

Global analysis of the Sivers functions at NLO+NNLL in QCD (2009.10710)

John Terry

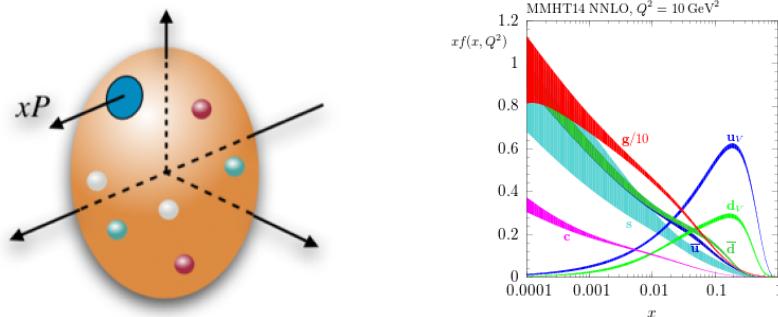
In collaboration with Miguel Echevarria and Zhong-bo Kang

November 22, 2020

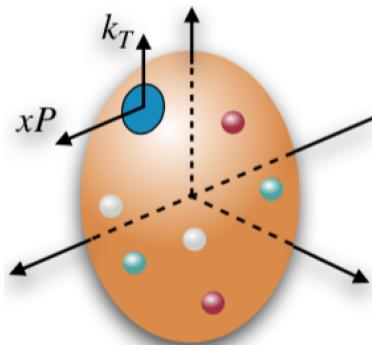


UCLA Mani L. Bhaumik Institute
for Theoretical Physics

Motivation for Transverse Momentum Distributions (TMDs)



PDFs, momentum densities along collinear direction.

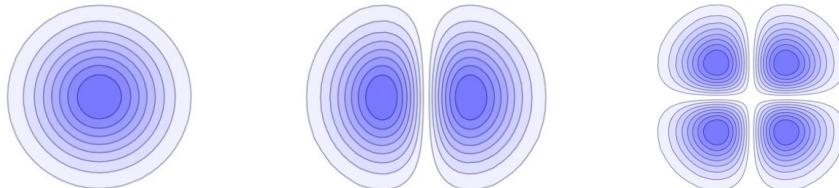
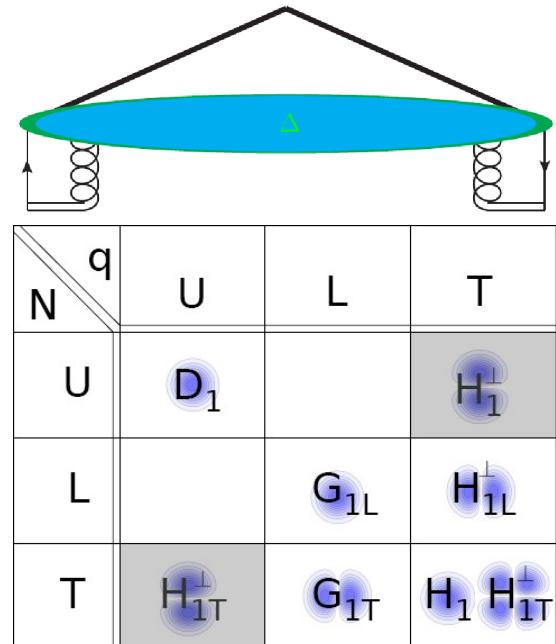
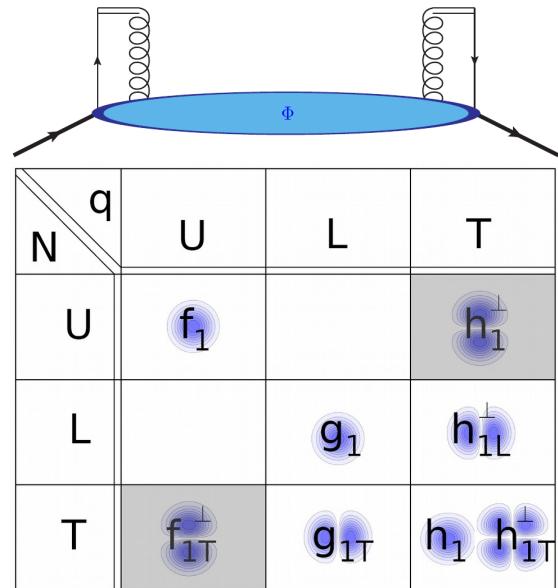


Goals of the TMD community:

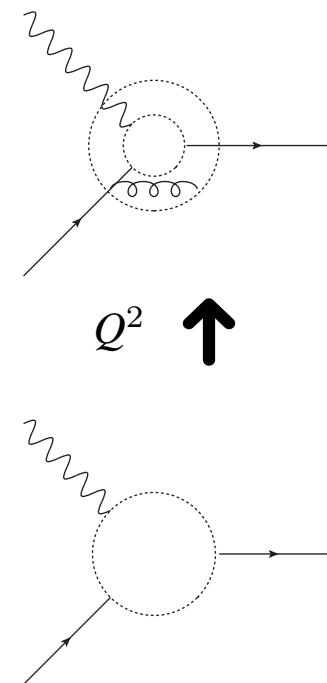
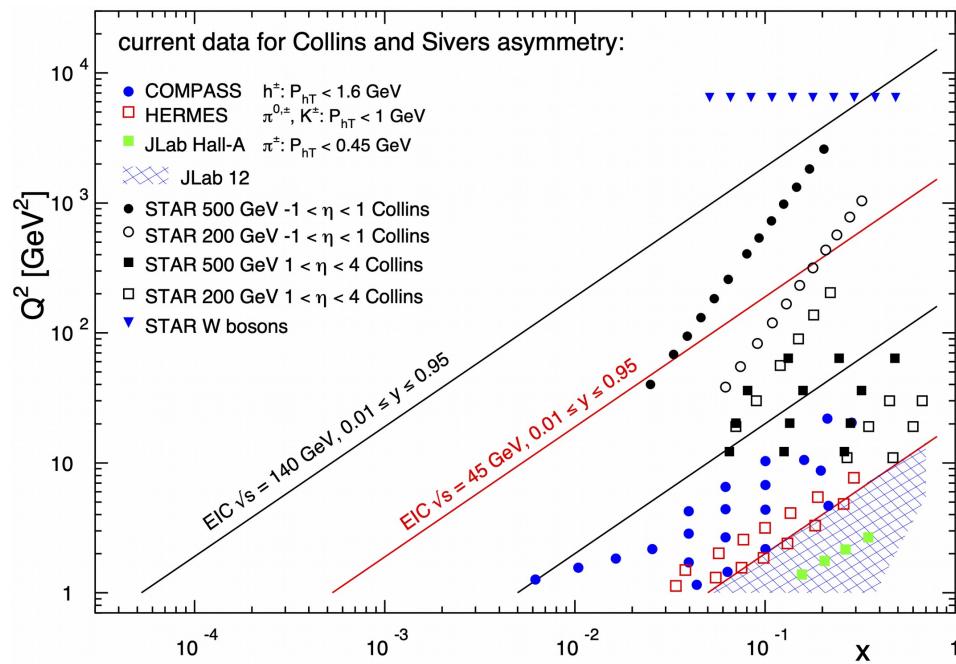
- Increase the precision at which we understand this substructure.
- Understand the evolution of TMDs in Q .
- Understand correlations between the spin and the transverse momentum.

