Jets,TMDs and resummation

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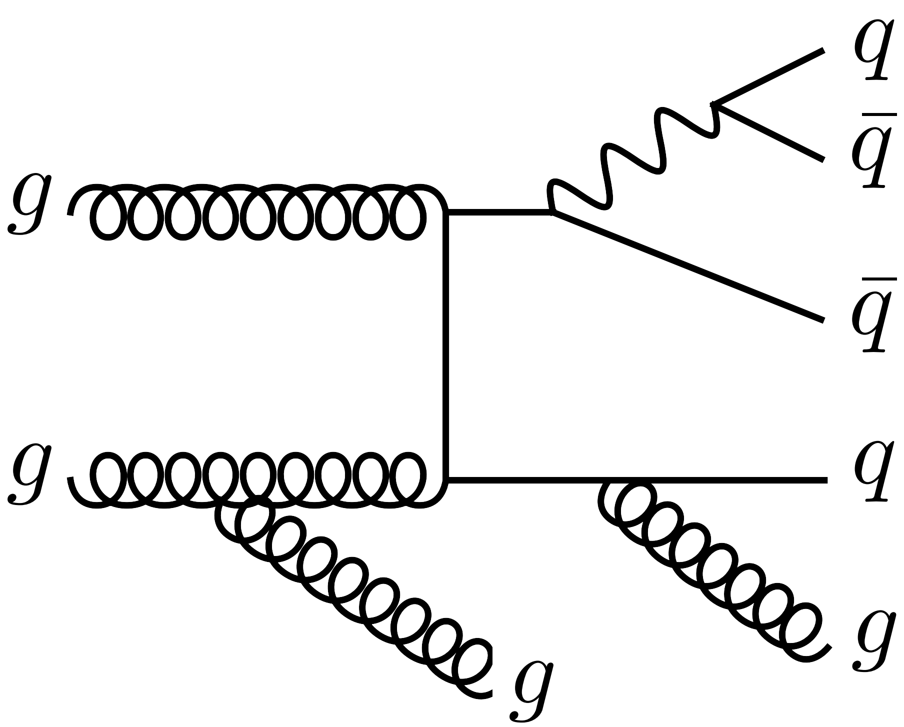
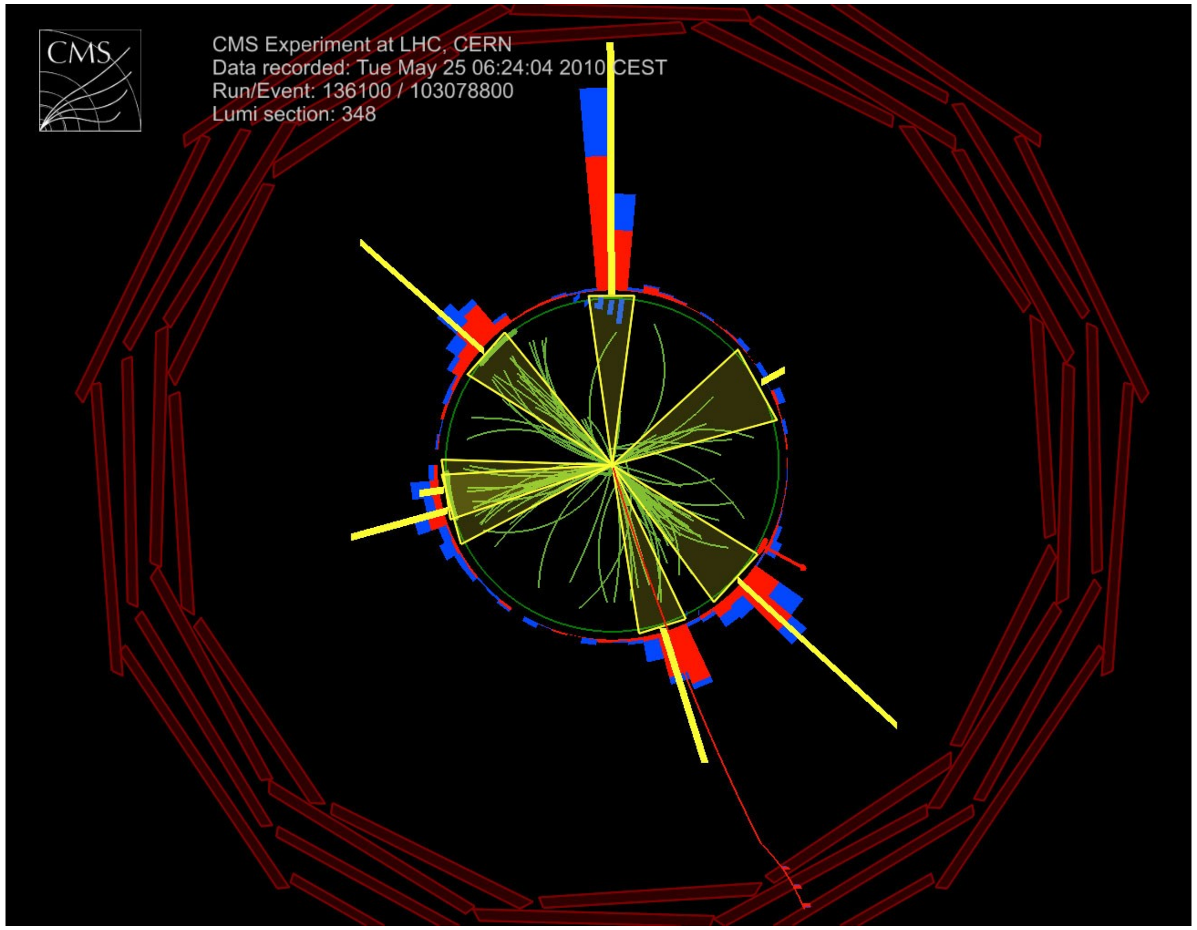
What is a jet?

- Energetic quarks and gluons radiate and hadronize
  → Produce sprays of collimated hadrons





Jet definitions

- Jet definition should be infrared safe (this was not always so)

Cone

Sterman-Weinberg

UA1

Midpoint

SISCone

XCone

1980 1990 2000 2010

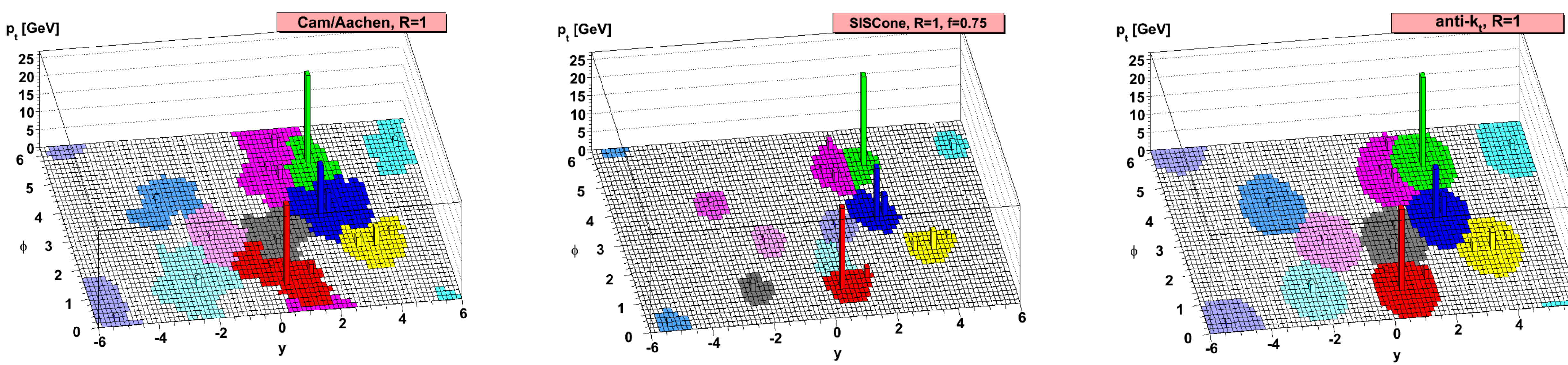
Clustering

Jade

k_T

anti-k_T

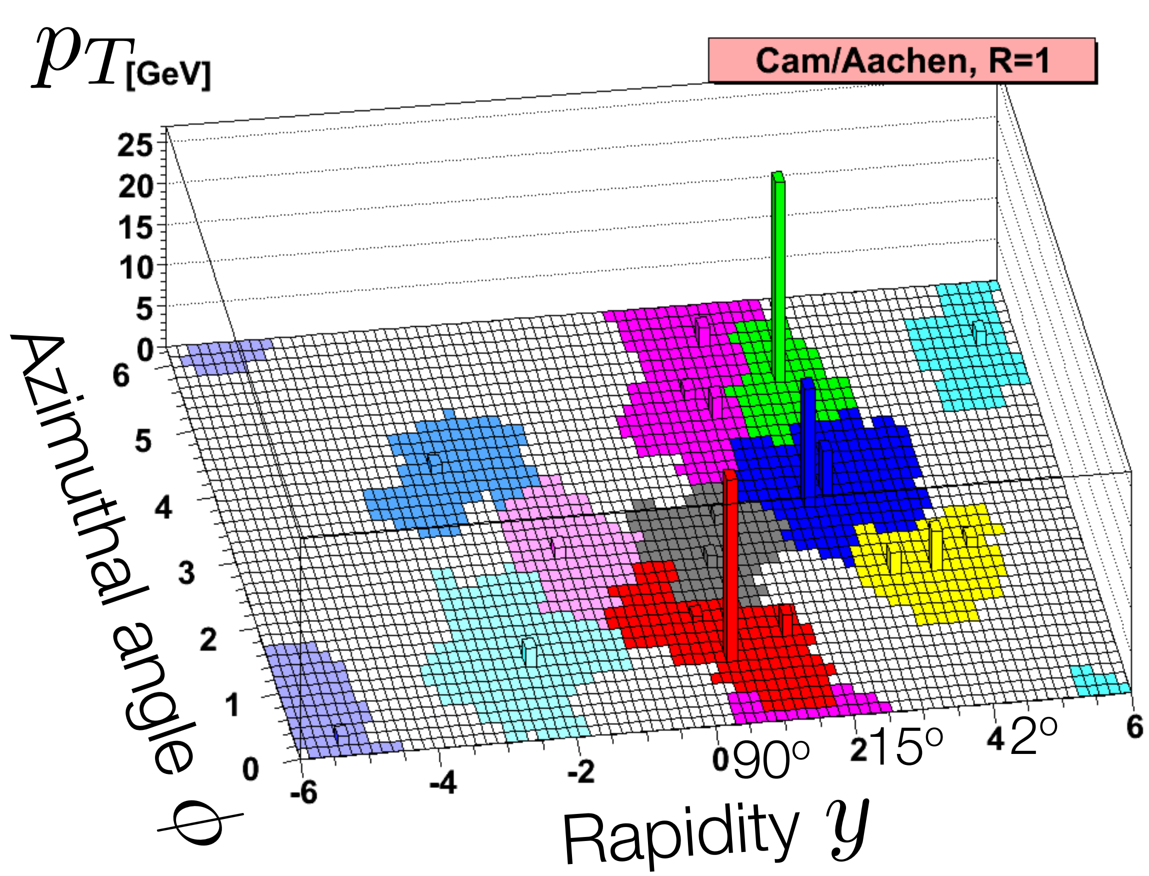
Cambridge/Aachen



[Cacciari, Salam, Soyez]

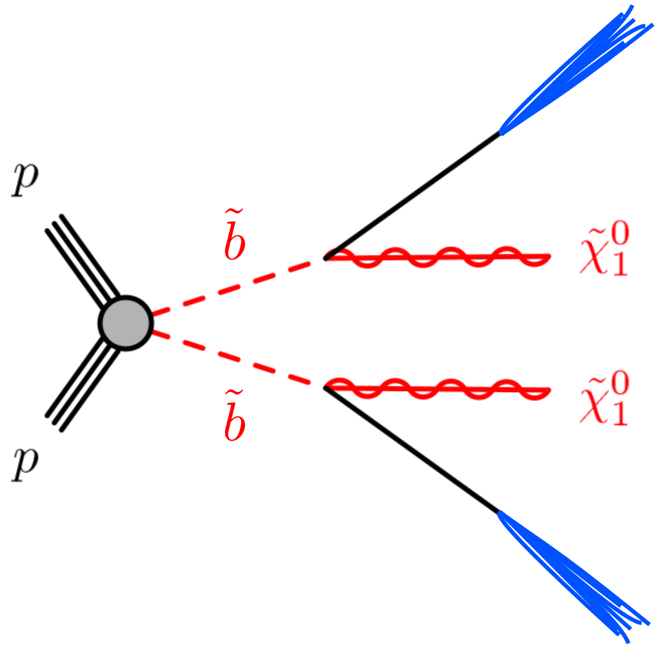
Jet clustering algorithms

- Determine distance between "particles"
  - Cambridge/Aachen: $\sqrt{(\Delta y)^2 + (\Delta \phi)^2}$
- Combine nearest "particles": $p_i, p_j \to p_i + p_j$
- Repeat until all distances larger than jet "radius" *R*

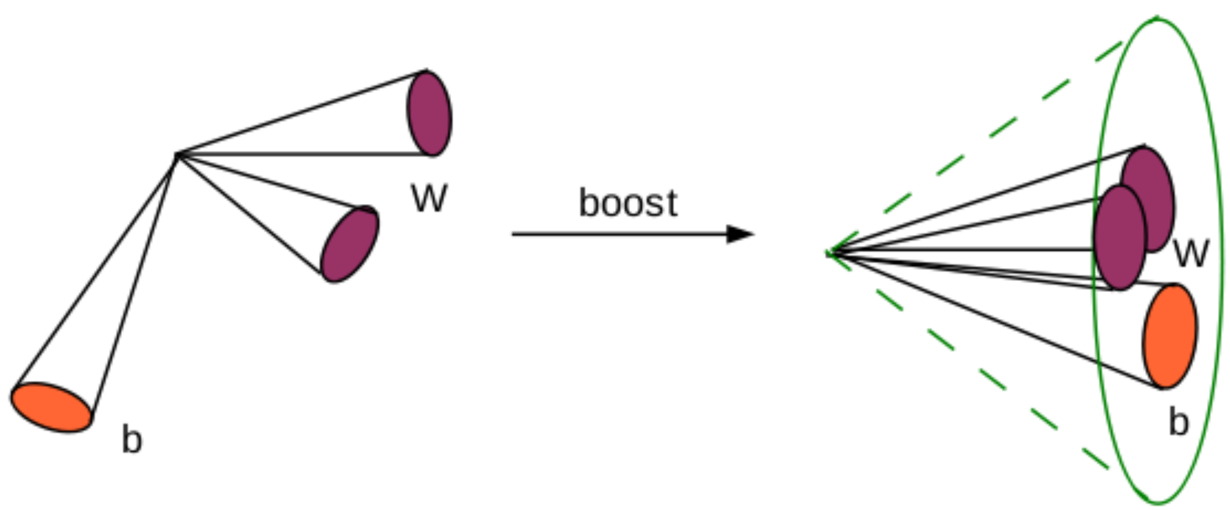


Why do jets matter?

- Jets enter in many LHC analyses
  E.g. supersymmetry searches

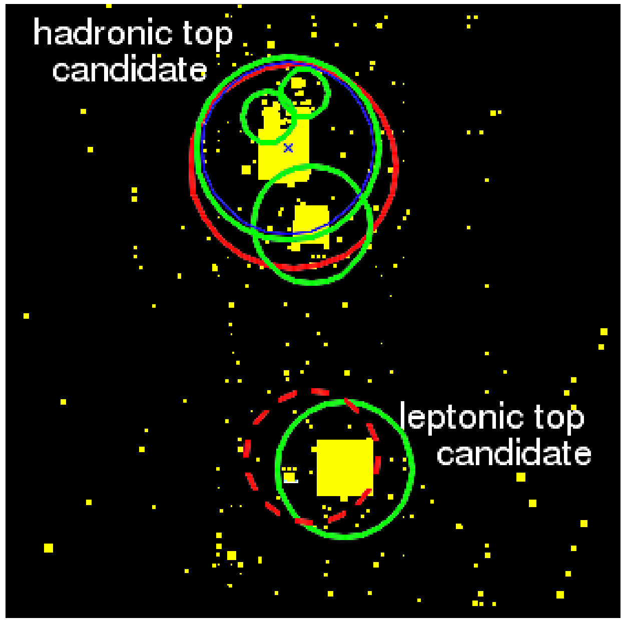


- Jets are essential to tag heavy particles at high energies



3 jets

3 subjets



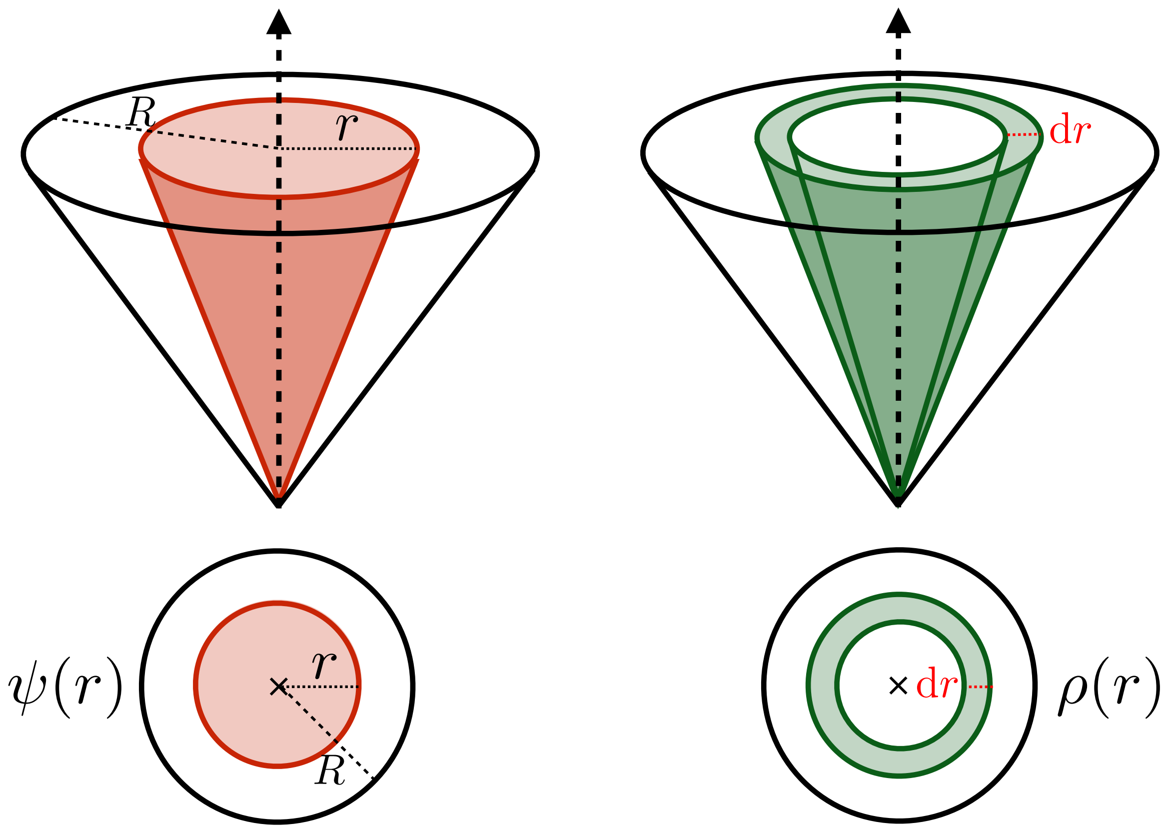
[ATLAS-CONF-2013-052]

