Spatial autocorrelation study for laser beam quality estimation

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- **Motivation**

- **Analytical definition of spatial autocorrelation index**

- **Autocorrelation estimation and GPT electron beam emittance evaluation for:**
  - Meshed beam
  - Real laser spots

- **Conclusions**

Motivation

➢ Motivation of this study: *High Brightness electron beam*

High beam charge

$$B[A/m^2] = \frac{Ne}{V_{6D}} \propto \frac{Q}{\varepsilon_{nx}\varepsilon_{ny}\sigma_t\sigma_\gamma}$$
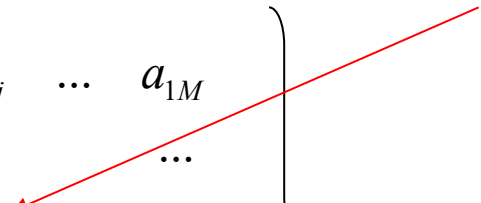
Low emittance

• Contributions to emittance degradations come from electromagnetic fields' non-linearity which can be reduced using a transversally and longitudinally uniform beam.

➢ Aim of this work: *To find an additional parameter able to evaluate the transverse laser beam uniformity*

➢ **Given a beam spot, represented by a matrix NxM, we can evaluate:**

main sample

$$\left\{ \begin{matrix} a_{11} & ... & a_{1j} & ... & a_{1M} \\ ... & & ... & & ... \\ a_{i1} & & a_{ij} & & a_{1M} \\ ... & & ... & & ... \\ a_{N1} & & a_{Nj} & & a_{NM} \end{matrix} \right\}$$

Non uniformity $\Longrightarrow$ **Standard deviation σ_a**

How non uniformity is distributed $\Longrightarrow$ **Index of spatial autocorrelation Λ**

main sample

$$\left\{ \begin{array}{ccccc} a_{i-h\,j-h} & \dots & a_{i-h\,j} & \dots & a_{i-h\,j+h} \\ \dots & & \dots & & \dots \\ a_{i\,j-h} & & a_{ij} & & a_{ij+h} \\ \dots & & \dots & & \dots \\ a_{i+h\,j-h} & & a_{i+h\,j} & & a_{i+h\,j+h} \end{array} \right\}$$

h

Samples at distance h around a_{ij}
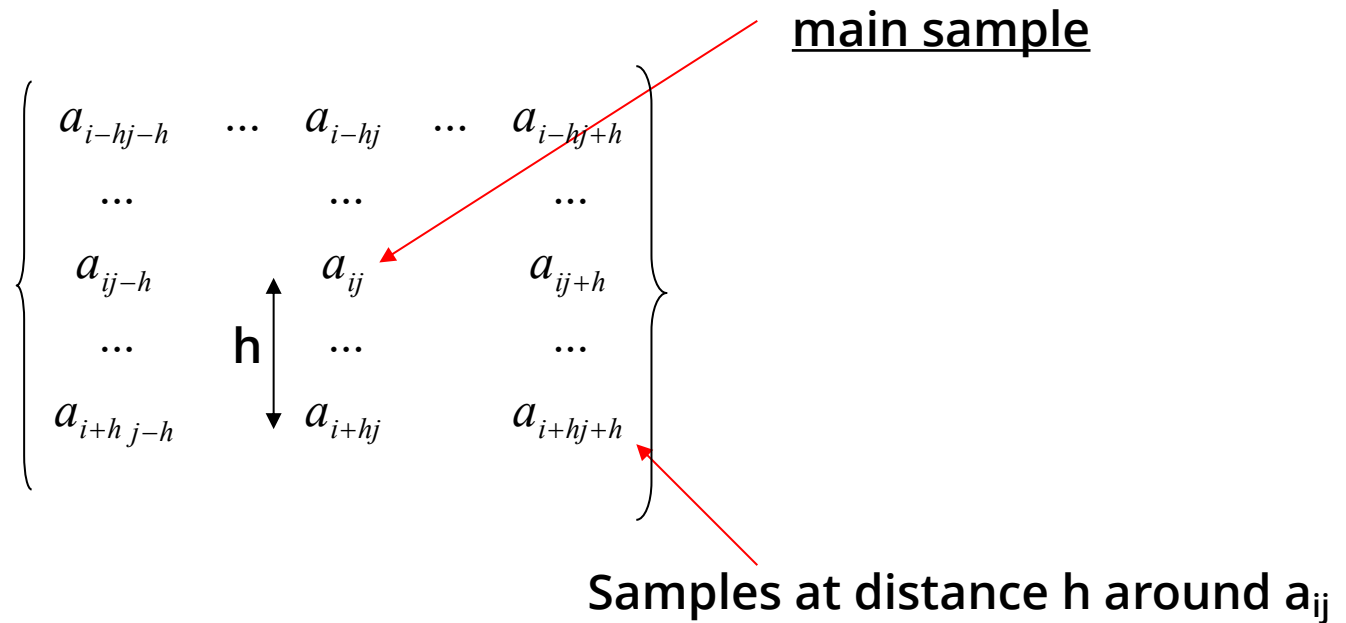
a_{ijh} **is the mean of the samples localized around the main sample a_{ij}:**

