qiu, W. Vogelsang, and F. Yuan, Phys. Rev. D73, 094017 (2006)
I. Scimemi, A. Tarasov, and A. Vladimirov, JHEP 05, 125 (2019)

Integrated observables

$$\frac{d\sigma}{dx\,dz\,dQ\,d\boldsymbol{q}_T} = H(Q) f(x, \boldsymbol{k}_{1T}) \otimes d(z, z\boldsymbol{k}_{2T}) + Y(x, z, \boldsymbol{q}_T, Q) + O(m/Q)$$

- *Unpolarized*

$$\int d^2\boldsymbol{q}_T \frac{d\sigma}{dx\,dz\,dQ\,d\boldsymbol{q}_T} = \int_x^1 \frac{d\xi}{\xi} \int_z^1 \frac{d\zeta}{\zeta^2} H(Q, x/\xi\,, z/\zeta) f(\xi) d(\zeta) + O(m/Q)$$

Integrated observables

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- *Unpolarized, approximated*

$$\int d^2\boldsymbol{q}_T \frac{d\sigma}{dx\, dz\, dQ\, d\boldsymbol{q}_T} \approx H(Q) f(x) \otimes d(z)$$

$$\int d^2 k_{1T}\, f(x, k_{1T}) \approx f(x)$$

$$\int d^2 k_{2T}\, d(z, zk_{2T}) \approx d(x)$$

Integrated observables

$$\frac{d\sigma}{dx\, dz\, dQ\, d\boldsymbol{q}_T} = H(Q) f(x, \boldsymbol{k}_{1T}) \otimes d(z, z\boldsymbol{k}_{2T}) + Y(x, z, \boldsymbol{q}_T, Q) + O(m/Q)$$

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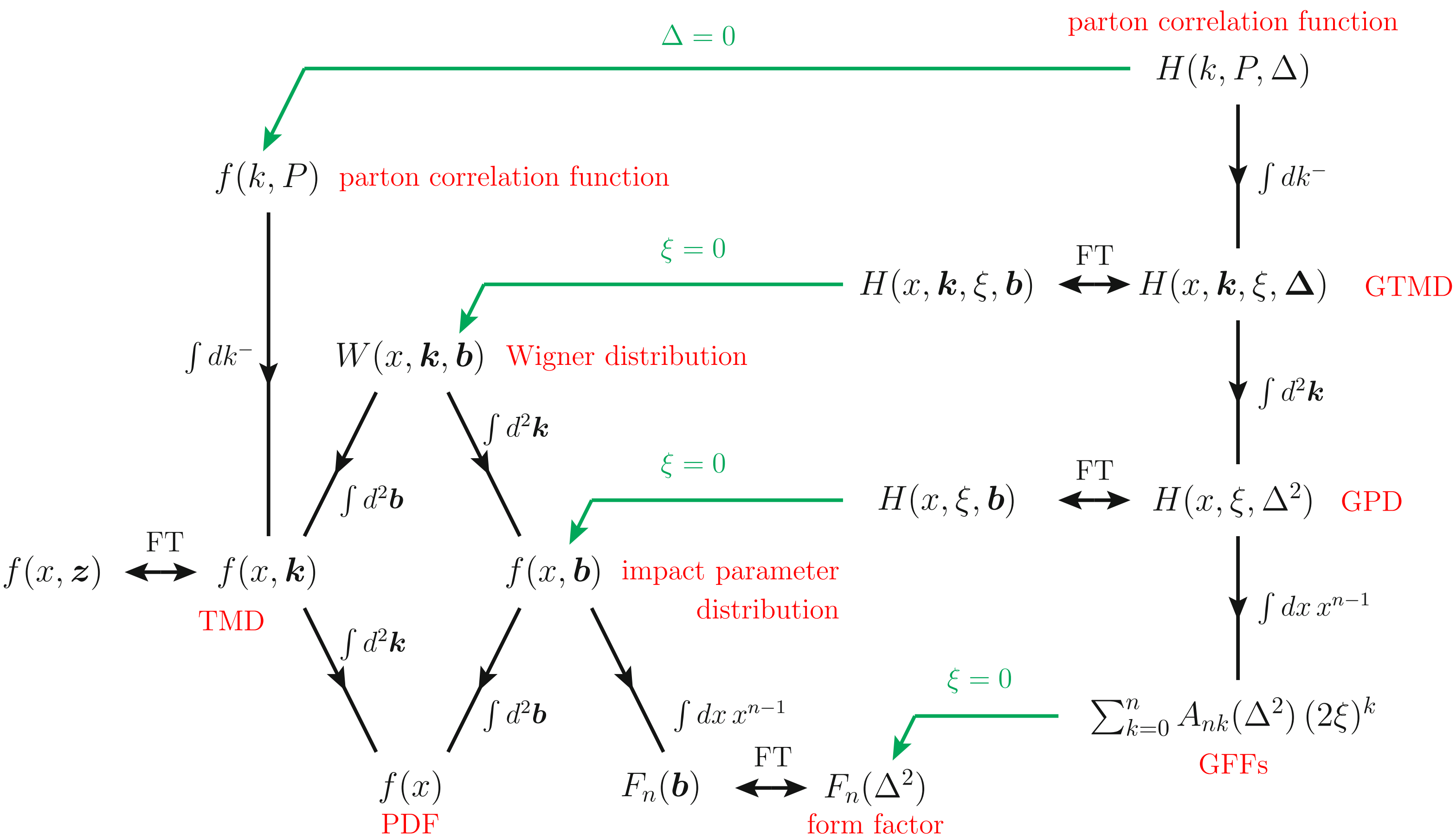
$$+O(\alpha_s(k_c))$$

