Direct detection of WIMPs of arbitrary spin

Paolo Gondolo

University of Utah

Tokyo Institute of Technology

IPMU, University of Tokyo

The effective theory of nuclear scattering
for a WIMP of arbitrary spin

Gondolo, Jeong, Kang, Scopel, Tomar, PRD, to appear [arxiv:2008.05120]

The phenomenology of nuclear scattering
for a WIMP of arbitrary spin

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Previous work

Effective operators for one-nucleon interactions

Nonrelativistic WIMP-nucleon contact operators classified

Barger et al 2008, Fan et al 2010, Fitzpatrick et al 2012, Dent et al 2015

$$\mathcal{O}_1 = 1_\chi 1_N \qquad \mathcal{O}_7 = \vec{S}_N \cdot \vec{v}^\perp_{\chi N}$$

$$\mathcal{O}_3 = -i\vec{S}_N \cdot \left(\frac{\vec{q}}{m_N} \times \vec{v}^\perp_{\chi N}\right) \qquad \mathcal{O}_8 = \vec{S}_\chi \cdot \vec{v}^\perp_{\chi N}$$

$$\mathcal{O}_4 = \vec{S}_\chi \cdot \vec{S}_N \qquad \mathcal{O}_9 = -i\vec{S}_\chi \cdot \left(\vec{S}_N \times \frac{\vec{q}}{m_N}\right)$$

$$\mathcal{O}_5 = -i\vec{S}_\chi \cdot \left(\frac{\vec{q}}{m_N} \times \vec{v}^\perp_{\chi N}\right) \qquad \mathcal{O}_{10} = -i\vec{S}_N \cdot \frac{\vec{q}}{m_N}$$

$$\mathcal{O}_6 = \left(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}\right)\left(\vec{S}_N \cdot \frac{\vec{q}}{m_N}\right) \qquad \mathcal{O}_{11} = -i\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}$$

and more in Barger et al. 2008, Fan et al. 2010, Dent et al 2015

To leading order in q and v, only O_1 and O_4 appear, which are the spin-independent and spin-dependent terms, respectively.

Previous work

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Complete for WIMPs of spin 0 and 1/2

Complete for WIMPs of spin 1 after clarification

But WIMPs of higher spin are interesting

To leading order in q and v, only O_1 and O_4 appear, which are the spin-independent and spin-dependent terms, respectively.

Anomalous multipole moments

Relativistically, higher multipole moments receive contributions from lower multipole moments, and any difference is called anomalous. Thus a particle with nonzero 2^p moment and zero lower moments has an anomalous multipole moment 2^p.

Dirac fermions have g=2 plus an anomalous magnetic dipole moment

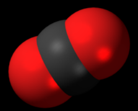
W bosons have charge e, magnetic dipole moment $\mu_W = e(1+\kappa+\lambda)/(2m_W)$, and electric quadrupole moment $Q_W = -e(\kappa-\lambda)/m_W^2$. Any dipole or quadrupole moment in addition to these is called anomalous.

A spin-1 dark matter particle may have zero charge, zero dipole moment, and a nonzero anomalous quadrupole moment: quadrupolar dark matter

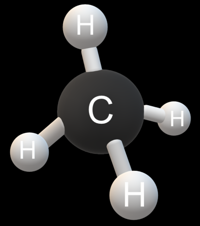
Molecular dark matter

For composite dark matter, the vanishing of the lower moments may be due to internal symmetry.

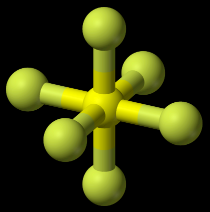
Quadrupolar molecules like CO_2, CS_2, C_6H_6 have zero net charge, zero permanent dipole moment, and nonzero permanent quadrupole moment



Octupolar molecules like CH_4, CF_4 have zero net charge, zero permanent dipole and quadrupole moments, and nonzero permanent octupole moment by tetrahedral symmetry



Hexadecapolar molecules like SF_6, CF_4 have zero net charge, zero permanent dipole, quadrupole and octupole moments, and nonzero permanent hexadecapole moment by octahedral symmetry



For dark matter, a dynamics in the dark sector may lead to dark matter molecules with similar high symmetry. Under interactions with very short range, no significant induced moments would be generated and the interactions with nucleons may be due to the highest permanent moment only: multipolar dark matter

WIMPs of any spin

It is possible that dark matter interacts with ordinary matter through high-order multipole moments 2^p only.