Outline

- **Jet shape at NLL'**
  Cal, Ringer, WW - JHEP 1905 (2019) 143
- **TMDs from jets**
  Gutierrez-Reyes, Scimemi, WW, Zoppi
  Phys. Rev. Lett. 121 (2018), 162001, arXiv:1904.04259
- **Multi-differential resummation**
  Procura, WW, Zeune - JHEP 1502 (2015) 117
  Lustermans, Michel, Tackmann, WW - JHEP 1903 (2019) 124

1. Jet shape at NLL’

Jet shape definition



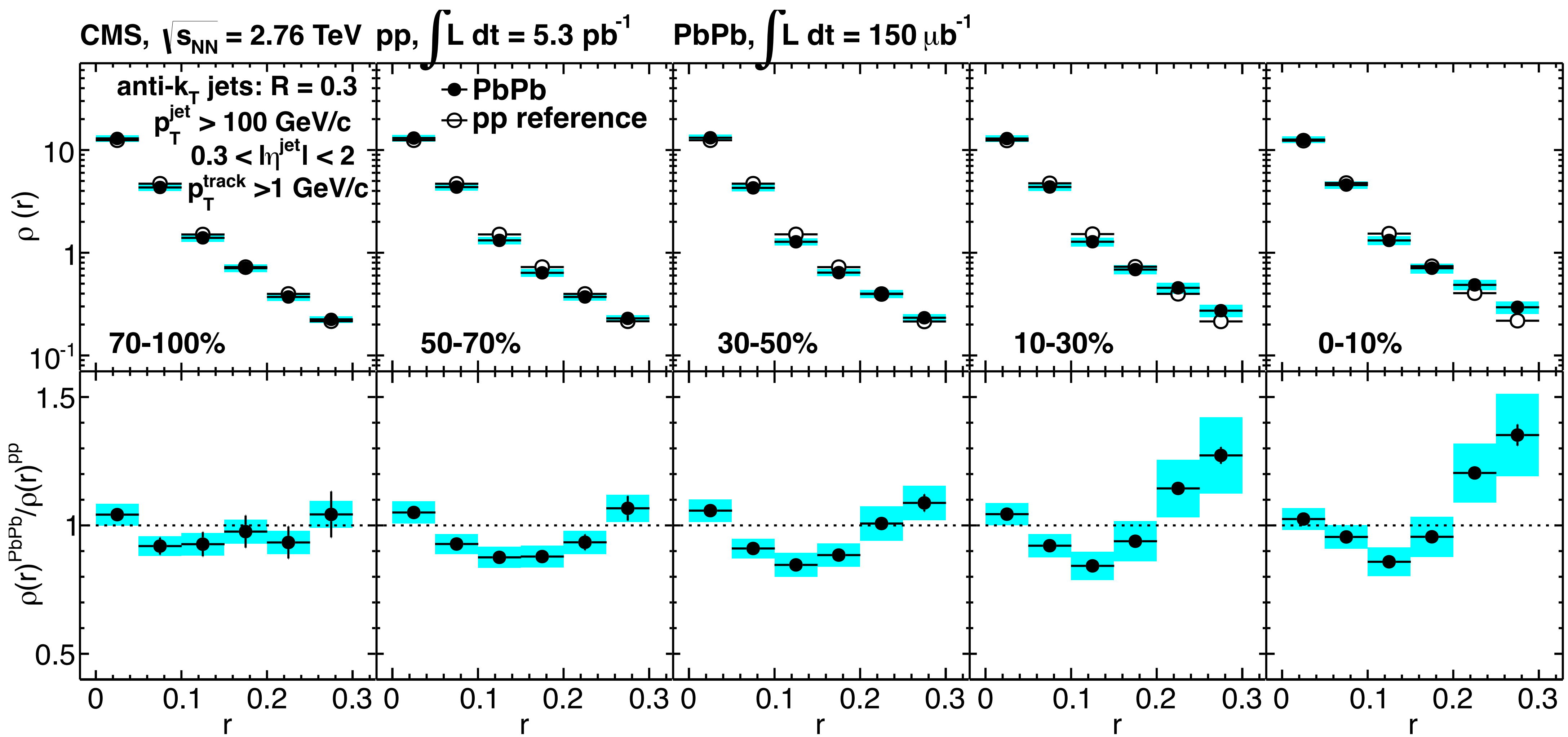
- Jet shape is average $z_r = p_T^{\text{subjet}} / p_T$

$$\psi(r) = \int_0^1 \mathrm{d}z_r \, z_r \, \frac{\mathrm{d}\sigma}{\mathrm{d}p_T \, \mathrm{d}\eta \, \mathrm{d}z_r} \Bigg/ \frac{\mathrm{d}\sigma}{\mathrm{d}p_T \, \mathrm{d}\eta} \qquad \rho(r) = \frac{\mathrm{d}\psi}{\mathrm{d}r}$$

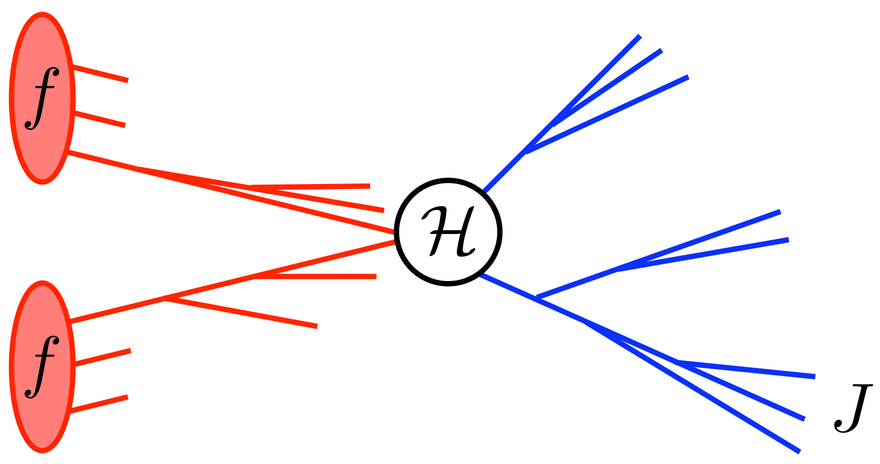
- Numerator & denominator integrated over jet kinematics p_T, η

Jet shape measurements

- Jet shape is classic jet substructure observable, measured in $pp, p\bar{p}, ep, e^+e^-$ and heavy ion collisions
- Constrain parton shower event generators [e.g. ATL-PHYS-PUB-2011-008]
- Study medium modification in heavy ion collisions



Factorization for inclusive sample of jets with $R \ll 1$



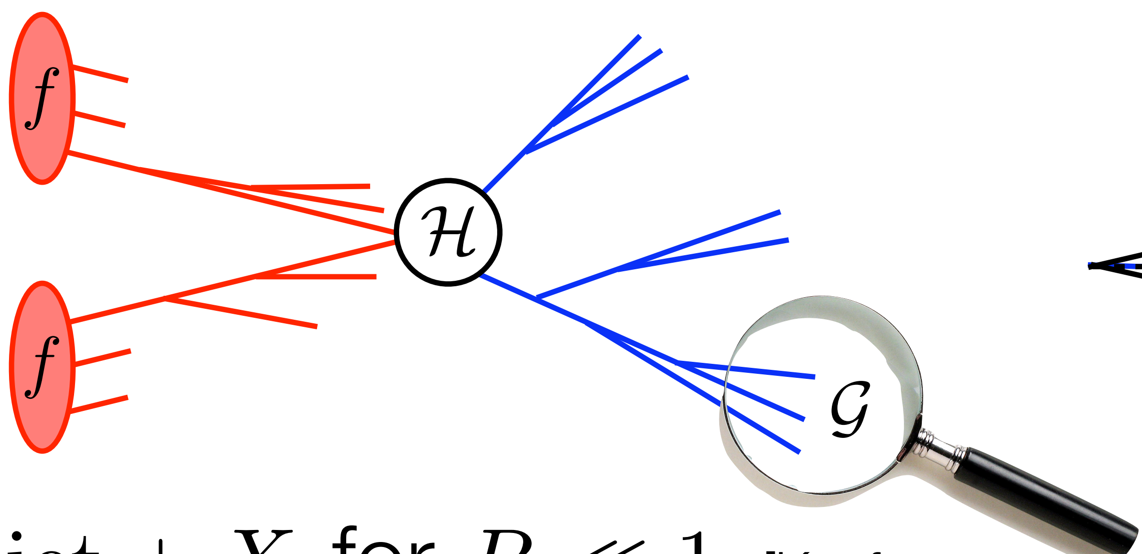
- $pp \to \text{jet} + X$ for $R \ll 1$ [Kaufmann et al, Kang et al, Dai et al]

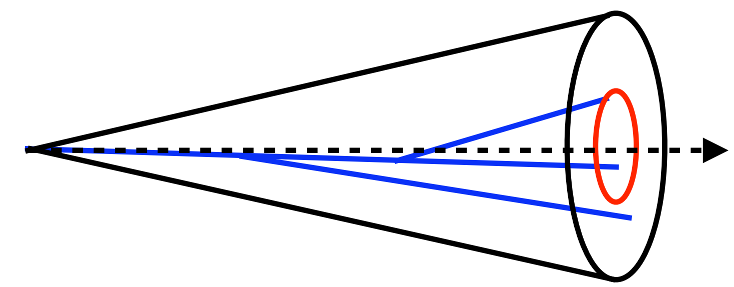
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\eta\,\mathrm{d}p_T} = \sum_{a,b,c} f_a(x_a) \otimes f_b(x_b) \otimes \mathcal{H}_{ab\to c}(x_a, x_b, \eta, p_T/z) \otimes J_c(z, p_T R)$$

- Resum logarithms of $\mu_J/\mu_{\mathcal{H}} \sim (p_T R)/p_T \sim R$ with DGLAP [see also Dasgupta et al]

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} J_i(z, p_T R, \mu) = \sum_j P_{ji}(z) \otimes J_j(z, p_T R, \mu)$$

Factorization for inclusive sample of jets with $R \ll 1$





- $pp \to \text{jet} + X$ for $R \ll 1$ [Kaufmann et al; Kang et al; Dai et al]

$$\frac{d\sigma}{d\eta \, dp_T \, dz_r} = \sum_{a,b,c} f_a(x_a) \otimes f_b(x_b) \otimes \mathcal{H}_{ab\to c}(x_a, x_b, \eta, p_T/z)$$

$$\otimes \boxed{\mathcal{G}_c(z, z_r, p_T R, r/R)}$$ Jet shape measurement

- Resum logarithms of $\mu_J/\mu_{\mathcal{H}} \sim (p_T R)/p_T \sim R$ with DGLAP

[see also Dasgupta et al]

$$\frac{d}{d\ln\mu} J_i(z, p_T R, \mu) = \sum_j P_{ji}(z) \otimes J_j(z, p_T R, \mu)$$

Separating jet production from jet shape

- At $\mathcal{O}(\alpha_s)$:

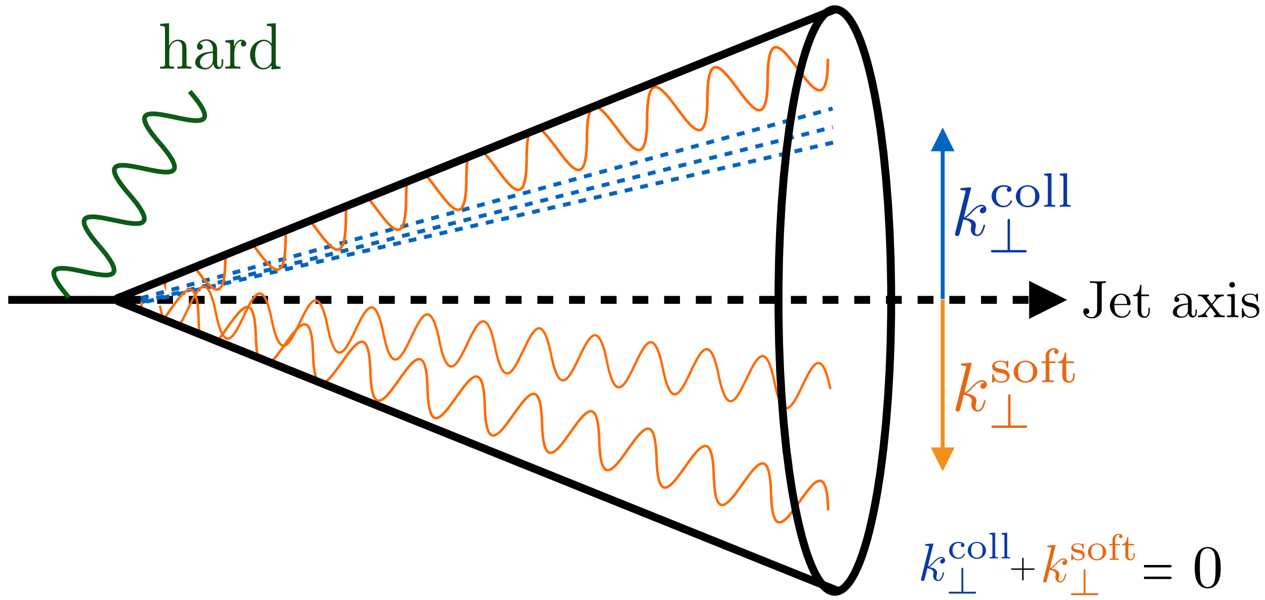
$$\mathcal{G} = \delta(1-z)\delta(1-z_r) + J^{(1)}(z)\,\delta(1-z_r) + \delta(1-z)\Delta\mathcal{G}^{(1)}(z_r)$$
$$= \underbrace{\left(\delta(1-z) + J^{(1)}(z)\right)}_{\text{jet production}}\underbrace{\left(\delta(1-z_r) + \Delta\mathcal{G}^{(1)}(z_r)\right)}_{\text{jet shape}} + \mathcal{O}(\alpha_s^2)$$

[Kaufmann et al; Cal, Ringer, WW]

- This is NOT a factorization of scales
- Jet shape has large logarithms for $r \ll R$. E.g. for quark jet

$$\psi_q(r) = 1 + \frac{\alpha_s C_F}{2\pi}\left(-2\ln^2\frac{r}{R} - 3\ln\frac{r}{R} - \frac{9}{2} + \frac{6r}{R} - \frac{3r^2}{2R^2}\right)$$

Factorization for jet shape with $r \ll R$



	$(k^-, k^+, k_\perp^\mu)$
hard(-collinear)	$p_T(1, R^2, R)$
collinear	$p_T(1, r^2, r)$
(collinear-)soft	$p_T(r/R, rR, r)$

[Kang, Ringer, WW]

- Hard emissions must be out of the jet. Only collinear radiation contributes to jet shape, but soft radiation displaces jet axis

$$\mathcal{G}_c(z, z_r, p_T R, r/R, \mu) = \sum_d H_{cd}(z, p_T R, \mu) \int d^2 k_\perp \, C_d(z_r, p_T r, k_\perp, \mu, \nu) \times S_d(-k_\perp, \mu, \nu R) \left[1 + \mathcal{O}\left(\frac{r}{R}\right)\right]$$

Resummation for $r \ll R$

- Resum logarithms of $\mu_C/\mu_H \sim \mu_S/\mu_H \sim \nu_S/\nu_C \sim r/R$ with renormalization group evolution in scales μ and ν

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} H_{cd}(z, p_T R, \mu) = \sum_e \int_z^1 \frac{\mathrm{d}z'}{z'} \gamma_{ce}^H\left(\frac{z}{z'}, p_T R, \mu\right) H_{ed}(z', p_T R, \mu)$$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C_d(z_r, p_T r, k_\perp, \mu, \nu) = \gamma_d^C(\mu, \nu/p_T)\, C_d(z_r, p_T r, k_\perp, \mu, \nu)$$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} S_d(k_\perp, \mu, \nu R) = \gamma_d^S(\mu, \nu R)\, S_d(k_\perp, \mu, \nu R)$$

$$\nu \frac{\mathrm{d}}{\mathrm{d}\nu} C_d(z_r, p_T r, k_\perp, \mu, \nu) = -\int \frac{\mathrm{d}^2 k'_\perp}{(2\pi)^2} \gamma_d^\nu(k_\perp - k'_\perp, \mu)\, C_d(z_r, p_T r, k'_\perp, \mu, \nu)$$

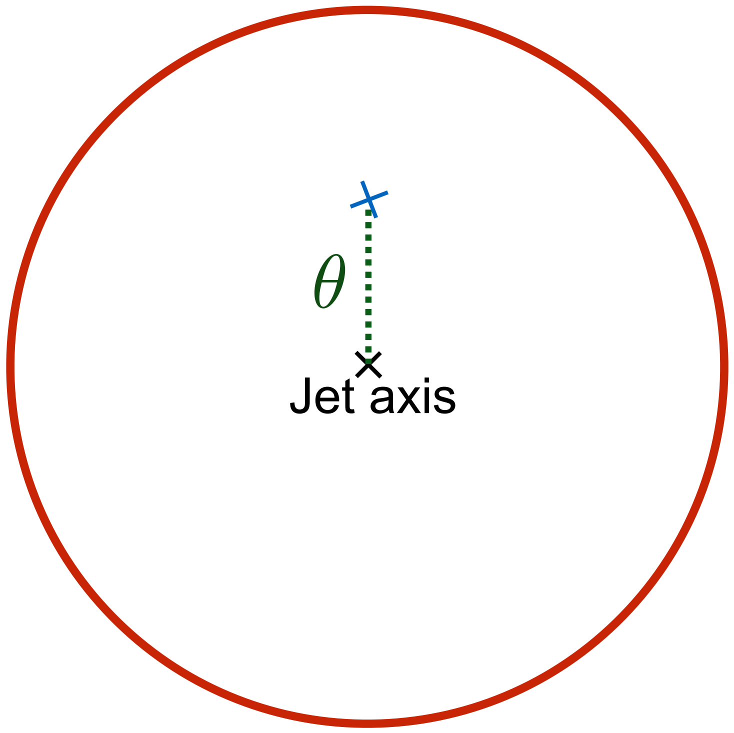
$$\nu \frac{\mathrm{d}}{\mathrm{d}\nu} S_d(k_\perp, \mu, \nu R) = \int \frac{\mathrm{d}^2 k'_\perp}{(2\pi)^2} \gamma_d^\nu(k_\perp - k'_\perp, \mu)\, S_d(k'_\perp, \mu, \nu R)$$