$k_c =$ transverse momentum cutoff

Tiers of transverse momentum dependence



M. Diehl, Eur.Phys.J.A 52 (2016) 6, 149

Weighted integrals

$$\int \mathrm{d}^2\mathbf{P}_{h\mathrm{T}}\,(P^{\alpha}_{h\mathrm{T}})^n\,\frac{\mathrm{d}\sigma^{\mathrm{SIDIS}}}{\mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}^2\mathbf{P}_{h\mathrm{T}}}$$

Quark \ Proton	Unpolarized	Longitudinally polarized	Transversely polarized
Unpolarized	$f_1(x,k_T)$	✖	$f_{1T}^{\perp}(x,k_T)$
Longitudinally polarized	✖	$g_{1L}(x,k_T)$	$g_{1T}(x,k_T)$
Transversely polarized	$h_1^{\perp}(x,k_T)$	$h_{1L}(x,k_T)$	$h_{1T}(x,k_T)$ $h_{1T}^{\perp}(x,k_T)$

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- T-odd effects

$$\int d^2\mathbf{k}_T\,\frac{k_T^2}{2M^2} f_{1\perp}(x,k_T) = f_{1\perp}^{(1)}(x) \approx \frac{1}{M}T(x)$$

Boer, Mulders, Pijlman, Nucl. Phys.B667, 201 (2003)

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- Lorentz-invariant relations

$$g_T(x)=g_1(x)+\frac{d}{dx}g_{1T}^{(1)}(x),$$

$$g_{1T}^{(1)}(x)\equiv\int d^2\boldsymbol{k}_T\,\frac{k_T^2}{2\,M^2}g_{1T}(x,k_T)$$

Mulders, Tangerman, Nucl. Phys. B461, 197 (1996)

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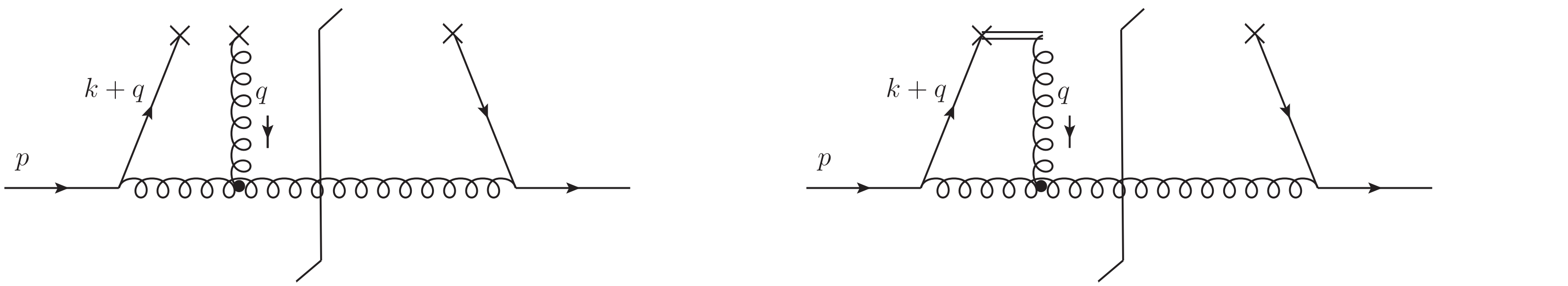
- *Which kind of evolution?*

Renormalization and regularization



J.-W. Qiu, TCR, B. Wang, Phys.Rev.D 101 (2020) 11, 116017

$$\left[f_{1\perp}^{(1)}(x)\right]^{\text{cutoff}} - \left[\frac{1}{M}T(x)\right]^{\overline{\text{MS}}\text{ renorm}} \sim \alpha_s^2(k_c)\ln^2\left(\frac{k_c^2}{m^2}\right)$$

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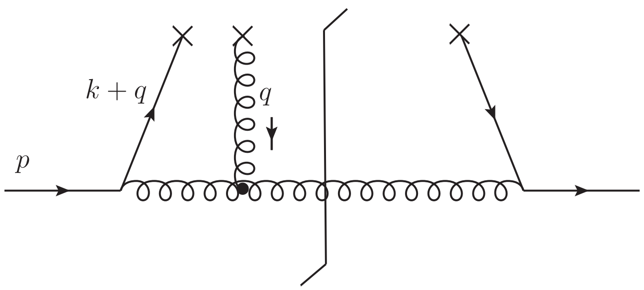