$$a_{ijh} = \frac{1}{(2h+1)^2 - 1}\left[\sum_{l=-h}^{h}\sum_{m=-h}^{h} a_{i+l\,j+m} - a_{ij}\right]$$

➤ **Non uniformity**

$$\text{var}(a) = \frac{1}{T} \sum_{i=1}^{N} \sum_{j=1}^{M} (a_{ij} - <a>)^2 \qquad \underline{\textbf{variance}}$$

$$<a> = \frac{1}{T} \sum_{i=1}^{N} \sum_{j=1}^{M} a_{ij} \qquad \underline{\textbf{mean}}$$

where T=NM.

$$\sigma_a = \sqrt{\text{var}(a)} \qquad \underline{\textbf{Standard deviation}}$$

Standard deviation σ_a <u>describes the contrast between spots</u> in an image: $\sigma_a \to 0$ means the image is uniform

➢ **How non uniformity is distributed**

The index Λ of spatial autocorrelation is defined as:

$$\Lambda(a,h) = \frac{\mathrm{cov}(a,h)}{\sigma_a^2}$$

with $-1 \leq \Lambda \leq 1$

where cov(a,h) is the covariance matrix, defined as:

$$\mathrm{cov}(a,h) = \frac{1}{T} \sum_{i=1}^{N} \sum_{j=1}^{M} (a_{ij} - <a>) \cdot (a_{ijh} - <a>)$$

**The quantity covariance answers the question whether
a sample and its neighbour are at the same time different or not
from the mean**

Autocorrelation estimation of meshed beam

➢ The charge distribution extracted from the cathode has been modelled as a sine and cosine function having a frequency n and a charge intensity δ