(dipole $p = 1$, quadrupole $p = 2$, octupole $p = 3$, hexadecapole $p = 4$, etc)

For a permanent multipole moment 2^p, rotational considerations require that the spin of a dark matter particle, which may not be elementary, is $j_\chi \geq p/2$.

(dipole $j_\chi \geq 1/2$, quadrupole $j_\chi \geq 1$, octupole $j_\chi \geq 3/2$, hexadecapole $j_\chi \geq 2$, etc)

Formalism

Building nonrelativistic interaction operators

Only five single-nucleon (current) densities

$$n^\tau(\mathbf{r}) = \sum_N \delta(\mathbf{r} - \mathbf{r}_N)\, t_N^\tau$$ number density

$$\boldsymbol{\Sigma}^\tau(\mathbf{r}) = \sum_N \boldsymbol{\sigma}_N\, \delta(\mathbf{r} - \mathbf{r}_N)\, t_N^\tau$$ spin density

$$\mathbf{j}_c^\tau(\mathbf{r}) = \sum_N \mathbf{v}_{c,N}(\mathbf{r})\, t_N^\tau$$ current density

$$\boldsymbol{\Phi}^\tau(\mathbf{r}) = \sum_N \boldsymbol{\sigma}_N \times \mathbf{v}_{c,N}(\mathbf{r})\, t_N^\tau$$ spin-orbit density

$$\Omega^\tau(\mathbf{r}) = \sum_N \boldsymbol{\sigma}_N \cdot \mathbf{v}_{c,N}(\mathbf{r})\, t_N^\tau$$ helicity density

$$\mathbf{v}_{c,N}(\mathbf{r}) = \frac{\delta(\mathbf{r} - \mathbf{r}_N)\,(-i\overrightarrow{\boldsymbol{\nabla}}) + (i\overleftarrow{\boldsymbol{\nabla}})\,\delta(\mathbf{r} - \mathbf{r}_N)}{2m_N}$$

The scattering cross section

WIMP-nucleus scattering amplitude

$$\mathcal{M}_{NR} = \sum_\tau \Big[\ell_M^\tau(\mathbf{q})\, F_M^\tau(\mathbf{q}) + \boldsymbol{\ell}_\Sigma^\tau(\mathbf{q}) \cdot \mathbf{F}_\Sigma^\tau(\mathbf{q}) + \boldsymbol{\ell}_\Delta^\tau(\mathbf{q}) \cdot \mathbf{F}_\Delta^\tau(\mathbf{q}) + \boldsymbol{\ell}_\Phi^\tau(\mathbf{q}) \cdot \mathbf{F}_\Phi^\tau(\mathbf{q}) + \ell_\Omega^\tau(\mathbf{q})\, F_\Omega^\tau(\mathbf{q}) \Big]$$

where $F_M^\tau(\mathbf{q}) = \int \langle \mathrm{f} | n^\tau(\mathbf{r}) | \mathrm{i} \rangle\, e^{i\mathbf{q}\cdot\mathbf{r}}\, d^3x,\ \mathbf{F}_\Sigma^\tau(\mathbf{q}) = \int \langle \mathrm{f} | \boldsymbol{\Sigma}^\tau(\mathbf{r}) | \mathrm{i} \rangle\, e^{i\mathbf{q}\cdot\mathbf{r}}\, d^3x, \ldots$

(further split vector form factors into transverse and longitudinal w.r.t. $\mathbf{q}$)

WIMP-nucleus unpolarized scattering cross section

$$\frac{d\sigma}{dE_R} = \frac{m_T}{2\pi v^2} \sum_{AB} \sum_{\tau\tau'} R_{AB}^{\tau\tau'}(q,v)\, F_{AB}^{\tau\tau'}(q)$$

WIMP response functions
contain WIMP-nucleon interactions

nucleus response functions
A and B equal to $M, \Sigma', \Sigma'', \tilde{\Phi}', \Phi'', \Delta$

The event rate

The scattering rate per unit target mass (recoil spectrum)

$$\frac{dR}{dE_R} = \frac{1}{m_T} \frac{\rho_\chi}{m_\chi} \int_{v>v_{\min}} v^2 \frac{d\sigma}{dE_R} \frac{f(\mathbf{v})}{v} \, d^3\mathbf{v}$$

The event rate per unit target mass (actually measured)

$$\frac{dR}{dE} = \int_0^\infty \mathcal{G}(E, E_R) \, \frac{dR}{dE_R} \, dE_R$$

Recoil energy

Measured energy

Effective energy response function:
probability of actually detecting an event that occurred

