# Breakdown of ChPT for the axion hot dark matter bound

*L. Di Luzio (Padua), G. Martinelli (Rome), GP* [2101.10330]

To appear in *Phys. Rev. Lett.*

**Gioacchino Piazza**

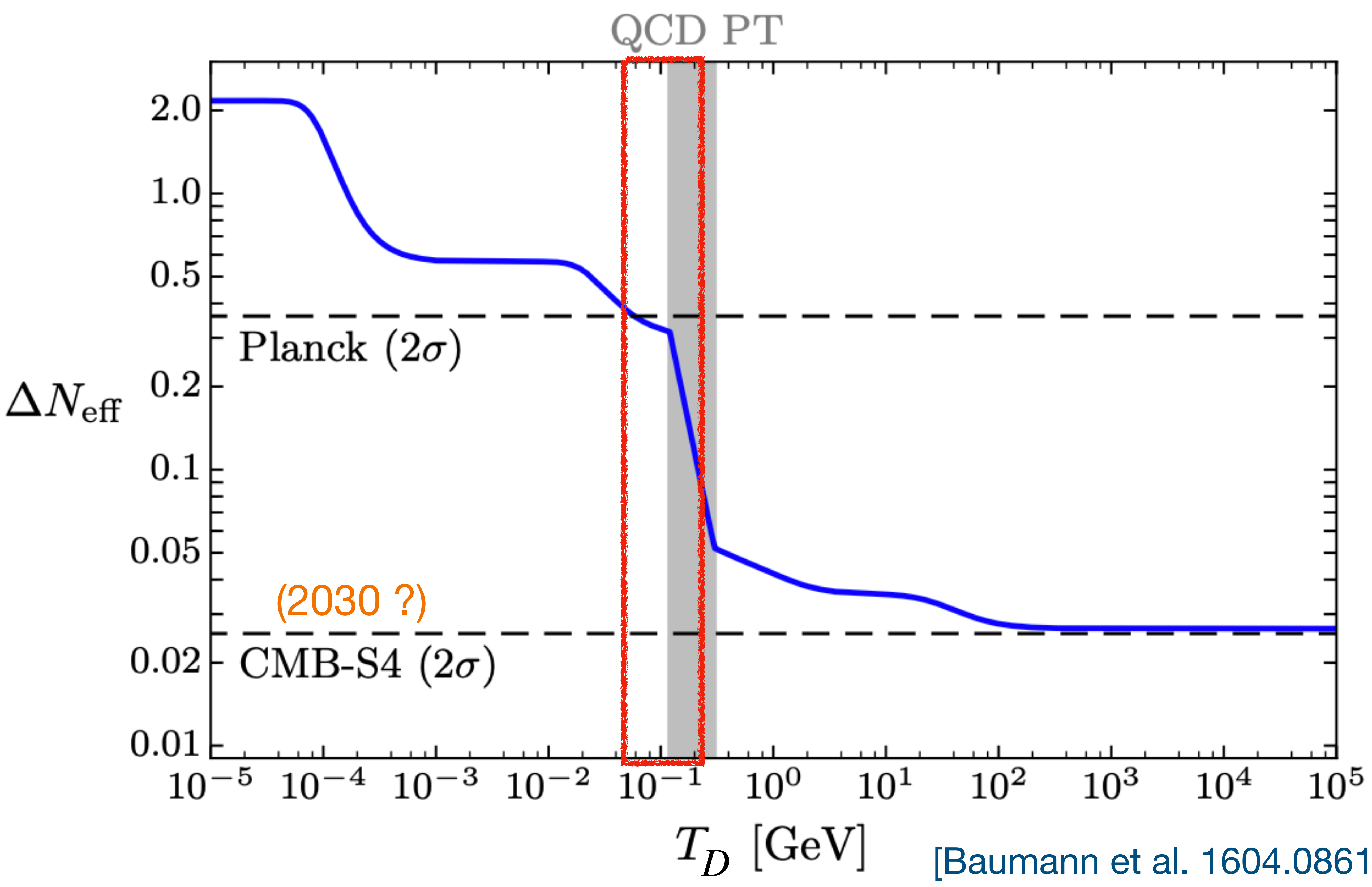
IJCLab, Pôle Théorie, CNRS and Paris Saclay U.

## 16th Patras Workshop on Axions, WIMPs and WISPs

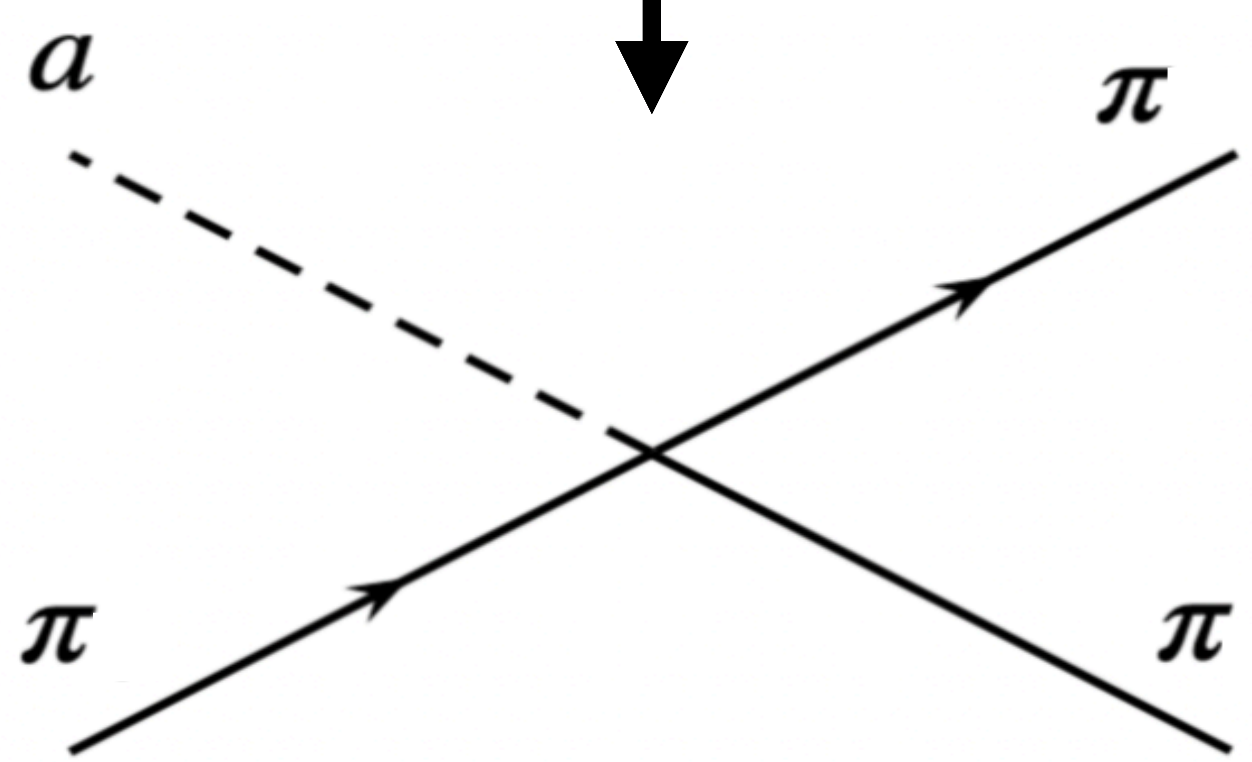
This project has received funding from the European Union’s Horizon 2020 research and innovation programme under the Marie Curie grant No 860881-HIDDeN

# A possible discovery channel for the axion!

- A thermal axion population behave as extra dark radiation ($\Delta N_{\rm eff}$). In the range $m_a \in [0.1, 1]\,\mathrm{eV}$: relevant thermalization channel




[Baumann et al. 1604.08614]




- with

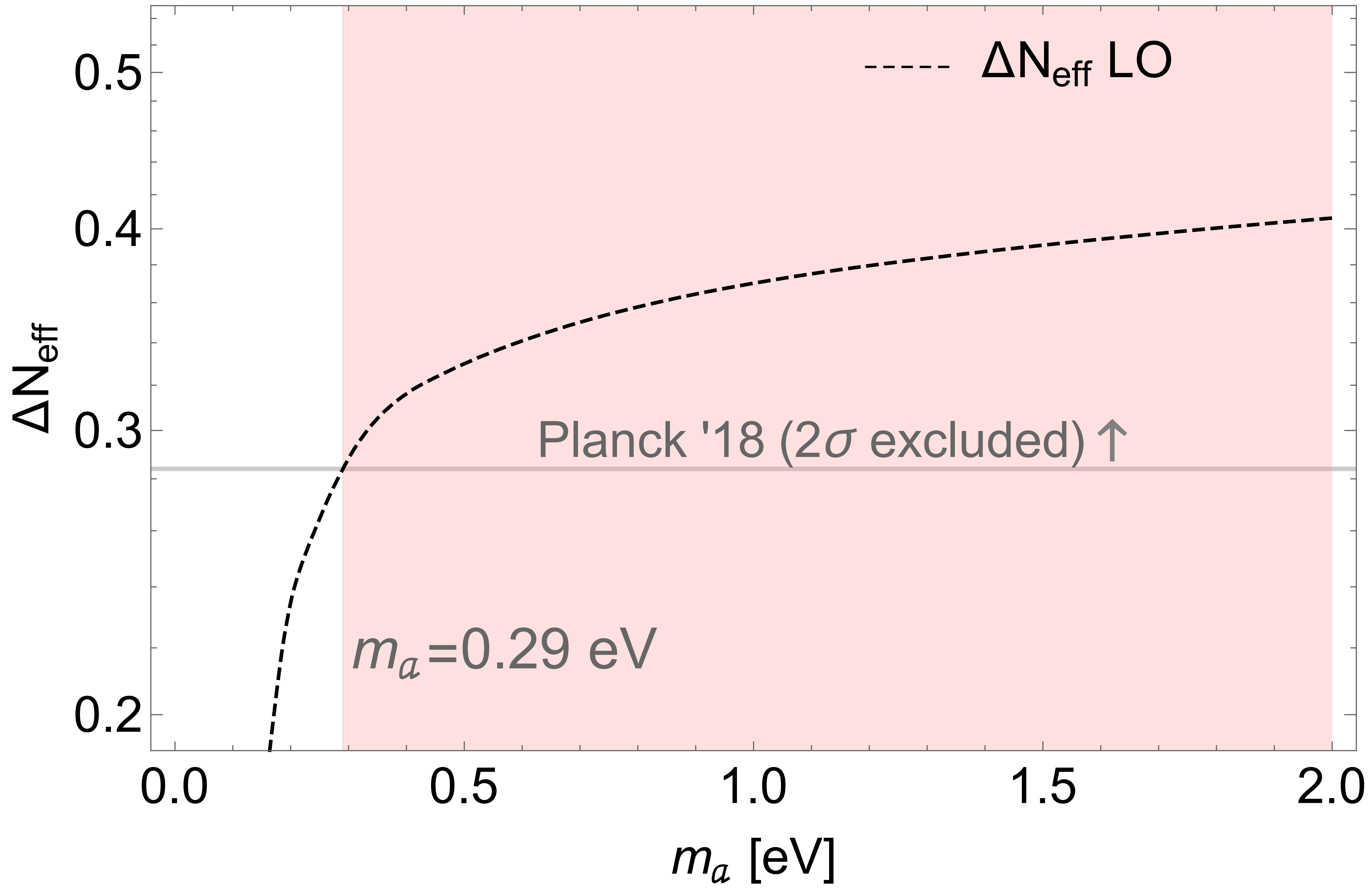
$$T_D \sim [50\ \mathrm{MeV}\,,\ 200\ \mathrm{MeV}]$$