How do spin degrees of freedom affect the TMDs? TMDPDFs

TMDFFs

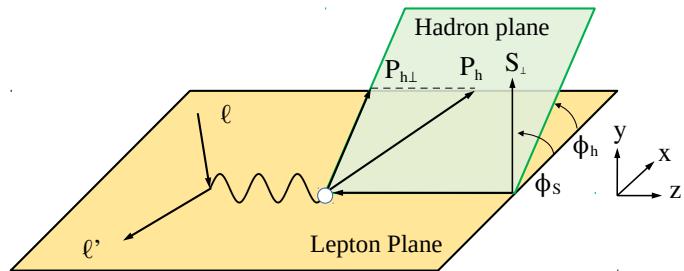
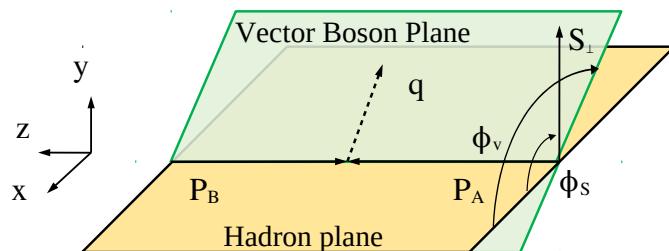


How do these correlations change in Q^2 ?



- DGLAP evolution provides information on how the x_B dependence of the distributions evolve in Q .
- TMD evolution provides information on how the k_\perp dependence of the distributions evolve in Q .

The Sivers asymmetry



Drell-Yan
W/Z Boson Production

$$= f_{q/p}(x_B, k_\perp) - \sin(\phi_s - \phi_k) \frac{k_\perp}{M} f_{1T,q/p}^\perp(x_B, k_\perp)$$

$$A_{\text{UT}}^{\sin(\phi_h - \phi_s)} \sim \frac{d\sigma(\mathbf{S}_\perp) - d\sigma(-\mathbf{S}_\perp)}{d\sigma(\mathbf{S}_\perp) + d\sigma(-\mathbf{S}_\perp)} \sim \frac{F_{UT}^{\sin(\phi_h - \phi_s)}}{F_{UU}}$$

$$\frac{F_{UT}^{\sin(\phi_h - \phi_s)}}{F_{UU}} \sim \frac{f_{1T,q/p}^\perp(x_B, k_\perp)}{f_{q/p}(x_B, k_\perp)}.$$

Unpolarized structure function in SIDIS

Unpolarized structure function in the TMD formalism

$$F_{UU} (x_B, z_h, P_{h\perp}, Q) = \int_0^\infty \frac{bdb}{2\pi} J_0 \left(\frac{b P_{h\perp}}{z_h} \right) \tilde{F}_{UU} (x_B, z_h, b, Q)$$

$$\begin{aligned} \tilde{F}_{UU} (x_B, z_h, b, Q) \equiv & \textcolor{orange}{H}(Q^2) \sum_q e_q^2 \textcolor{blue}{C}_{q \leftarrow i} \otimes f_{i/p} (x_B, \mu_b) \hat{C}_{j \leftarrow q} \otimes \textcolor{green}{D}_{h/j} (z_h, \mu_b) \\ & \times \exp \left(- \textcolor{violet}{S}_{pert} - \textcolor{cyan}{S}_{NP} \right) \end{aligned}$$

$$\mu_b \sim 1/b$$

- DGLAP evolution from initial scale to μ_b .
- DGLAP evolution from initial scale to μ_b .
- Perturbative TMD evolution from μ_b to Q .
- Non-perturbative TMD evolution from initial scale to Q .

Parameterization for Unpolarized Structure Functions

Coefficient functions are taken at NLO.

$$\left[C_{q \leftarrow i} \otimes f_{i/p} \right] (x, b, Q) = \int_x^1 \frac{d\hat{x}}{\hat{x}} C_{q \leftarrow i} \left(\frac{x}{\hat{x}}, b, Q \right) f_{i/p} (\hat{x}, Q) .$$

Perturbative evolution is taken at NNLL

$$S_{\text{pert}}(b, Q) = \int_{\mu_b}^Q \frac{d\mu'}{\mu'} \left[\gamma^Y + \Gamma_{\text{cusp}} \ln \left(\frac{Q^2}{\mu'^2} \right) \right] + D(b; \mu_b) \ln \left(\frac{Q^2}{\mu_b^2} \right) .$$

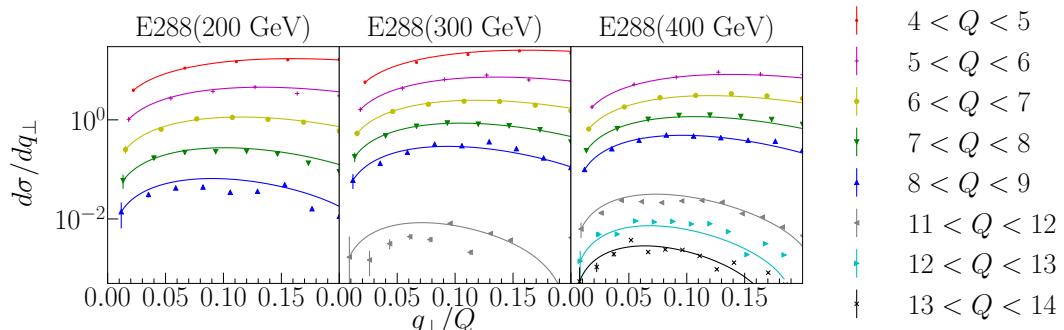
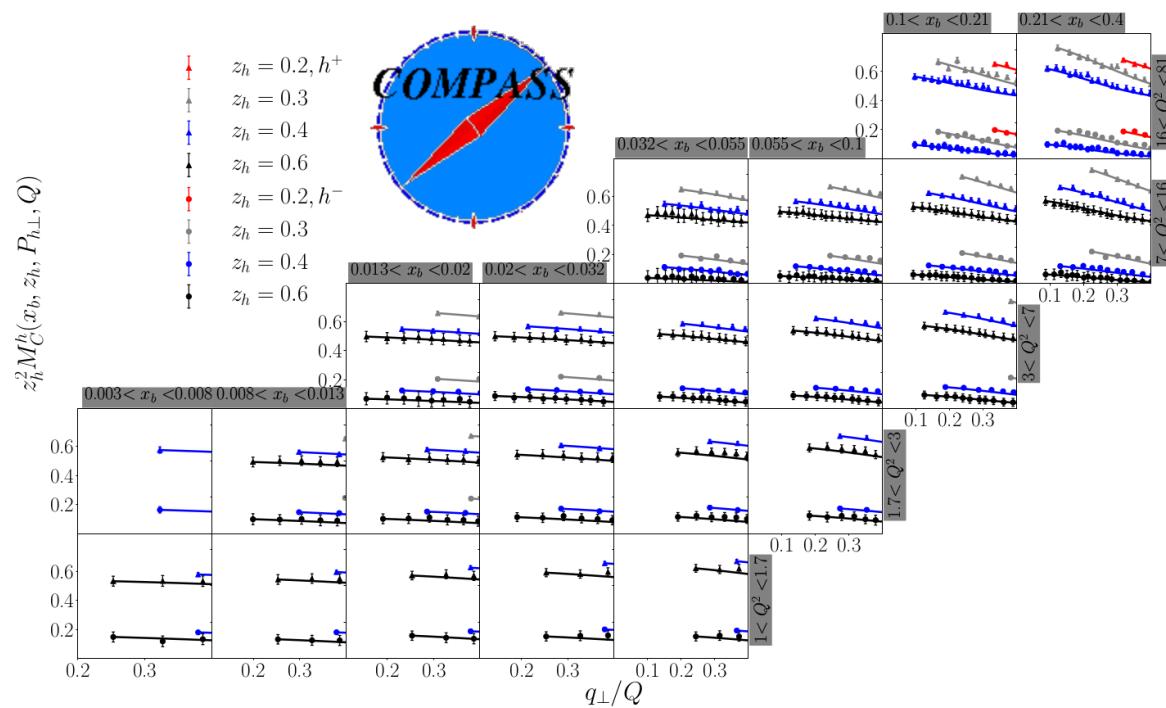
$$S_{\text{NP}}^f(b; Q_0, Q) = g_1^f b^2 + \frac{g_2}{2} \ln \frac{Q}{Q_0} \ln \frac{b}{b_*}$$

