Unitarity and higher-order CP asymmetries

A systematic perturbative approach

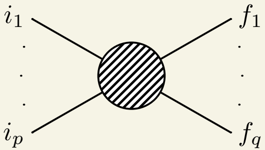
Tomáš Blažek & **Peter Maták**

Based on arXiv:2102.05914, arXiv:2104.06395

Faculty of Mathematics, Physics and Informatics
Comenius University in Bratislava

14-18 June 2021
16th Patras Workshop on Axions, WIMPs and WISPs

Boltzmann equation in the FLRW universe



$$\dot{n}_{i_k} + 3Hn_{i_k} = -\gamma_{fi} + \gamma_{fi} + \dots \tag{1}$$

← reaction rates

$$\gamma_{fi} = \int \prod_{\{i\}} [d\mathbf{p}_i] f_i(p_i) \int \prod_{\{f\}} [d\mathbf{p}_f] (2\pi)^4 \delta^{(4)}(p_f - p_i) |M_{fi}|^2 \tag{2}$$

← M_{fi} extracted from $S_{fi} = \delta_{fi} + \mathrm{i}T_{fi} = \delta_{fi} + (2\pi)^4 \delta^{(4)}(p_f - p_i) M_{fi}$

← possible on-shell intermediate states subtracted to avoid double-counting E. W. Kolb and S. Wolfram, Nucl. Phys. B 172 (1980) 224

Unitarity and CPT constraints

← unitarity, $1 - \mathrm{i}T^\dagger = (1 + \mathrm{i}T)^{-1}$, or $\mathrm{i}T^\dagger = \mathrm{i}T - (\mathrm{i}T)^2 + (\mathrm{i}T)^3 - \ldots$ leads to

$$|T_{fi}|^2 = -\mathrm{i}T_{if}\mathrm{i}T_{fi} + \sum_n \mathrm{i}T_{in}\mathrm{i}T_{nf}\mathrm{i}T_{fi} - \ldots \tag{3}$$

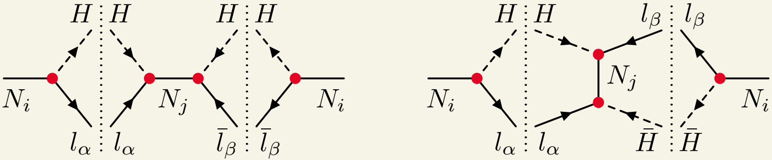
← CP asymmetry of the $i \to f$ reaction T. Blažek, P. Maták, Phys. Rev. D 103, L091302 (2021)

$$\begin{aligned}\Delta|T_{fi}|^2 &= \sum_n (\mathrm{i}T_{in}\mathrm{i}T_{nf}\mathrm{i}T_{fi} - \mathrm{i}T_{if}\mathrm{i}T_{fn}\mathrm{i}T_{ni}) \\ &- \sum_{n,m} (\mathrm{i}T_{in}\mathrm{i}T_{nm}\mathrm{i}T_{mf}\mathrm{i}T_{fi} - \mathrm{i}T_{if}\mathrm{i}T_{fm}\mathrm{i}T_{mn}\mathrm{i}T_{ni}) \\ &+ \ldots \quad \Rightarrow \quad \sum_f \Delta|T_{fi}|^2 = 0\end{aligned} \tag{4}$$

LO asymmetries in leptogenesis

← CP asymmetric processes with right-handed neutrinos

$$\mathcal{L}_{\text{int}} \supset -\mathcal{Y}_{\alpha i}\bar{N}_i P_L l_\alpha H + \text{H.c.} \tag{5}$$



$$\Delta|T_{fi}|^2 = \sum_n (\mathrm{i}T_{in}\mathrm{i}T_{nf}\mathrm{i}T_{fi} - \text{m.t.}) - \dots \tag{6}$$

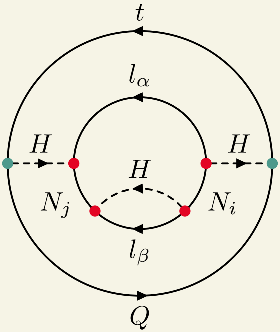
← cyclic permutations lead to

$$\Delta\gamma^{\text{eq}}_{N_i\to lH} = \Delta\gamma^{\text{eq}}_{lH\to\bar{l}\bar{H}} = \Delta\gamma^{\text{eq}}_{\bar{l}\bar{H}\to N_i} \tag{7}$$

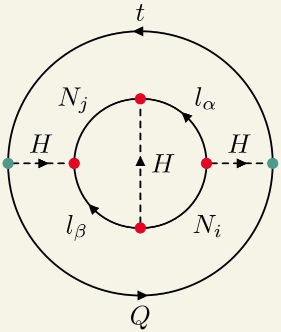
← at $\mathcal{O}((\mathrm{i}T)^3)$ order equivalent to cyclic notation of E. Roulet, L. Covi, and F. Vissani, Phys. Lett. B 424, 101 (1998)

Higher-order corrections and N_iQ scattering

$$\mathcal{L}_{\text{int}} \supset -\mathcal{Y}_{\alpha i}\bar{N}_i P_L l_\alpha H + \mathcal{Y}_t \bar{t} P_L Q H + \text{H.c.} \tag{8}$$