# The LO HDM bound (naively)

- Using **leading order** *axion-pion EFT* (ChPT)

$$\Gamma_a(T) = 0.212 \left(\frac{C_{a\pi}}{f_a f_\pi}\right)^2 T^5 h_{\rm LO}(m_\pi/T)$$

$c^0_{u,\,d} = 0$




$$f_a \gtrsim 2 \times 10^7 \text{ GeV}$$

$$m_a \lesssim 0.29 \text{ eV}$$

see also:

[Chang, Choi, hep-ph/9306216]
[Hannestad, Mirizzi, Raffelt, hep-ph/0504059]
[Melchiorri, Mena, Slosar, arXiv:0705.2695]
[Hannestad, Mirizzi, Raffelt, Wong, arXiv:0803.1585]
[Hannestad, Mirizzi, Raffelt, Wong, arXiv:1004.0695]
[Di Valentino, Giusarma, Lattanzi, Mena, Melchiorri, Silk, arXiv:1507.08665]

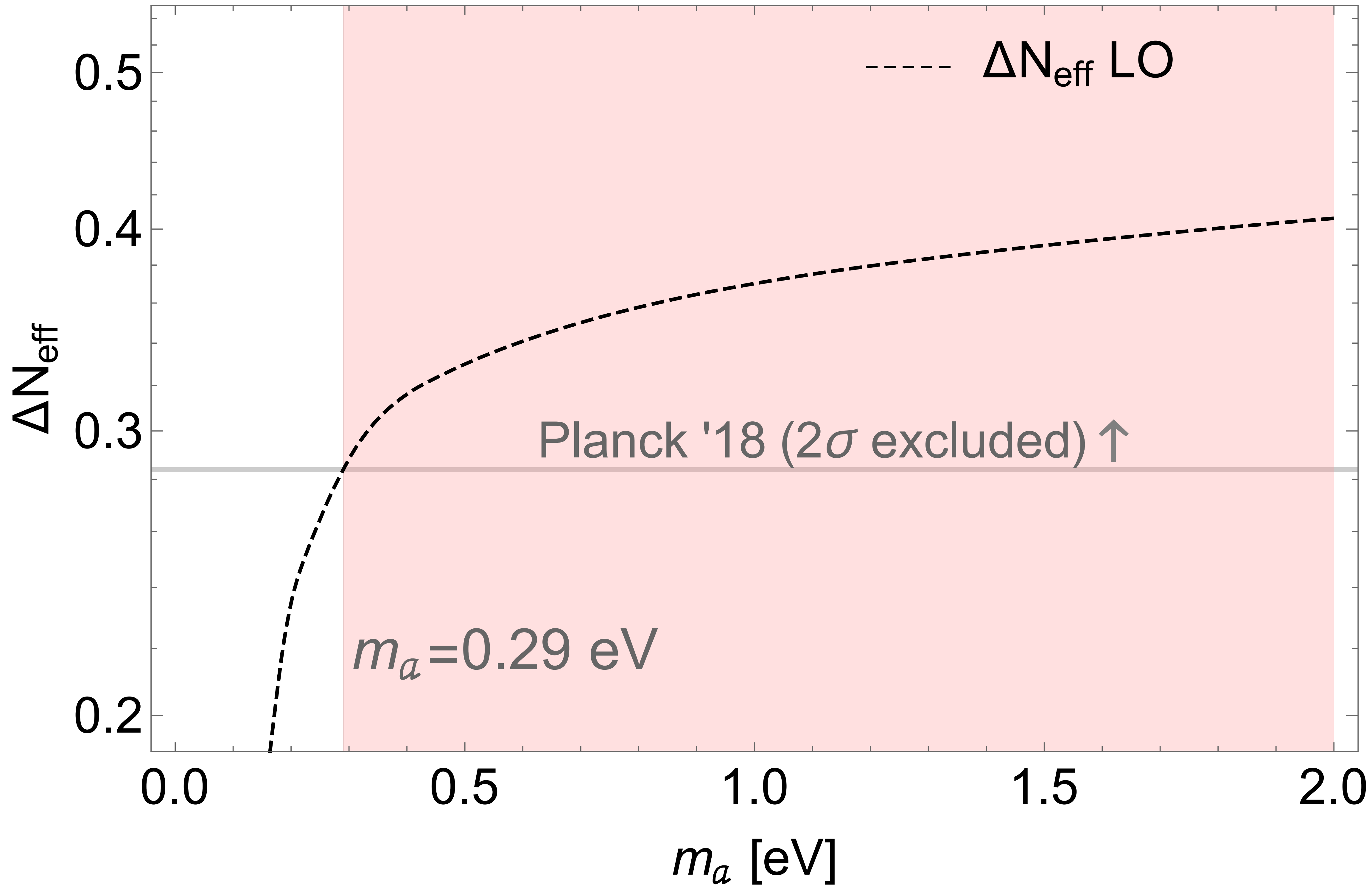
⋮

# Motivation

- Using **leading order** *axion-pion EFT* (ChPT)

$$\Gamma_a(T) = 0.212 \left(\frac{C_{a\pi}}{f_a f_\pi}\right)^2 T^5 h_{\rm LO}(m_\pi/T)$$

$c^0_{u,\,d} = 0$




$$f_a \gtrsim 2 \times 10^7 \text{ GeV}$$

$$m_a \lesssim 0.29 \text{ eV}$$

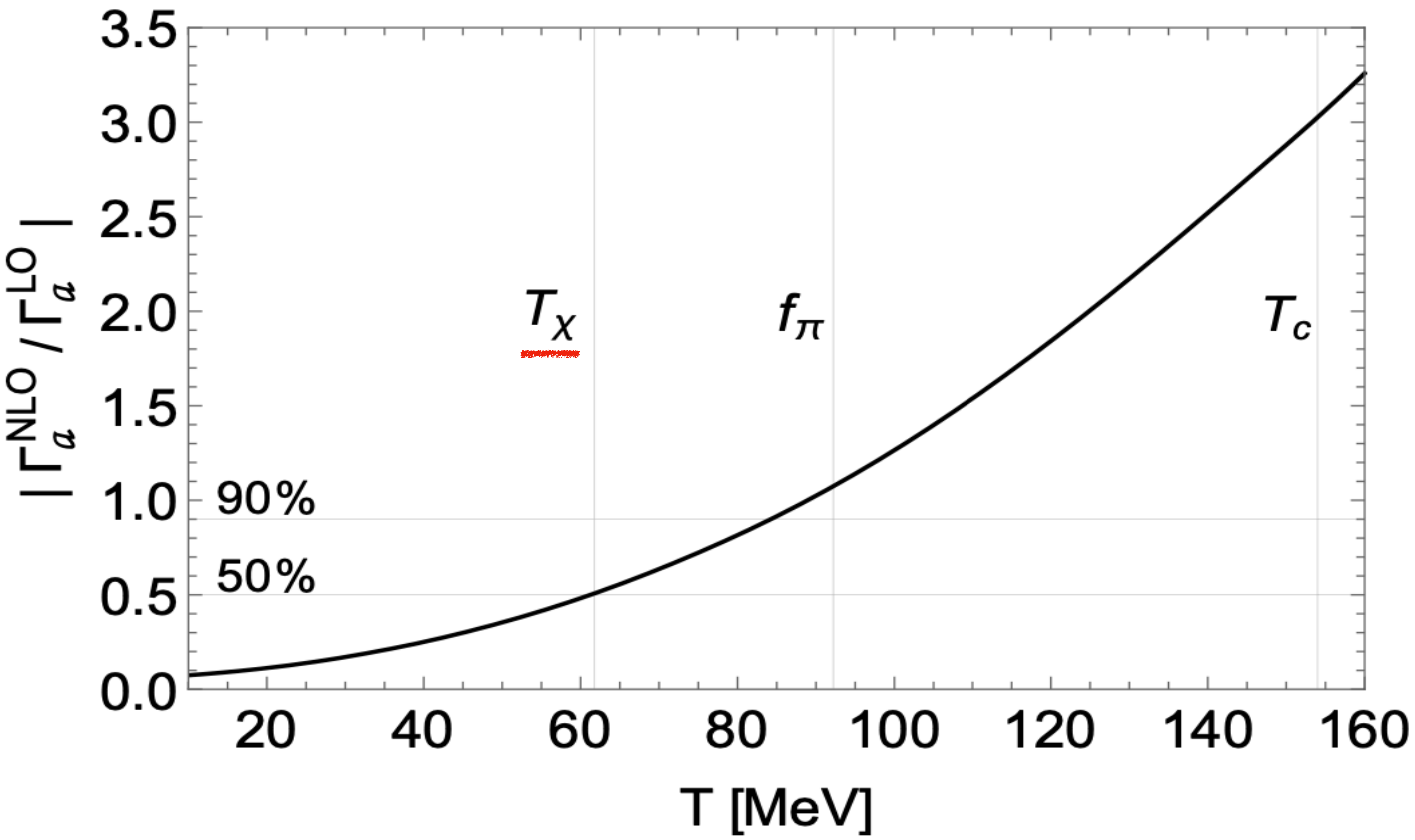
The mean energy of $a$-$\pi$ scattering at $T \simeq 100$ MeV is $\sqrt{s} \sim 620$ MeV
while
ChPT is valid for $\sqrt{s} \lesssim 500$ MeV

# NLO Thermalization rate

$$\Gamma_a(T) = \left(\frac{C_{a\pi}}{f_a f_\pi}\right)^2 \left\{ \underbrace{T^5\ 0.212\ h_{\rm LO}(m_\pi/T)}_{\rm LO} - \underbrace{\frac{T^7}{f_\pi^2}\ 0.619\ h_{\rm NLO}(m_\pi/T)}_{\rm NLO} \right\}$$

Correction of $\sim 50\,\%$ already at $T_\chi \equiv 62\ \mathrm{MeV}$

Convergence problem!