WIMP-nucleus scattering

An arbitrary WIMP-nucleon interaction potential can be expanded in irreducible operators and multipole moments

In position space:

$$H_{\chi N} = \frac{p_\chi^2}{2m_\chi} + \frac{p_N^2}{2m_N} + V_{\chi N}$$

$$\begin{aligned} V_{\chi N} = {} & V_M\left(r, S_\chi\right) + S_N \cdot V_\Sigma\left(r, S_\chi\right) + \left[V_\Delta\left(r, S_\chi\right) \cdot v_{\chi N}\right]_{\rm sym} \\ & + S_N \cdot \left[V_\Phi\left(r, S_\chi\right) \times v_{\chi N}\right]_{\rm sym} + S_N \cdot \left[V_\Omega\left(r, S_\chi\right) v_{\chi N}\right]_{\rm sym} \end{aligned}$$

$$V_M\left(r, S_\chi\right) = \sum_{s=0}^{2j_\chi} (-1)^s \, \overline{(S_\chi \cdot \nabla)^s} \, V_{M,s,s}(r)$$

In momentum space: matrix elements between WIMP plane waves correspond to the Fourier transform of the position space interaction potential

Irreducible tensors

Decomposition into irreducible tensors

$$A_{ij} = \frac{A}{3}\,\delta_{ij} + A_{[ij]} + \overline{A_{ij}}$$

$$A = \sum_k A_{kk}$$

$$A_{[ij]} = \frac{1}{2}\,(A_{ij} - A_{ji})$$

$$\overline{A_{ij}} = \frac{1}{2}\,(A_{ij} + A_{ji}) - \frac{A}{3}\,\delta_{ij}$$

J=0 *J*=1 *J*=2

definite angular momentum J

Combine irreducible tensors using addition of angular momentum

Operator basis

The WIMP-nucleon interaction is a linear combination of basis operators

$$\widehat{\mathcal{O}}_{\chi N} = \sum_{X \tau s l} c^{\tau}_{X,s,l}(q)\, \widehat{\mathcal{O}}_{X,s,l}\, t^{\tau}_{N}.$$

$\mathcal{O}_{X,s,l}$

X = nuclear current

s = number of WIMP spin operators S_χ

l = power of momentum transfer $\tilde{q} = q/m_N$

$$
\begin{aligned}
\mathcal{O}_{M,s,s} &= \overline{(i\,\vec{\tilde{q}}\cdot\vec{S}_\chi)^s} && (s \geq 0),\\
\mathcal{O}_{\Omega,s,s} &= \overline{(i\,\vec{\tilde{q}}\cdot\vec{S}_\chi)^s}\ (\vec{v}^{\,+}_{\chi N}\cdot\vec{S}_N) && (s \geq 0),\\
\mathcal{O}_{\Sigma,s,s-1} &= \overline{(i\,\vec{\tilde{q}}\cdot\vec{S}_\chi)^{s-1}(\vec{S}_N\cdot\vec{S}_\chi)} && (s \geq 1),\\
\mathcal{O}_{\Sigma,s,s} &= \overline{(i\,\vec{\tilde{q}}\cdot\vec{S}_\chi)^{s-1}(i\,\vec{\tilde{q}}\times\vec{S}_N\cdot\vec{S}_\chi)} && (s \geq 1),\\
\mathcal{O}_{\Sigma,s,s+1} &= \overline{(i\,\vec{\tilde{q}}\cdot\vec{S}_\chi)^s}\ (i\,\vec{\tilde{q}}\cdot\vec{S}_N) && (s \geq 0),\\
\mathcal{O}_{\Delta,s,s-1} &= \overline{(i\,\vec{\tilde{q}}\cdot\vec{S}_\chi)^{s-1}(\vec{v}^{\,+}_{\chi N}\cdot\vec{S}_\chi)} && (s \geq 1),\\
\mathcal{O}_{\Delta,s,s} &= \overline{(i\,\vec{\tilde{q}}\cdot\vec{S}_\chi)^{s-1}(i\,\vec{\tilde{q}}\times\vec{v}^{\,+}_{\chi N}\cdot\vec{S}_\chi)} && (s \geq 1),\\
\mathcal{O}_{\Delta,s,s+1} &= \overline{(i\,\vec{\tilde{q}}\cdot\vec{S}_\chi)^s}\,(i\,\vec{\tilde{q}}\cdot\vec{v}^{\,+}_{\chi N}) && (s \geq 0),\\
\mathcal{O}_{\Phi,s,s-1} &= \overline{(i\,\vec{\tilde{q}}\cdot\vec{S}_\chi)^{s-1}(\vec{v}^{\,+}_{\chi N}\times\vec{S}_N\cdot\vec{S}_\chi)} && (s \geq 1),\\
\mathcal{O}_{\Phi,s,s} &= \overline{(i\,\vec{\tilde{q}}\cdot\vec{S}_\chi)^{s-1}(\vec{v}^{\,+}_{\chi N}\cdot\vec{S}_\chi)}\ (i\,\vec{\tilde{q}}\cdot\vec{S}_N) && (s \geq 1),\\
\mathcal{O}_{\Phi,s,s+1} &= \overline{(i\,\vec{\tilde{q}}\cdot\vec{S}_\chi)^s}\ (i\,\vec{\tilde{q}}\times\vec{v}^{\,+}_{\chi N}\cdot\vec{S}_N) && (s \geq 0).
\end{aligned}
$$