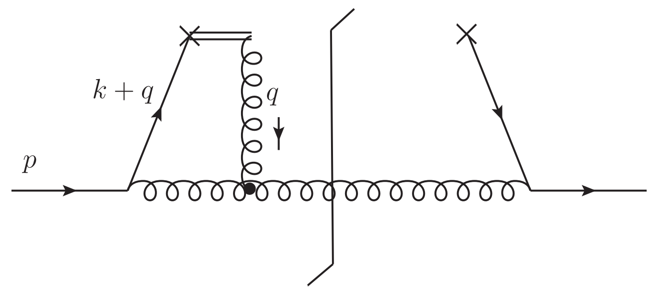

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$$\alpha_s^2(k_c) \sim 1/\ln^2(k_c^2/m^2)$$

Difference unsuppressed by asymptotic freedom

Phenomenology

- TMD / higher-twist frequently used interchangeably:

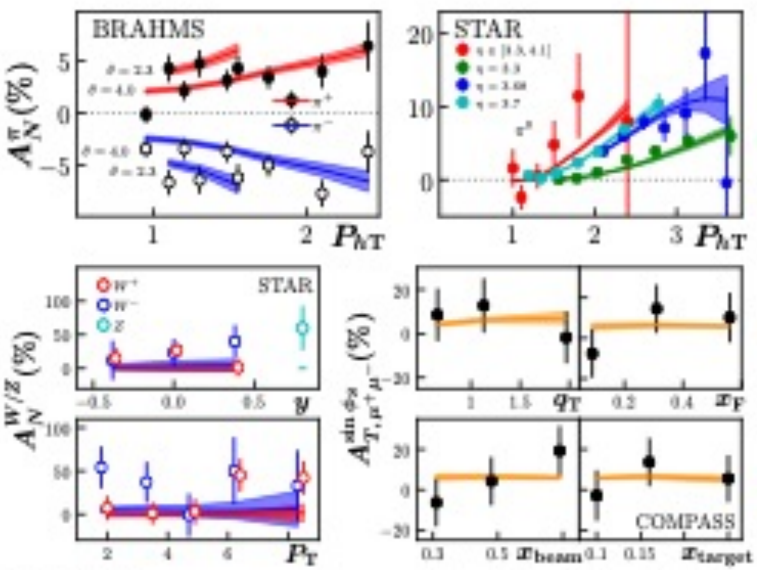

FIG. 5. Theory compared to experiment for A_N^{π} and $A_{\mathrm{DY}}^{\mathrm{Siv}}$.

J. Cammarota et al, Phys.Rev.D 102 (2020) 5, 054002

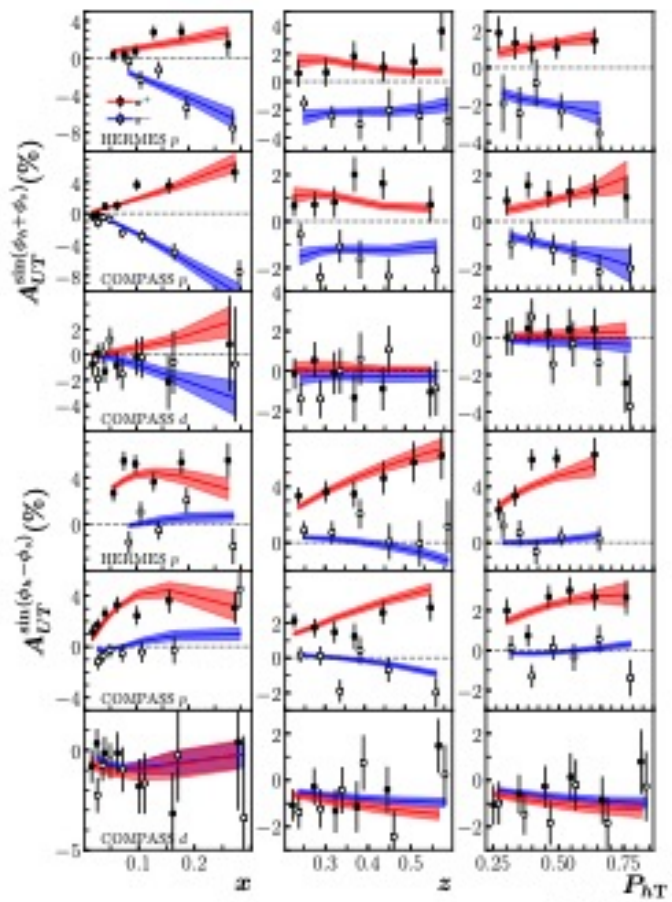

FIG. 4. Theory compared to experiment for $A_{\mathrm{SIDIS}}^{\mathrm{Col/Siv}}$.

Discussion

- Ultraviolet divergences in transversely-integrated quantities are related to:
  - Identifying intrinsic versus process-specific effects
  - Evolution

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- For weighted asymmetries, asymptotic freedom does not always suppress errors to the naïve number density interpretation

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- When access to the intrinsic transverse momentum is the objective:
  - Limit transverse momentum in weighted integrals and use TMD evolution

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- How to merge collinear higher twist and TMD in integrated quantities?

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