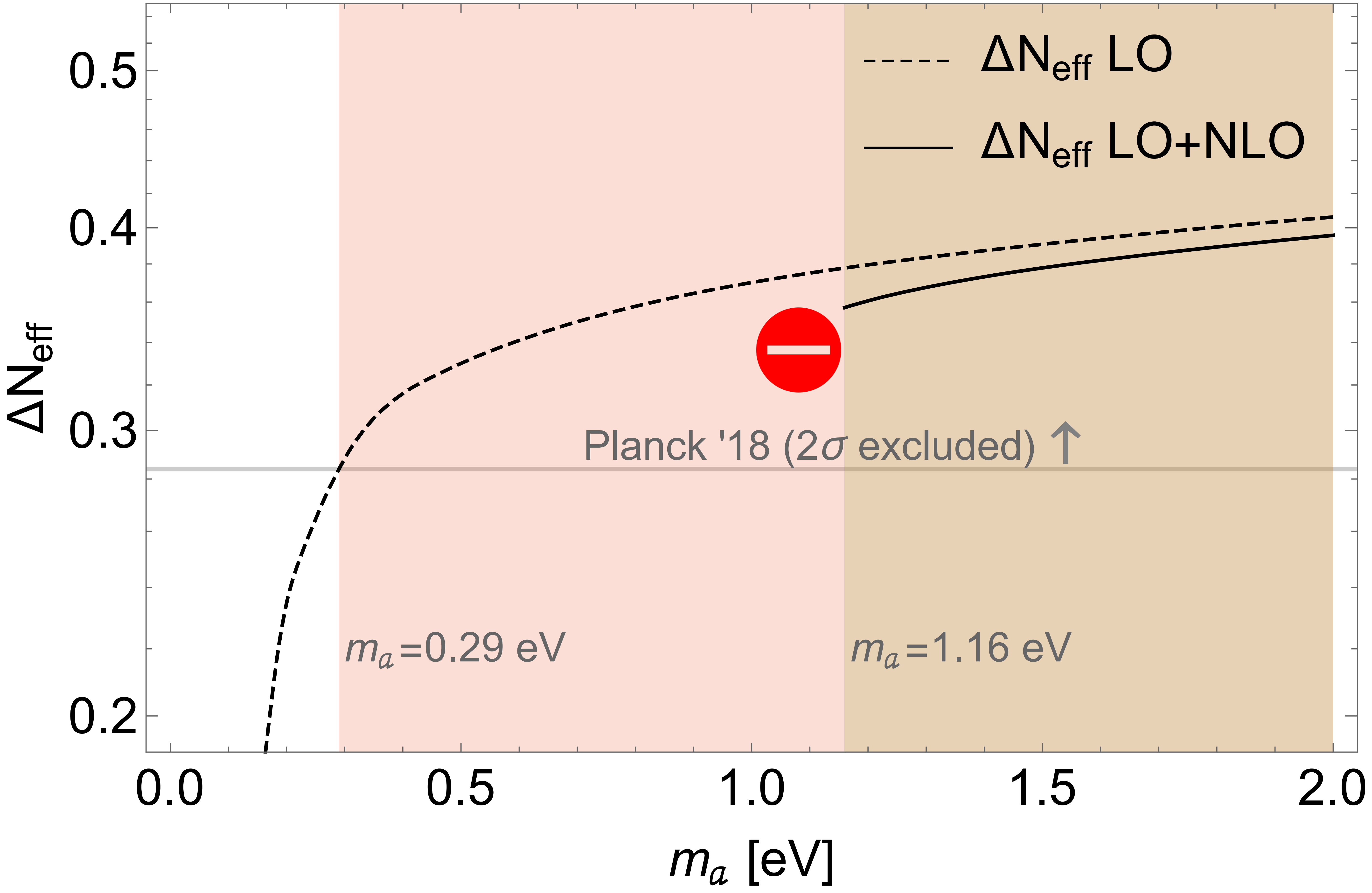

[Di Luzio, Martinelli, GP, **2101.10330**]

# $\Delta N_{\text{eff}}$ including NLO correction

The region where ChPT fails corresponds to $m_a < 1.16$ eV. Already this value yields a too large contribution to $\Delta N_{\text{eff}}$.

However, in the relevant mass range, $T_{\text{D}}$ and the HDM bound cannot be reliably extracted.

[Di Luzio, Martinelli, GP, **2101.10330**]




A reliable computation is needed to set targets for future CMB surveys (lattice QCD, …?)

# Thanks for the attention!

# Backup

# Axion-Pion Effective Lagrangian: Leading Order

$E \lesssim 1$ GeV

$$\mathcal{L}_a^{\chi} = \frac{f_\pi^2}{4} Tr\left[(D^\mu U)^\dagger D_\mu U + U\chi^\dagger + \chi U^\dagger\right] + \frac{\partial^\mu a}{f_a}\frac{1}{2} Tr\left[c_q \sigma^a\right] J_\mu^a$$

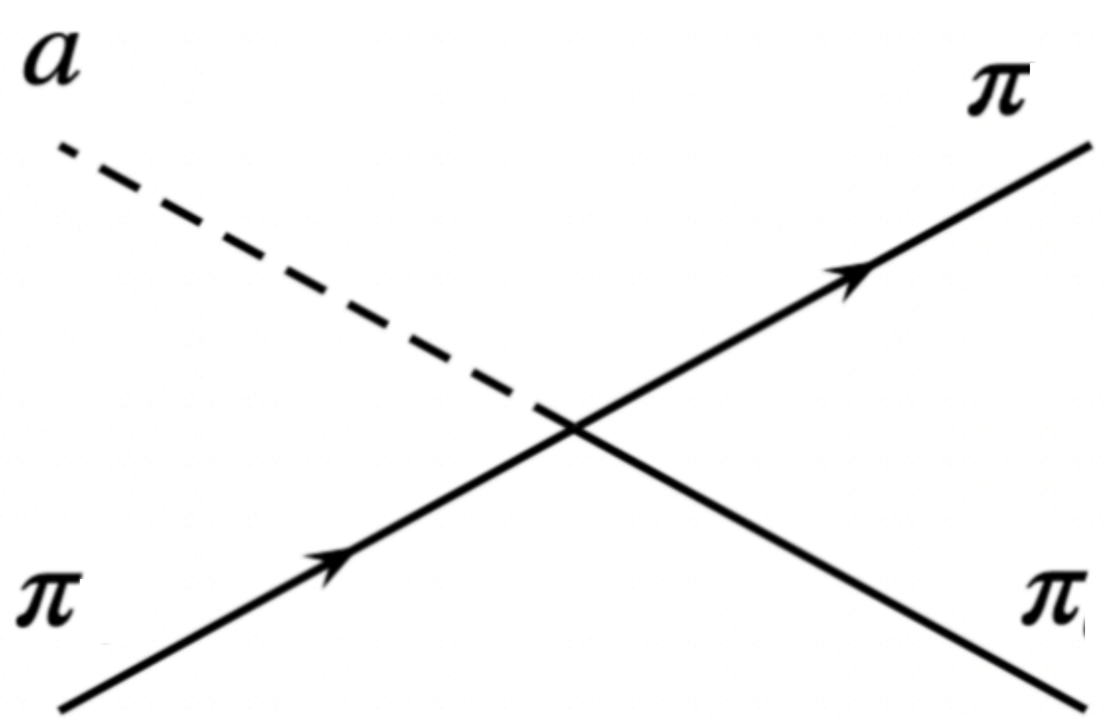
[Georgi, Kaplan, Randall, Phys. Lett. B **169** (1986)]

$$\begin{cases} U = e^{i\pi^a\sigma^a/f_\pi} \\ \chi = 2B_0\, e^{i\frac{a}{2f_a}Q_a} M_q e^{i\frac{a}{2f_a}Q_a} \end{cases}$$

$$J_\mu^a = \frac{i}{4} f_\pi^2 Tr\left[\sigma^a\{U, (D^\mu U)^\dagger\}\right]$$

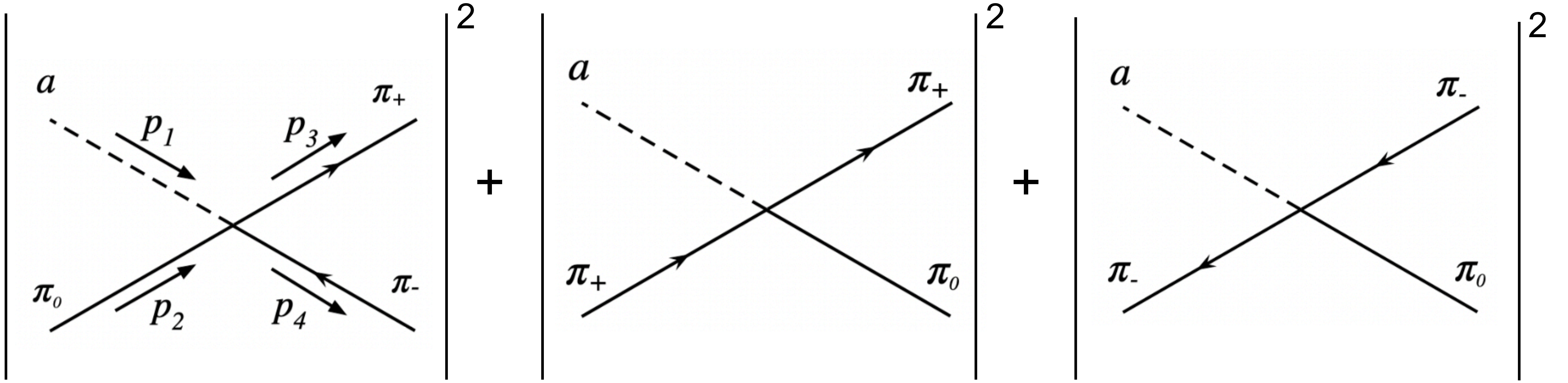
$$\mathcal{L}_{a\pi}^{(\mathrm{LO})} = \frac{C_{a\pi}}{f_a f_\pi}\partial_\mu a\left(2\partial_\mu\pi_0\pi_+\pi_- - \pi_0\partial_\mu\pi_+\pi_- - \pi_0\pi_+\partial_\mu\pi_-\right)$$

$$C_{a\pi} = \frac{1}{3}\left(\frac{m_d - m_u}{m_u + m_d} + c_d^0 - c_u^0\right)$$



