Weyl-Invariant Gravity and the Nature of Dark Matter

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Basic Assumptions

- Weyl-invariant (WI) version of general relativity (GR). WI: freedom to **locally rescale** our fundamental units
- The standard model (SM) of particle physics
- A distinction between active & passive gravitational masses
- G & M_{ac} weakly vary over galactic and super-galactic scales

Weyl-Transformation

$$g_{\mu\nu} \rightarrow \Omega^2(x) g_{\mu\nu}$$
$$\mathcal{L}_M \rightarrow \Omega^{-4}(x) \mathcal{L}_M$$
$$\phi \rightarrow \Omega^{-1}(x) \phi$$

Note that Ω(x) is an arbitrary function

Can be used in the WI extension of GR to generate new solutions.

Weyl-Invariant Scalar-Tensor (WIST) Theory

$$\mathcal{I}_{EH} = \int \left[R/(16\pi G) + \mathcal{L}_M \right] \sqrt{-g} d^4x = \int \left(\frac{1}{2} m_P^2 R + \mathcal{L}_M \right) \sqrt{-g} d^4x$$

$$\mathcal{I}_{WIST} = \int \left(\frac{1}{6} |\phi|^2 R + \phi_{,\mu} \phi^{*,\mu} + \mathcal{L}_M(|\phi|, \{\psi\}) \right) \sqrt{-g} d^4x$$

Deser 1970

Field Equations

$$\frac{\delta \mathcal{I}_{WIST}}{\delta g^{\mu\nu}} = 0 \Rightarrow \qquad \frac{|\phi|^2}{3} G_{\mu\nu} = T_{M,\mu\nu} + \Theta_{\mu\nu}$$

$$3\Theta_{\mu\nu} \equiv \phi^*_{\mu;\nu}\phi - 2\phi^*_\mu \phi_\nu - g_{\mu\nu}(\phi^* \Box \phi - \frac{1}{2}\phi^*_\alpha \phi^\alpha) + c.c.$$

$$\phi^* \frac{\partial \mathcal{L}_M}{\partial \phi^*} + \phi \frac{\partial \mathcal{L}_M}{\partial \phi} = T_M$$

Linearly perturbed metric assuming baryons only (G is a constant):

$$ds^2 = -(1-2\Phi_b)dt^2 - dtdx^i v_i + (1+2\Phi_b)dx_i dx^i$$

Adding DM potential:

$$\Omega(x) = 1 + \Phi_{DM}(x)$$

$$ds^2 \rightarrow \Omega^2 ds^2 =$$

$$-[1-2(\Phi_b+\Phi_{DM})]dt^2 - dtdx^i v_i + [1+2(\Phi_b+\Phi_{DM})]dx_i dx^i$$

Summary

- DM phenomena on galactic and galaxy cluster scales could be explained by a WI version of GR, i.e. a `new' symmetry of gravitation
- This entails weak spatial (and possibly temporal as well) variation of GM_{ac} of $O(\phi_{DM})$ that act as an effective ρ_{DM}

Backup Material

Machianity of Brans-Dicke Theory

$$(3 + 2\omega_{BD})\Box\Phi_{BD} = 8\pi T_M$$

But there is a (reasonable) `cost':

$$GM_{ac} \propto m_P^{-2} M_{ac} \propto \phi^{-2} \phi = \phi^{-1} \propto 1 + \Phi_{DM}(x)$$

e.g $$\Phi_{DM} = \Phi_{NFW}(r) \propto r^{-1} \ln(1 + r/r_s)$$

Why is the gravitational potential linear whereas the overdensity is non-linear ?