$$\rho(i,j) = \rho_0(1 + \delta \cos k_n i)(1 + \delta \cos k_n j)$$

where

$$k_n = \frac{2\pi n}{R}$$

with R is the beam radius, ρ_0 is the normalization constant.
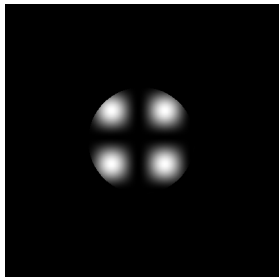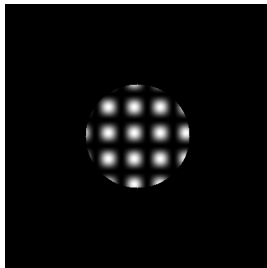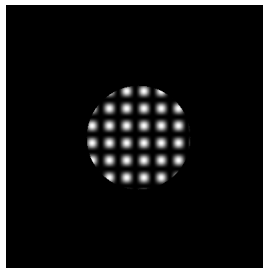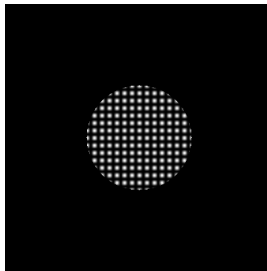
n=1	n=2	n=3	n=4	n=5
n=6	n=7	n=8	n=9	n=10
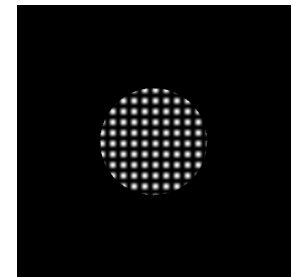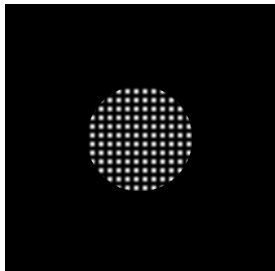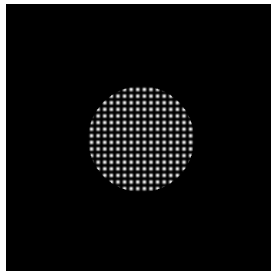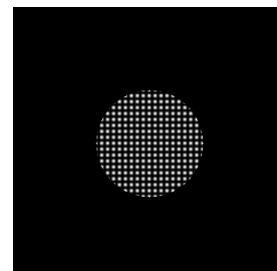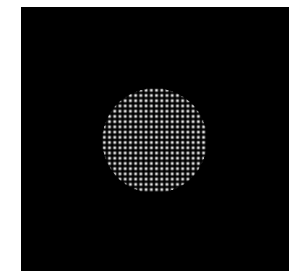
Mean=0.30
σ=0.29 (n=1)

mean distance
of the non
homogeneity
<u>**h*×pixel**</u>
<u>**size(µm)**</u>
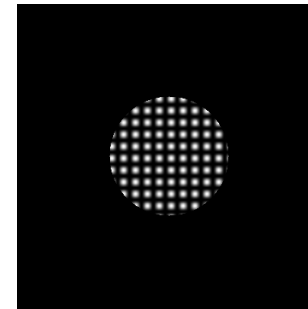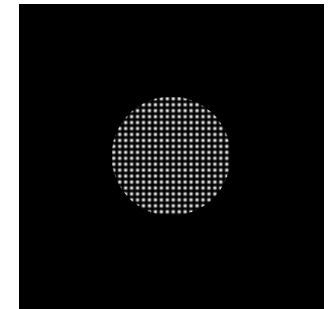
Mean=0.25
σ=0.28 (n=5)