# Leading order scattering amplitude

$$\mathcal{L}_{a\pi}^{(\mathrm{LO})} = \frac{C_{a\pi}}{f_a f_\pi} \partial_\mu a \Big( 2\partial_\mu \pi_0 \pi_+ \pi_- - \pi_0 \partial_\mu \pi_+ \pi_- - \pi_0 \pi_+ \partial_\mu \pi_- \Big)$$




$$\sum |\mathcal{M}|^2_{\mathrm{LO}} = \left( \frac{C_{a\pi}}{f_a f_\pi} \right)^2 \frac{9}{4} \left[ s^2 + t^2 + u^2 - 3m_\pi^4 \right]$$

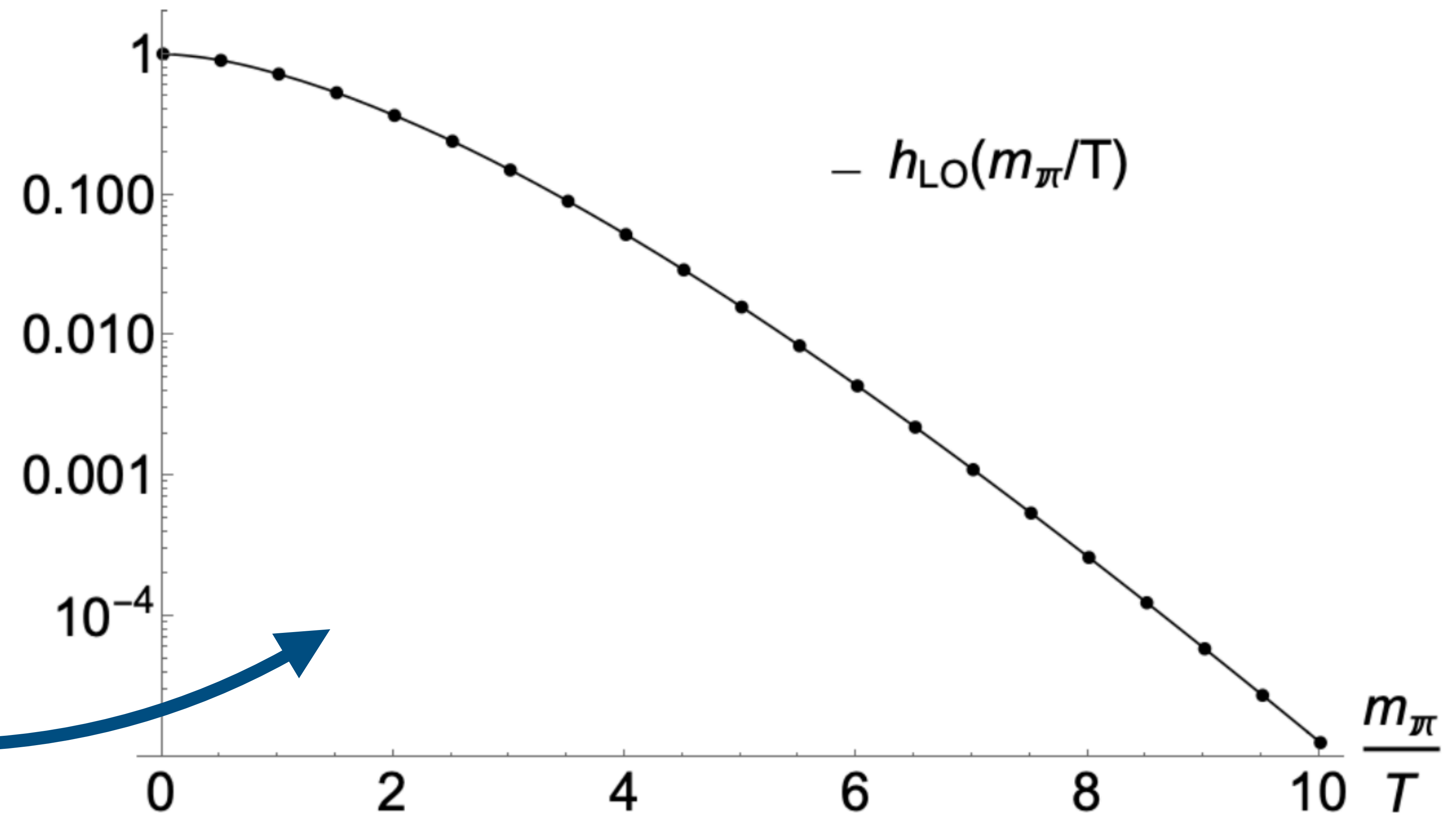
# Thermal scattering rate

$$\Gamma = \frac{1}{n_a^{\rm eq}} \int \frac{d^3\mathbf{p}_1}{(2\pi)^3 2E_1} \frac{d^3\mathbf{p}_2}{(2\pi)^3 2E_2} \frac{d^3\mathbf{p}_3}{(2\pi)^3 2E_3} \frac{d^3\mathbf{p}_4}{(2\pi)^3 2E_4} \sum |\mathcal{M}|^2$$
$$(2\pi)^4 \delta^4 \left(p_1 + p_2 - p_3 - p_4\right) f_1 f_2 (1 \pm f_3)(1 \pm f_4)$$

$$\sum |\mathcal{M}|^2_{\rm LO} = \left(\frac{C_{a\pi}}{f_a f_\pi}\right)^2 \frac{9}{4} \left[s^2 + t^2 + u^2 - 3m_\pi^4\right]$$

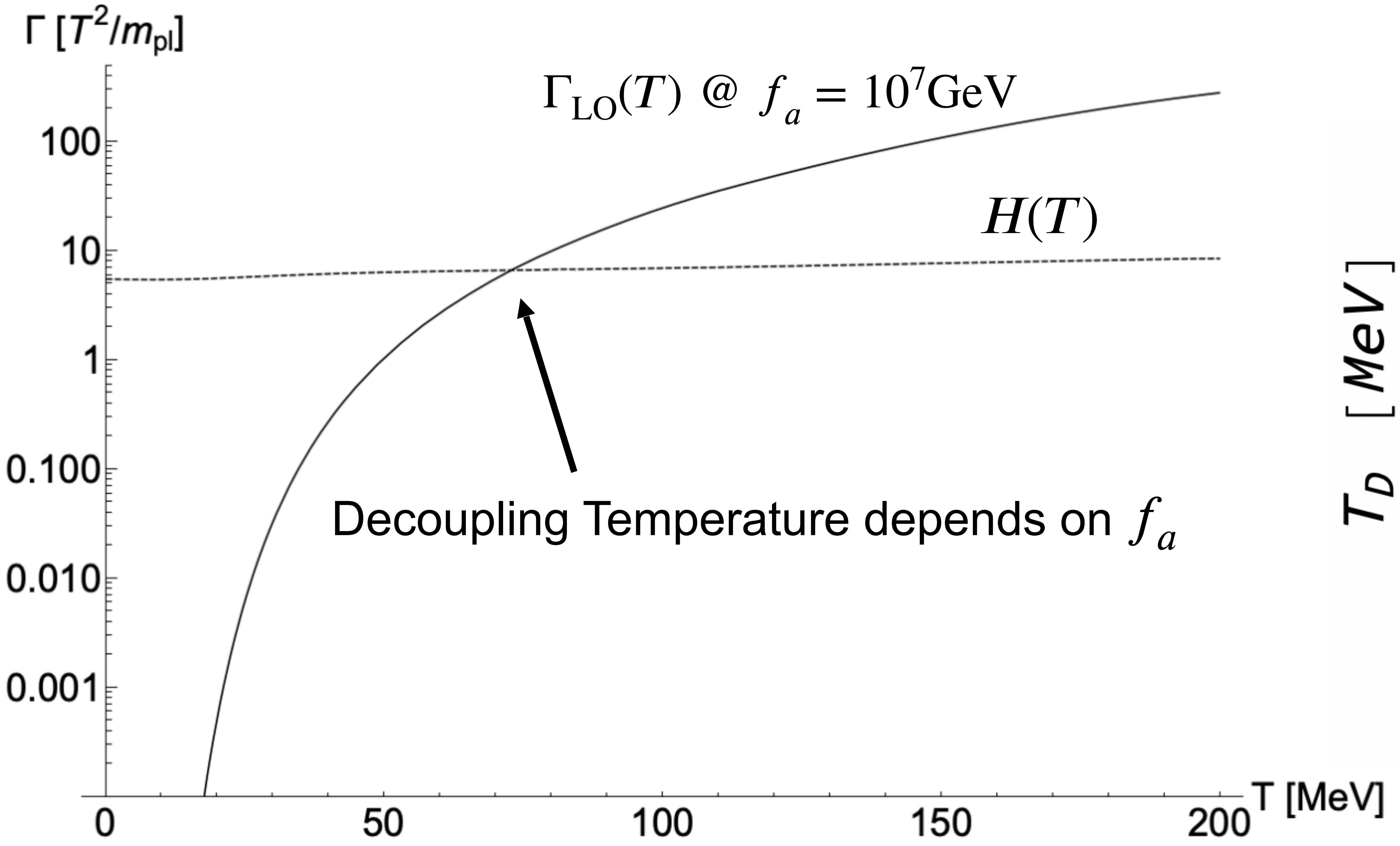
Numerically integrating:

$$\Gamma(T) = 0.212 \left(\frac{C_{a\pi}}{f_a f_\pi}\right)^2 T^5 h_{\rm LO}(m_\pi/T)$$