$$S_{\text{NP}}^D(z, b; Q_0, Q) = g_1^D \frac{b^2}{z^2} + \frac{g_2}{2} \ln \frac{Q}{Q_0} \ln \frac{b}{b_*}$$

$$g_1^f = 0.106 \text{GeV}^2 \quad g_1^D = 0.042 \text{GeV}^2 \quad g_2 = 0.84 \quad Q_0^2 = 2.4 \text{GeV}^2$$

$$g_1^\pi = 0.082 \text{GeV}^2$$

Unpolarized data

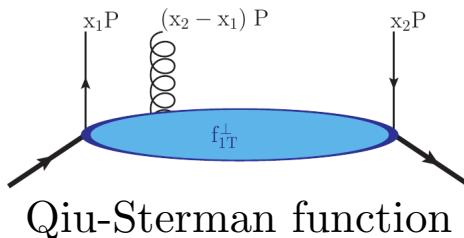


Polarized structure functions

Polarized structure function in the TMD formalism

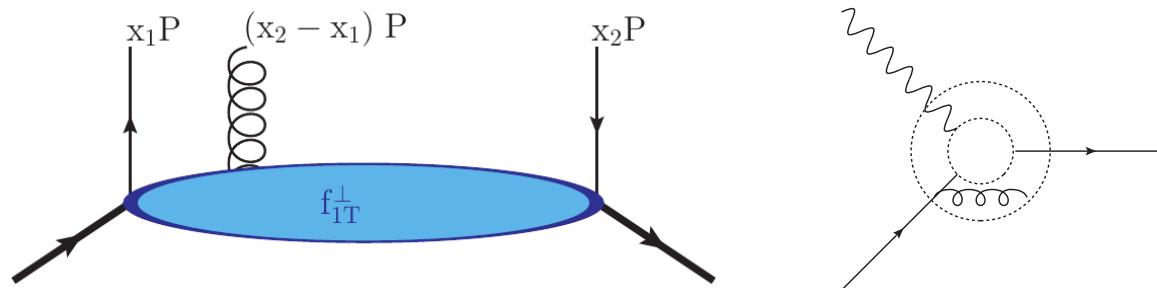
$$F_{UT}^{\sin(\phi_h - \phi_s)}(x_B, z_h, P_{h\perp}, Q) = \int_0^\infty \frac{b^2 db}{4\pi} J_1\left(\frac{b P_{h\perp}}{z_h}\right) \tilde{F}_{UT}^{\sin(\phi_h - \phi_s)}(x_B, z_h, b, Q)$$

$$\begin{aligned} \tilde{F}_{UT}^{\sin(\phi_h - \phi_s)}(x_B, z_h, b, Q) &\equiv H(Q^2) \sum_q e_q^2 \bar{C}_{q \leftarrow i} \otimes T_{F,i/p}(x_1 = x_B, x_2 = x_B, \mu_{b_*}) \\ &\quad \times \hat{C}_{j \leftarrow q} \otimes D_{h/j}(z_h, \mu_{b_*}) \exp(S_{pert} + S_{NP}^\perp) \end{aligned}$$



- DGLAP evolution for Qiu-Sterman function is not well-understood.
- This evolution is treated phenomenologically in one of two schemes.

DGLAP evolution of the Qiu-Sterman function



- Scheme 1.) Kang and Qiu showed in 2009 that at large x_B , the evolution can be treated as a diagonal evolution.

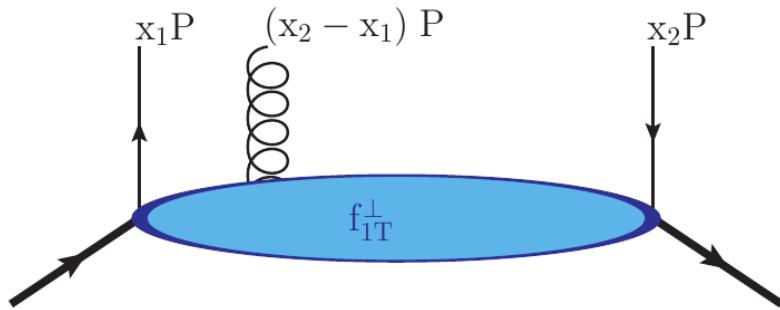
$$\frac{\partial T_{F,q/p}(x, x, \mu)}{\partial \ln \mu^2} = \frac{\alpha_s(\mu^2)}{2\pi} [P_{q \leftarrow q}^T \otimes T_{F,q/p}] (x, \mu)$$

$$P_{q \leftarrow q}^T(x) = P_{q \leftarrow q}(x) - \eta \delta(1-x), \quad \eta = N_c$$

- Scheme 2.) For phenomenological applications, it is common to treat the evolution to be the same as for the collinear PDF. Can model this behavior with $\eta = 0$.

$$P_{q \leftarrow q}^T(x) = P_{q \leftarrow q}(x), \quad \eta = 0$$

Parameterization for the Qiu-Sterman Function



Initial condition

$$T_{F,q/p}(x, Q_0) = N_q \frac{(\alpha_q + \beta_q)^{(\alpha_q + \beta_q)}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}} x^{\alpha_q} (1-x)^{\beta_q} f_{q/A}(x, Q_0)$$

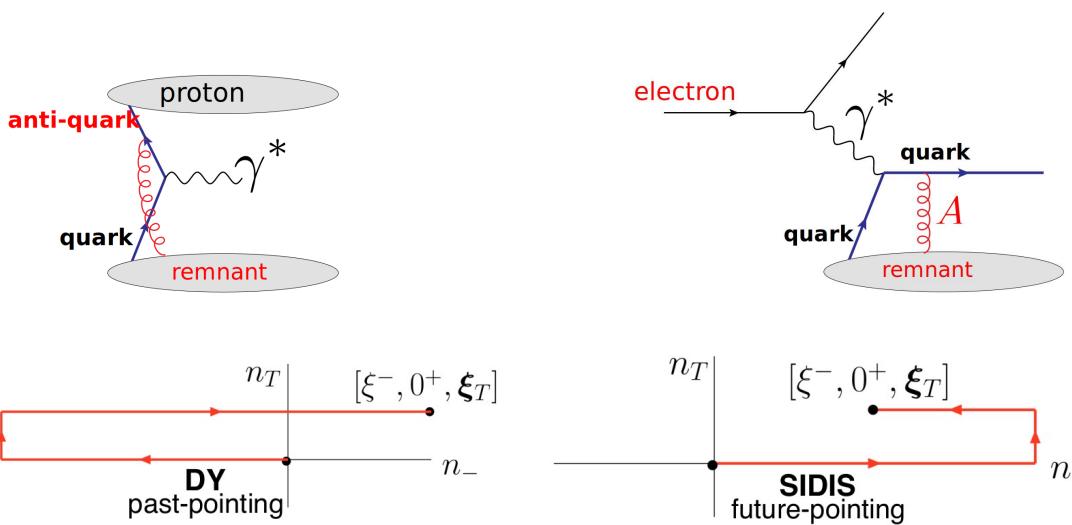
fit parameters: $\alpha_u, \alpha_d, N_u, N_d$ are the valence fit parameters and $\alpha_{sea}, N_{\bar{u}}, N_{\bar{d}}, N_s$, and $N_{\bar{s}}$ are the sea fit parameters and $\beta_q = \beta$ is the same for all flavors. We can probe $u, d, s, \bar{u}, \bar{d}, \bar{s}$ Sivers function.

