Axion quality from gauge flavour symmetries

Based in part on arXiv:2102.05055 (L. Darmé, EN) and on work in progress (Grillli Di Cortona, L. Darmé, C. Smarra)

Enrico Nardi

INFN
Istituto Nazionale di Fisica Nucleare
Laboratori Nazionali di Frascati

PATRAS 2021 - June 9th, 2021
Basic ingredients of the PQ solution

[Peccei, Quinn (1977), Weinberg (1978), Wilczek (1978)]
Basic ingredients of the PQ solution

[Peccei, Quinn (1977), Weinberg (1978), Wilczek (1978)]

- A scalar potential invariant under a global $U(1)$: $\Phi \rightarrow e^{i\xi} \Phi$, $\delta V(\Phi) = 0$
Basic ingredients of the PQ solution

[Peccei, Quinn (1977), Weinberg (1978), Wilczek (1978)]

• A scalar potential invariant under a global $U(1)$: $\Phi \rightarrow e^{i\xi} \Phi$, $\delta V(\Phi) = 0$

• $U(1)$ SSB: $\langle \Phi \rangle \sim v_a e^{i\alpha(x)/v_a}$. $\alpha(x): V(\alpha) = 0 \rightarrow$ shift symmt. $\alpha \rightarrow \alpha + \xi v_a$
Basic ingredients of the PQ solution

[Peccei, Quinn (1977), Weinberg (1978), Wilczek (1978)]

- A scalar potential invariant under a global $U(1)$: $\Phi \to e^{i\xi} \Phi$, $\delta V(\Phi) = 0$

- $U(1)$ SSB: $\langle \Phi \rangle \sim v_a e^{ia(x)/v_a}$. $a(x): V(a) = 0 \to$ shift symmt. $a \to a + \xi v_a$

- Couplings between the scalars and some quarks $\bar{Q}_L \Phi q_R \to \bar{Q}_L v_a q_R e^{ia(x)/v_a}$.

$U(1)$ is then enforced by identifying chiral PQ charges $X(Q_L) - X(q_R) = X(\Phi)$
Basic ingredients of the PQ solution

[Pececi, Quinn (1977), Weinberg (1978), Wilczek (1978)]

• A scalar potential invariant under a global U(1): \( \Phi \rightarrow e^{i\xi} \Phi, \quad \delta V(\Phi) = 0 \)

• U(1) SSB: \( \langle \Phi \rangle \sim v \ e^{ia(x)/v} \). \( a(x): V(a) = 0 \rightarrow \) shift symmt. \( a \rightarrow a + \xi v_a \)

• Couplings between the scalars and some quarks \( \bar{Q}_L \Phi q_R \rightarrow \bar{Q}_L v a \ q_R \ e^{ia(x)/v} \)

U(1) is then enforced by identifying chiral PQ charges \( X(Q_L) - X(q_R) = X(\Phi) \)

• The U(1) must have a mixed U(1)-SU(3)\(_c\) anomaly: \( \Sigma_q (X_Q - X_q) \neq 0 \)
Basic ingredients of the PQ solution

[Peccei, Quinn (1977), Weinberg (1978), Wilczek (1978)]

• A scalar potential invariant under a global $U(1)$: $\Phi \rightarrow e^{i\xi} \Phi$, $\delta V(\Phi) = 0$

• $U(1)$ SSB: $\langle \Phi \rangle \sim v_a e^{i a(x)/v_a}$. $a(x)$: $V(a) = 0 \rightarrow$ shift symmt. $a \rightarrow a + \xi v_a$

• Couplings between the scalars and some quarks $\bar{Q}_L \Phi q_R \rightarrow \bar{Q}_L v_a q_R e^{i a(x)/v_a}$

$U(1)$ is then enforced by identifying chiral PQ charges $X(\bar{Q}_L) - X(q_R) = X(\Phi)$

• The $U(1)$ must have a mixed $U(1)$-$SU(3)_c$ anomaly: $\Sigma_q (X_Q - X_q) \neq 0$

• Redefining the quark fields in the real mass basis $\bar{Q}_L v_a q_R$:

$\Theta G\tilde{G} \rightarrow (a(x)/v_a + \Theta) G\tilde{G} \rightarrow (a(x)/v_a) G\tilde{G}$
Basic ingredients of the PQ solution