Mean=0.25
σ=0.28 (n=10)
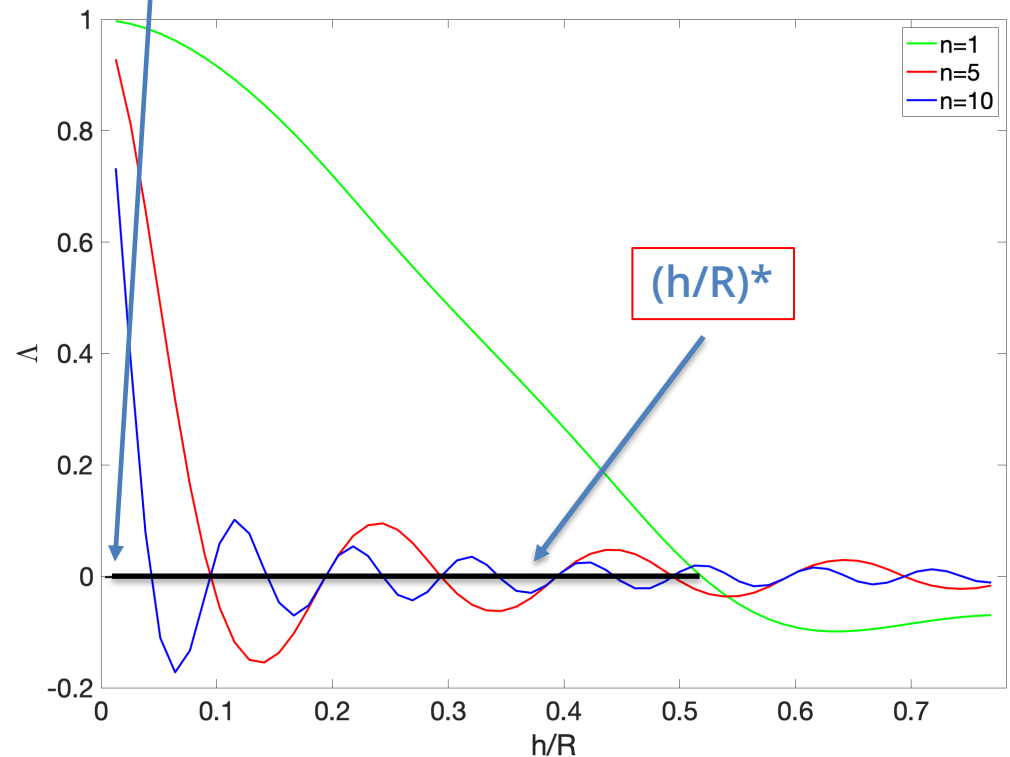
✓ Case n=1:
(h/R)*=0.5
R=78 pixel
Camera Pixel size=6.45 **µm**/pixel

<u>h*(µm)=0.5xRx(6.45**µm**/pixel)=</u>**256 µm**
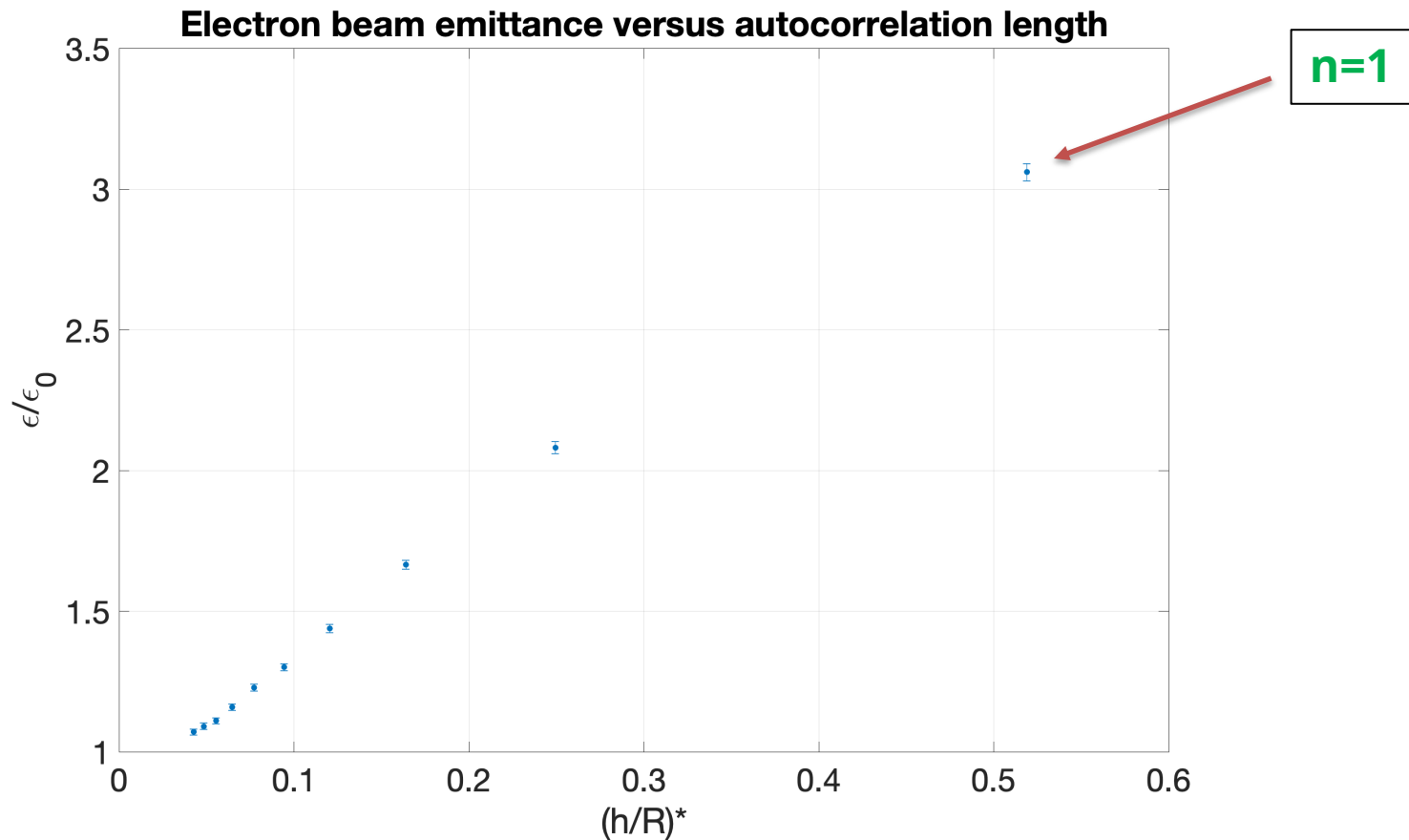
(h/R)*