Resummation orders

		Fixed-order	β	γ_μ	γ_ν	NGLs
$\ln R$	LL	tree	1-loop	1-loop	-	-
	NLL	1-loop	2-loop	2-loop	-	-
	NNLL	2-loop	3-loop	3-loop	-	-
$\ln(r/R)$	LL	tree	1-loop	1-loop	-	-
	NLL	tree	2-loop	2-loop	1-loop	LL
	NLL$'$	1-loop	2-loop	2-loop	1-loop	LL
	NNLL	1-loop	3-loop	3-loop	2-loop	NLL

- Single logarithms $\alpha_s^n \ln^n R$, double logarithms $\alpha_s^n \ln^{2n}(r/R)$

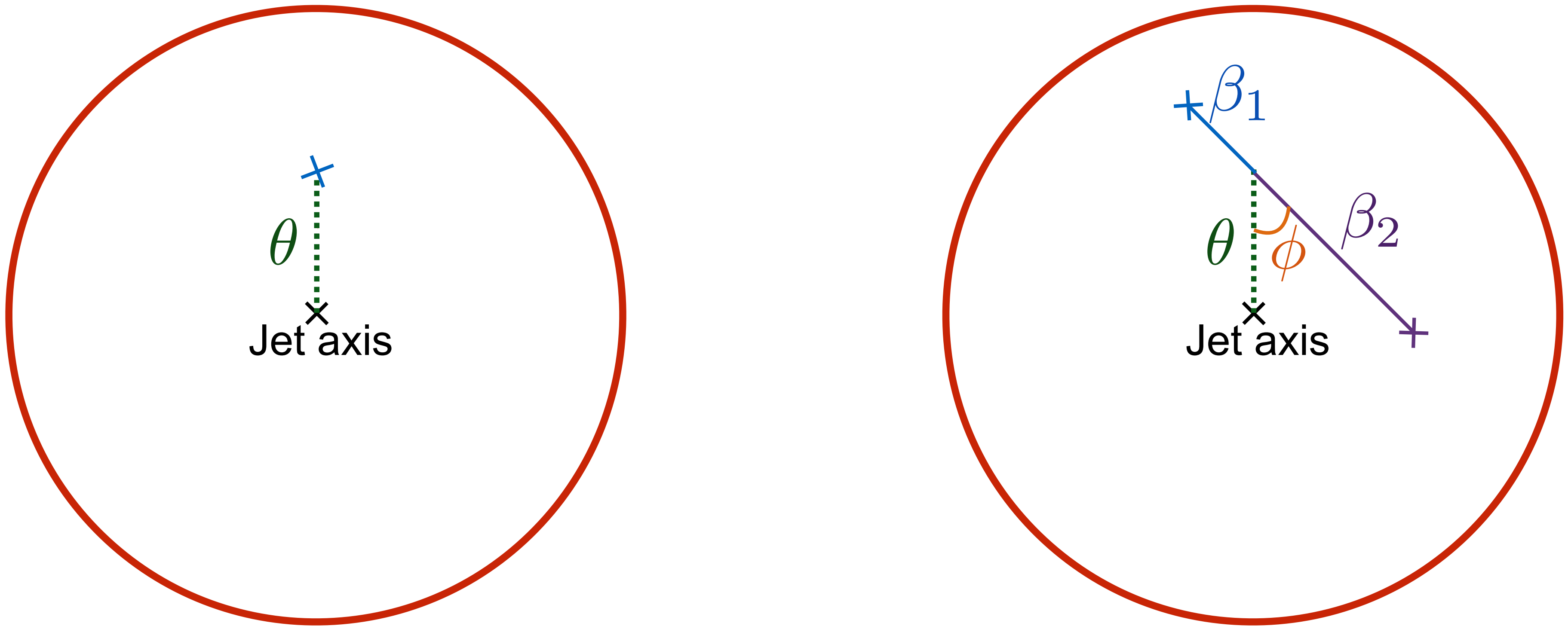
Collinear function



- At tree level, parton is in/out depending on recoil $\theta = k_\perp / p_T$

$$C_d^{(0)} = \delta(1 - z_r)\,\Theta(\theta < r)$$

Collinear function



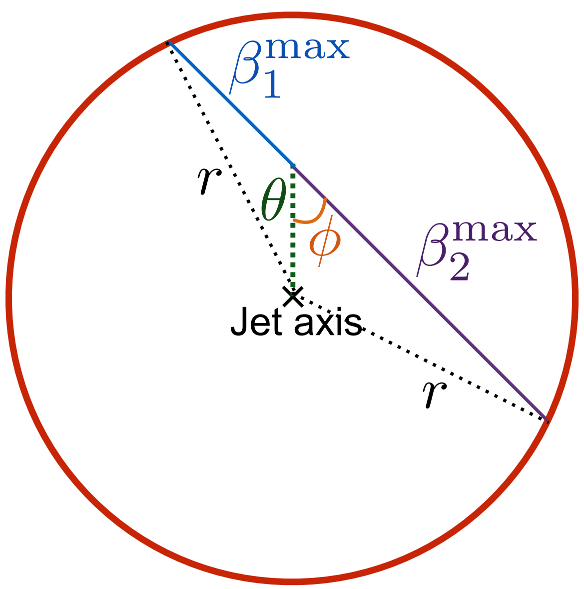
- At tree level, parton is in/out depending on recoil $\theta = k_\perp / p_T$

$$C_d^{(0)} = \delta(1 - z_r)\,\Theta(\theta < r)$$

- At $\mathcal{O}(\alpha_s)$, determining which partons are in/out involves nontrivial ϕ dependence, due to recoil

Collinear function at $\mathcal{O}(\alpha_s)$

- Quark jet with $\theta < r$



$$C_q^{(\theta<r)} = \frac{\alpha_s C_F}{2\pi^2} \int_0^{2\pi} \mathrm{d}\phi \left\{ \delta(1-z_r) \left[\left(\frac{1}{\eta} + \ln\frac{\nu}{2p_T} + \frac{3}{4} \right) \left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{p_T^2 (\beta_1^{\max})^2} \right) \right.\right.$$
$$\left. - \ln^2(1-\widetilde{\beta}) + 2\ln\widetilde{\beta}\ln(1-\widetilde{\beta}) - \frac{3}{2}\ln\widetilde{\beta} + 2\mathrm{Li}_2(1-\widetilde{\beta}) - \frac{\widetilde{\beta}}{2} - \frac{\pi^2}{3} + 2 \right]$$
$$+ \Theta(z_r > \widetilde{\beta}) \left[-(1+z_r^2) \left(\frac{\ln(1-z_r)}{1-z_r} \right)_+ + \ln\left(\frac{z_r(1-\widetilde{\beta})}{\widetilde{\beta}} \right) \frac{1+z_r^2}{(1-z_r)_+} \right]$$
$$\left. + \Theta(z_r > 1-\widetilde{\beta}) \left[\frac{1+(1-z_r)^2}{z_r} \ln\left(\frac{z_r \widetilde{\beta}}{(1-z_r)(1-\widetilde{\beta})} \right) \right] \right\}$$

$$\widetilde{\beta} = \frac{\beta_2^{\max}}{\beta_1^{\max} + \beta_2^{\max}}$$

- Residual ϕ integral, but $1/\epsilon, 1/\eta$ can be calculated analytically
- Simplifies when averaging over z_r

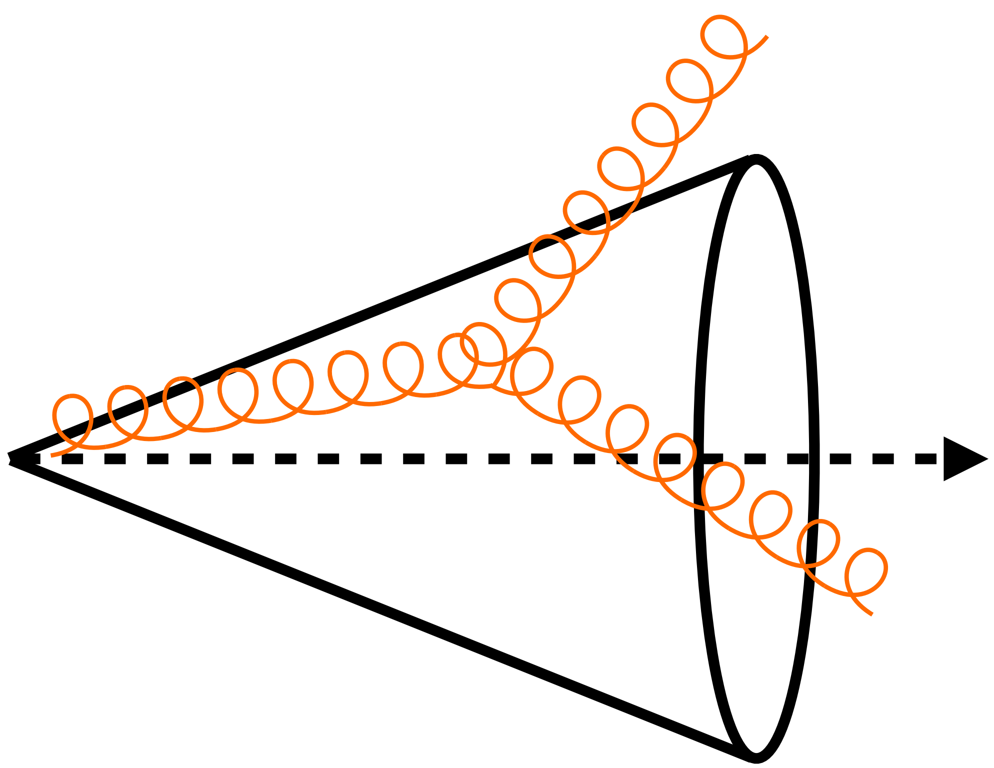
Soft function

- Only soft radiation inside jet recoils jet axis. Up to $\mathcal{O}(\alpha_s)$,

$$S_q(k_\perp,\mu,\nu R)=\delta^2(k_\perp)+\frac{\alpha_s C_F}{2\pi^2}\left[-\frac{1}{\mu^2}\left(\frac{\ln(k_\perp^2/\mu^2)}{k_\perp^2/\mu^2}\right)_+ +\frac{1}{\mu^2}\,\frac{1}{(k_\perp^2/\mu^2)}_+\ln\frac{\nu^2R^2}{4\mu^2}-\frac{\pi^2}{12}\delta(\vec{k}_\perp^{\,2})\right]$$

- Nonglobal logarithms [Dasgupta, Salam]

$$-\frac{\alpha_s^2 C_F C_i}{24\pi}\,\frac{1}{(p_T R)^2}\left(\frac{\ln(k_\perp^2/(p_T R)^2)}{k_\perp^2/(p_T R)^2}\right)_+$$



Soft function and nonglobal logarithms

- Only soft radiation inside jet recoils jet axis. Up to $\mathcal{O}(\alpha_s)$,

$$S_q(k_\perp,\mu,\nu R) = \delta^2(k_\perp) + \frac{\alpha_s C_F}{2\pi^2}\left[-\frac{1}{\mu^2}\left(\frac{\ln(k_\perp^2/\mu^2)}{k_\perp^2/\mu^2}\right)_+ + \frac{1}{\mu^2}\frac{1}{(k_\perp^2/\mu^2)}_+ \ln\frac{\nu^2 R^2}{4\mu^2} - \frac{\pi^2}{12}\delta(\vec{k}_\perp^{\,2})\right]$$

- Nonglobal logarithms [Dasgupta, Salam]

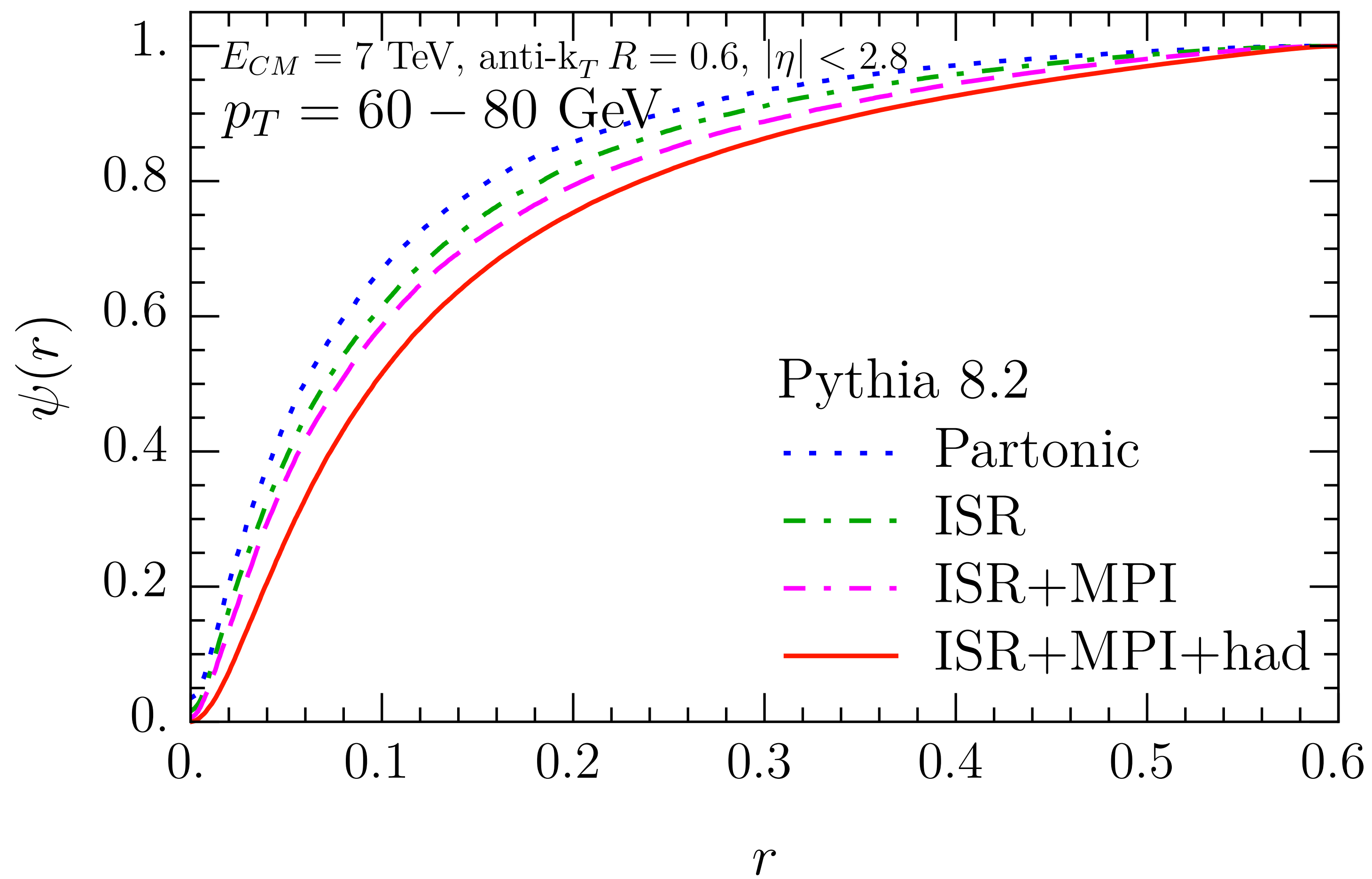
$$\int d^2k_\perp \Theta(k_\perp < p_T r) \times -\frac{\alpha_s^2 C_F C_i}{24\pi}\frac{1}{(p_T R)^2}\left(\frac{\ln(k_\perp^2/(p_T R)^2)}{k_\perp^2/(p_T R)^2}\right)_+ = -\frac{\alpha_s^2 C_F C_i}{12}\ln^2\frac{R}{r}$$

- Upon integrating with the collinear function, this is the same as the hemisphere case [Banfi, Dasgupta, Khelifa-Kerfa, Marzani]

- Extends to leading nonglobal logs:

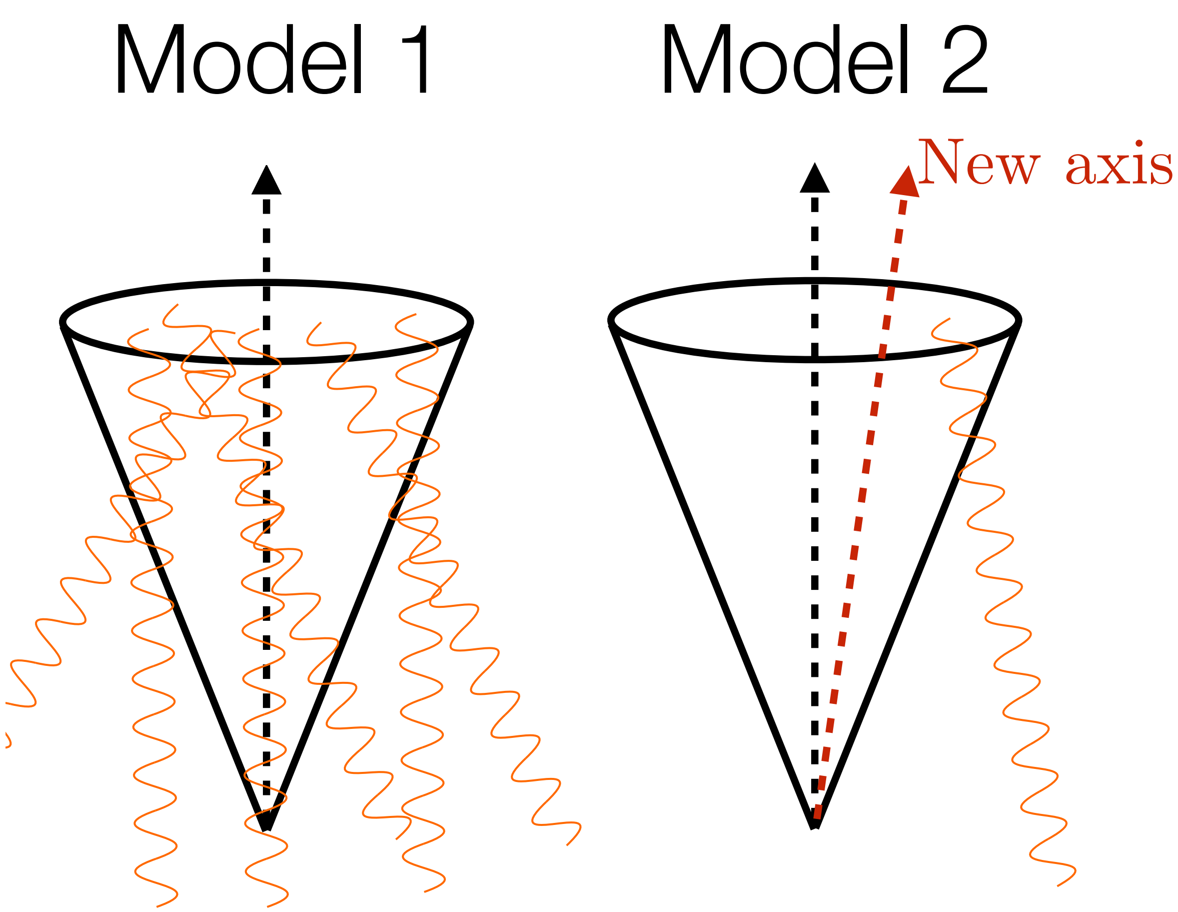
$$S_q^{\rm NG}(\widehat{L}) = 1 - \frac{\pi^2}{24}\widehat{L}^2 + \frac{\zeta_3}{12}\widehat{L}^3 + \frac{\pi^4}{34560}\widehat{L}^4 + \ldots \qquad \widehat{L} = \frac{\alpha_s N_c}{\pi}\ln\frac{R}{r}$$