$$S_{NP}^\perp(b; Q_0, \mu) = g_1^s b^2 + \frac{g_2}{2} \ln \frac{\mu}{Q_0} \ln \frac{b}{b_*}$$

Fit to low energy data

Collaboration	Scattering event	Number of points	Year
	$e + P \rightarrow e + h^+$	34	2017
	$e + P \rightarrow e + h^-$	31	2017
	$e + D \rightarrow e + \pi^+$	12	2008
	$e + D \rightarrow e + \pi^+$	12	2008
	$e + D \rightarrow e + K^+$	13	2008
	$e + D \rightarrow e + K^0$	7	2008
	$e + D \rightarrow e + K^-$	11	2008
	$\pi^- + P \rightarrow \gamma^*$	15	2017
	$e + P \rightarrow e + K^-$	14	2009
	$e + P \rightarrow e + K^+$	14	2009
	$e + P \rightarrow e + \pi^0$	13	2009
	$e + P \rightarrow e + \pi^+$	14	2009
	$e + P \rightarrow e + \pi^-$	14	2009
	$e + P \rightarrow e + \pi^-$	14	2009
	$e + P \rightarrow e + \pi^+$	4	2011
	$e + P \rightarrow e + \pi^-$	4	2011
	$P + P \rightarrow W/Z$	17	2015

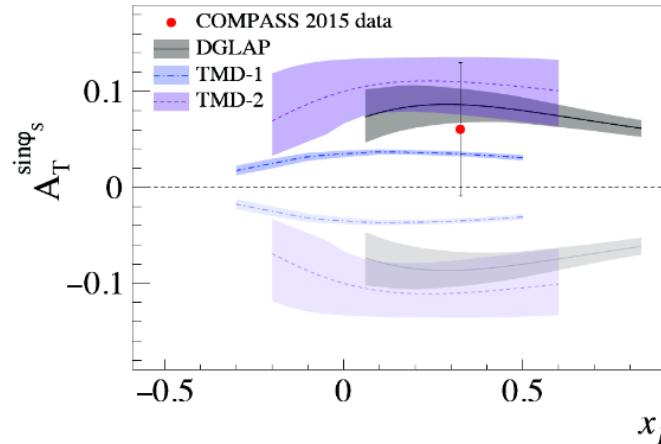
Signs change of the Sivers function



Repulsion between anti-quark and remnant in di-hadron collisions.

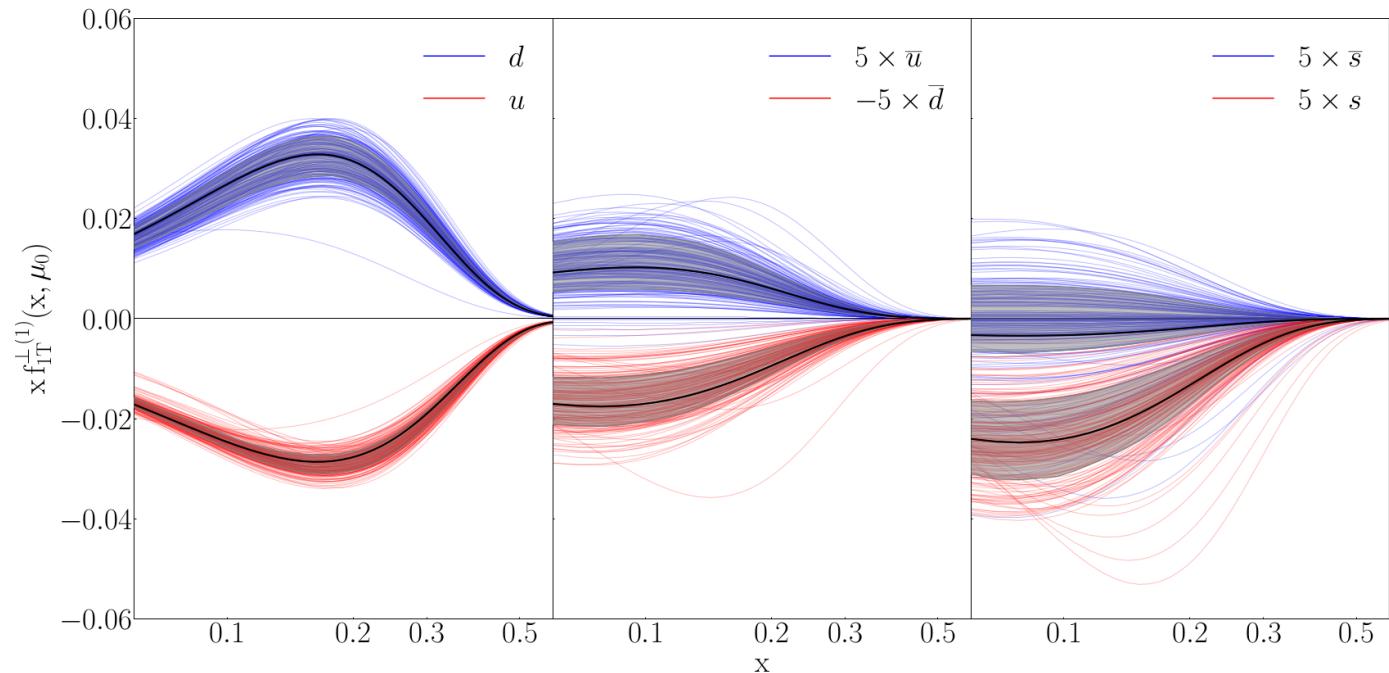
Attraction between quark and remnant in SIDIS.