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GPT Parameters:

- E_{RF}= 115MV/m
- Working RF phase=30°
- Laser pulse length=2ps - rms (Gaussian profile)
- Laser radius =500 µm (Flat top profile)
- E= 5MeV - Electron beam energy
- Bunch charge = 50pC
- ε_{intr} = 0.55 µm/mm (normalized intrinsic emittance)
- $I_{picco} \simeq$ 14.5 A
- Particles number=50000
- Mesh number: $N_x=N_{y=}80$, $N_z=50$

Ideal laser spot

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✓ **ε_0= 0.55 µm/mm** (value for the ideal laser spot image)

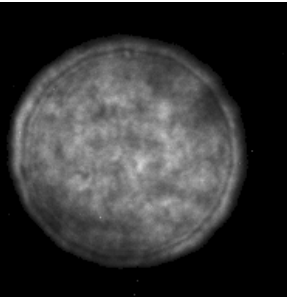✓ From the GPT simulation we have extrapolated the beam emittance value at about 1 cm from the photocathode surface
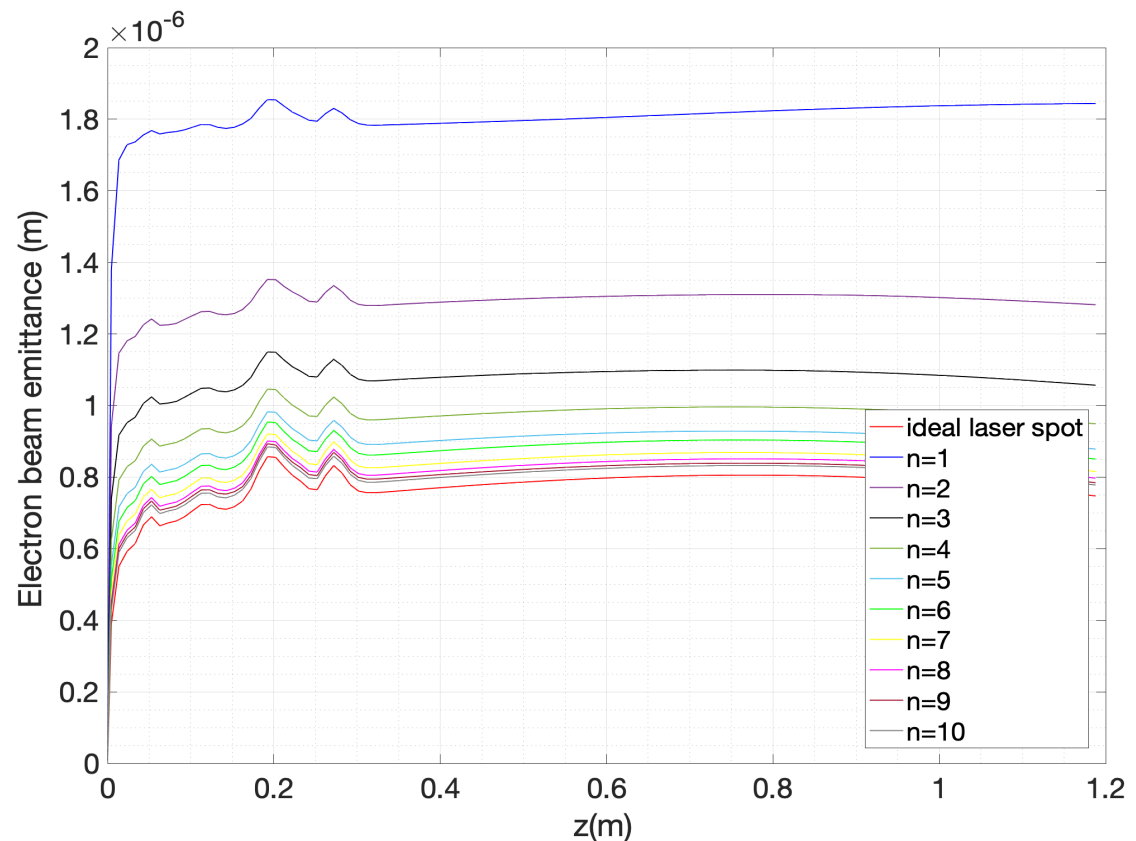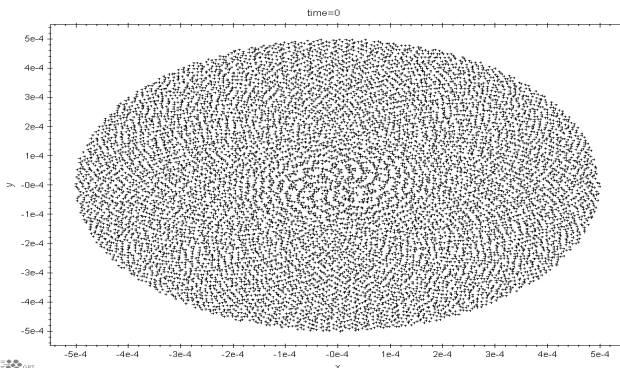
Electron beam emittance versus autocorrelation length

n=1

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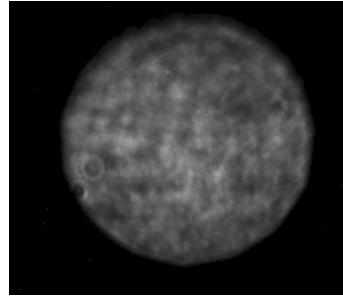
Autocorrelation estimation of real laser spots
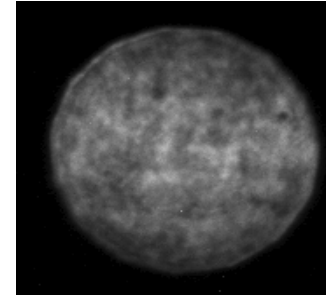
Laser 2
Mean= 0.39
σ=0.14