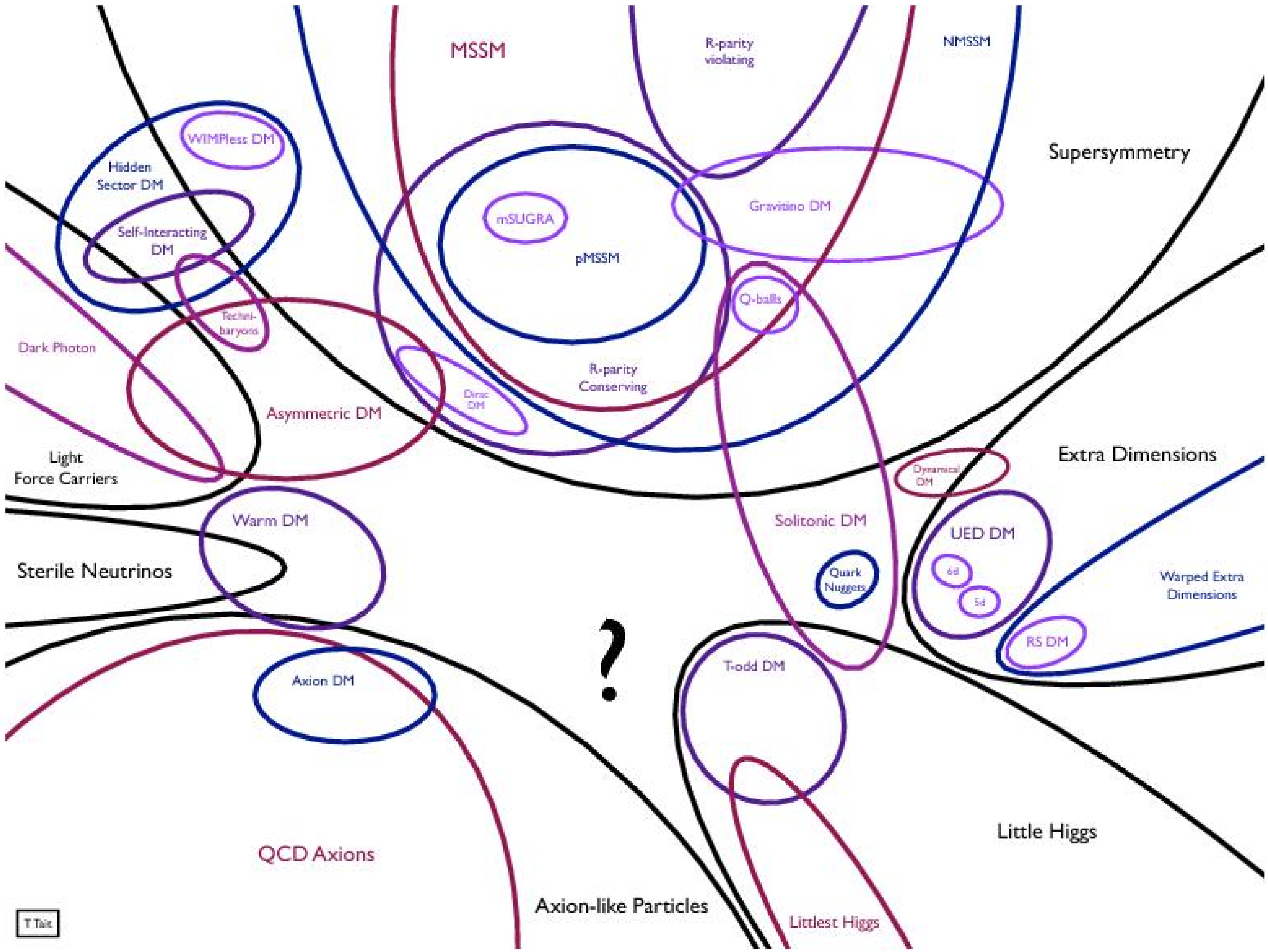
$$\nabla^2 \Phi = 4\pi G \delta\rho = 4\pi G \rho \frac{\delta\rho}{\rho} = \frac{3H^2}{2} \frac{\delta\rho}{\rho}$$

$$\Rightarrow \qquad \left(\frac{l_H}{l_{gal}}\right)^2 \Phi = \frac{\delta\rho}{\rho}$$

Slightly modified baryon density

$$\rho_b \to \rho_b/(1+\Phi_{DM})^4 \approx \rho_b(1-4\Phi_{DM})$$

Dark Matter Venn Diagram



Mannheim-Kazanas Metric

$$ds^2_{MK} = -(1 - \frac{\beta}{r} + \gamma r - \kappa r^2)dt^2 + \frac{dr^2}{1 - \frac{\beta}{r} + \gamma r - \kappa r^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

Mannheim & Kazanas, 1989
MS , arXiv:1702.08472
Li & Modesto, 2020