$$f_{1T,q/p}^{\perp,\text{DY}} = -f_{1T,q/p}^{\perp,\text{SIDIS}}$$



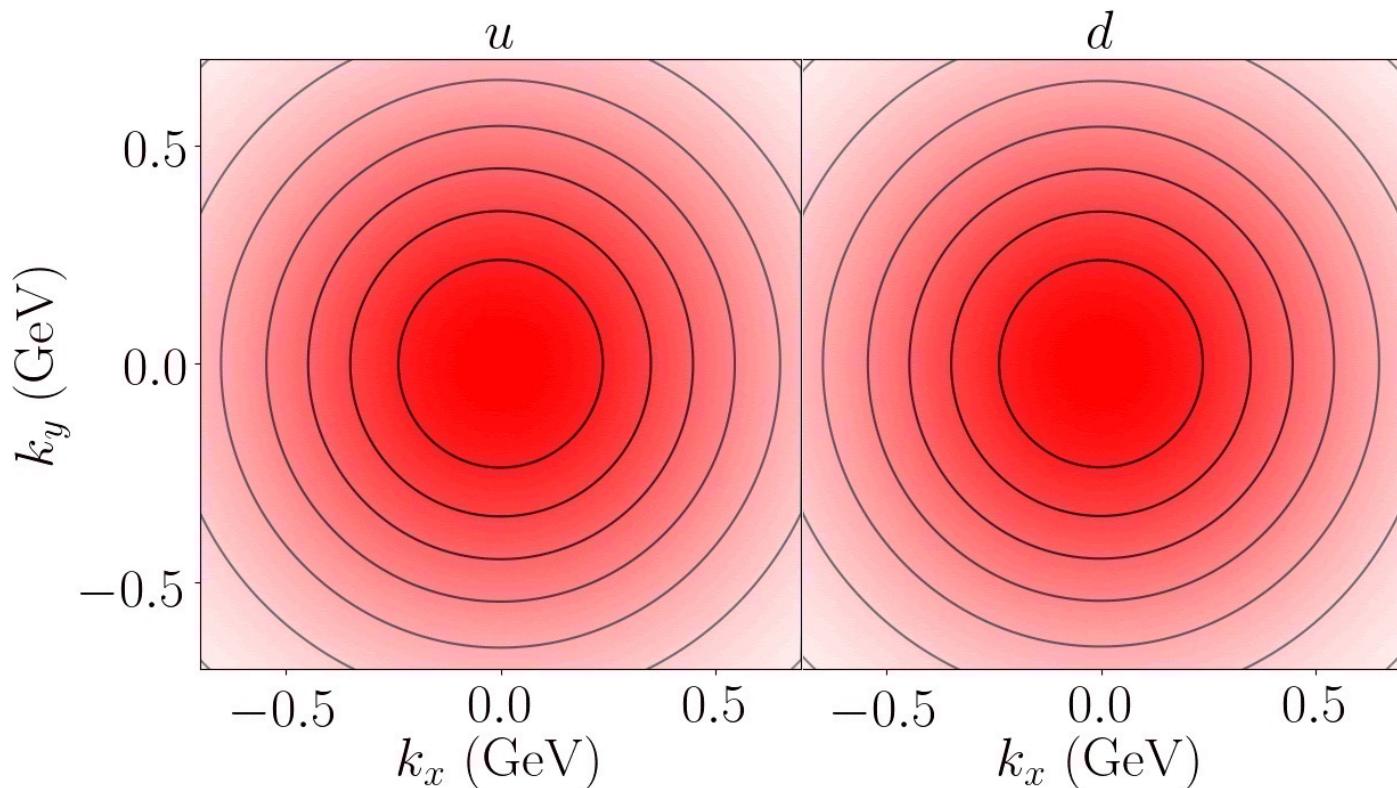
Fit results without RHIC data

$\chi^2/\text{d.o.f} = 1.032$ for 11 parameters and 226 points.



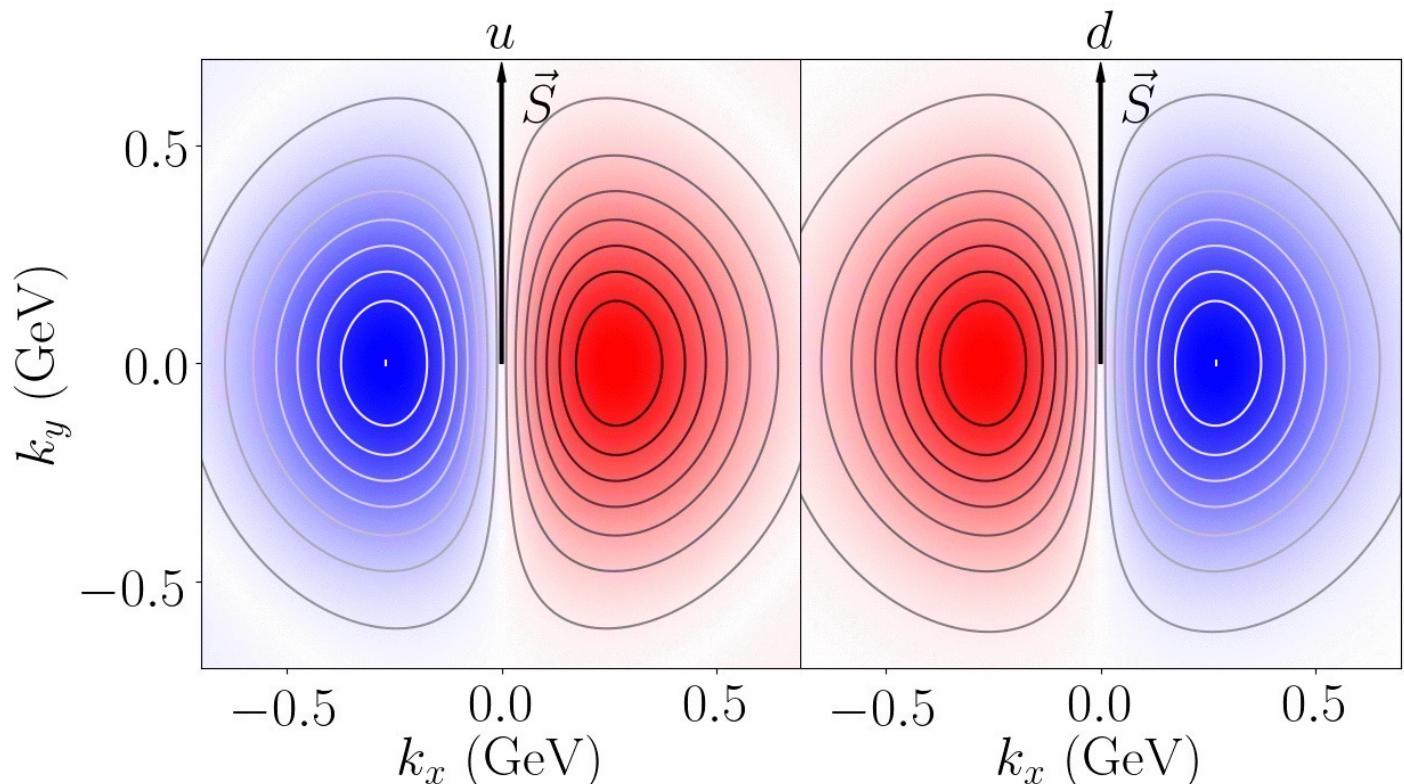
Unpolarized TMD PDF

$$f_{q/p}(x_B, k_\perp, Q) \quad x_B = 0.2 \quad Q^2 \text{ from } 1.9 \rightarrow 100 \text{ GeV}^2$$