[Peccei, Quinn (1977), Weinberg (1978), Wilczek (1978)]

• A scalar potential invariant under a global U(1): $\Phi \to e^{i\xi} \Phi$, $\delta V(\Phi) = 0$

• U(1) SSB: $\langle \Phi \rangle \sim v_a e^{ia(x)/v_a}$. $a(x)$: $V(a) = 0 \to$ shift symmt. $a \to a + \xi v_a$

• Couplings between the scalars and some quarks $\bar{Q}_L \Phi q_R \to \bar{Q}_L v_a q_R e^{ia(x)/v_a}$

U(1) is then enforced by identifying chiral PQ charges $X(Q_L) - X(q_R) = X(\Phi)$

• The U(1) must have a mixed U(1)-SU(3)$_c$ anomaly: $\Sigma_q (X_Q - X_q) \neq 0$

• Redefining the quark fields in the real mass basis $\bar{Q}_L v_a q_R$:

$\Theta G\bar{G} \to (a(x)/v_a + \Theta) G\bar{G} \to (a(x)/v_a) G\bar{G}$

• Non-pert. (instanton related) QCD effects generate a potential $V_{QCD}(a) = -(m_\pi f_\pi)^2 \cos(a/v_a)$ that drives $\langle a/v_a \rangle \to 0$ at the minimum
But where does the U(1) symmetry come from?
The PQ "origin" and "quality" problems ...
But where does the U(1) symmetry come from?
The PQ "origin" and "quality" problems ...

- **Origin:** U(1)$_{PQ}$ is **anomalous:** is not a symmetry of the theory.

  The generating functional $Z \sim \int [DA_\mu D\Phi] D\psi D\bar{\psi} \exp(iS)$ is not invariant under a PQ transformation. U(1)$_{PQ}$ cannot be "imposed"

- In benchmark axion models, $\Phi$ is a complex **scalar**, and a **gauge singlet**. Renormalizable terms $\mu^3\Phi$, $\mu^2\Phi^2$, $\mu\Phi^3$, $\lambda\Phi^4$ do not break gauge or Lorentz and cannot be forbidden. However, they would destroy U(1)$_{PQ}$ and the axion solution!
But where does the U(1) symmetry come from? The PQ "origin" and "quality" problems ...

- **Origin:** U(1)$_{PQ}$ is anomalous: is not a symmetry of the theory.
  
  The generating functional $Z \sim \int [DA_\mu D\Phi] D\psi D\bar{\psi} \exp(iS)$ is not invariant under a PQ transformation. U(1)$_{PQ}$ cannot be "imposed"

- In benchmark axion models, $\Phi$ is a complex scalar, and a gauge singlet. Renormalizable terms $\mu^3\Phi$, $\mu^2\Phi^2$, $\mu\Phi^3$, $\lambda\Phi^4$ do not break gauge or Lorentz and cannot be forbidden. However, they would destroy U(1)$_{PQ}$ and the axion solution!

- **moreover:** Non-pt. quantum gravity effects do not respect global symmt.

  Controlled solutions [Euclid. wormholes] do generate: $e^{-S_{wh}} M_P^3 \Phi + \text{h.c.}$
  
  Safe suppression requires $S_{wh} > 190$, while typically $S_{wh} \sim \log(M_P/v_a) \sim 15$

  [Kallosh et al. ‘95, Alonso & Urbano ‘17, Alvey & Escudero ‘20]
But where does the U(1) symmetry come from?
The PQ "origin" and "quality" problems ...

• **Origin**: \( \text{U(1)}_{\text{PQ}} \) is anomalous: is not a symmetry of the theory.

  The generating functional \( Z \sim \int [DA_\mu D\Phi] D\psi D\bar{\psi} \exp(iS) \) is not invariant under a PQ transformation. \( \text{U(1)}_{\text{PQ}} \) cannot be "imposed"

• In benchmark axion models, \( \Phi \) is a complex scalar, and a gauge singlet. Renormalizable terms \( \mu^3 \Phi, \mu^2 \Phi^2, \mu \Phi^3, \lambda \Phi^4 \) do not break gauge or Lorentz and cannot be forbidden. However, they would destroy \( \text{U(1)}_{\text{PQ}} \) and the axion solution!

• moreover: Non-pt. quantum gravity effects do not respect global symmt.