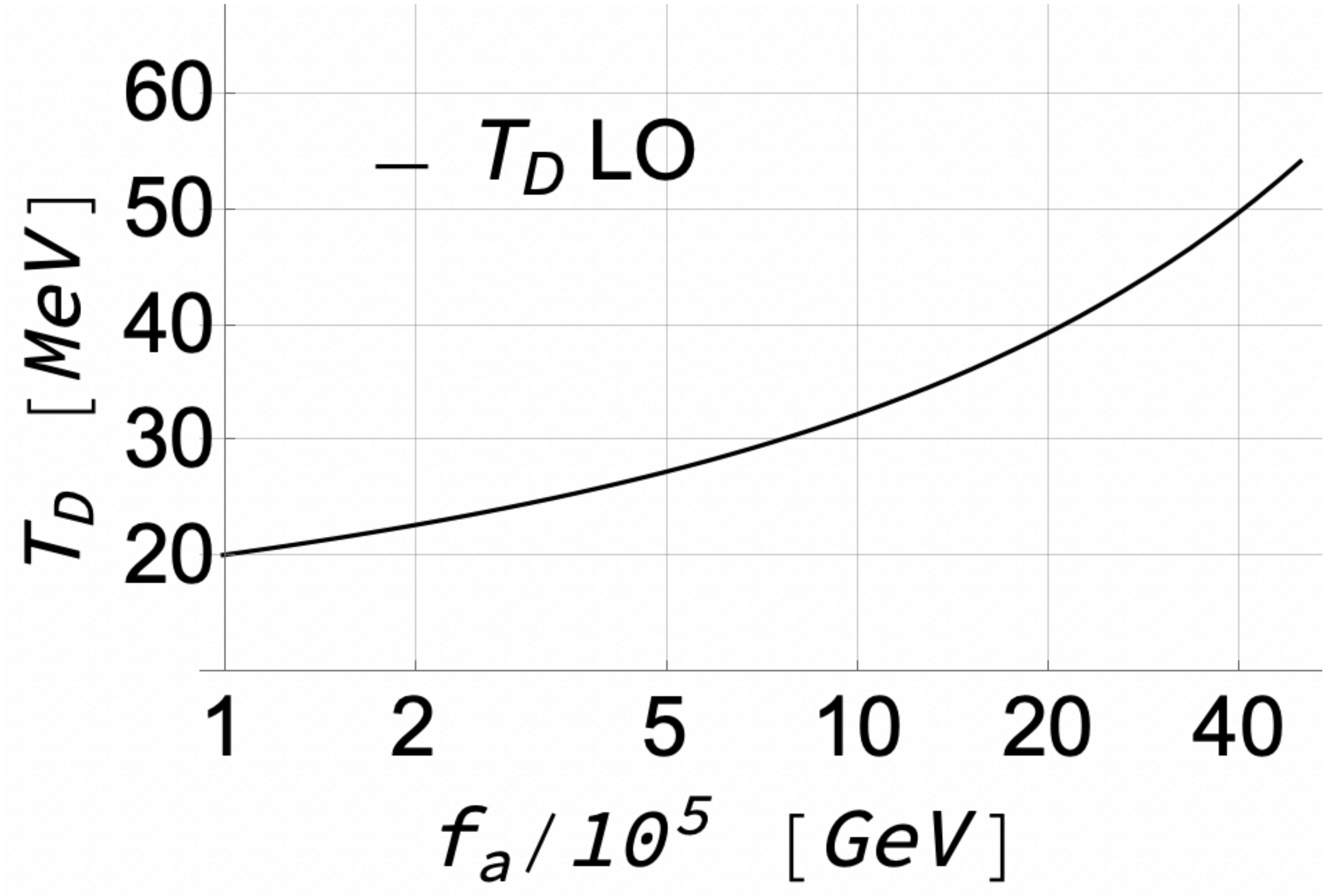



see also:
[Chang, Choi, hep-ph/9306216]
[Hannestad, Mirizzi, Raffelt, hep-ph/0504059]

# Γ vs H



Γ [T²/m_pl]
Γ_LO(T) @ f_a = 10⁷GeV
H(T)
Decoupling Temperature depends on f_a
100
10
1
0.100
0.010
0.001
0
50
100
150
200
T [MeV]

— T_D LO
T_D [MeV]
60
50
40
30
20
1
2
5
10
20
40
f_a / 10⁵ [GeV]

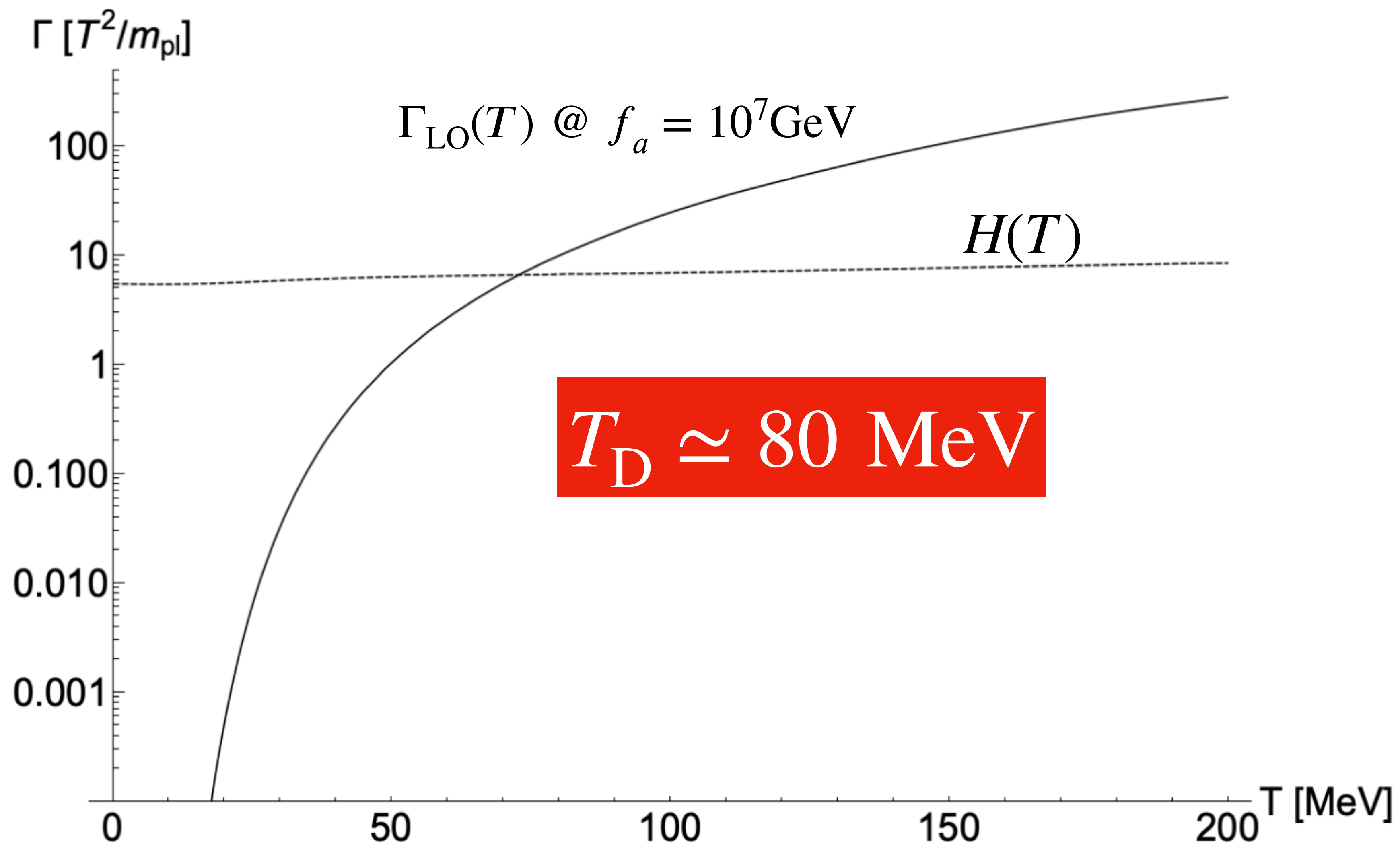
# ChPT validity

The mean energy of $\pi, a$ at
$T \simeq 80$ MeV is
$\langle E \rangle \equiv \rho/n \simeq 305$ MeV, 220 MeV

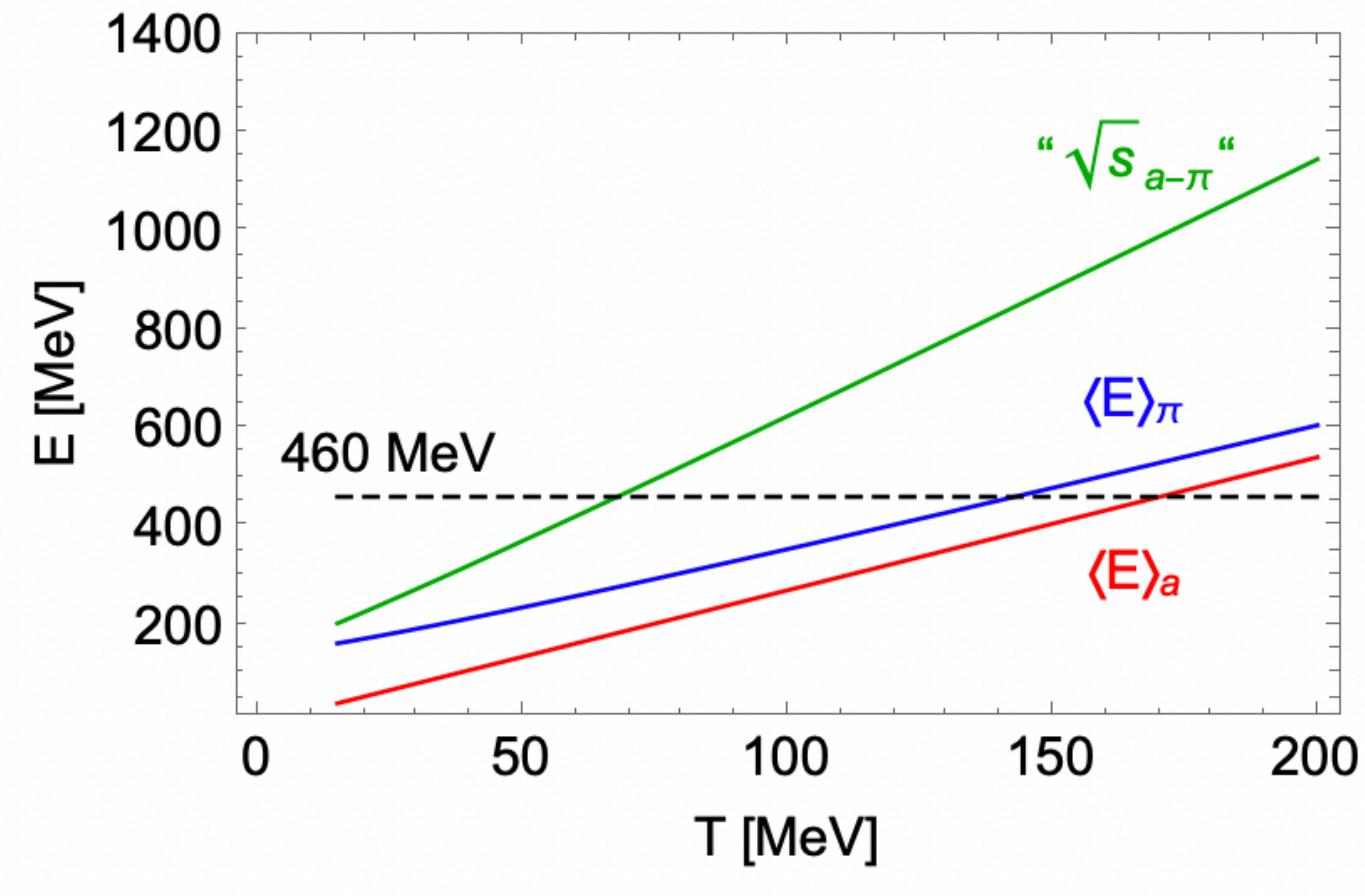
BUT

ChPT is valid for $E \lesssim 460$ MeV

[Donoghue et al., PhysRevD.86.014025]