The scattering cross section

The WIMP response functions $R^{\tau\tau'}_{AB}(v, q)$ are complicated but calculable

$$R^{\tau\tau'}_{M}\left(v^{+2}_{\chi T},\widetilde{q}^2\right) = v^{+2}_{\chi T}\, R^{\tau\tau'}_{\Delta}\left(v^{+2}_{\chi T},\widetilde{q}^2\right) + \sum_{s=0}^{2j_\chi} B_{j_\chi,s} c^{\tau}_{M,s,s} c^{\tau'*}_{M,s,s} \widetilde{q}^{2s}$$

$$R^{\tau\tau'}_{\Phi''}\left(v^{+2}_{\chi T},\widetilde{q}^2\right) = \frac{1}{4} c^{\tau}_{\Phi,0,1} c^{\tau'*}_{\Phi,0,1}\widetilde{q}^2 + \frac{1}{4}\sum_{s=1}^{2j_\chi} B_{j_\chi,s}\widetilde{q}^{2s-2}\left(c^{\tau}_{\Phi,s,s-1} - c^{\tau}_{\Phi,s,s+1}\widetilde{q}^2\right)\left(c^{\tau'*}_{\Phi,s,s-1} - c^{\tau'*}_{\Phi,s,s+1}\widetilde{q}^2\right)$$

$$R^{\tau\tau'}_{\Phi'' M}\left(v^{+2}_{\chi T},\widetilde{q}^2\right) = -c^{\tau}_{\Phi,0,1} c^{\tau'*}_{M,0,0} + \sum_{s=1}^{2j_\chi} B_{j_\chi,s}\widetilde{q}^{2s-2}\left(c^{\tau}_{\Phi,s,s-1} - c^{\tau}_{\Phi,s,s+1}\widetilde{q}^2\right) c^{\tau'*}_{M,s,s},$$

$$R^{\tau\tau'}_{\tilde{\Phi}'}\left(v^{+2}_{\chi T},\widetilde{q}^2\right) = \sum_{s=1}^{2j_\chi} B_{j_\chi,s}\frac{s+1}{8s}\widetilde{q}^{2s-2}\left(c^{\tau}_{\Phi,s,s-1} c^{\tau'*}_{\Phi,s,s-1} + c^{\tau}_{\Phi,s,s} c^{\tau'*}_{\Phi,s,s}\widetilde{q}^2\right),$$

$$R^{\tau\tau'}_{\Sigma''}\left(v^{+2}_{\chi T},\widetilde{q}^2\right) = v^{+2}_{\chi T}\, R^{\tau\tau'}_{\tilde{\Phi}'}\left(v^{+2}_{\chi T},\widetilde{q}^2\right) + \frac{1}{4} c^{\tau}_{\Sigma,0,1} c^{\tau'*}_{\Sigma,0,1}\widetilde{q}^2 + \sum_{s=1}^{2j_\chi}\frac{1}{4} B_{j_\chi,s}\widetilde{q}^{2s-2}\left(c^{\tau}_{\Sigma,s,s-1} - c^{\tau}_{\Sigma,s,s+1}\widetilde{q}^2\right)\left(c^{\tau'*}_{\Sigma,s,s-1} - c^{\tau'*}_{\Sigma,s,s+1}\widetilde{q}^2\right),$$