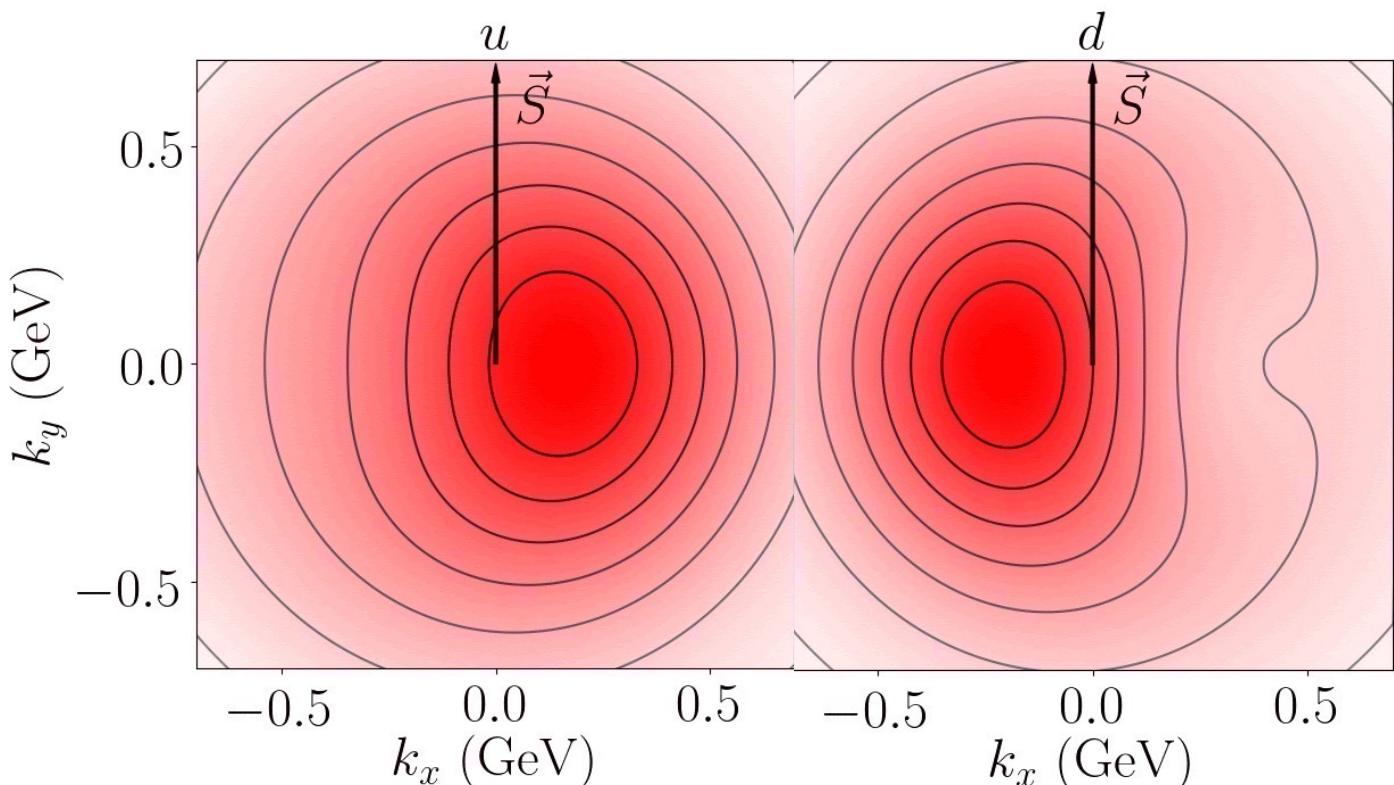
Sivers function

$$-\sin(\phi_s - \phi_k) \frac{k_\perp}{M} f_{1T,q/p}^\perp(x_B, k_\perp, Q) \quad x_B = 0.2 \quad Q^2 \text{ from } 1.9 \rightarrow 100 \text{ GeV}^2$$