$$\langle E \rangle (T) \sim \frac{\rho(T)}{n(T)}$$




The bound extracted from ChPT is not reliable

# Axion-Pion scattering: Next-to-Leading Order

## Ingredients

Tree-level graph from NLO Lagrangian and loop amplitudes from LO Lagrangian contributes to the same Order

$$
\begin{aligned}
\mathcal{L}_{\rm NLO} = & \frac{l_1}{4}\left\{\mathrm{Tr}\left[D_\mu U\left(D^\mu U\right)^\dagger\right]\right\}^2 + \frac{l_2}{4}\mathrm{Tr}\left[D_\mu U\left(D_\nu U\right)^\dagger\right]\mathrm{Tr}\left[D^\mu U\left(D^\nu U\right)^\dagger\right] \\
& + \frac{l_3}{16}\left[\mathrm{Tr}\left(\chi U^\dagger + U\chi^\dagger\right)\right]^2 + \frac{l_4}{4}\mathrm{Tr}\left[D_\mu U\left(D^\mu \chi\right)^\dagger + D_\mu \chi\left(D^\mu U\right)^\dagger\right] \\
& + l_5\left[\mathrm{Tr}\left(f^R_{\mu\nu} U f_L^{\mu\nu} U^\dagger\right) - \frac{1}{2}\mathrm{Tr}\left(f^L_{\mu\nu} f_L^{\mu\nu} + f^R_{\mu\nu} f_R^{\mu\nu}\right)\right] \\
& + i\frac{l_6}{2}\mathrm{Tr}\left[f^R_{\mu\nu} D^\mu U\left(D^\nu U\right)^\dagger + f^L_{\mu\nu}\left(D^\mu U\right)^\dagger D^\nu U\right] \\
& - \frac{l_7}{16}\left[\mathrm{Tr}\left(\chi U^\dagger - U\chi^\dagger\right)\right]^2 + \frac{h_1+h_3}{4}\mathrm{Tr}\left(\chi\chi^\dagger\right) + \frac{h_1-h_3}{16}\left\{\left[\mathrm{Tr}\left(\chi U^\dagger + U\chi^\dagger\right)\right]^2\right. \\
& \left. + \left[\mathrm{Tr}\left(\chi U^\dagger - U\chi^\dagger\right)\right]^2 - 2\,\mathrm{Tr}\left(\chi U^\dagger \chi U^\dagger + U\chi^\dagger U\chi^\dagger\right)\right\} - 2h_2\,\mathrm{Tr}\left(f^L_{\mu\nu} f_L^{\mu\nu} + f^R_{\mu\nu} f_R^{\mu\nu}\right)
\end{aligned}
$$

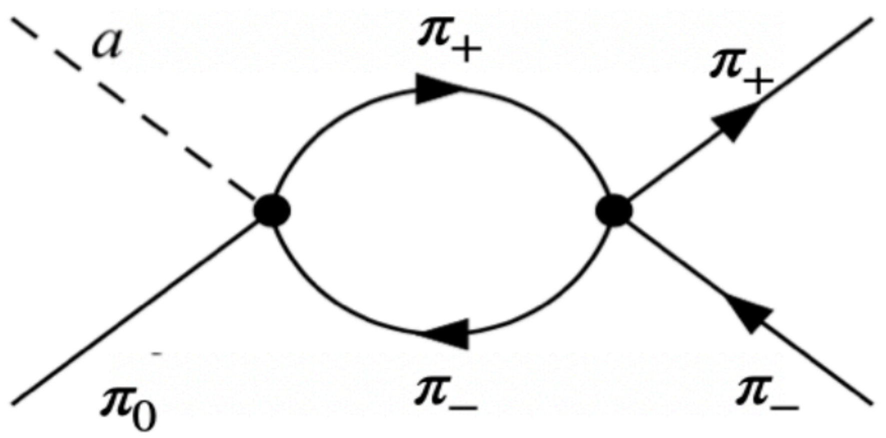
NLO Lagrangian

+

NLO chiral axial current $J^a_\mu$

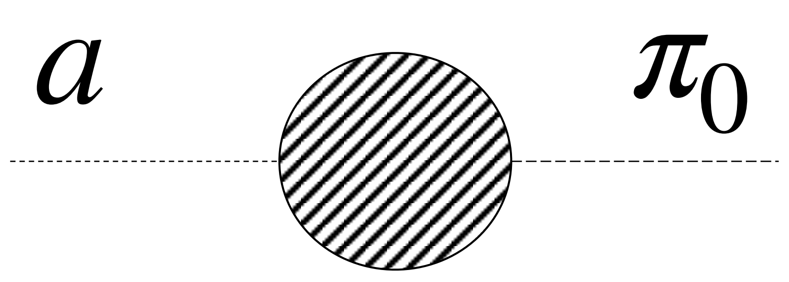
$$\mathcal{L}^\chi_a \supset \frac{\partial^\mu a}{f_a}\mathrm{Tr}\frac{1}{2}\left[c_q\sigma^a\right]J^a_\mu$$