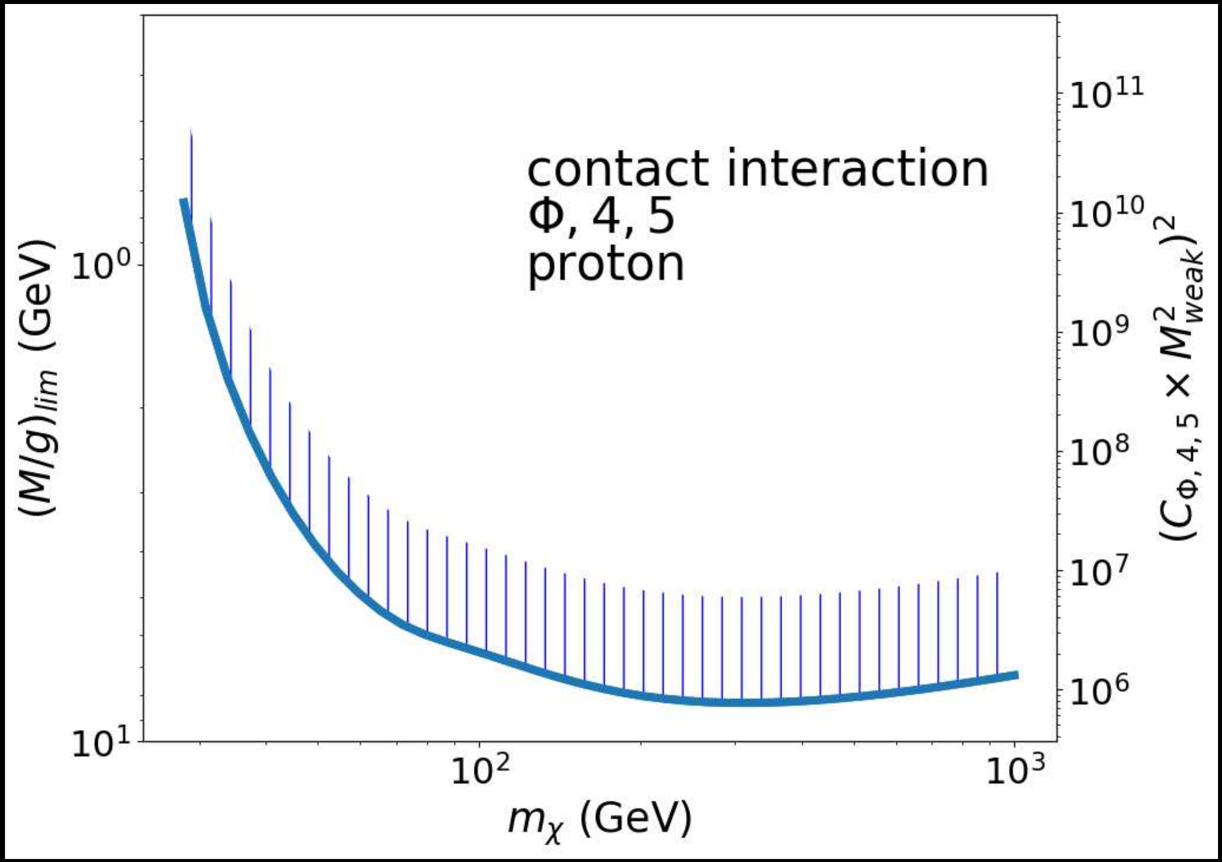
$$R^{\tau\tau'}_{\Sigma'}\left(v^{+2}_{\chi T},\widetilde{q}^2\right) = \frac{1}{2} v^{+2}_{\chi T}\, R^{\tau\tau'}_{\Phi''}\left(v^{+2}_{\chi T},\widetilde{q}^2\right) + \sum_{s=0}^{2j_\chi}\frac{1}{8} B_{j_\chi,s}\, c^{\tau}_{\Omega,s,s} c^{\tau'*}_{\Omega,s,s} v^{+2}_{\chi T}\widetilde{q}^{2s} + \sum_{s=1}^{2j_\chi}\frac{1}{8} B_{j_\chi,s}\frac{s+1}{s}\widetilde{q}^{2s-2}\left(c^{\tau}_{\Sigma,s,s-1} c^{\tau'*}_{\Sigma,s,s-1} + c^{\tau}_{\Sigma,s,s} c^{\tau'*}_{\Sigma,s,s}\widetilde{q}^2\right),$$