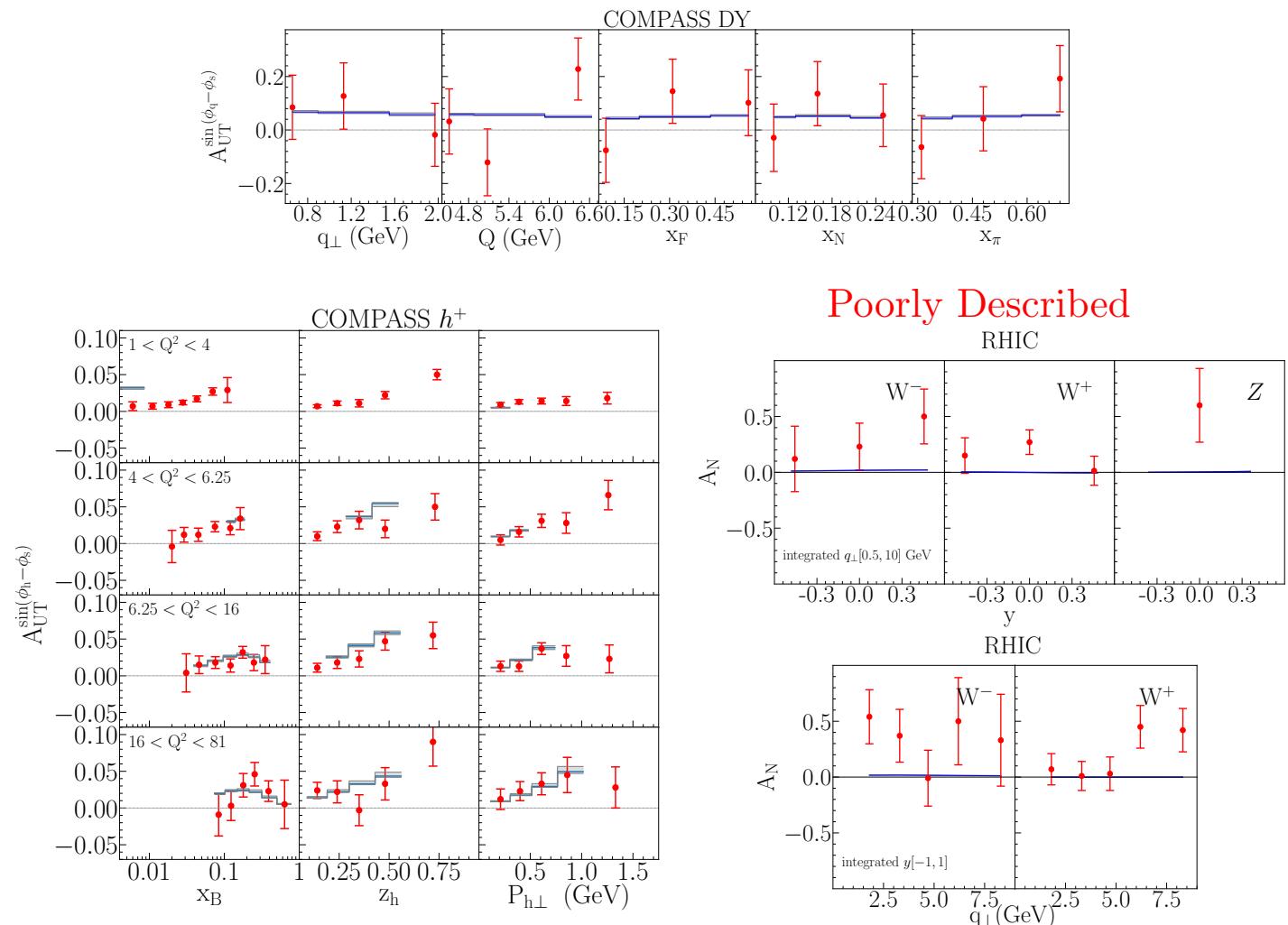


Spin-dependent TMD PDF

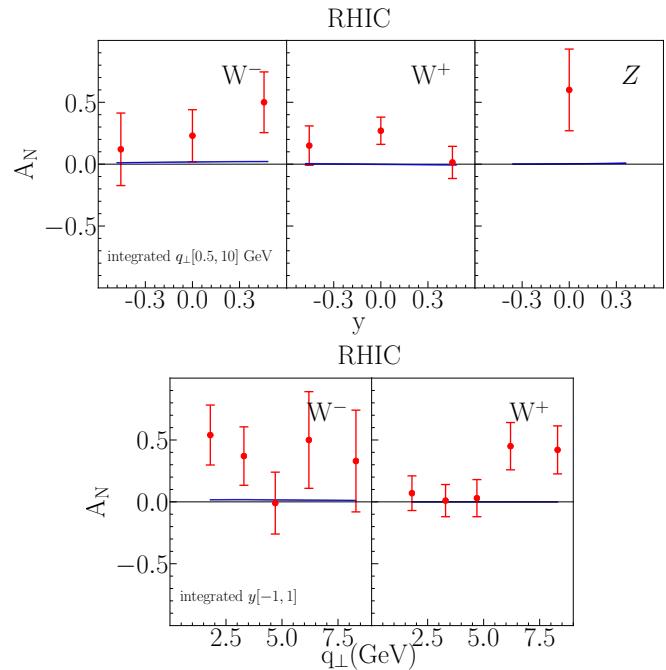
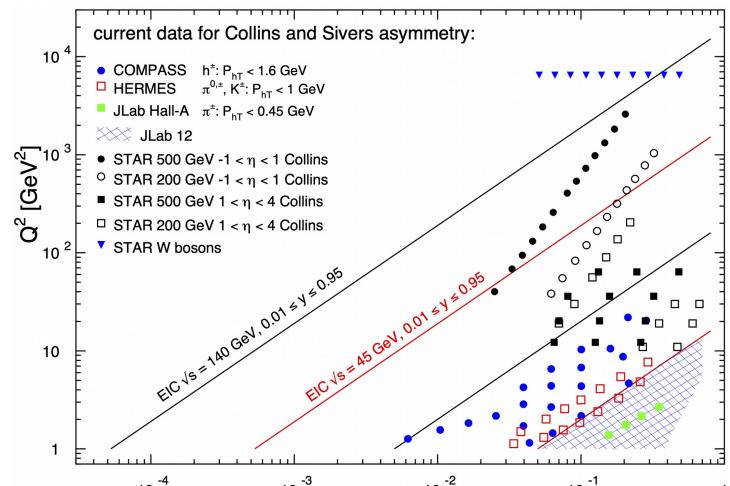
$$f_{q/p}(x_B, k_\perp, Q; \vec{S}_\perp) = f_{q/p}(x_B, k_\perp, Q) - \sin(\phi_s - \phi_k) \frac{k_\perp}{M} f_{1T,q/p}^\perp(x_B, k_\perp, Q)$$
$$x_B = 0.2 \quad Q^2 \text{ from } 1.9 \rightarrow 100 \text{ GeV}^2$$



Description of the data (RHIC not included in fit)



Is the evolution to blame?



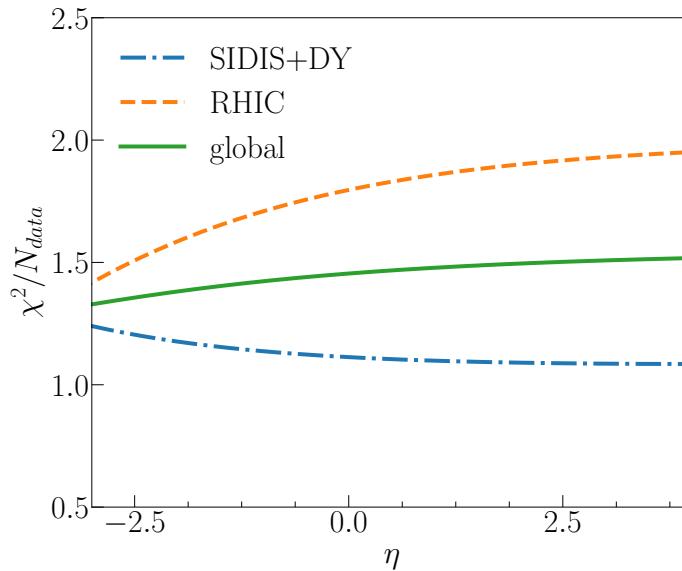
- COMPASS and DY data are only available up to about 70 GeV 2 .
- Vector boson production data is available at M_V^2 .

Dependence on the DGLAP evolution

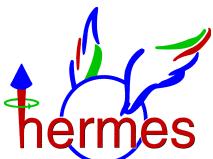
$$P_{q \leftarrow q}^T(x) = P_{q \leftarrow q}(x) - \eta \delta(1-x),$$

$$\Delta\mathcal{T} = \frac{1}{N_{\text{set}}} \sum_{i=1}^{N_{\text{set}}} \left| \frac{\text{Scheme}_1(\eta = N_C) - \text{Scheme}_2(\eta = 0)}{\text{Scheme}_1(\eta = N_C)} \right| \times 100$$

- For SIDIS and COMPASS DY data $\Delta\mathcal{T} \sim 1\%$.
- For RHIC data $\Delta\mathcal{T} \sim 50\%$.

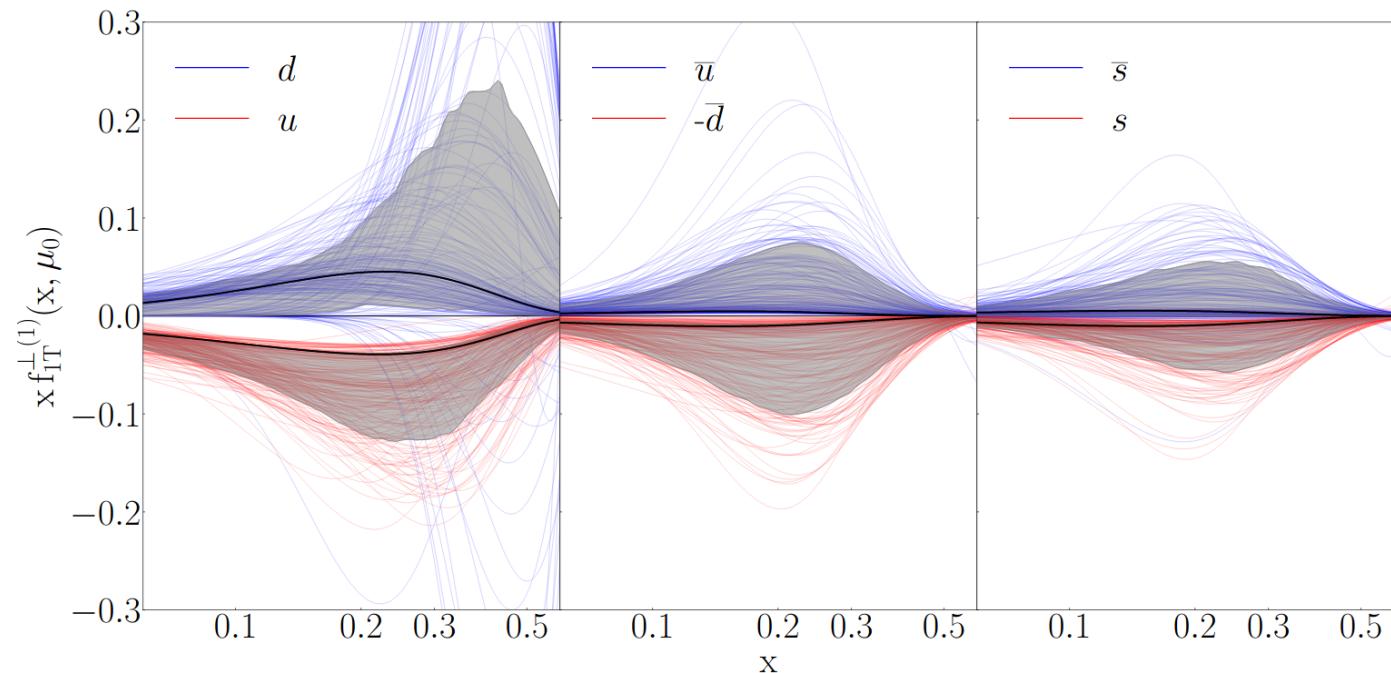


Global weighted fit ($\omega = 226/17$)

