Axion quality from gauge flavour symmetries

Based in part on arXiv:2102.05055 (L. Darmé, EN) **and on work in progress** (Grillli Di Cortona, L. Darmé, C. Smarra)

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- Non-pert. (instanton related) QCD effects generate a potential $V_{QCD}(a) = -(m_\pi f_\pi)^2 \cos(a/v_a)$ that drives $\langle a/v_a\rangle \to 0$ at the minimum

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The generating functional $Z \sim \int [DA_\mu D\Phi] \, D\psi D\bar{\psi} \, \exp(iS)$

is not invariant under a PQ transformation. $U(1)_{PQ}$ cannot be "imposed"

- In benchmark axion models, Φ is a complex scalar, and a gauge singlet.

Renormalizable terms $\mu^3\Phi$, $\mu^2\Phi^2$, $\mu\Phi^3$, $\lambda\Phi^4$ do not break gauge or Lorentz and cannot be forbidden. However, they would destroy $U(1)_{PQ}$ and the axion solution !

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- moreover: Non-pt. quantum gravity effects do not respect global symmt.

  Controlled solutions [Euclid. wormholes] do generate : $e^{-S_{wh}} M_P^3 \Phi + h.c.$

  Safe suppression requires $S_{wh} > 190$, while typically $S_{wh} \sim Log(M_P/v_a) \sim 15$

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- **Quality:** Effective ~~PQ~~ opts.: ~~PQ~~ vacuum eng. density $< 10^{-10}\, V_{QCD}(a)$

For $f_a \sim 10^{10}$ GeV and effective scale M_P, this implies Eff. Opt. Dim. $\gtrsim 10$

[Barr & Seckel '92, Kamionkowski & March-Russel '92, Holman et al. '92, Ghigna et al. '92]

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- Discrete gauge symm. $\mathbb{Z}_n$: $\Phi \to e^{i\,2\pi/n}\,\Phi$; 1^{st} ~~PQ~~ opt. $\Lambda^{4-n}\,\Phi^n$

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- Non-Abelian $SU(N)_L \times SU(N)_R$, $a(x) \in Y_{n\times n}$ "Orbital mode" $Y = U\,\hat{Y}\,V^\dagger\,e^{ia/v_a}$

  $N > 4$, $\mathcal{L}_{ren}$ has an automatic rephasing symm. $V = V(YY^\dagger)$ $Y \to e^{i\xi}\,Y$.

  1st ~~PQ~~ opt. Λ^{4-N} det Y dim = N. This requires again $N \geq 10$

  (Fong, EN '14 [in SU(3)xSU(3)], Di Luzio, Ubaldi, EN '17)

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  Gauge invariants are constructed with Kronecker δ and Levi-Civita ε

δ-invariants involve $Y^\dagger Y$:

They are obviously all Hermitian ⇒ accidental U(1): $Y \to e^{i\xi}\, Y$

ε-invariants (non-Hermitian): there is none $\varepsilon_{\alpha\beta\ldots\sigma}\, Y_{\alpha i}\, Y_{\beta j} \ldots Y_{\sigma r} = 0.$

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Note: for a $Y_{n\mathrm{x}n}$ square matrix $\varepsilon_{\alpha\beta\ldots\sigma}\, \varepsilon_{ij\ldots r}\, Y_{\alpha i}\, Y_{\beta j} \ldots Y_{\sigma r} \propto \det Y \neq 0$

Such automatic exact U(1) symmetries are peculiar of local `rectangular' symmetries

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Operators for which $\langle O \rangle = 0$ do not break the symmetries of the minimum, the vacuum can enjoy a larger symmetry than the Lagrangian.

Scalar bosons associated with these symmetries remain massless [Georgi & Pais '75]

The axion can remain protected even if $V(Y,Z)$ breaks $U(1)_{PQ}$

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The non-Abelian local G_F thus is a flavour symmetry !

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We are currently studying flavour groups that we would never have considered, had it not been for the axion !