Nonperturbative effects



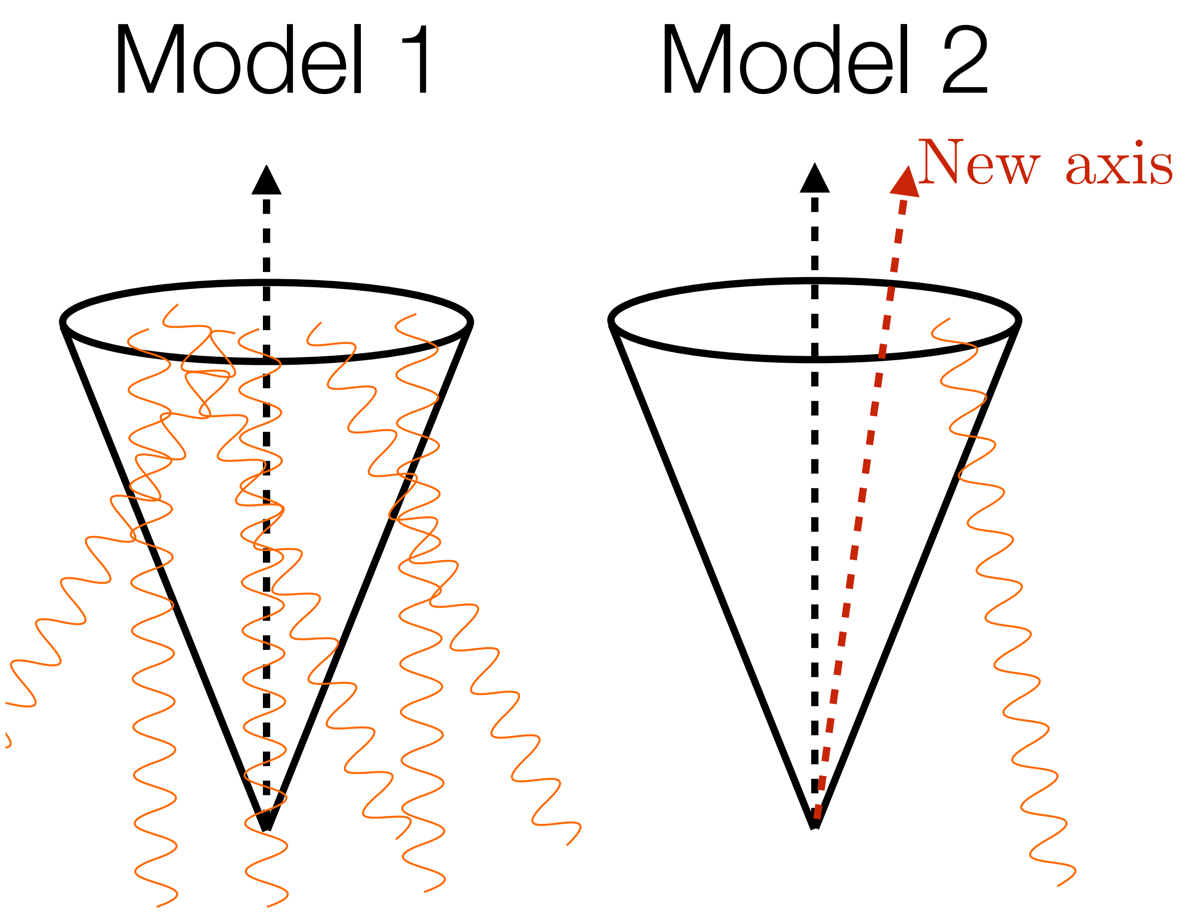
- Significant effects from soft radiation: initial-state radiation, multi-parton interactions and hadronization effects

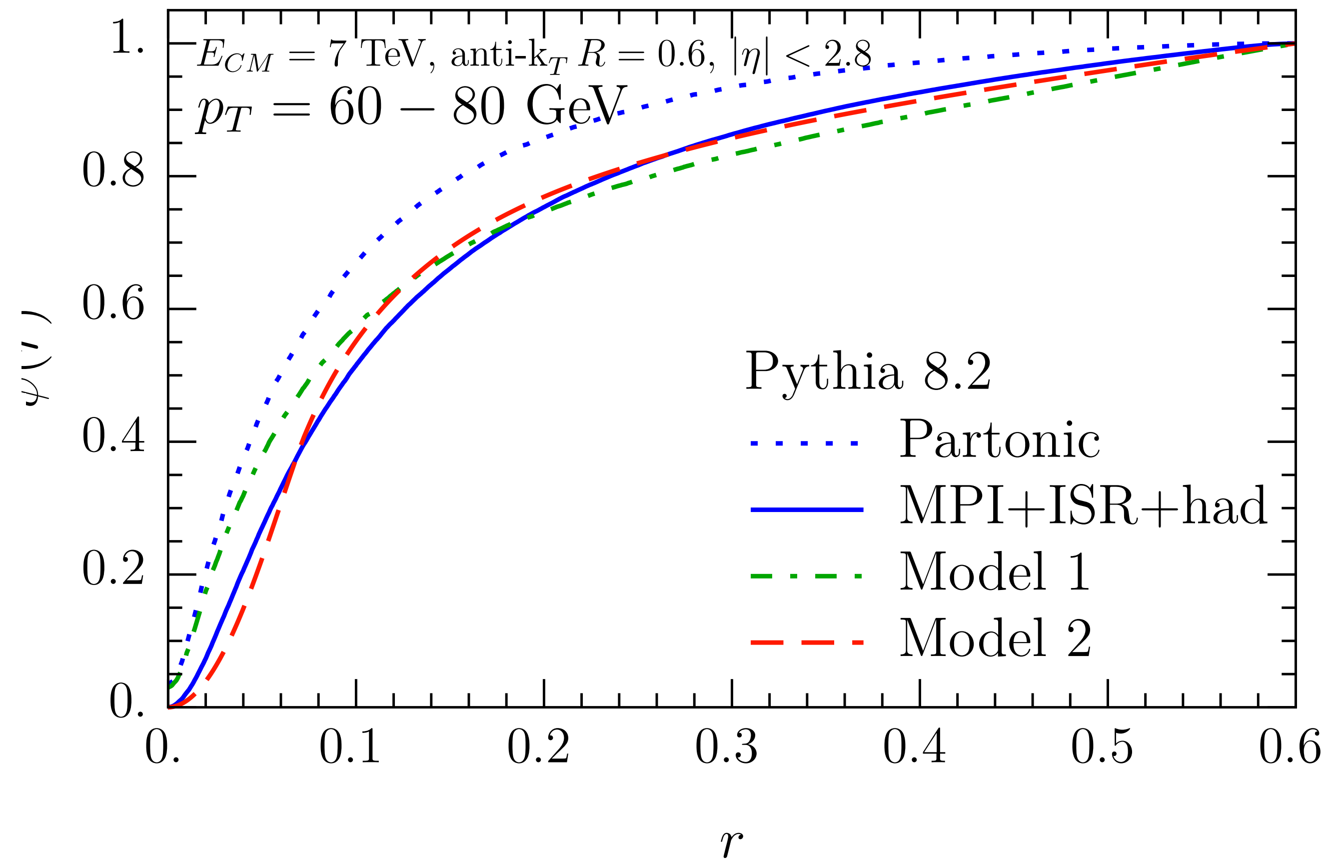
Nonperturbative model



- 1: uniform contamination, $\psi(r) \to \frac{1}{1+f}\,\psi(r) + \frac{f}{1+f}\left(\frac{r}{R}\right)^2$
- 2: localized contamination, also displaces jet axis

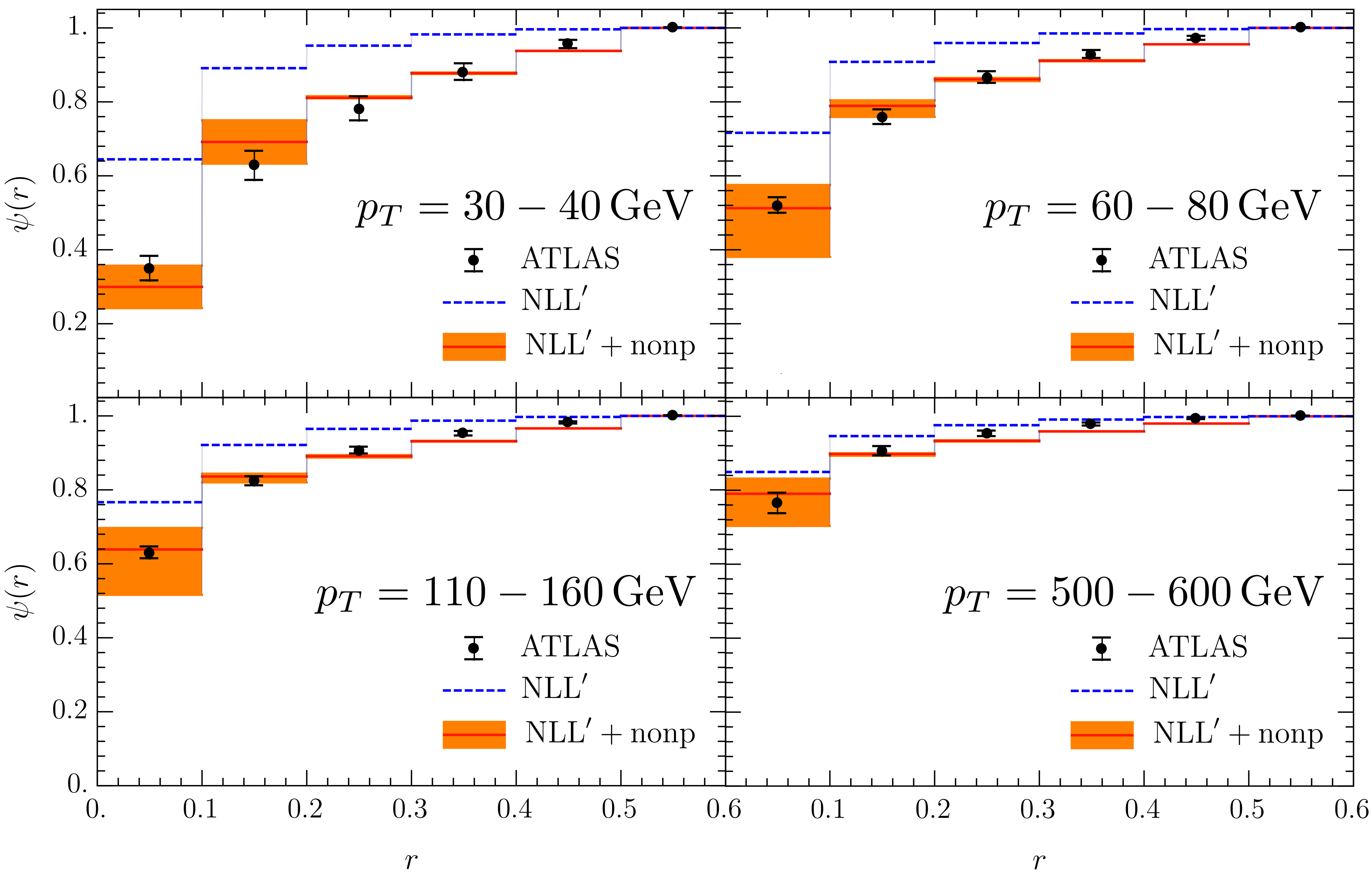
Nonperturbative model





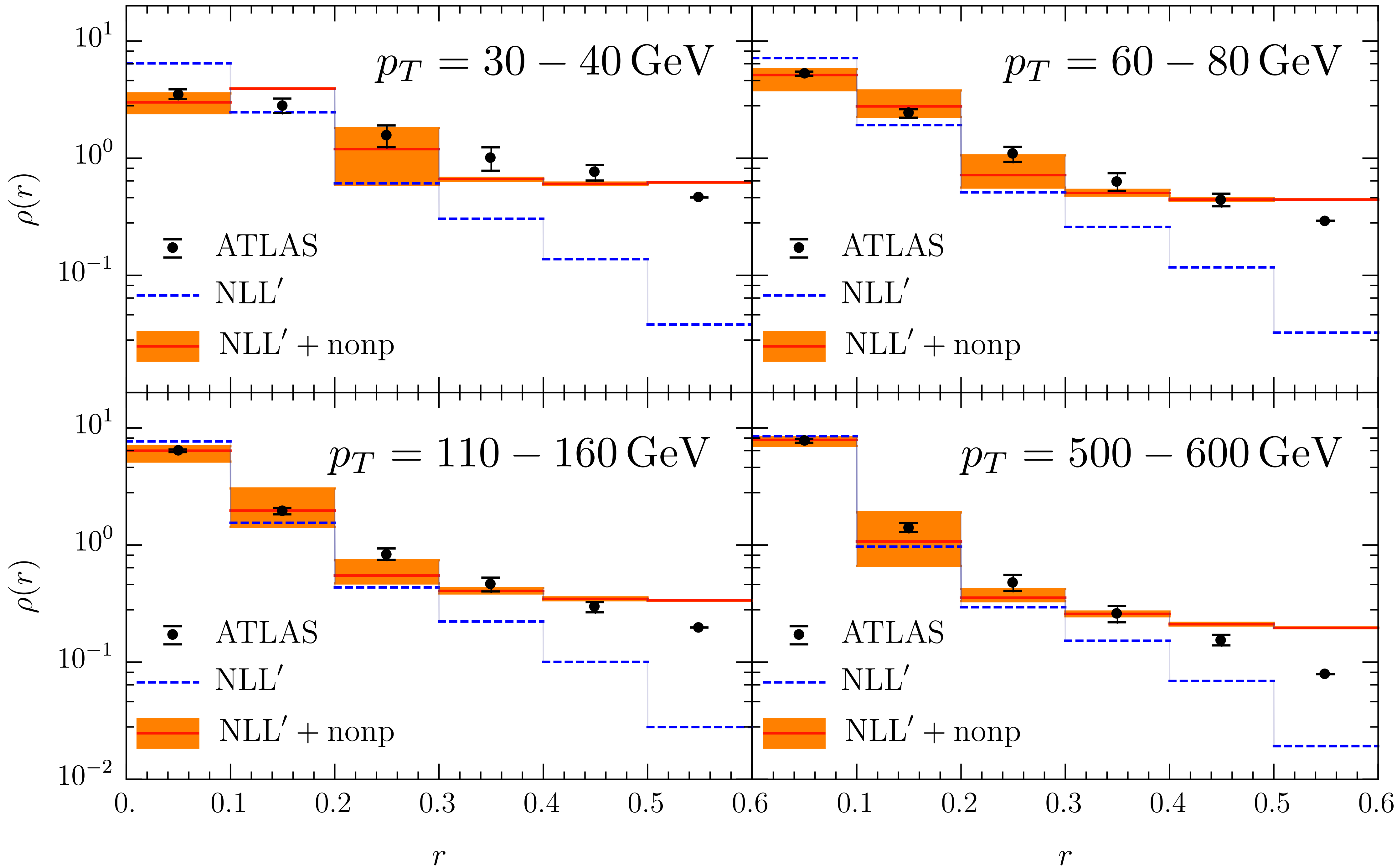
- 1: uniform contamination, $\psi(r) \to \frac{1}{1+f}\,\psi(r) + \frac{f}{1+f}\left(\frac{r}{R}\right)^2$
- 2: localized contamination, also displaces jet axis
- Model 2 agrees better, used when comparing to LHC data

ATLAS integrated jet shape



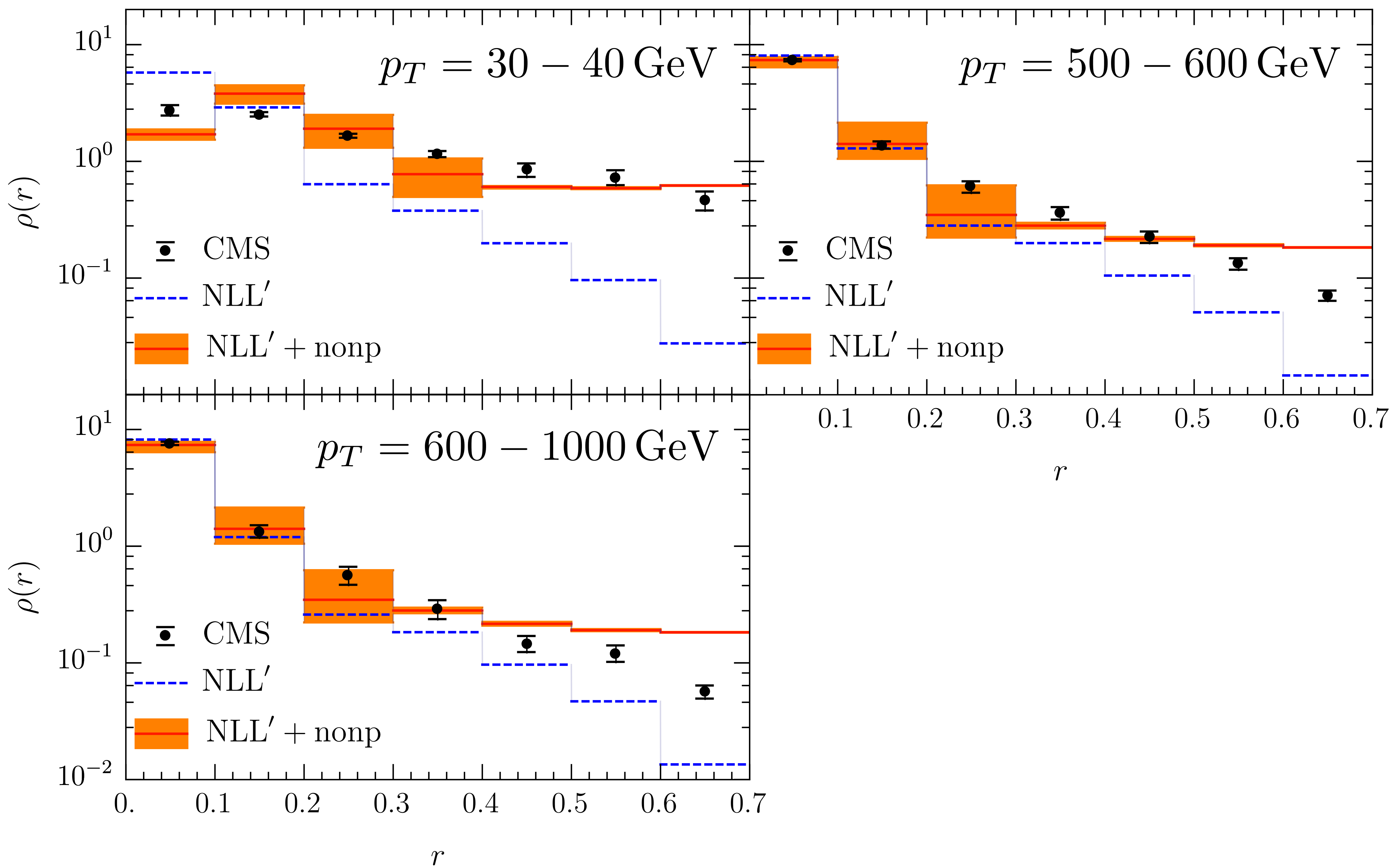
- Good agreement. Perturbative uncertainty largest for small r
  Nonperturbative effects $\propto 1/p_T$