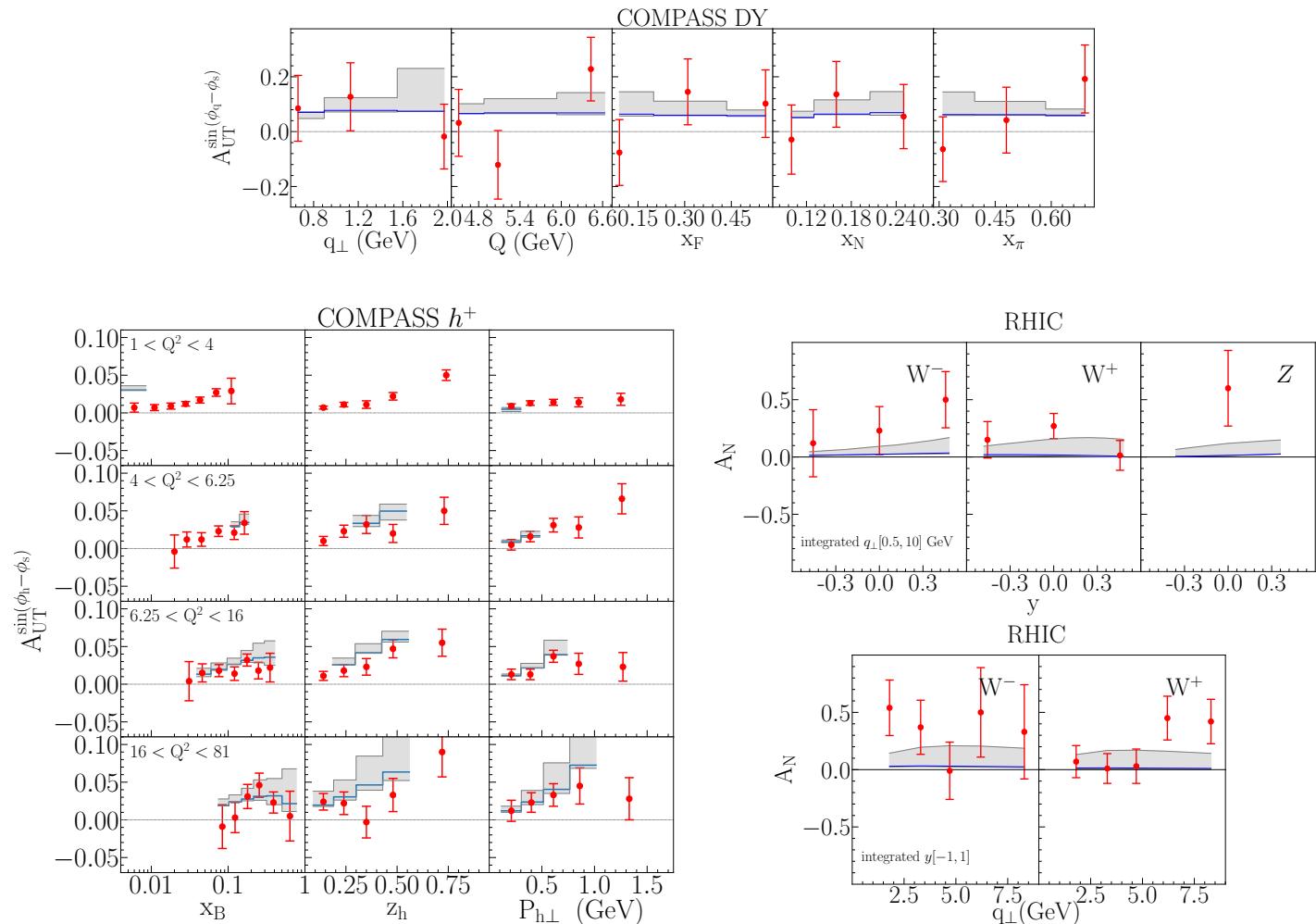
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	$e + P \rightarrow e + \pi^-$	14	2009
	$e + P \rightarrow e + \pi^+$	4	2011
	$e + P \rightarrow e + \pi^-$	4	2011
	$P + P \rightarrow W/Z$	$17 \times \omega$	2015

Fit results with RHIC data

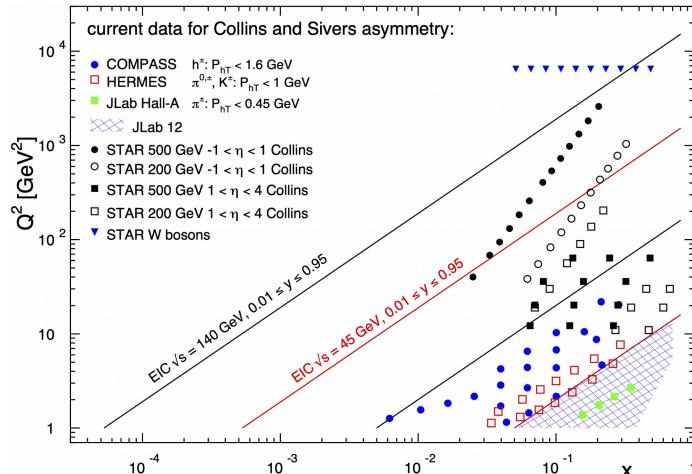
$\chi^2/\text{d.o.f} = 1.482$ for 11 parameters and 243 points.



Description of the data (with RHIC data included)



Conclusion



- We understand the low Q^2 data very well.
- Describing RHIC data is challenging.
- The treatment of the DGLAP evolution has a large impact.
- We require additional data in the intermediate Q^2 region to understand the evolution and reduce the experimental error.
- Polarized anti-proton data at $Q^2 = 100 \text{ GeV}^2$ from AMBER can help here.
- Future re-analysis of the RHIC data could help us better understand these effects.

Thank you!

Backup

Back

b-space

$$F_{UU}(x_B, z_h, P_{h\perp}, Q) = H^{\text{DIS}}(Q) \sum_q e_q^2 \int d^2 \mathbf{k}_\perp d^2 \mathbf{p}_\perp \delta^2(z_h \mathbf{k}_\perp + \mathbf{p}_\perp - \mathbf{P}_{h\perp}) \\ \times f_{q/p}(x_B, k_\perp^2, Q) D_{h/q}(z_h, p_\perp^2, Q),$$

$$F_{UU}(x_B, z_h, P_{h\perp}, Q) = H^{\text{DIS}}(Q) \sum_q e_q^2 \int_0^\infty \frac{b db}{2\pi} J_0\left(\frac{b P_{h\perp}}{z_h}\right) \\ \times f_{q/p}(x_B, b, Q) D_{h/q}(z_h, b, Q),$$

DGLAP evolution

$$f_{1T,q/p}^\perp(x, b, Q) = \left[\bar{C}_{q \leftarrow i} \otimes T_{F i/p} \right](x, b, Q),$$

$$f_{1T,q/p}^{\perp q}(N, b, Q) = \bar{C}_{q \leftarrow i}(N, Q) T_{F i/p}(N, Q),$$

$$f_{1T,q/p}^{\perp q}(x, b, Q) = \frac{1}{\pi} \int_0^\infty dz \text{Im} \left[e^{i\phi} x^{-c - ze^{i\phi}} f_{1T,q/p}^{\perp q}(c + ze^{i\phi}, b, Q) \right],$$

$$\frac{\partial T_{F q/p}(x, x; \mu)}{\partial \ln \mu^2} = \frac{\alpha_s(\mu^2)}{2\pi} \left[P_{q \leftarrow q}^T \otimes T_{F q/p} \right](x; \mu).$$

$$\frac{\partial}{\partial \ln \mu^2} T_{F q/p}(N, \mu) = \frac{\alpha_s(\mu^2)}{2\pi} \gamma(N) T_{F q/p}(N, \mu).$$