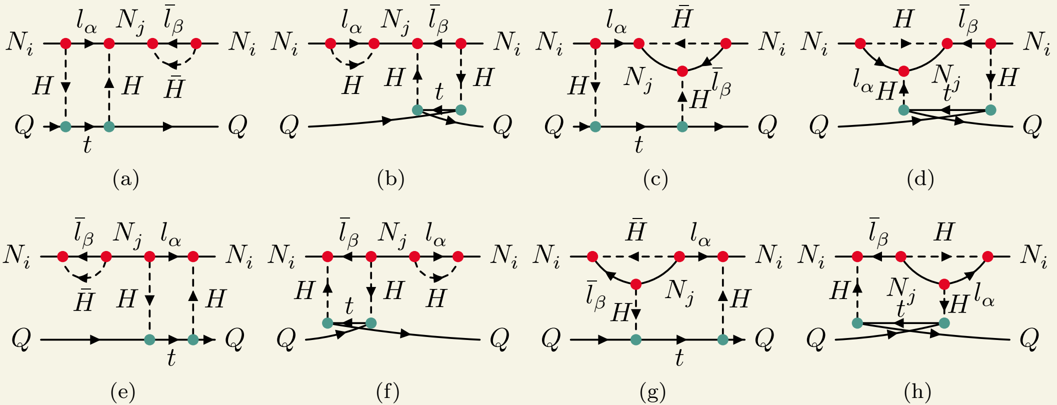
(a)



(b)

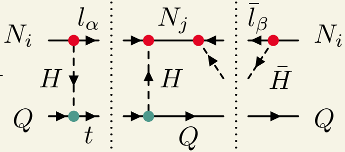
Diagram (b) considered in J. Racker, J. High Energy Phys. 02 (2019) 042

Higher-order corrections and $N_i Q$ scattering



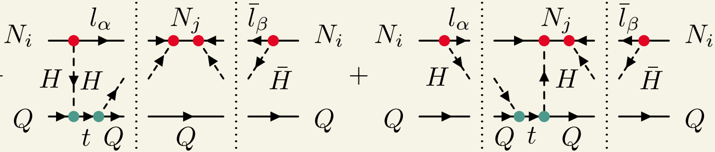
Higher-order corrections and N_iQ scattering

$$\Delta\gamma^{(a)}_{N_iQ\to lt} \leftarrow$$



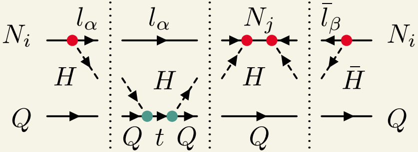
$$- \text{m.t.} \quad (9a)$$

$$\Delta\gamma^{(a)}_{N_iQ\to lHQ} \leftarrow$$



$$(9b)$$

$$-$$



$$- \text{m.t.}$$

Higher-order corrections and N_iQ scattering

$\leftarrow$ unitarity and CPT constraints

$$\Delta\gamma^{\rm eq}_{N_iQ\to lt} + \Delta\gamma^{\rm eq}_{N_iQ\to lHQ} + \Delta\gamma^{\rm eq}_{N_iQ\to \bar{l}\bar{H}Q} + \Delta\gamma^{\rm eq}_{N_iQ\to \bar{l}QQ\bar{t}} = 0 \tag{10}$$

$\leftarrow$ $N_iQ \to \bar{l}QQ\bar{t}$ was not considered in the literature within the classical Boltzmann approach, even though it is of the same order as the contributions of $N_iQ \to lt$ E. Nardi, J. Racker, E. Roulet, JHEP 09 (2007) 090 and $N_iQ \to lHQ$ J. Racker, J. High Energy Phys. 02 (2019) 042

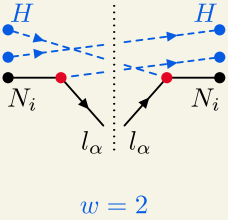
$\leftarrow$ $N_iQ \to \bar{l}QQ\bar{t}$ reaction rate corresponds to a part of the Q Pauli blocking factor in $N_i \to \bar{l}Q\bar{t}$

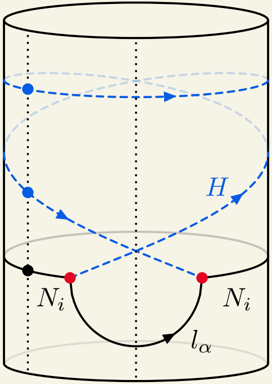
$\leftarrow$ sum of cuttings for each forward diagram

$$\Delta\gamma^{\rm (a)}_{N_iQ\to lt} + \Delta\gamma^{\rm (a)}_{N_iQ\to lHQ} \tag{11}$$

is IR finite (CP symmetric part in J. Racker, J. High Energy Phys. 02 (2019) 042)

Thermal corrections





$$\sum_{w=0}^{\infty} \left(\mathrm{e}^{-E/T}\right)^{w} = \frac{\mathrm{e}^{E/T}}{\mathrm{e}^{E/T}-1} \qquad (12)$$

$$=1 + f_{\mathrm{BE}}$$

← summing over winding numbers leads to correct statistical factors for intermediate states

← lepton number source-term derived in T. Blažek, P. Maták, arXiv:2104.06395

Summary

- ← Simplified diagrammatic treatment of CP asymmetries and cancelations of IR divergences based on

$$|T_{fi}|^2 = -\mathrm{i}T_{if}\mathrm{i}T_{fi} + \sum_n \mathrm{i}T_{in}\mathrm{i}T_{nf}\mathrm{i}T_{fi} - \ldots$$

  has been introduced T. Blažek, P. Maták, Phys. Rev. D 103, L091302 (2021).

- ← Unnatural splitting of amplitudes into couplings and imaginary kinematics (Cutkosky rules) is avoided.

- ← Systematic diagrammatic procedure is needed to obtain a complete form of Boltzmann collision integral and not to forget anything.

- ← Statistical factors due to on-shell intermediate states are formally represented by the cuts of multiple spectator lines on a cylindrical surface T. Blažek, P. Maták, arXiv:2104.06395.

- ← For long and technical questions, do not hesitate to use `peter.matak@fmph.uniba.sk`!

Thank you for your attention!