$$R^{\tau\tau'}_{\Delta}\left(v^{+2}_{\chi T},\widetilde{q}^2\right) = \sum_{s=1}^{2j_\chi} B_{j_\chi,s}\frac{s+1}{2s}\widetilde{q}^{2s-2}\left(c^{\tau}_{\Delta,s,s-1} c^{\tau'*}_{\Delta,s,s-1} + c^{\tau}_{\Delta,s,s} c^{\tau'*}_{\Delta,s,s}\widetilde{q}^2\right),$$

$$R^{\tau\tau'}_{\Delta\Sigma'}\left(v^{+2}_{\chi T},\widetilde{q}^2\right) = -\sum_{s=1}^{2j_\chi} B_{j_\chi,s}\frac{s+1}{2s}\widetilde{q}^{2s-2}\left(c^{\tau}_{\Delta,s,s} c^{\tau'*}_{\Sigma,s,s-1} + c^{\tau}_{\Delta,s,s-1} c^{\tau'*}_{\Sigma,s,s}\right),$$

Phenomenology

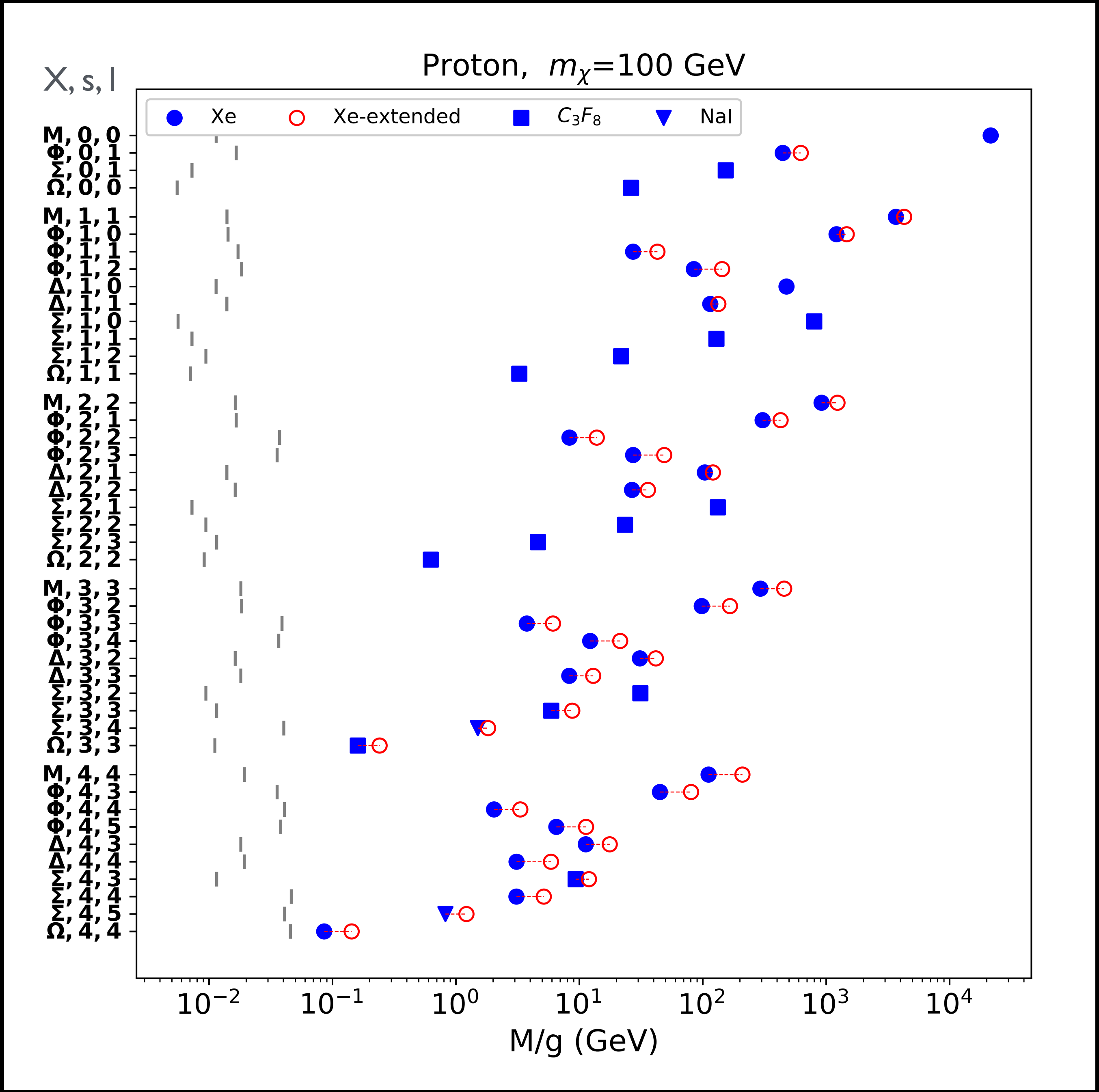
Limits on single operators



Placing limits on the ratio (mediator mass)/(coupling constant) M/g instead of the cross section allows a parallel treatment of interactions that vanish at zero momentum exchange $q = 0$.

The coupling $c_{X,s,l}$ associated to each operator $\mathcal{O}_{X,s,l}$ is $c_{X,s,l} = g^2/M^2$.

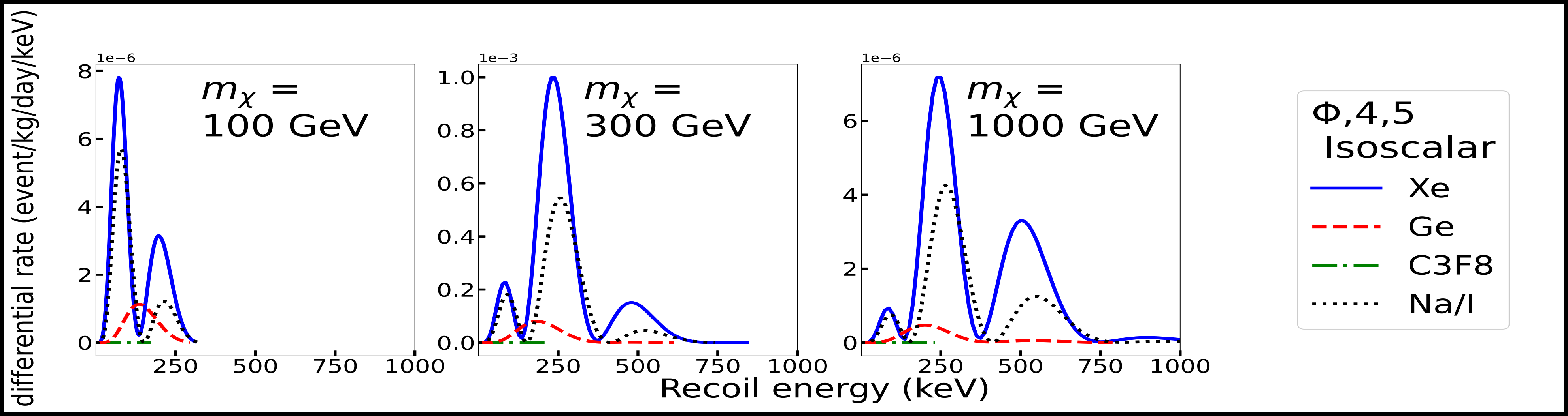
Limits on single operators



Bounds on M/g for all 44 operators with dark matter spin up to 2 (coupling to proton only).