  Controlled solutions [Euclid. wormholes] do generate: \( e^{-S_{\text{wh}}} \, M_P^3 \, \Phi + \text{h.c.} \)

  Safe suppression requires \( S_{\text{wh}} > 190 \), while typically \( S_{\text{wh}} \sim \log(M_P/v_a) \sim 15 \)

  [Kallosh et al. '95, Alonso & Urbano '17, Alvey & Escudero '20]

• **Quality**: Effective PQ opts.: PQ vacuum eng. density < \( 10^{-10} \) \( V_{\text{QCD}}(a) \)

  For \( f_a \sim 10^{10} \) GeV and effective scale \( M_P \), this implies Eff. Opt. Dim. \( \approx 10 \)

  [Barr & Seckel '92, Kamionkowski & March-Russel '92, Holman et al. '92, Ghigna et al. '92]
A sample of proposed solutions
A sample of proposed solutions

$U(1)_{PQ}$ should arise automatically as a consequence of first principles. SSB requires VEVs $\Rightarrow$ Lorentz singlets. Rely on local gauge symmetries.
A sample of proposed solutions

U(1)$_{\text{PQ}}$ should arise automatically as a consequence of first principles. SSB requires VEVs $\Rightarrow$ Lorentz singlets. Rely on local gauge symmetries.

- **Discrete gauge symm.** $\mathbb{Z}_n$: $\Phi \rightarrow e^{i\frac{2\pi}{n}} \Phi$; 1$^\text{st}$ PQ opt. $\wedge^{4-n} \Phi^n$

Requires $\mathbb{Z}_{10}$ or larger [Krauss & Wilczek '89, Dias & al. '03, Carpenter & al. '09, Harigaya & al. '13]
U(1)$_{PQ}$ should arise automatically as a consequence of first principles. SSB requires VEVs $\Rightarrow$ Lorentz singlets. Rely on local gauge symmetries

- Discrete gauge symm. $\mathbb{Z}_n$: $\Phi \rightarrow e^{i \frac{2\pi}{n}} \Phi$; $1^{st}$ PQ opt. $\Lambda^{4-n} \Phi^n$

Requires $\mathbb{Z}_{10}$ or larger [Krauss & Wilczek '89, Dias & al. '03, Carpenter & al. '09, Harigaya & al. '13]

- Local gauge U(1) + 2 scalars with gauge charges $q_1, q_2$ relatively prime

$1^{st}$ PQ operator: $\Lambda^{4-q_1-q_2} (\Phi_1^\dagger)^{q_2} (\Phi_2)^{q_1}$ Requires $q_1 + q_2 \geq 10$

[Barr & Seckel '92]
U(1)_{PQ} should arise automatically as a consequence of first principles. SSB requires VEVs ⇒ Lorentz singlets. Rely on local gauge symmetries.

- **Discrete gauge symm.** $\mathbb{Z}_n$: $\Phi \rightarrow e^{i \frac{2\pi}{n}} \Phi$; 1st PQ opt. $\Lambda^{4-n} \Phi_n$

  Requires $\mathbb{Z}_{10}$ or larger [Krauss & Wilczek '89, Dias & al. '03, Carpenter & al. '09, Harigaya & al. '13]

- **Local gauge** $U(1) + 2$ scalars with gauge charges $q_1, q_2$ relatively prime
  
  1st PQ operator: $\Lambda^{4-q_1-q_2} (\Phi_1^\dagger)^{q_2} (\Phi_2)^{q_1}$ Requires $q_1 + q_2 \geq 10$

  [Barr & Seckel '92]

- **Non-Abelian** $SU(N)_L \times SU(N)_R$, $a(x) \in Y_{n \times n}$ "Orbital mode" $Y = U \hat{Y} V^\dagger e^{ia/v_a}$

  $N > 4$, $\mathcal{L}_{\text{ren}}$ has an automatic rephasing symm. $V = V(YY^\dagger)$ $Y \rightarrow e^{i\xi} Y$.

  1st PQ opt. $\Lambda^{4-N} \det Y$ dim = $N$. This requires again $N \geq 10$

  (Fong, EN '14 [in SU(3)xSU(3)], Di Luzio, Ubaldi, EN '17)
Can we do any better? `Rectangular' gauge symmt.