+




1-loop amplitudes from LO

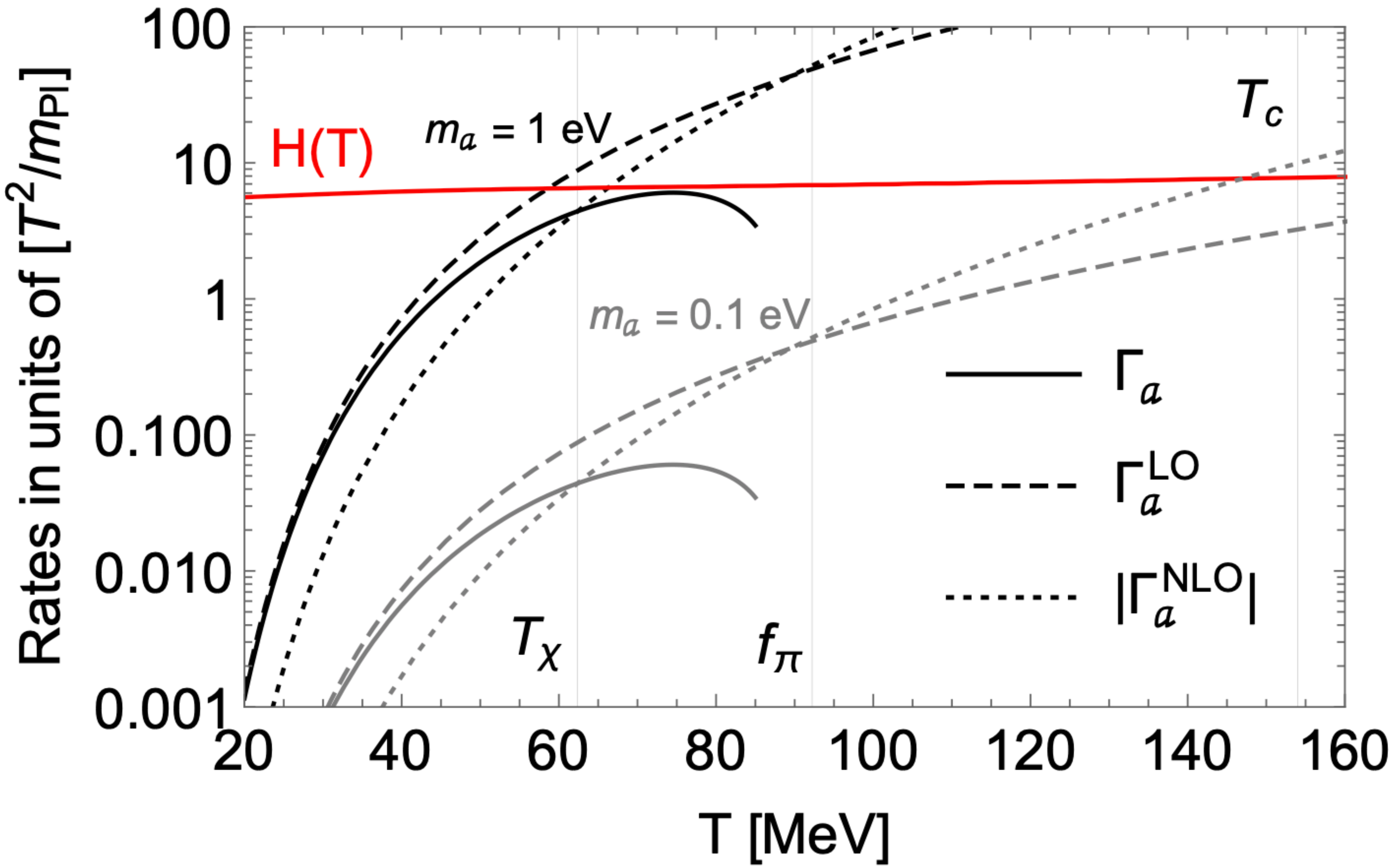
+




$a - \pi_0$ diagonalization

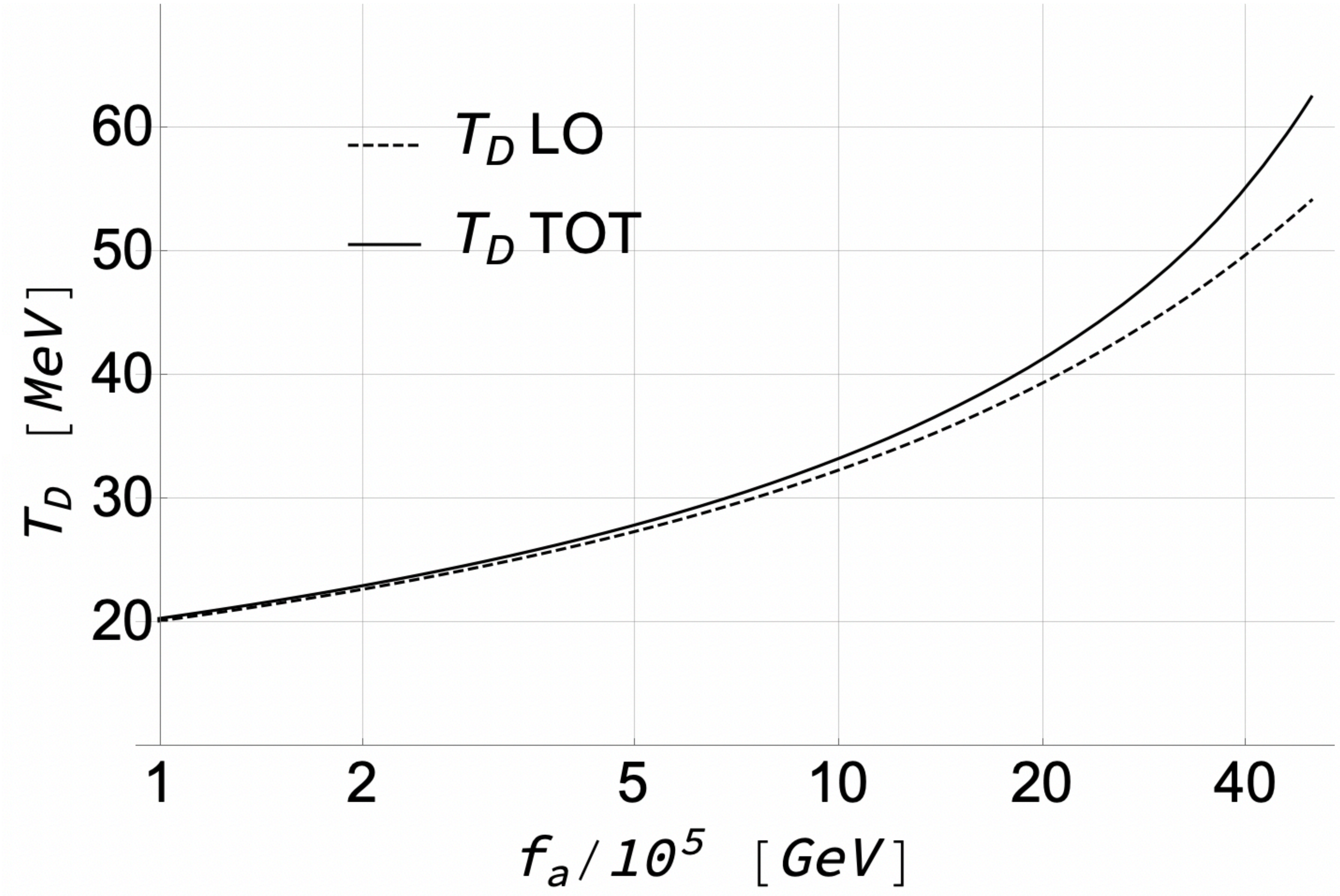
# $\Gamma$ vs H , **NLO**

- $m_a = 1$ eV: the most conservative HDM bound
- $m_a = 0.1$ eV: typical reach of future CMB-S4 experiments
- $T_\chi \sim 62$ MeV: boundary of validity of the chiral expansion