ATLAS differential jet shape



- Nonperturbative effects particularly important in tail (not the region where r/R resummation is important)

CMS differential jet shape



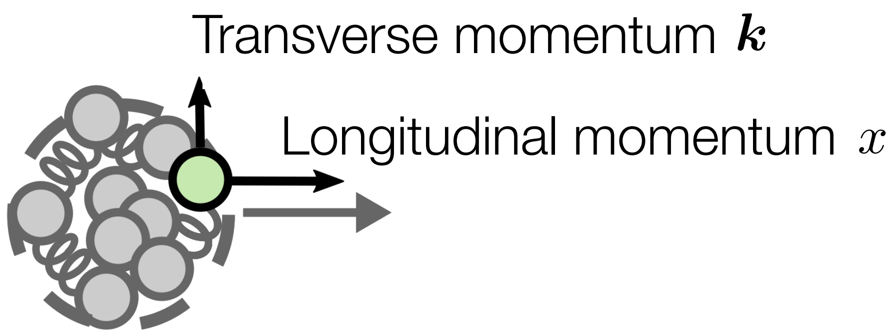
- Similar level of agreement
  Slightly larger R and nonperturbative effects

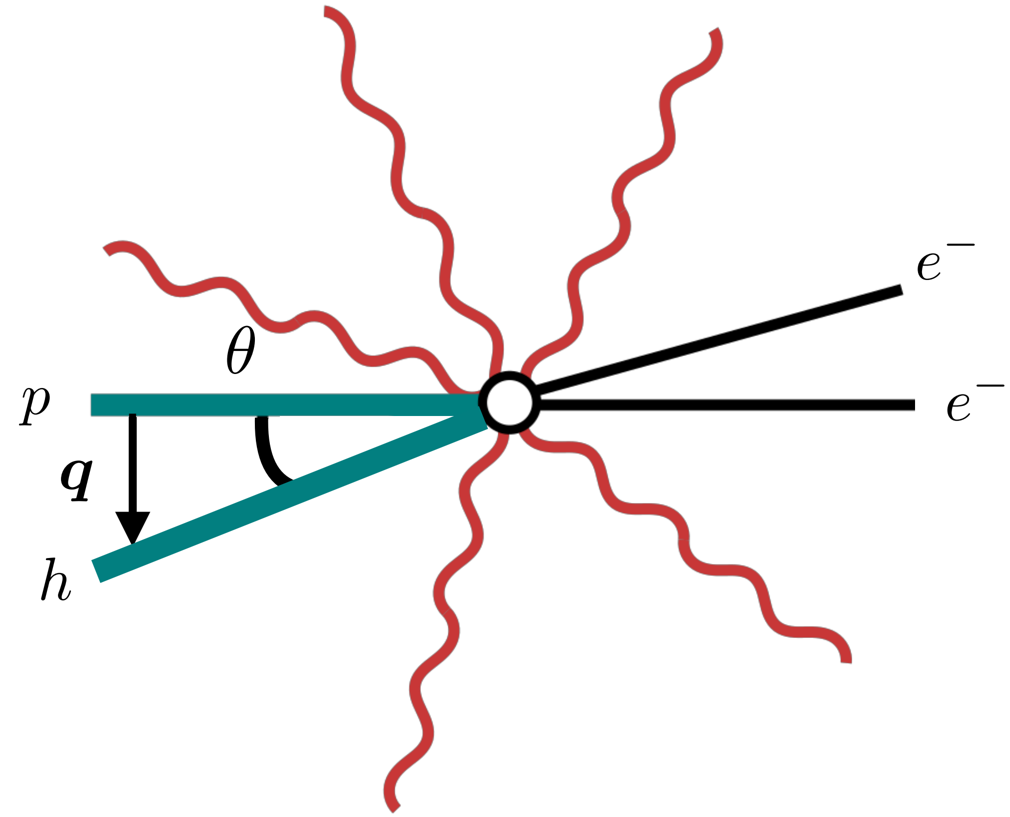
Conclusions

- First jet shape calculation beyond LL: recoil of soft radiation
  - Collinear function with recoil is more complicated
  - Nonglobal logarithms are fortunately same as for the hemisphere case
- Good agreement with data when using nonperturbative model
  → Can grooming help to reduce this sensitivity?

2. TMDs from Jets

TMDs from SIDIS

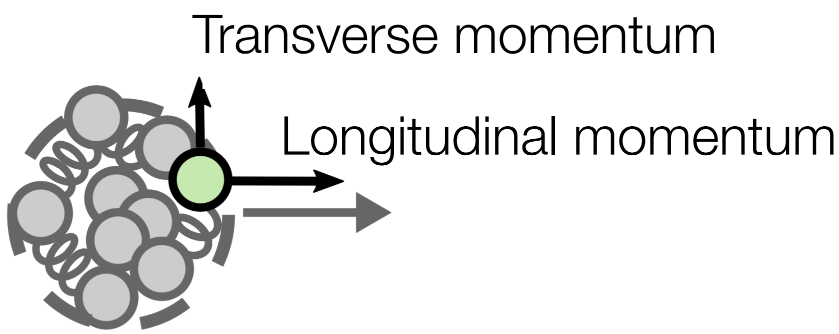


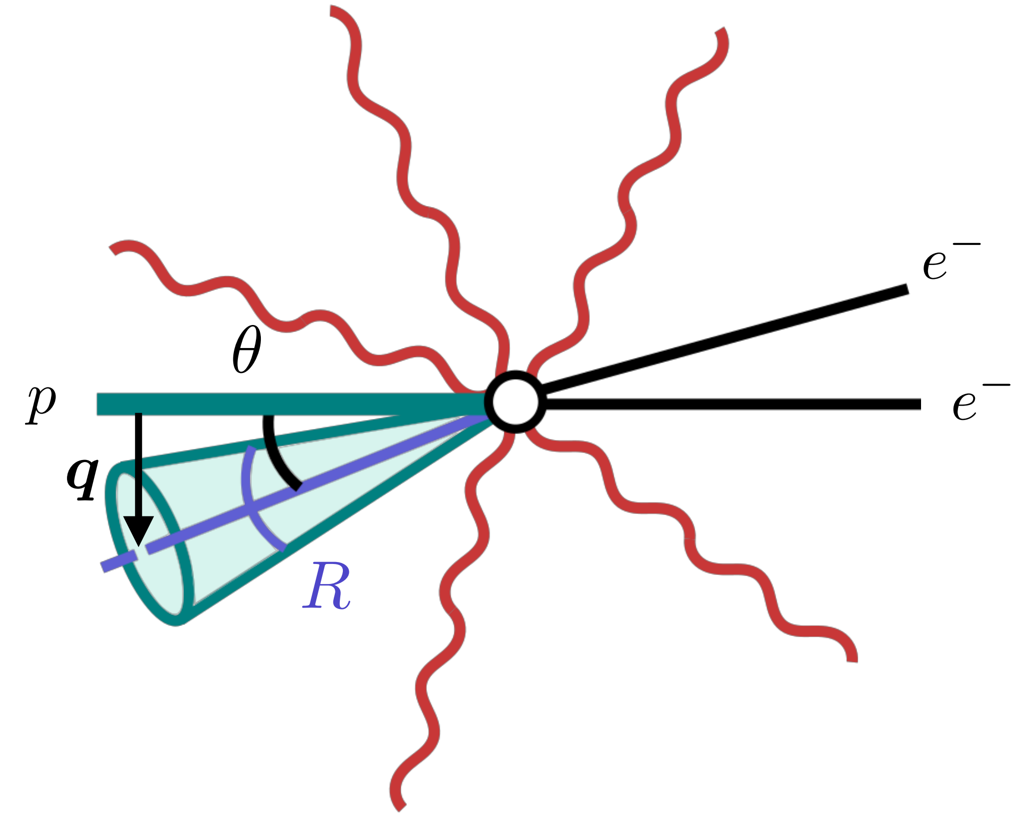


$$\frac{\mathrm{d}\sigma_{ep\to ehX}}{\mathrm{d}Q^2\,\mathrm{d}x\,\mathrm{d}z\,\mathrm{d}\boldsymbol{q}} = \sum_q H_q^{\mathrm{DIS}}(x,Q^2)\int\frac{\mathrm{d}\boldsymbol{b}}{(2\pi)^2}\,e^{-\mathrm{i}\boldsymbol{b}\cdot\boldsymbol{q}}F_q(x,\boldsymbol{b})\,D_{q\to h}(z,\boldsymbol{b})$$

- F_i describes transverse momentum $\boldsymbol{k}$ of parton *i* in terms of Fourier conjugate variable $\boldsymbol{b}$
- $D_{i\to h}$ encodes transverse momentum $\boldsymbol{k}' = \boldsymbol{p}_h/z$ of hadron *h* fragmenting from parton *i*. Total trans. momentum $\boldsymbol{q} = \boldsymbol{k} + \boldsymbol{k}'$

TMDs from SIDIS with jets

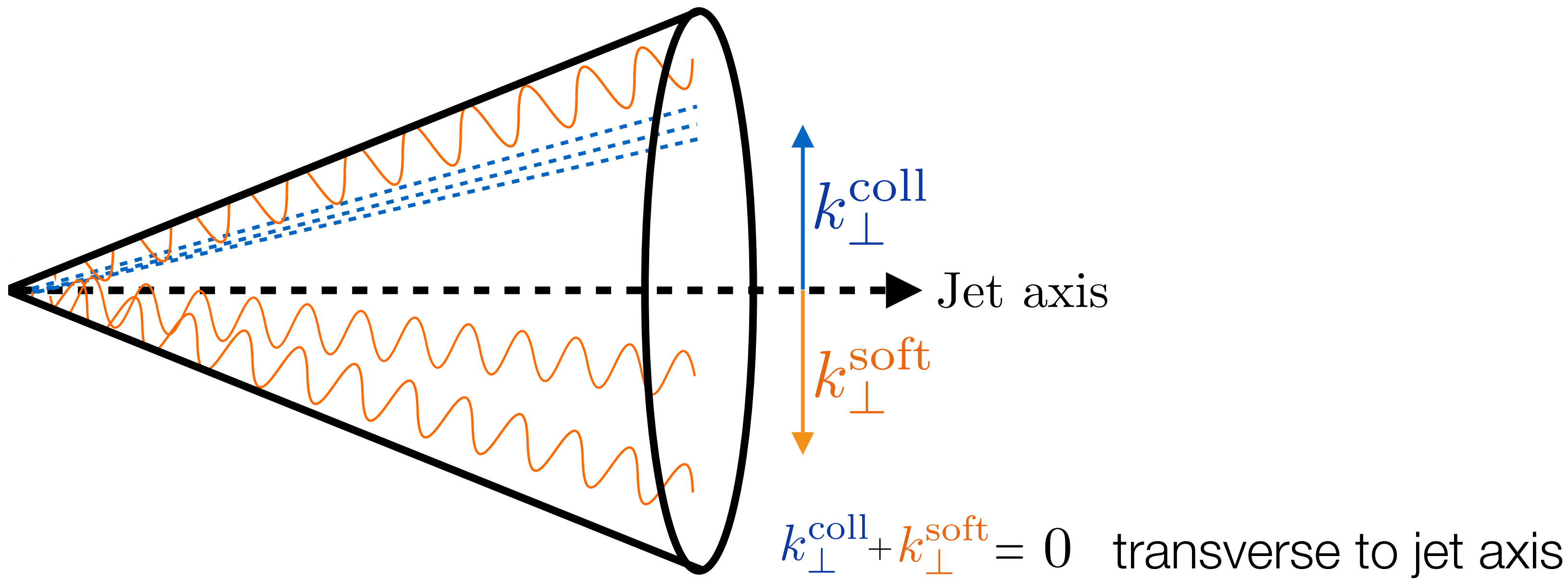




$$\frac{\mathrm{d}\sigma_{ep\to eJX}}{\mathrm{d}Q^2\,\mathrm{d}x\,\mathrm{d}z\,\mathrm{d}\boldsymbol{q}} = \sum_q H_q^{\mathrm{DIS}}(x,Q^2)\int\frac{\mathrm{d}\boldsymbol{b}}{(2\pi)^2}\, e^{-\mathrm{i}\boldsymbol{b}\cdot\boldsymbol{q}} F_q(x,\boldsymbol{b})\, J_q(z,\boldsymbol{b},QR)$$

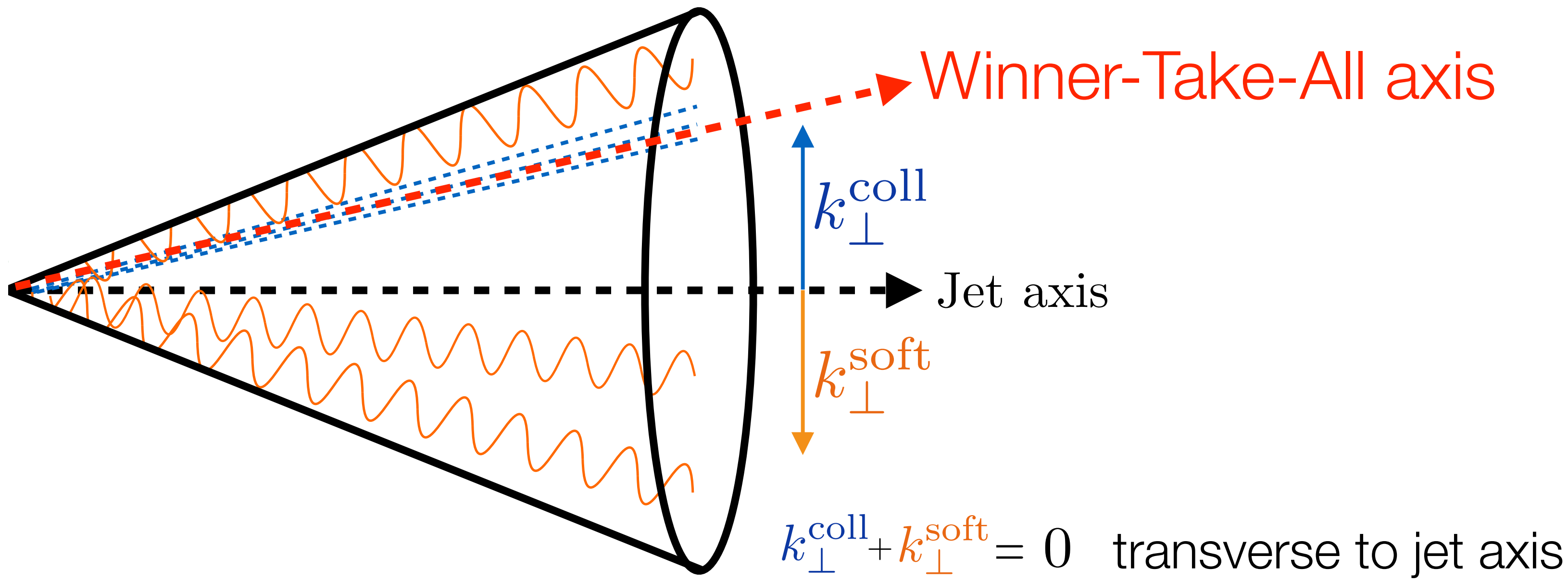
- F_i describes transverse momentum $\boldsymbol{k}$ of parton *i*, in terms of Fourier conjugate variable $\boldsymbol{b}$
- J_q encodes transverse momentum $\boldsymbol{k}' = \boldsymbol{p}_J/z$ of jet fragmenting from parton *i*, reducing nonperturbative sensitivity

Soft recoil



- Jet axis along total jet momentum, recoiled by soft radiation **inside** jet → nonglobal logarithms limit accuracy to NLL'

Soft recoil and the Winner-Take-All axis



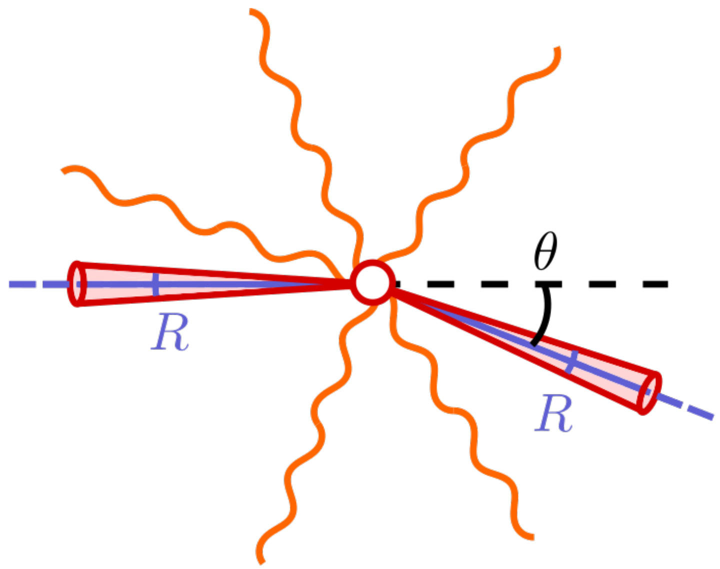
- Jet axis along total jet momentum, recoiled by soft radiation **inside** jet → nonglobal logarithms limit accuracy to NLL'
- Remove by Winner-Take-All clustering in jet algorithm → N[3]LL

$$E_r = E_1 + E_2$$

$$\hat{n}_r = \begin{cases} \hat{n}_1 & \text{if } E_1 > E_2 \\ \hat{n}_2 & \text{if } E_2 > E_1 \end{cases}$$

[Salam; Bertolini, Chan, Thaler]