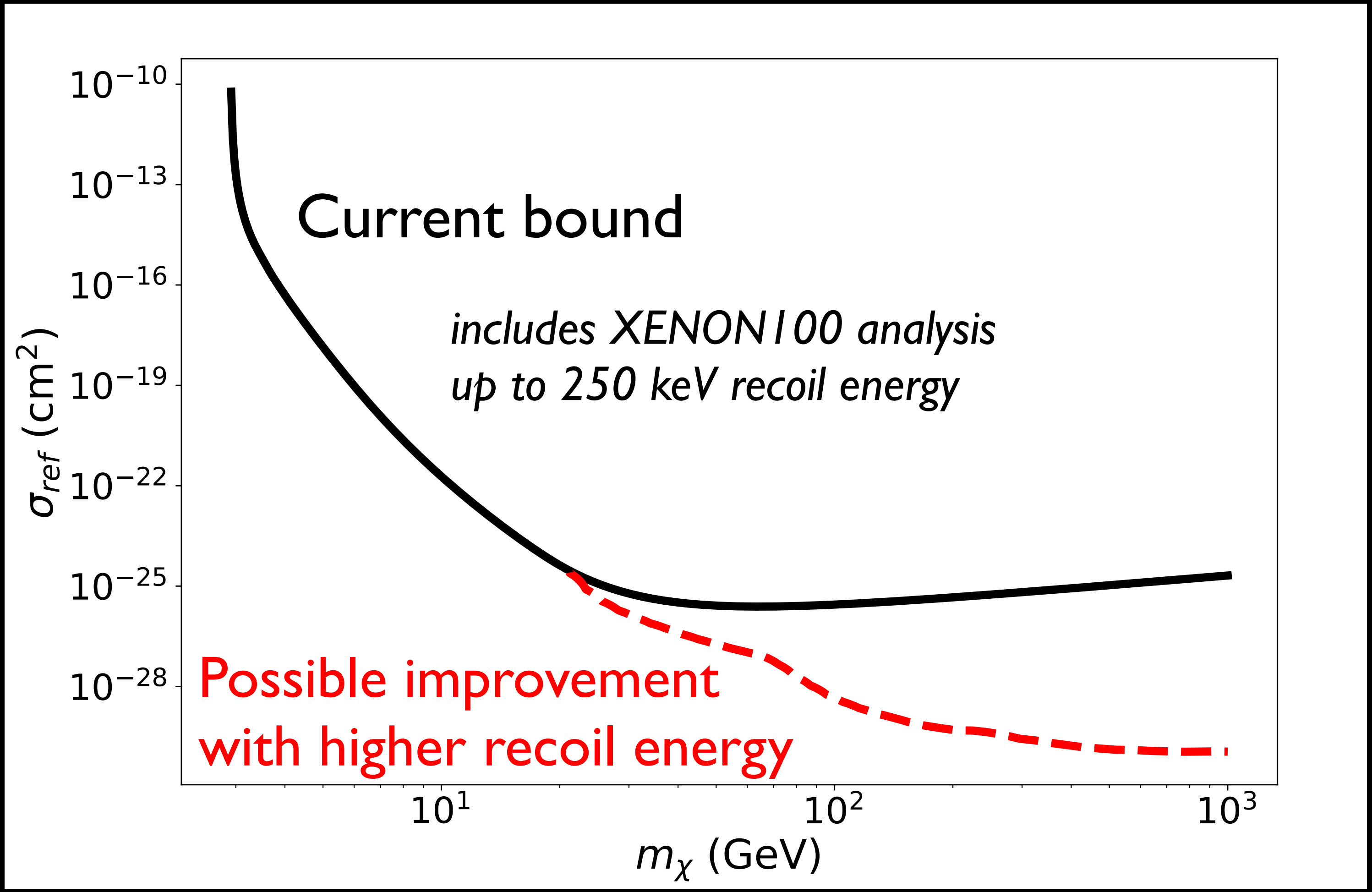
The coupling $c_{X,s,l}$ associated to each operator $\mathcal{O}_{X,s,l}$ is $c_{X,s,l} = g^2/M^2$.

Diffraction minima



Expected differential rates for the operator $\mathcal{O}_{\Phi,4,5}$ calculated for an isoscalar interaction $c^{\mathrm{p}}_{\Phi,4,5} = c^{\mathrm{n}}_{\Phi,4,5}$ and dark matter spin $s = 2$. The rates are normalized to the maximum rate allowed by current bounds.

Explore 100keV-1MeV recoil energies?



Possible improved bounds on the cross section $\sigma_{\rm ref} = c^2_{M,4,4}\mu^2_{\chi N}/\pi$ for the operator $\mathcal{O}_{M,4,4}$ from extending the XENON1T analysis to recoil energies higher than 250 keV.

More information is in the papers

arxiv:2008.05120, arxiv:2102.09778

The papers also contain

- Massless mediators and mediators of any mass
- The scaling of the differential rate with the spin and angular momentum of each operator
- The dependence of the differential rate on the nuclear structure functions
- The detailed mathematical derivation of the cross section formulas for linear combinations of operators of any spin
- The intricacies of relative velocities, current symmetrization, interaction potentials in position and momentum space
- A summary of useful formulas for multipole expansions of nuclear currents

Direct detection of WIMPs of any spin

It is possible that dark matter interacts with ordinary matter through high-order multipole moments 2^p only (multipolar dark matter, molecular dark matter, anomalous high-spin dark matter, …)

The cross section for scattering of high-spin dark matter off nuclei is complicated but calculable.

The phenomenology of direct detection of high-spin dark matter shows new patterns and strategies, for example more prominent diffraction peaks and minima, especially at higher recoil energies (100 keV - 1 MeV).