# $T_D$ vs $\mathrm{f_a}$

Decoupling temperature for the LO and LO+NLO case, as a function of $f_a$



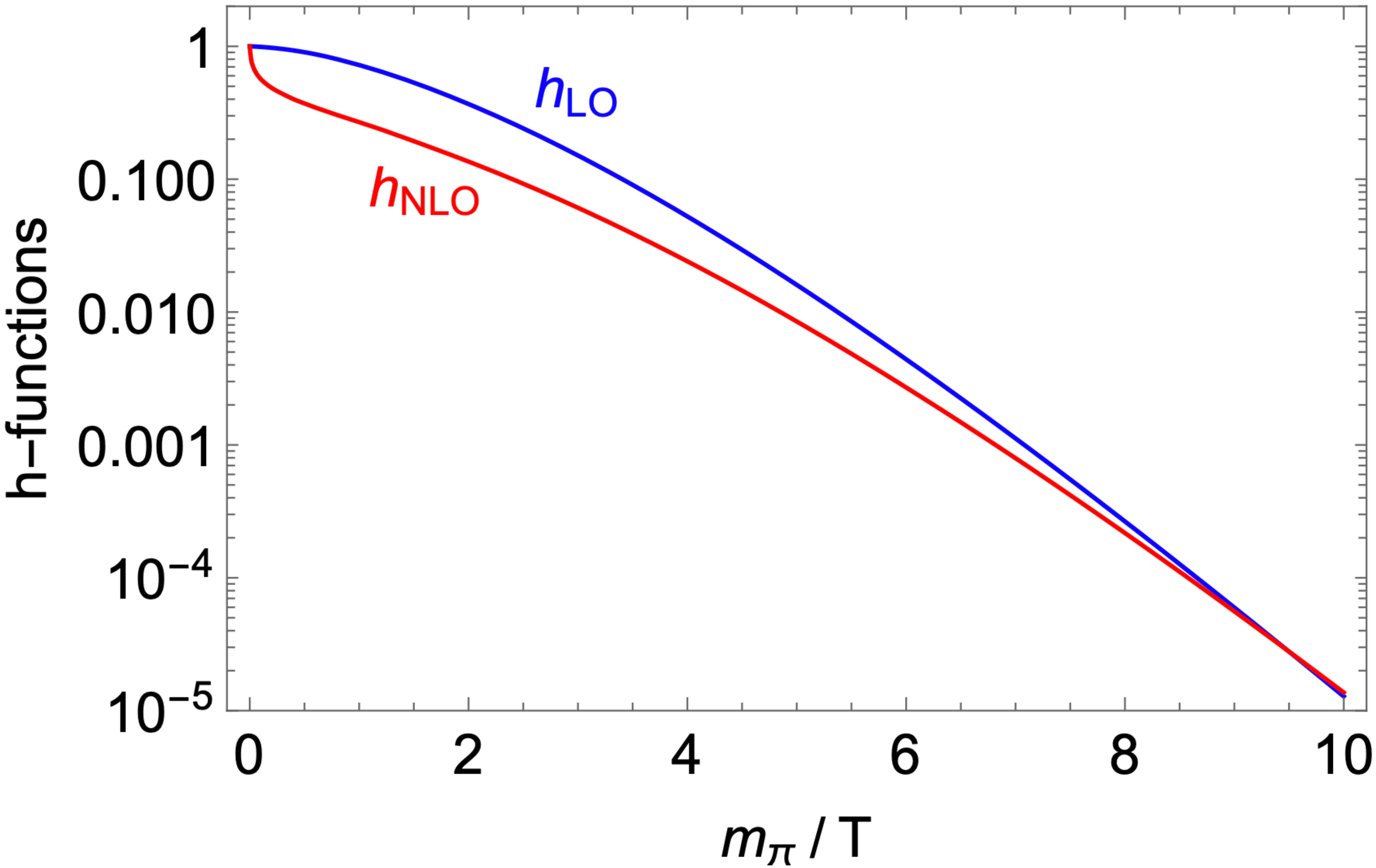

$$T_D < T_\chi$$

if

$$f_a \lesssim 4.9 \times 10^6 \text{ GeV}$$

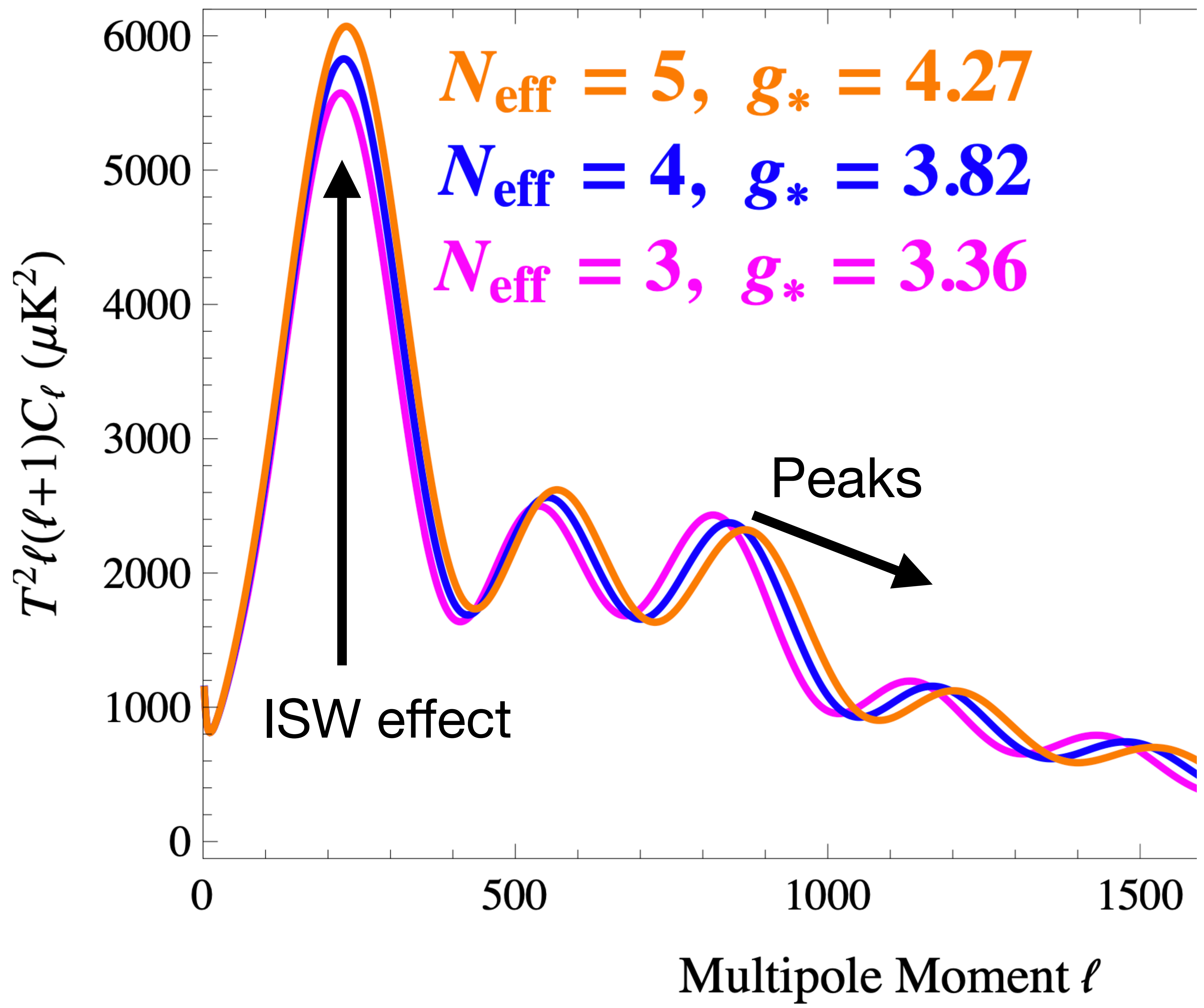
$$m_a \gtrsim 1.16 \text{ eV}$$

# *h* functions



# Effects of $N_{\text{eff}}$ on the CMB

- $N_{\text{eff}} \uparrow \Rightarrow H \uparrow$, time for photons diffusion in the plasma decreases, reducing Silk damping and restricting it to higher $\ell$. $\ell_{\text{dump}} \uparrow$
- $H \uparrow$ Acoustic oscillation length scale decreases, increasing the sound horizon. $\ell_{\text{sound}} \uparrow$
- Overall less dumping but more peaks dumped. $H \uparrow \Rightarrow \ell_s/\ell_d \uparrow$
- Also, gravitational red/blue shift increased on 1st peak scales (ISW)

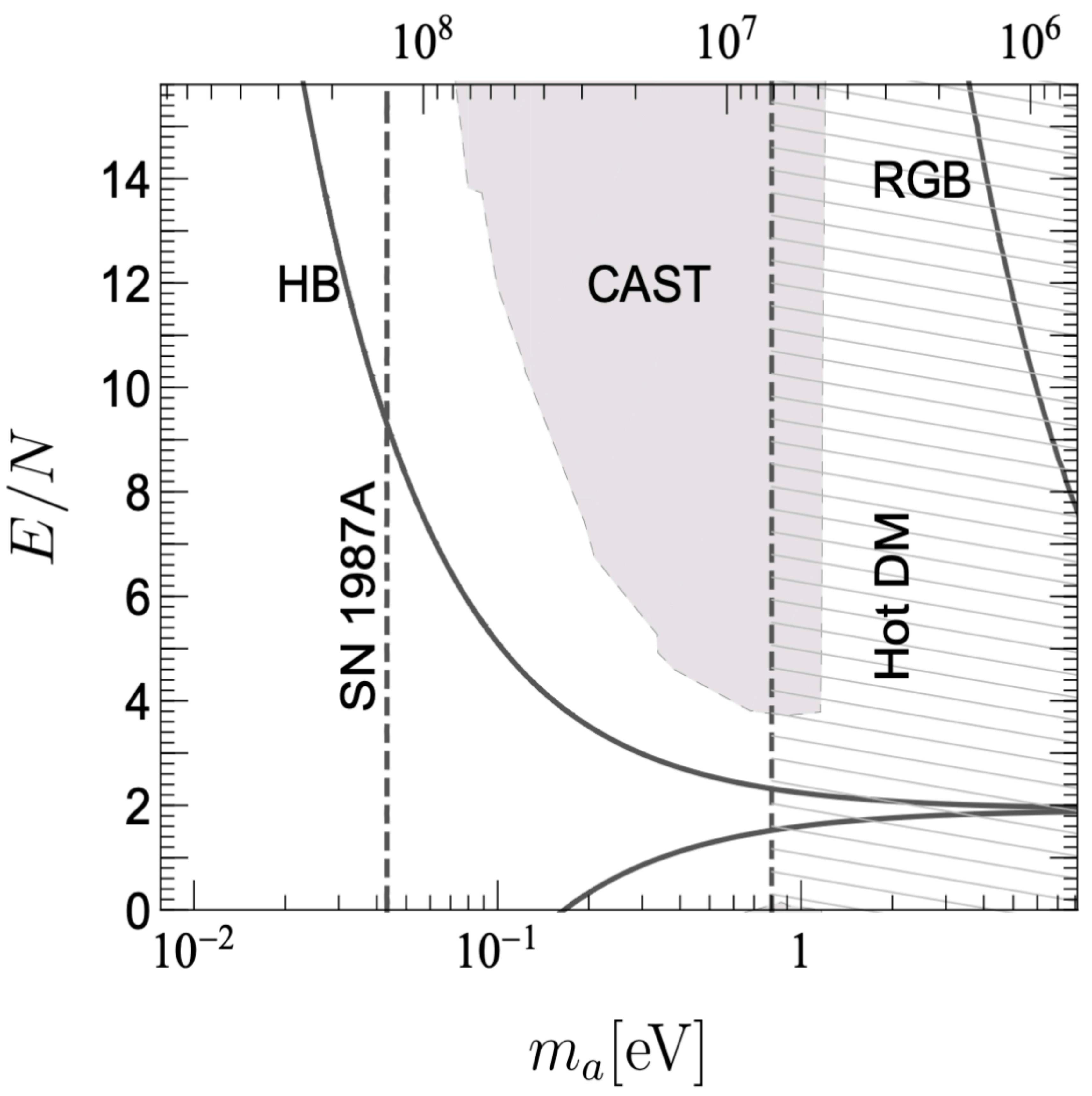



[Brust, Kaplan, Walters, arXiv:1303.5379]

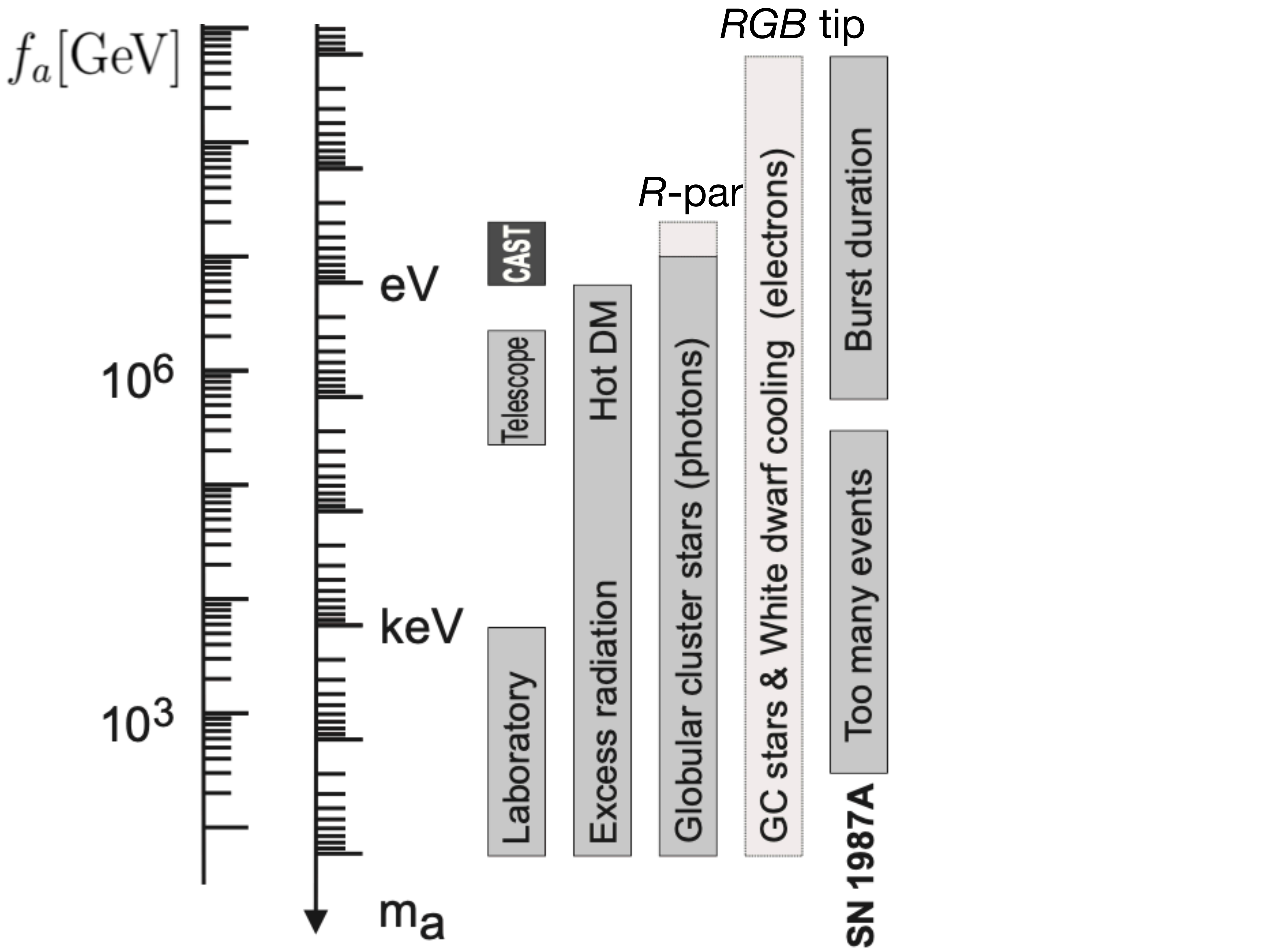
[Silk, *Astrophys.J.* 151 (1968)]
[Sachs, Wolfe, *Astrophys. J.* 147 (1967)]
[Bowen, Hansen, Melchiorri, Silk, Trotta, arXiv: astro-ph/0110636]
[Brust, Kaplan, Walters, arXiv:1303.5379]

# ASTRO Bounds

QCD axion




[Di Luzio et al., Phys. Rept. **870** (2020) ]




[Raffelt hep-ph/061135]

- $g_{ae}^0 = 0$ in KSVZ models
- SN bound not solid from astrophysics
  [Bar, Blum, D'Amico 1907.05020]
- $g_{a\gamma}$ can be accidentally suppressed ($N_Q > 1$)
  [Di Luzio, Mescia, Nardi, arXiv:1705.05370]