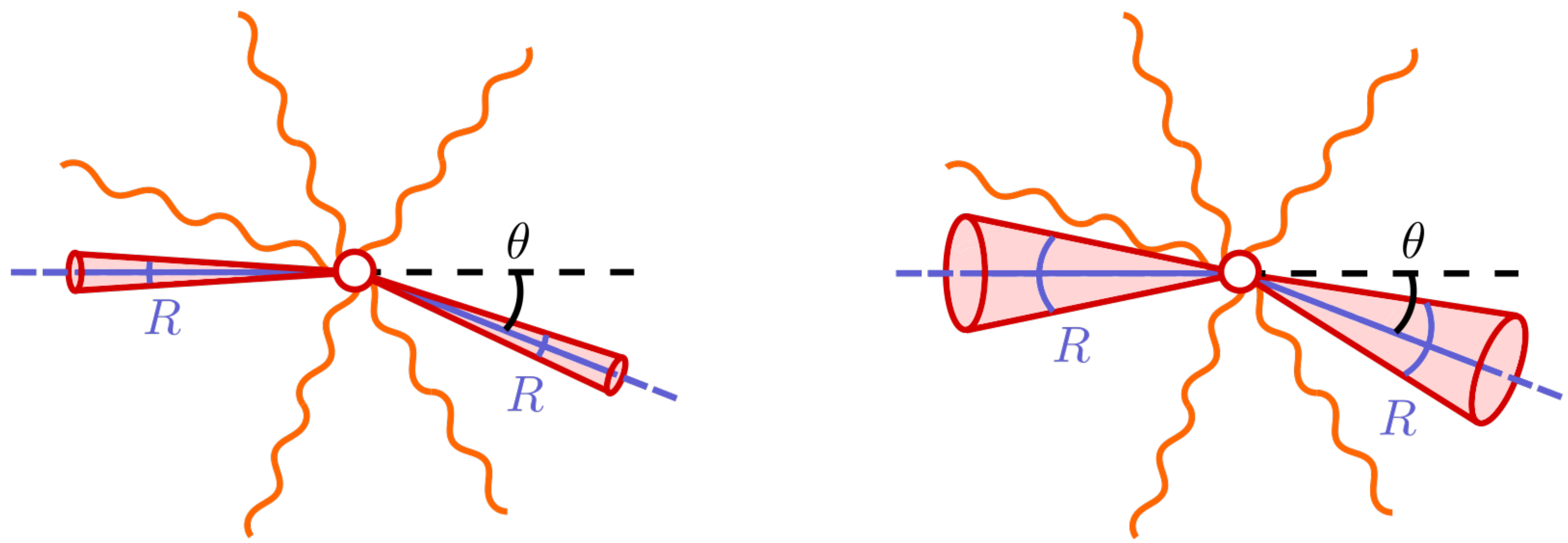
Angular decorrelation in e^+e^-



$$\frac{d\sigma_{e^+e^-\to JJX}}{dz_1\, dz_2\, d\boldsymbol{q}} = \sum_q H_q^{e^+e^-}(s) \int \frac{d\boldsymbol{b}}{(2\pi)^2}\, e^{-i\boldsymbol{b}\cdot\boldsymbol{q}} J_q(z_1, \boldsymbol{b}, \sqrt{s}R)\, J_q(z_2, \boldsymbol{b}, \sqrt{s}R)$$

- Angular decorrelation related to transverse momentum $\theta \approx \dfrac{2q_T}{\sqrt{s}}$
- $\theta \gg R$: no jet axis dependence

Angular decorrelation in e^+e^-



$$\frac{\mathrm{d}\sigma_{e^+e^-\to JJX}}{\mathrm{d}z_1\,\mathrm{d}z_2\,\mathrm{d}\boldsymbol{q}} = \sum_q H_q^{e^+e^-}(s)\int\frac{\mathrm{d}\boldsymbol{b}}{(2\pi)^2}\,e^{-\mathrm{i}\boldsymbol{b}\cdot\boldsymbol{q}} J_q(z_1,\boldsymbol{b},\sqrt{s}R)\,J_q(z_2,\boldsymbol{b},\sqrt{s}R)$$

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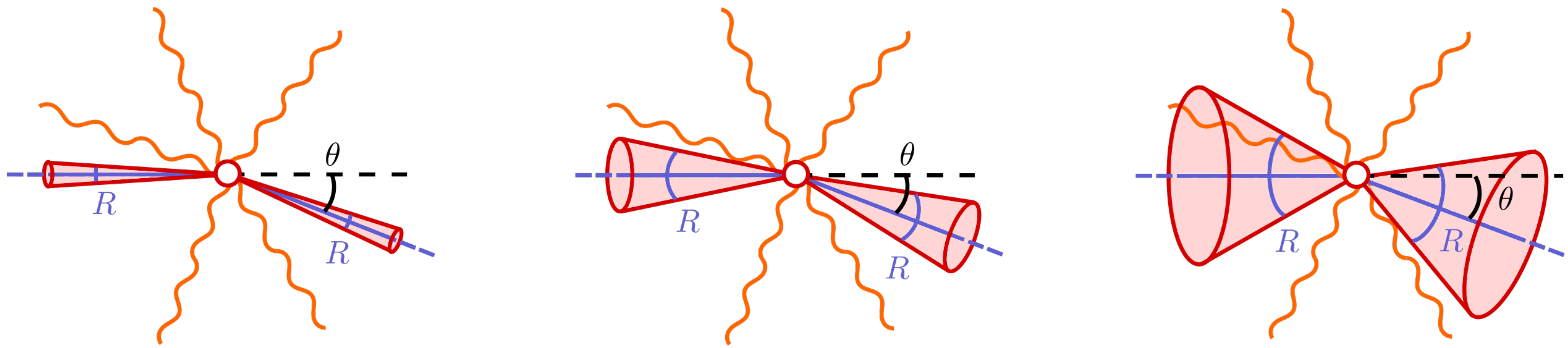

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- Angular decorrelation related to transverse momentum $\theta \approx \frac{2q_T}{\sqrt{s}}$
- $\theta \gg R$: no jet axis dependence
- $\theta \sim R$: finite terms in jet function depend on axis choice
- $\theta \ll R$: factorization only holds for Winner-Take-All

Jet function

- We have calculated the jet function at one loop for the standard jet axis and Winner-Take-All
- $\boldsymbol{k}^2 \sim \boldsymbol{b}^{-2} \gg ER$: axis dependence drops out, matches onto semi-inclusive jet functions $\mathscr{J}$, same coefficients $\mathbb{C}$ as TMD FFs

$$J_i^{\text{axis}}(z, \boldsymbol{b}, ER) = \sum_j \int \frac{\mathrm{d}z'}{z'} \left[(z')^2 \mathbb{C}_{i\to j}(z', \boldsymbol{b})\right] \mathscr{J}_j\left(\frac{z}{z'}, \frac{2z}{z'} ER\right)$$

[Echevarria, Scimemi, Vladimirov] [Kang et al, Dai et al]

- DGLAP evolution resums $\ln R$

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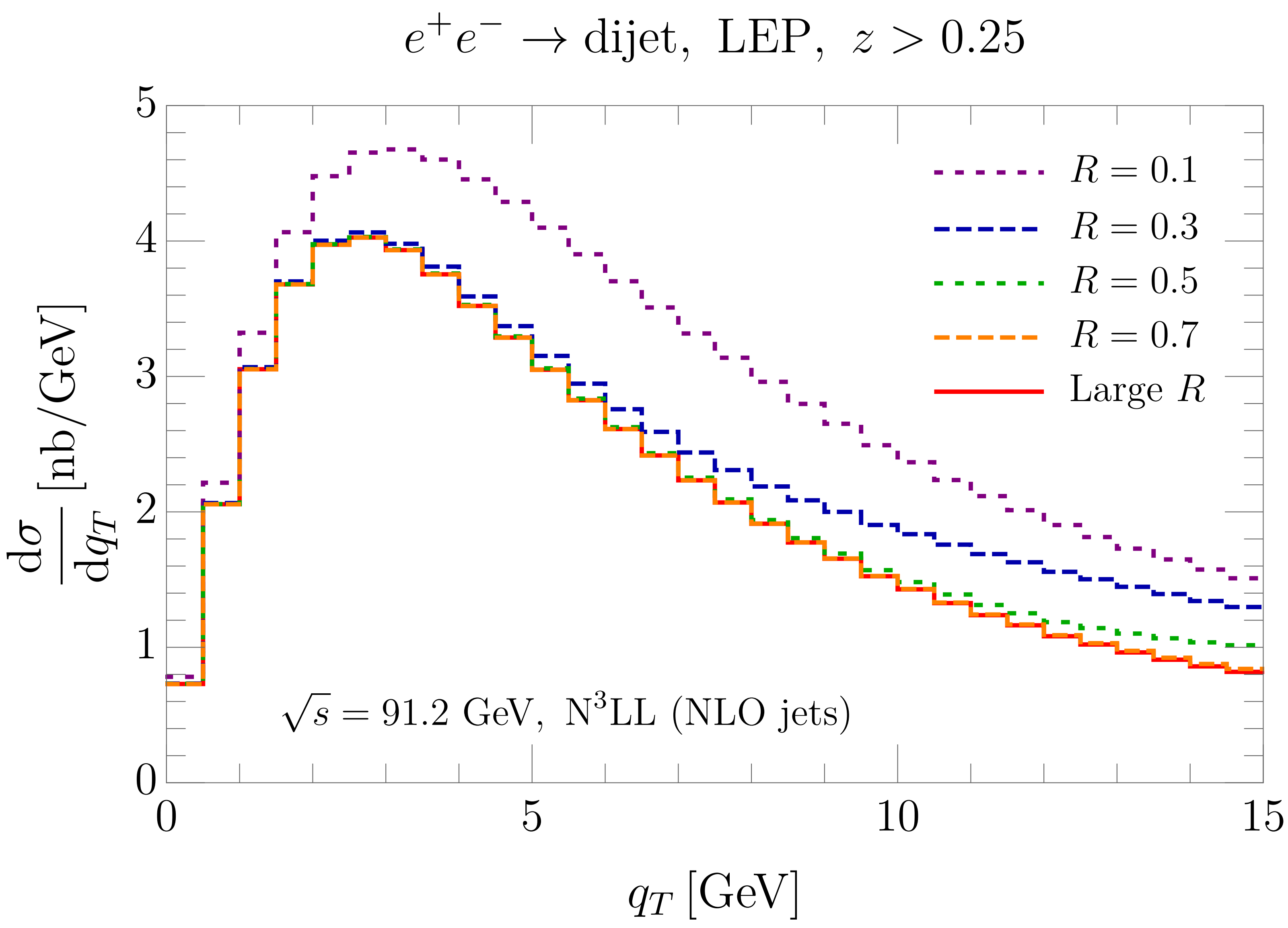
[Echevarria, Scimemi, Vladimirov] [Kang et al, Dai et al]

- DGLAP evolution resums $\ln R$
- $\boldsymbol{k}^2 \sim \boldsymbol{b}^{-2} \ll ER$ for WTA: jet radius dependence drops out

$$J_i^{\rm WTA}(z, \boldsymbol{b}, ER) = \delta(1-z)\, \mathscr{J}_i^{\rm WTA}(\boldsymbol{b})$$

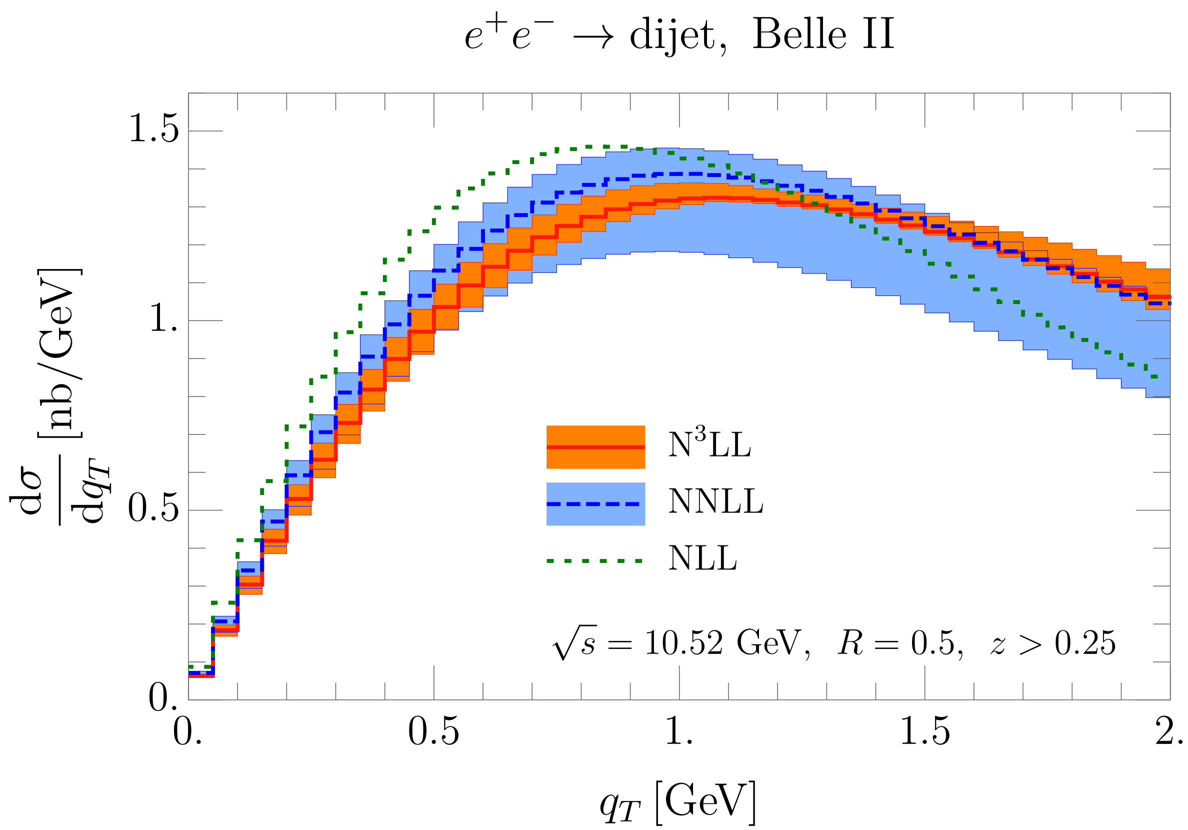
- Determined by known anomalous dimensions up to two-loop constant, which we extract for quarks from EVENT2 [Catani, Seymour]

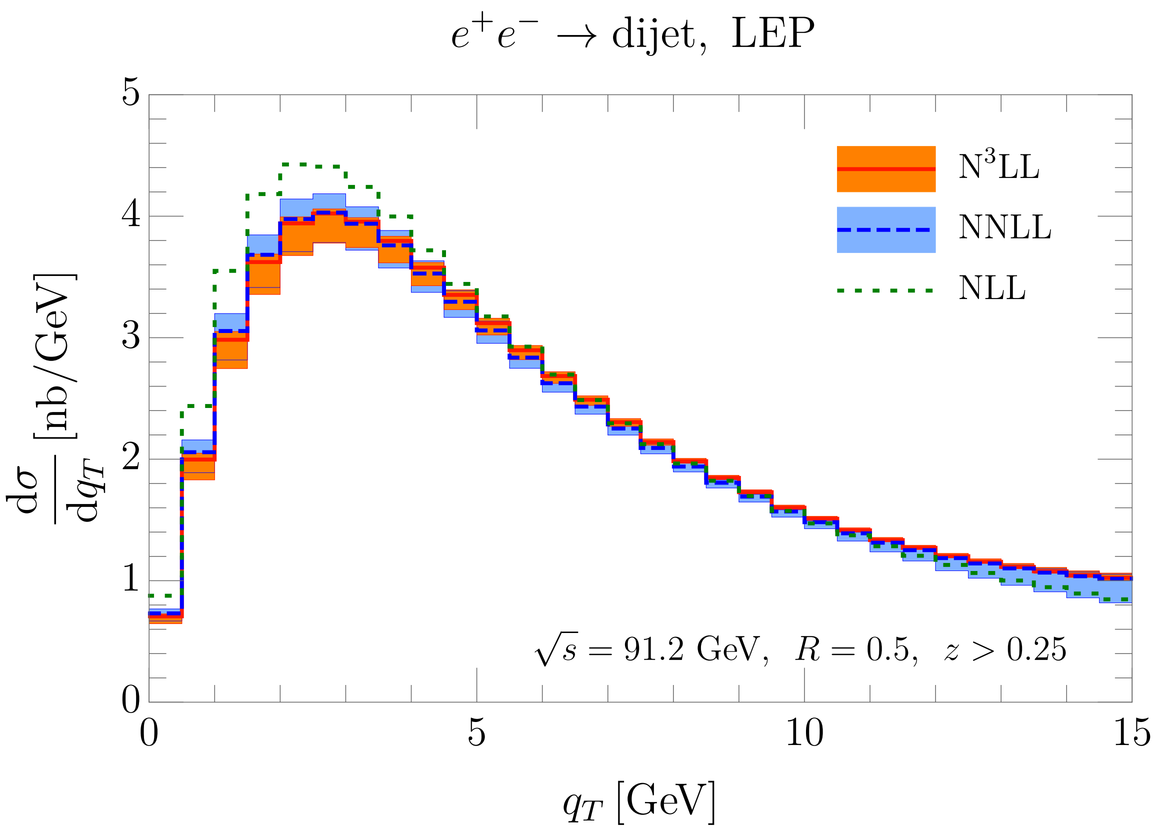
Jet radius dependence



- Even for medium values of *R*, we can use large *R* jet function
- Large *R* jet function determined at two loop → N^{3}LL

