The spectral lineshape of gradient-coupled bosonic dark matter in our galaxy

Arne Wickenbrock, Alexander Gramolin et al.

for the CASPEr collaboration

PATRAS conference, 18.06.2021
Nuclear Magnetic Resonance Meets Dark Matter

The Cosmic Axion Spin Precession Experiment (CASPERe)
Why is the lineshape relevant?

Improving limits by optimally filtering the power spectrum data in SHAFT

Talk by Alex Sushkov (Thursday)

Ciaran O’Hare: https://cajohare.github.io/AxionLimits/
CASPEr Boston result

Search for Axionlike Dark Matter Using Solid-State Nuclear Magnetic Resonance

Deniz Aybas, Janos Adam, Emmy Blumenthal, Alexander V. Gramolin, Dorian Johnson, Annalies Kleyheeg, Samer Alach, John W. Blanchard, Gary P. Centers, Antoine Garcon, Martin Engler, Nathan L. Figueroa, Marina Gil Sendra, Arne Wickenbrock, Matthew Lawson, Tao Wang, Teng Wu, Haosu Luo, Hamdi Mani, Philip Mauskopf, Peter W. Graham, Surjeet Rajendran, Derek F. Jackson Kimball, Dmitry Budker, and Alexander O. Sushkov

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Optimal filtering in CASPEr-E Boston required the gradient coupling lineshape
How to search for Axions (ALPs)?

**Axion (ALP) Interactions**

**Gravity**

- Gauge Fields
  - \(rac{a}{f_a} F_{\mu\nu} \tilde{F}^{\mu\nu} \)
  - \(rac{a}{f_a} G_{\mu\nu} \tilde{G}^{\mu\nu} \)

**Fermions**

- \( \psi^\mu \gamma^\mu \gamma^5 \psi \)

**Most Searches**

- (CASPER-E)
- (CASPER-Gradient, GNOME, QUAX)
CASPER – Gradient Mainz – idea

Detecting oscillating torque on nuclear xenon-129 spins from (0.01-4) MHz

\[ \frac{\partial \mu^a}{\partial f_a} \bar{\Psi} f \gamma^\mu \gamma_5 \Psi_f \]

• Polarized nuclear spins
• B field
• Axion gradient couples to spins
• Oscillating torque on spins
• Transversal magnetization
• Pickup (somehow)

Larmor Frequency = Axion mass
=> Resonant enhancement

In Mainz:
Casper Gradient Low field 0.01-4 MHz
Casper Gradient High Field 4-600 MHz

Talk by Hendrik Bekker (Thursday)
Two ways to search

(a) \( \omega_L = \gamma B_0 \)

(b) \( m = 1/2 \)
\( m = -1/2 \)
\( \omega_{RF} \)
\( \omega_{ALP} \)

(c) Signal X, Y

Perp. To B
Sensitivity plane

Parallel to B
Sensitivity axis

18.06.2021, Patras
Take home messages

1. **Gradient signal** = 3D pseudo magnetic field vector rotating in space with two independent phases
2. **Gradient line shape** fundamentally different (from the scalar lineshape)
3. Most power in the direction of movement (~ linear polarized signal)
4. Larger effect of annual and daily modulation
   1. On signal amplitude
   2. On signal frequency
5. Not new, but still: Virialized dark matter power spectra = modulated white noise
Derivation of the scalar lineshape

Axion field

\[ a(r, t) = \frac{a_0}{\sqrt{N}} \sum_{n=1}^{N} \cos \left( 2\pi \nu_n t - k_n \cdot r + \phi_n \right) \]

Axion field frequency class \( j \)

\[ a_j(t) = \frac{a_0}{\sqrt{N_j}} \sum_{i \in \Omega_j} \cos (\omega_j t + \phi_i) \propto \mathcal{R} \left[ \exp (i\omega_j t) \sum_{i \in \Omega_j} \exp (i\phi_i) \right] \]

Central limit theorem:

\[ \sum_{i \in \Omega_j} \exp (i\phi_i) = \sqrt{\frac{N_j}{2}} \sum_{i \in \Omega_j} \exp (i\phi_j) \]

\[ \propto N_j \]

Rayleigh distributed amplitude

Random phase

Exponentially distributed power!
Scalar lineshape – ALP velocity distribution

\[ \lambda(\nu) = f(\nu) \left. \frac{dv}{d\nu} \right|_{v_c = c\sqrt{2(\nu/\nu_a - 1)}} \]

Speed probability distribution Maxwell–Boltzmann (\(\sigma_v=156\, \text{km/s}\))

Power spectrum = Axion per frequency class distribution (\(\sigma_v=156\, \text{km/s}\))

18.06.21, Patras

Ciaran O’Hare PHYSICAL REVIEW D95, 063017 (2017)
Scalar lineshape – Power spectra, exponentially distributed in each bin.
Derivation of the gradient lineshape

Gradient Axion field

\[ \nabla a(r, t) = \sqrt{2 \rho_{DM}} \left( \sum_{n=1}^{N} \nabla_n \sin \left( 2\pi \nu_n t - k_n \cdot r + \phi_n \right) \right) \]

Gradient field frequency class \( j \), x-direction

\[ \nabla a_{j,x}(t) \propto \exp(i\omega_j t) \left( \sum_{i \in \Omega_j} v_{x,i} \exp(i\phi_i) \right) \]

Central limit theorem:

\[ \sum_{i \in \Omega_j} v_{x,i} \exp(i\phi_i) \propto \sqrt{N_j \left( \sigma_{v_x}^2 + \mu_{v_x}^2 \right)} \alpha_j \exp(i\phi_j) \]

Rayleigh distributed amplitude

Random phase, different for different directions!

Exponentially distributed power!

Variance and Mean \( v_x \) component for a given frequency
Gradient lineshape – ALP velocity distribution

\[ \lambda(\nu) = \left( \sigma_{v_x}^2 + \mu_{v_x}^2 \right) f(\nu) \frac{dv}{dv} \left|_{v = c \sqrt{2(v/v\alpha - 1)}} \right. \]

Scalar lineshape!
Function of speed \( v \)

Lineshapes:
- \(<a(\omega)^2>\>
- \(<\nabla_a ||(\omega)^2>\>
- \(<\nabla_a \perp(\omega)^2>\>

Constant speed \( v \)
Constant \( v_z \)
Scaling with lab velocity

Perpendicular to the direction of movement

In the direction of movement
Daily modulation

CASPEr Mainz $B_0$

Earth velocity

Strong daily intensity modulation

18.06.2021, Patras
Daily modulation

- CASPER Mainz $B_0$
- Earth velocity

Strong daily intensity modulation
And
Daily frequency modulation! (10%-15% FWHM)
Take home messages

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5. Not new, but still:
   Virialized dark matter power spectra = modulated white noise
Wavy Dark Matter summer in Mainz/Germany August 2022

Mon, 08th
PATRAS
Mainz

Tue, 09th
PATRAS
Mainz

Wed, 10th
PATRAS
Mainz

Thu, 11th
PATRAS
Mainz

Fri, 12th
PATRAS
Mainz

Mon, 15th
MITP
Mainz

Tue, 16th
MITP
Mainz

Wed, 17th
MITP
Mainz

Thu, 18th
MITP
Mainz

Fri, 19th
MITP
Mainz

Bad Honnef
Summer school
Scientific foundations and experimental searches

Ultra-light Dark Matter

Mainz Institute of Theoretical Physics - Workshop
Searches for Wave-Like Dark Matter with Quantum Networks
Proposal!
Thank you