[Darmé & EN (2021)]
Can we do any better? `Rectangular’ gauge symmt.

• Take a local \( SU(m) \times SU(n) \) (\( m > n \)) and a scalar multiplet \( Y_{ai} \sim (m,\bar{n}) \)

  Gauge invariants are constructed with Kronecker \( \delta \) and Levi-Civita \( \varepsilon \)

  \( \delta \)-invariants involve \( Y^\dagger Y \):

  They are obviously all Hermitian \( \Rightarrow \) accidental \( U(1) \): \( Y \to e^{i\xi} Y \)

  \( \varepsilon \)-invariants (non-Hermitian): there is none

  \( \varepsilon_{\alpha\beta...\sigma} Y_{ai} Y_{bj} ... Y_{sr} = 0 \).

  Some \( SU(n) \) index \( (i,j,...,r) \) must coincide: \( \varepsilon \) opts. vanish symmetrically

  Already for \( SU(3) \times SU(2) \), \( V(Y) \) enjoys automatically an exact global \( U(1) \)

[Darmé & EN (2021)]
Can we do any better? `Rectangular' gauge symmt.

[Darmé & EN (2021)]

- Take a local $SU(m) \times SU(n)$ ($m > n$) and a scalar multiplet $Y_{ai} \sim (m, \bar{n})$

  **Gauge invariants** are constructed with Kronecker $\delta$ and Levi-Civita $\epsilon$

  $\delta$-invariants involve $Y^\dagger Y$:
  
  They are obviously **all Hermitian** $\Rightarrow$ accidental $U(1)$: $Y \rightarrow e^{i\xi} Y$

  $\epsilon$-invariants (non-Hermitian): **there is none** $\epsilon_{\alpha\beta...\sigma} Y_{ai} Y_{\beta j} ... Y_{\sigma r} = 0$.

  Some $SU(n)$ index $(i,j,...,r)$ must coincide: $\epsilon$ opts. vanish symmetrically

  Already for $SU(3) \times SU(2)$, $V(Y)$ enjoys **automatically** an **exact** global $U(1)$

  **Note:** for a $Y_{n \times n}$ square matrix $\epsilon_{\alpha\beta...\sigma} \epsilon_{ij...r} Y_{ai} Y_{\beta j} ... Y_{\sigma r} \propto \det Y \neq 0$

  Such **automatic exact** $U(1)$ symmetries are peculiar of local `rectangular' symmetries
Vacuum values of PQ breaking operators
Vacuum values of PQ breaking operators

• Generally more scalars are needed in order to have all $m_q \neq 0$

Take e.g. gauge $G_F = SU(3)_L \times SU(2)_R$ with $Y_{ai} \sim (3, \bar{2})$ and $Z_a \sim (3, 1)$

• Two mixed invariants

$I_\varepsilon = \varepsilon_{\alpha\beta\gamma} \varepsilon_{ij} Y_{ai} Y_{\beta j} Z_\gamma \neq 0$ preserves $U(1)_\varepsilon$  \underline{$U(1)_\delta$}

$I_\delta = \varepsilon_{ij} (Z^t Y)_i (Z^t Y)_j \neq 0$ preserves $U(1)_\delta$  \underline{$U(1)_\varepsilon$}

However, it can be shown that if $\langle I_\varepsilon \rangle \neq 0$, then $\langle I_\delta \rangle = 0$ (and vice versa)

On the vacuum either one, $U(1)_\varepsilon$ or $U(1)_\delta$ remain preserved
Vacuum values of PQ breaking operators

• Generally more scalars are needed in order to have all $m_q \neq 0$

Take e.g. gauge $G_F = SU(3)_L \times SU(2)_R$ with $Y_{ai} \sim (3, \tilde{2})$ and $Z_a \sim (3, 1)$

• Two mixed invariants
  \[ I_\varepsilon = \varepsilon_{\alpha \beta \gamma} \varepsilon_{ij} Y_{ai} Y_{\beta j} Z_\gamma \neq 0 \] preserves $U(1)_\varepsilon$ \( U(1)_\delta \)
  \[ I_\delta = \varepsilon_{ij} (Z^t Y)_i (Z^t Y)_j \neq 0 \] preserves $U(1)_\delta$ \( U(1)_\varepsilon \)

However, it can be shown that if $\langle I_\varepsilon \rangle \neq 0$, then $\langle I_\delta \rangle = 0$ (and viceversa)

On the vacuum either one, $U(1)_\varepsilon$ or $U(1)_\delta$ remain preserved

Operators for which $\langle O \rangle = 0$ do not break the symmetries of the minimum, the vacuum can enjoy a larger symmetry than the Lagrangian.