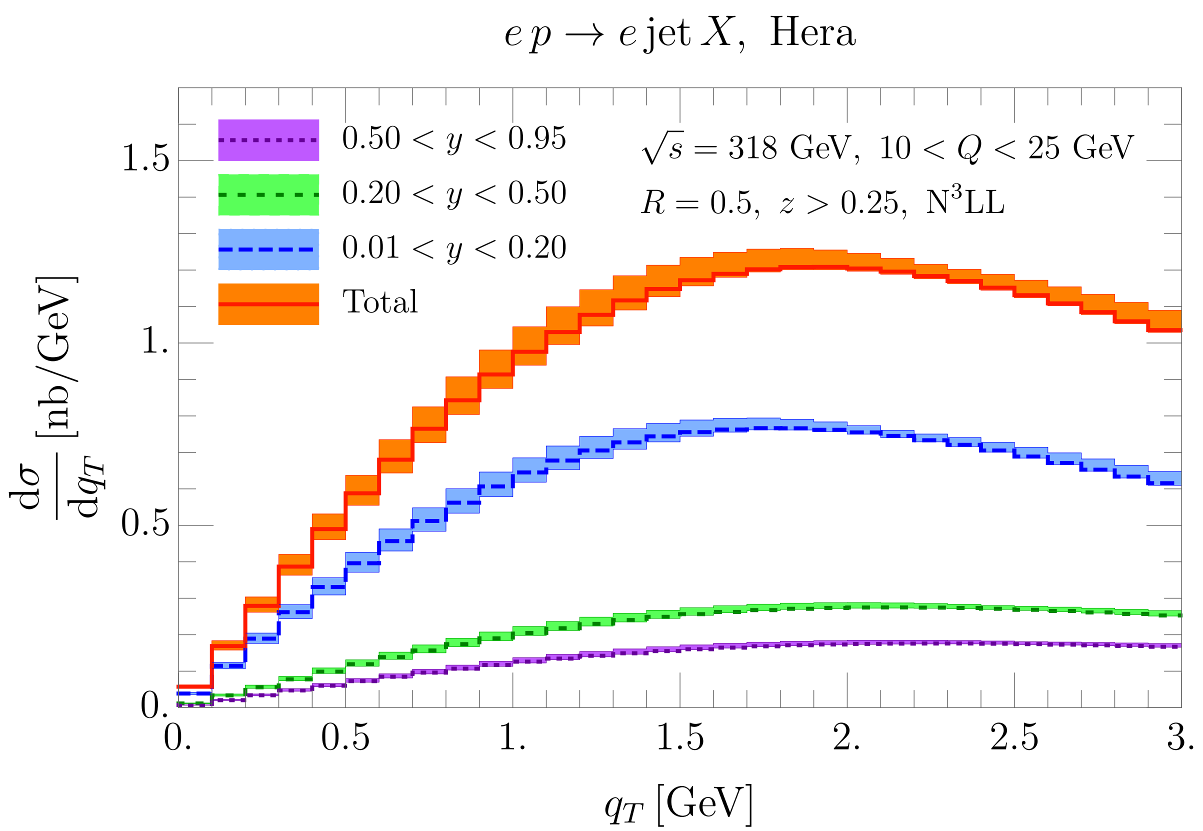
e^+e^- results

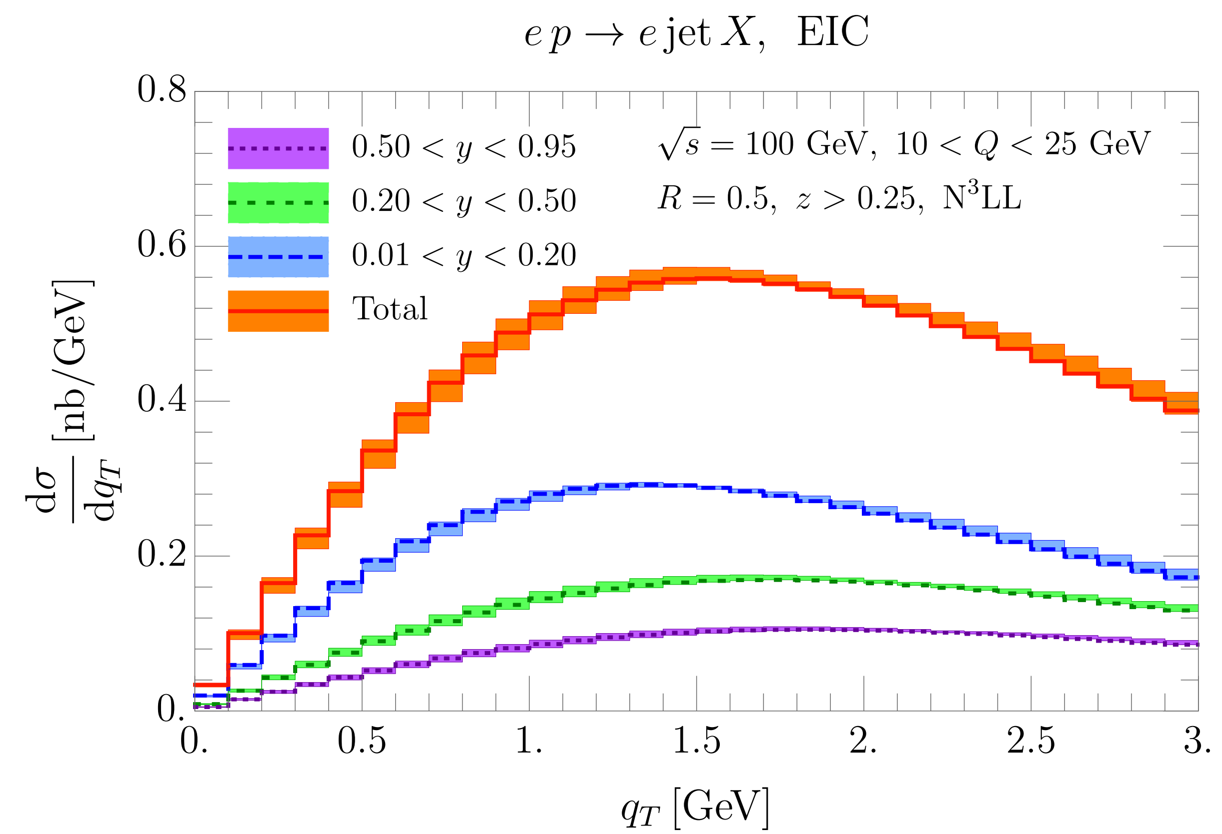




- Implemented in arTeMiDe [Scimemi, Vladimirov]
- Good perturbative convergence: uncertainty bands overlap and reduce at higher orders

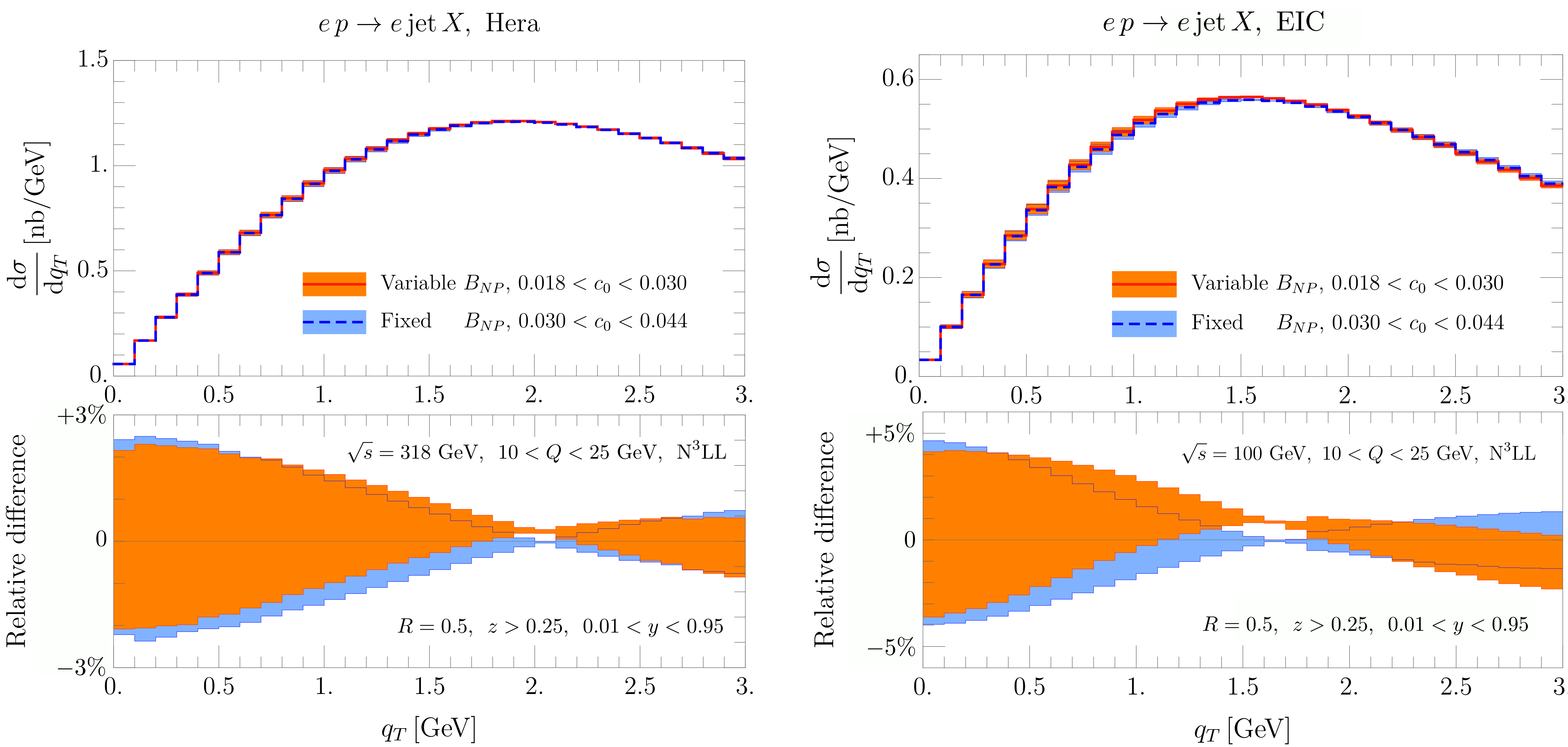
SIDIS results





- Transverse momentum distribution for Hera and EIC for different elasticity intervals

Sensitivity to nonperturbative parameters



- Vary nonperturbative parameters of TMD PDFs within current uncertainty from Drell-Yan extraction [Bertone, Scimemi, Vladimirov]

Conclusions

- The transverse momentum distribution of partons in a proton can be extracted from SIDIS using jets
  - Calculable *z* dependence → reduced nonpert. uncertainty
  - Perturbative ingredients available for N^3LL
- Potential experimental issue: angular resolution. Could be remedied with charged particle jets

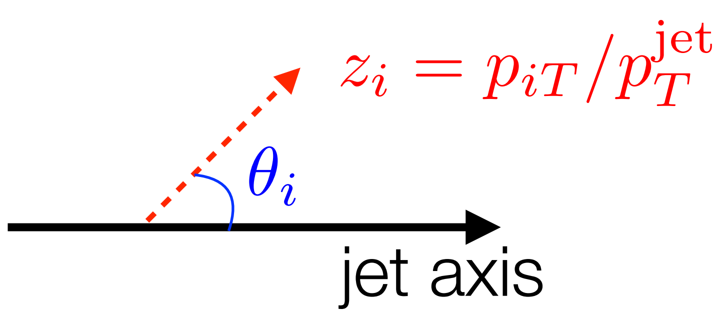
3. Multi-differential resummation

Why multi-differential resummation?

- The classic: threshold and transverse mom. resummation
  [Laenen, Sterman, Vogelsang; Li; Lustermans, WW, Zeune; Marzani, Theeuwes; Muselli, Forte, Ridolfi, …]
- LHC analyses involve multiple cuts
  → describe correlations beyond accuracy of parton shower
- Ratio observables require double-differential resummation
  E.g. N-subjettiness, energy correlation functions, planar flow, …
- Toy example: ratio of two angularities e_α / e_β

$$e_\alpha = \sum_{i \in \text{jet}} z_i \left(\frac{\theta_i}{R}\right)^\alpha$$

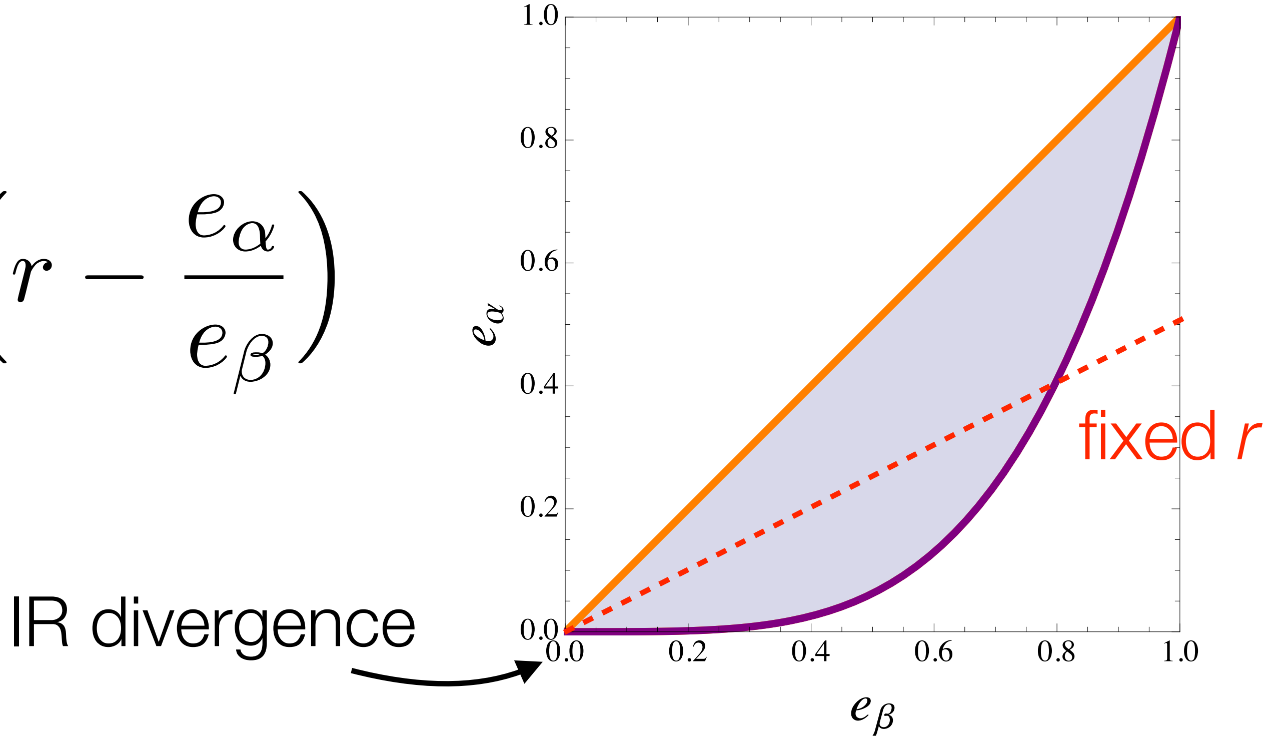
[Berger, Kucs, Sterman; Almeida et al.]



Ratios require double-differential resummation

- Ratio $r = e_\alpha / e_\beta$ is not IR safe
  [Soyez, Salam, Kim, Dutta, Cacciari]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}r} = \int \mathrm{d}e_\alpha\, \mathrm{d}e_\beta\, \frac{\mathrm{d}^2\sigma}{\mathrm{d}e_\alpha\, \mathrm{d}e_\beta}\, \delta\left(r - \frac{e_\alpha}{e_\beta}\right)$$



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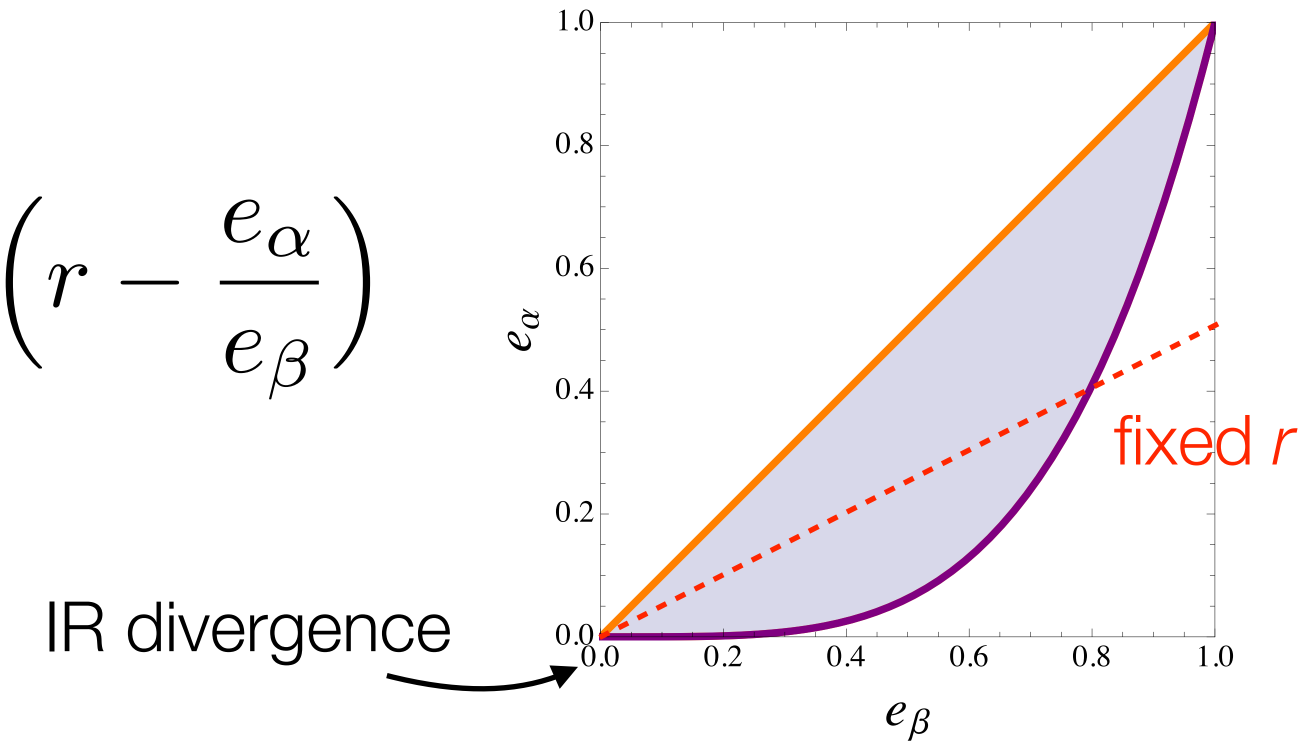


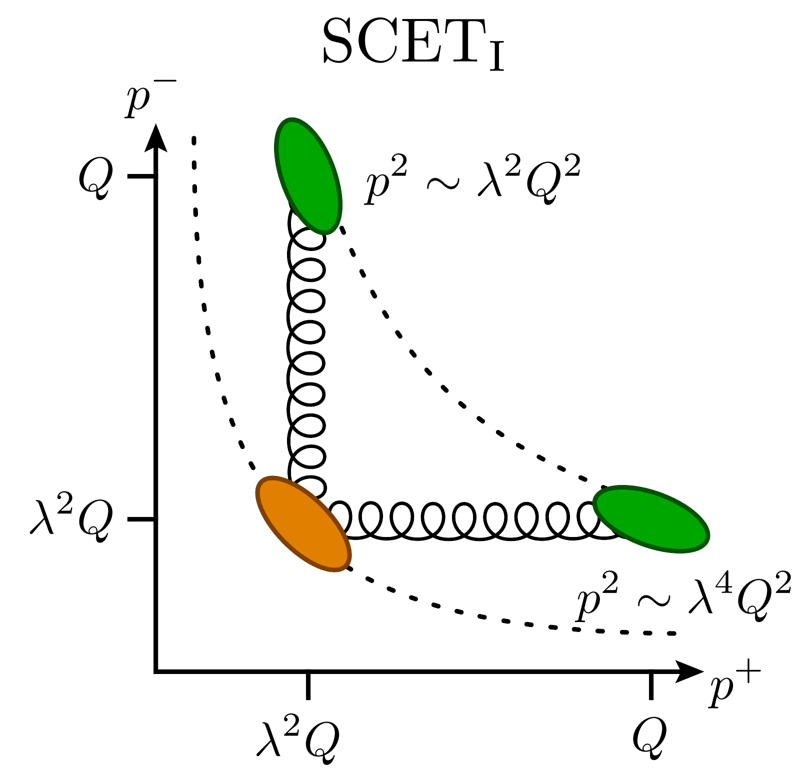
- IR region is Sudakov suppressed [Larkoski, Thaler]
  Requires simultaneous resummation of $\ln e_\alpha, \ln e_\beta$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}r} = \sqrt{\alpha_s} \, \frac{\sqrt{C_F \beta}}{\alpha - \beta} \, \frac{1}{r} + \ldots$$

can't get this from fixed-order

Simple example: beam thrust $\mathcal{T}$ and q_T in Drell-Yan

	$\mathrm{SCET_I}$
n-collinear	$Q(\lambda^2, 1, \lambda)$
$\bar{n}$-collinear	$Q(1, \lambda^2, \lambda)$
soft	$Q(\lambda^2, \lambda^2, \lambda^2)$