Laser 3
Mean= 0.25
σ=0.07

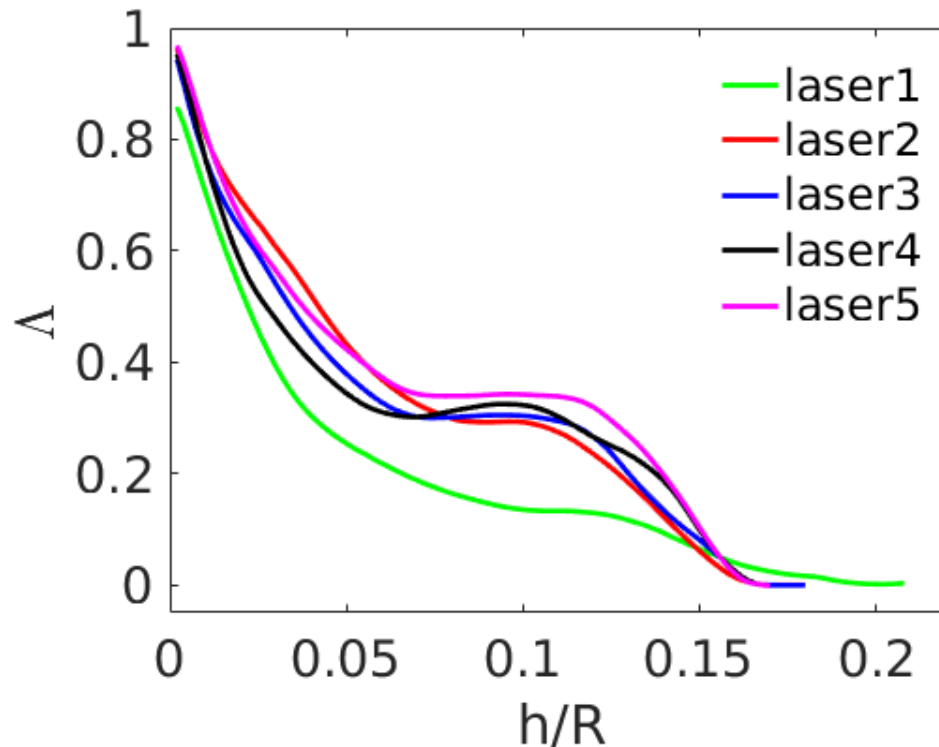**Laser 4**
Mean= 0.32
σ=0.10

Laser 5
Mean=0.33
σ=0.13

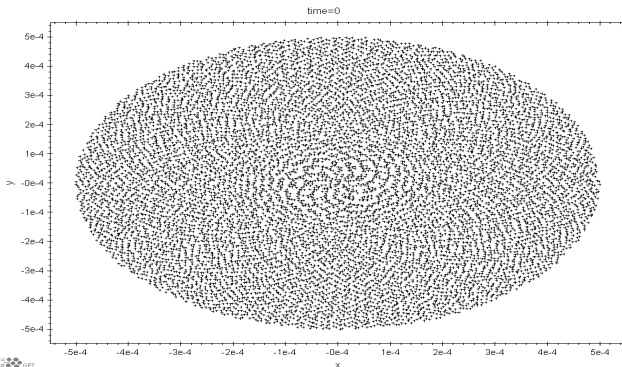**Laser 1**
Mean= 0.135
σ= 0.05

GPT Parameters:
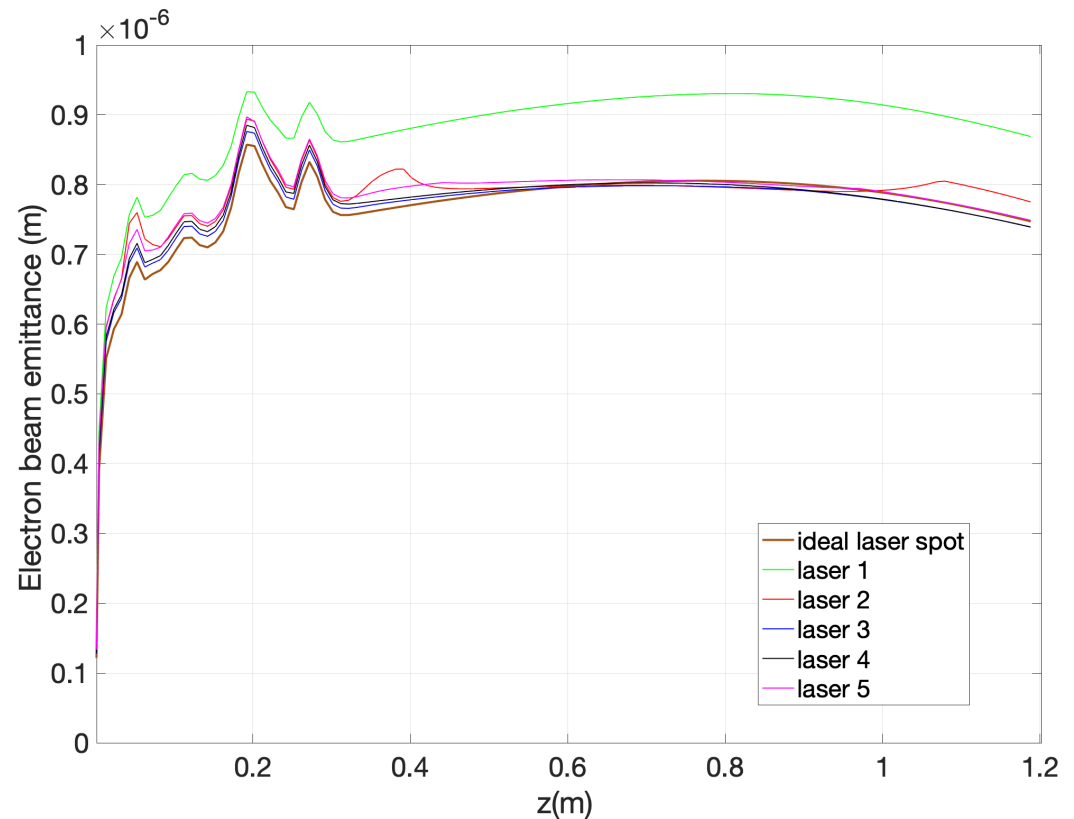
- E_{RF}= 115MV/m
- Working RF phase=30°
- Laser pulse length=2ps - rms (Gaussian profile)
- Laser radius =500 μm (Flat top profile)
- E= 5MeV - Electron beam energy
- Bunch charge = 50pC
- ε_{intr} = 0.55 μm/mm (normalized intrinsic emittance)
- $I_{picco} \simeq$14.5 A
- Particles number=50000
- Mesh number: N_x=N_y=80, N_z=50

Ideal laser spot

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✓ <u>ε_0 = 0.55±0.02 μm/mm</u> (value for the ideal laser spot)

✓ From the GPT simulation we have extrapolated the beam emittance value at about 1 cm from the photocathode surface

Real laser spot	ε (μm)	$\varepsilon / \varepsilon_0$	(h/R)*
Laser 1	0.62±0.02	1.13±0.06	0.218
Laser 2	0.59±0.02	1.08±0.06	0.166
Laser 3	0.58±0.02	1.04±0.06	0.168
Laser 4	0.58±0.02	1.06±0.06	0.166
Laser 5	0.59±0.02	1.08±0.06	0.166

• The standard deviation determines the contrast while the autocorrelation index determines how the non-uniformity are distributed

• They describe the laser beam quality, concerning the uniformity, and they give an idea of the emittance growth due to the laser beam degradation

• The parameter (h/R)* is a good estimator of the beam quality since it is strictly correlated with beam emittance at the emission!

➢ Future directions:

- o experimental emittance measurements with masks
- o systematic study with larger laser dataset

Thank you for your attention