Scalar bosons associated with these symmetries remain massless \cite{Georgi & Pais '75}

The axion can remain protected even if $V(Y,Z)$ breaks $U(1)_{PQ}$
The “PQ quality - flavour” connection
Can symmetries of this type be promoted to realistic PQ symmetries? Can we learn something beyond `axion issues`?
The “PQ quality - flavour” connection

Can symmetries of this type be promoted to realistic PQ symmetries? Can we learn something beyond `axion issues'? 

**Origin & quality of acdct. symmt.** <= non-Abelian `rectangular’ gauge group $G_F$ acting on some set of scalar multiplets.
The “PQ quality - flavour” connection

Can symmetries of this type be promoted to realistic PQ symmetries? Can we learn something beyond `axion issues'? 

Origin & quality of accidt. symmt. ≤ non-Abelian `rectangular’ gauge group $G_F$ acting on some set of scalar multiplets.

Promoting U(1) to a PQ symmt. requires a mixed QCD anomaly.

$\Rightarrow$ Quarks must transform under the accidt. U(1) symmt.

$\Rightarrow$ Hence they must couple to the scalar multiplets

$\Rightarrow$ Hence they must also transform under $G_F$
The “PQ quality - flavour” connection

Can symmetries of this type be promoted to realistic PQ symmetries? Can we learn something beyond `axion issues'? 

Origin & quality of accidt. symmt. <= non-Abelian `rectangular’ gauge group $G_F$ acting on some set of scalar multiplets.

Promoting $U(1)$ to a PQ symmt. requires a mixed QCD anomaly.

=> Quarks must transform under the accidt. $U(1)$ symmt.

=> Hence they must couple to the scalar multiplets

=> Hence they must also transform under $G_F$

The non-Abelian local $G_F$ thus is a flavour symmetry!
The “PQ quality - flavour” connection
The “PQ quality - flavour” connection

The axion: \( \Theta_{QCD} \to 0 \) to CDM to ... SM flavour puzzle ??
The "PQ quality - flavour" connection

The axion: from $\Theta_{QCD} \to 0$ to CDM to ... SM flavour puzzle ??

Any non-Abelian gauge symmt. generating a $U(1)_{PQ}$ IS a flavour symmetry!
The “PQ quality - flavour” connection

The axion: from $\Theta_{QCD} \to 0$ to CDM to ... SM flavour puzzle ??

Any non-Abelian gauge symmt. generating a $U(1)_{PQ}$ IS a flavour symmetry!

The guiding principle is that a PQ symmetry of the required high quality must arise automatically from $G_F$ and the field content.
The “PQ quality - flavour” connection

The axion: \( \Theta_{\text{QCD}} \rightarrow 0 \) to CDM to ... SM flavour puzzle ?

Any non-Abelian gauge symmm. generating a \( U(1)_{\text{PQ}} \) IS a flavour symmetry!

The guiding principle is that a PQ symmetry of the required high quality must arise automatically from \( G_F \) and the field content.

General features of \( G_F \) symmetries suited for \( U(1)_{\text{PQ}} \) protection:

• Not all the quarks transform in the same way under \( G_F \)
• Some Yukawa quarks originate from different operators
• Mass hierarchies seem to arise rather naturally
The “PQ quality - flavour” connection

The axion: $\Theta_{\text{QCD}} \rightarrow 0$ to CDM to ... SM flavour puzzle ??

Any non-Abelian gauge symmt. generating a $U(1)^{\text{PQ}}$ is a flavour symmetry!

The guiding principle is that a PQ symmetry of the required high quality must arise automatically from $G_F$ and the field content.

General features of $G_F$ symmetries suited for $U(1)^{\text{PQ}}$ protection:

• Not all the quarks transform in the same way under $G_F$
• Some Yukawa quarks originate from different operators
• Mass hierarchies seem to arise rather naturally

We are currently studying flavour groups that we would never have considered, had it not been for the axion!