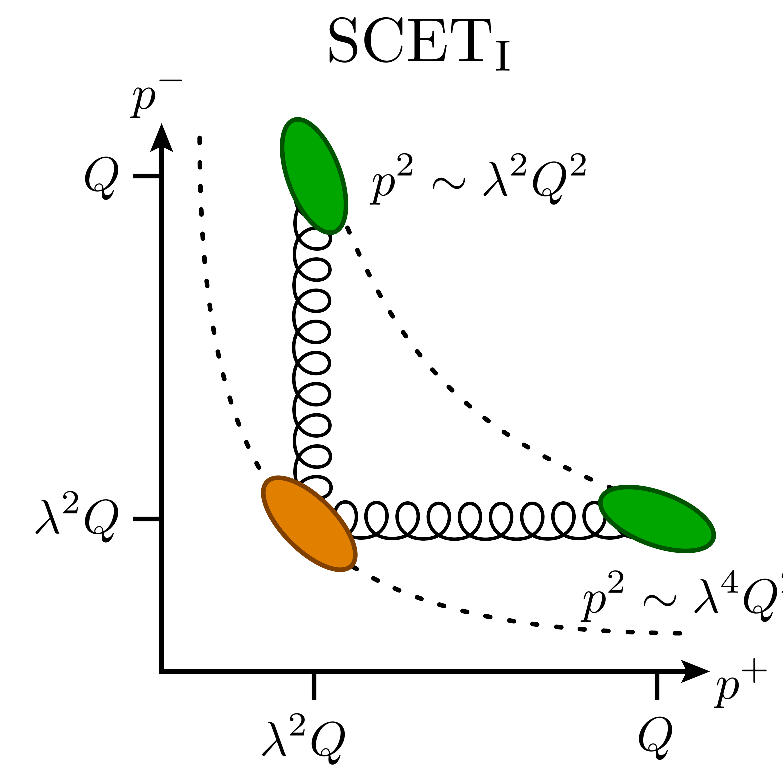
- Light cone coordinates

$$p^\mu = (p^+, p^-, p_\perp^\mu) = (p^0 - p^3, p^0 + p^3, p_\perp^\mu)$$

- Measurement determines modes:
  - $\mathcal{T} = \sum_i \min\{p_i^+, p_i^-\}$ described by $\mathrm{SCET_I}$ with $\lambda^2 = \mathcal{T}/Q$

Simple example: beam thrust $\mathcal{T}$ and q_T in Drell-Yan

	$\mathrm{SCET_I}$	$\mathrm{SCET_{II}}$
n-collinear	$Q(\lambda^2, 1, \lambda)$	$Q(\lambda^2, 1, \lambda)$
$\bar{n}$-collinear	$Q(1, \lambda^2, \lambda)$	$Q(1, \lambda^2, \lambda)$
soft	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q(\lambda, \lambda, \lambda)$



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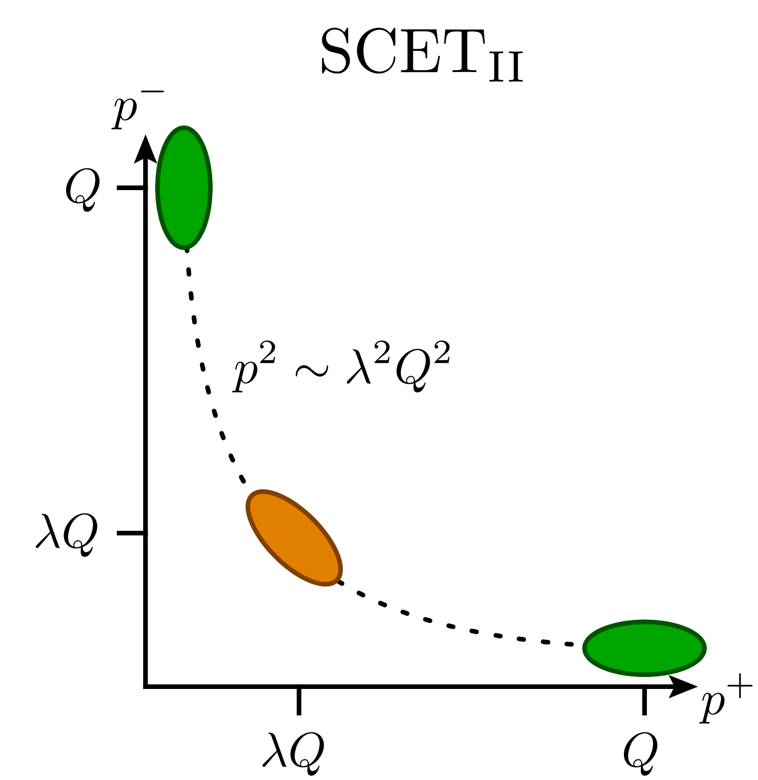
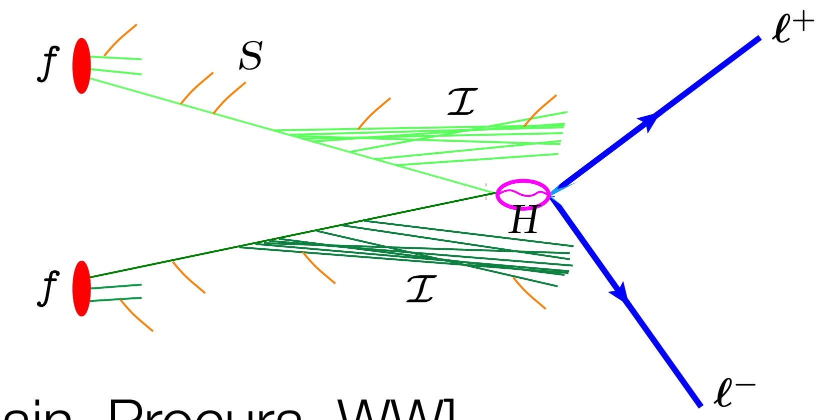
  - $\vec{q}_T = \sum_i \vec{p}_{i\perp}$ is $\mathrm{SCET_{II}}$ with $\lambda = q_T/Q$



Beam thrust and transverse momentum resummation



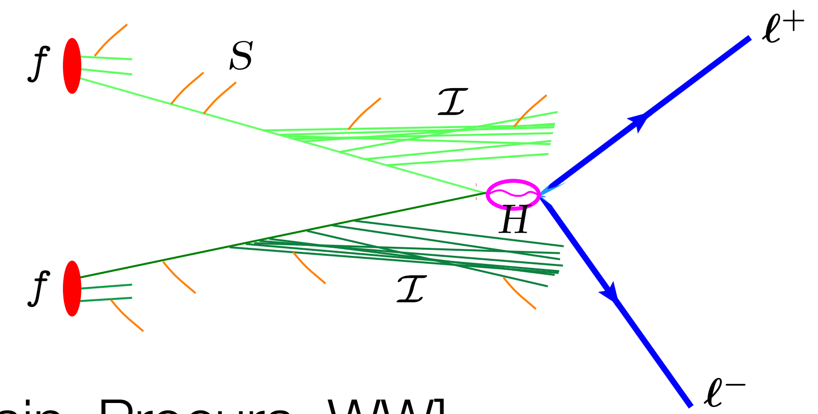
- Make $\mathcal{T}$ or q_T factorization formulas more differential:

$$\frac{d\sigma}{d\vec{q}_T\, d\mathcal{T}} = H(Q) B(Q\mathcal{T}, \vec{q}_T) \otimes B(Q\mathcal{T}, \vec{q}_T) \otimes S(\mathcal{T})$$ [Jain, Procura, WW]

- Structure dictated by power counting

$$\mathrm{SCET_I}: \ \vec{q}_T = \vec{q}_T^{\,\mathrm{coll}} + \vec{q}_T^{\,\mathrm{coll}} + \mathcal{O}(\lambda^2) \qquad \to \qquad Q\mathcal{T} \sim \vec{q}_T^{\,2}$$

Beam thrust and transverse momentum resummation



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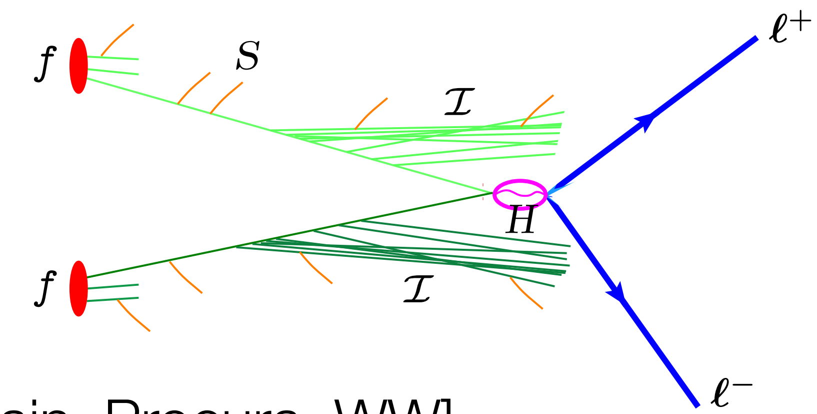
Beam thrust and transverse momentum resummation



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$$\frac{d\sigma}{d\vec{q}_T\, d\mathcal{T}} = H(Q) B(\vec{q}_T) \otimes B(\vec{q}_T) \otimes S_+(\mathcal{T}, \vec{q}_T) \otimes S_+(\mathcal{T}, \vec{q}_T) \otimes S(\mathcal{T})$$ [Procura, WW, Zeune]

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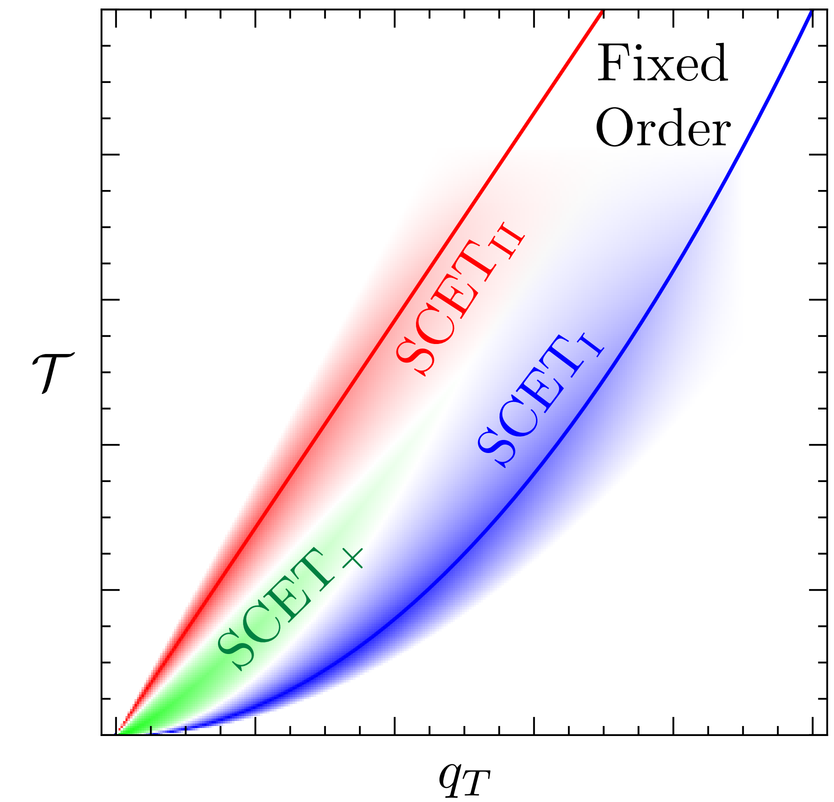
$$\mathrm{SCET_{II}}: \; \mathcal{T} = \mathcal{T}^{\mathrm{soft}} + \mathcal{O}(\lambda^2) \quad \rightarrow \quad \mathcal{T}^2 \sim \vec{q}_T^{\,2}$$

- Intermediate regime requires extra collinear-soft functions S_+

Consistency relations



- Between SCET$_{\rm I}$ and SCET+

$$B(Q\mathcal{T}, \vec{q}_T) = B(\vec{q}_T) \otimes S_+(\mathcal{T}, \vec{q}_T) \times \left[1 + \mathcal{O}\left(\frac{q_T^2}{Q\mathcal{T}}\right)\right]$$

- Between SCET$_{\rm II}$ and SCET+

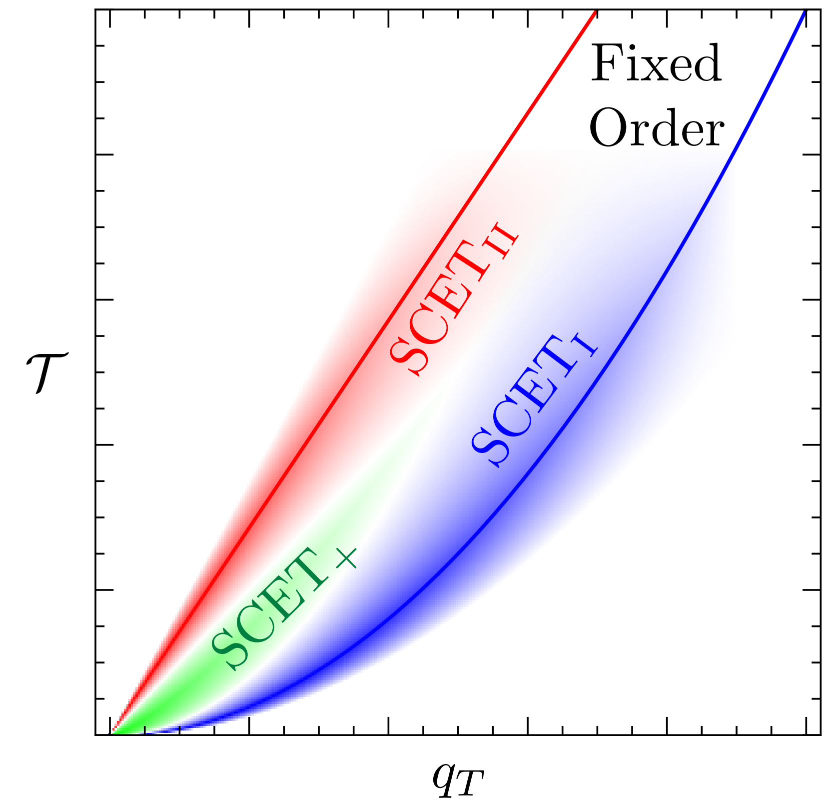
$$S(\mathcal{T}, \vec{q}_T) = S_+(\mathcal{T}, \vec{q}_T) \otimes S_+(\mathcal{T}, \vec{q}_T) \otimes S(\mathcal{T}) \left[1 + \mathcal{O}\left(\frac{\mathcal{T}^2}{q_T^2}\right)\right]$$

- Verified for anomalous dimensions and NLO ingredients

Scales choices and matching

- Natural scales connect regimes:

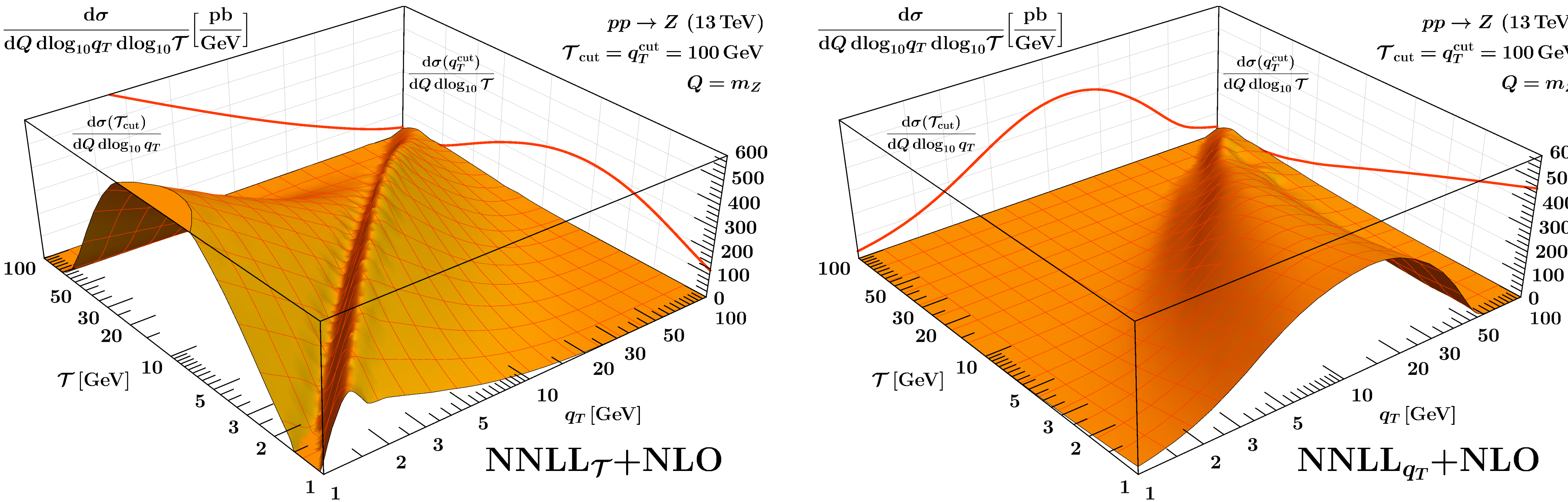
$$
\begin{aligned}
\mu_H &= Q\,, \\
\mu_B &= q_T\,, \quad \nu_B = Q\,, \\
\mu_{S_+} &= q_T\,, \quad \nu_{S_+} = q_T^2/\mathcal{T}\,, \\
\mu_S &= \mathcal{T}
\end{aligned}
$$



- Turn resummation off using profile scales [Ligeti, Stewart, Tackmann]
- Combining different regimes: [Lustermans, Michel, Tackmann, WW]

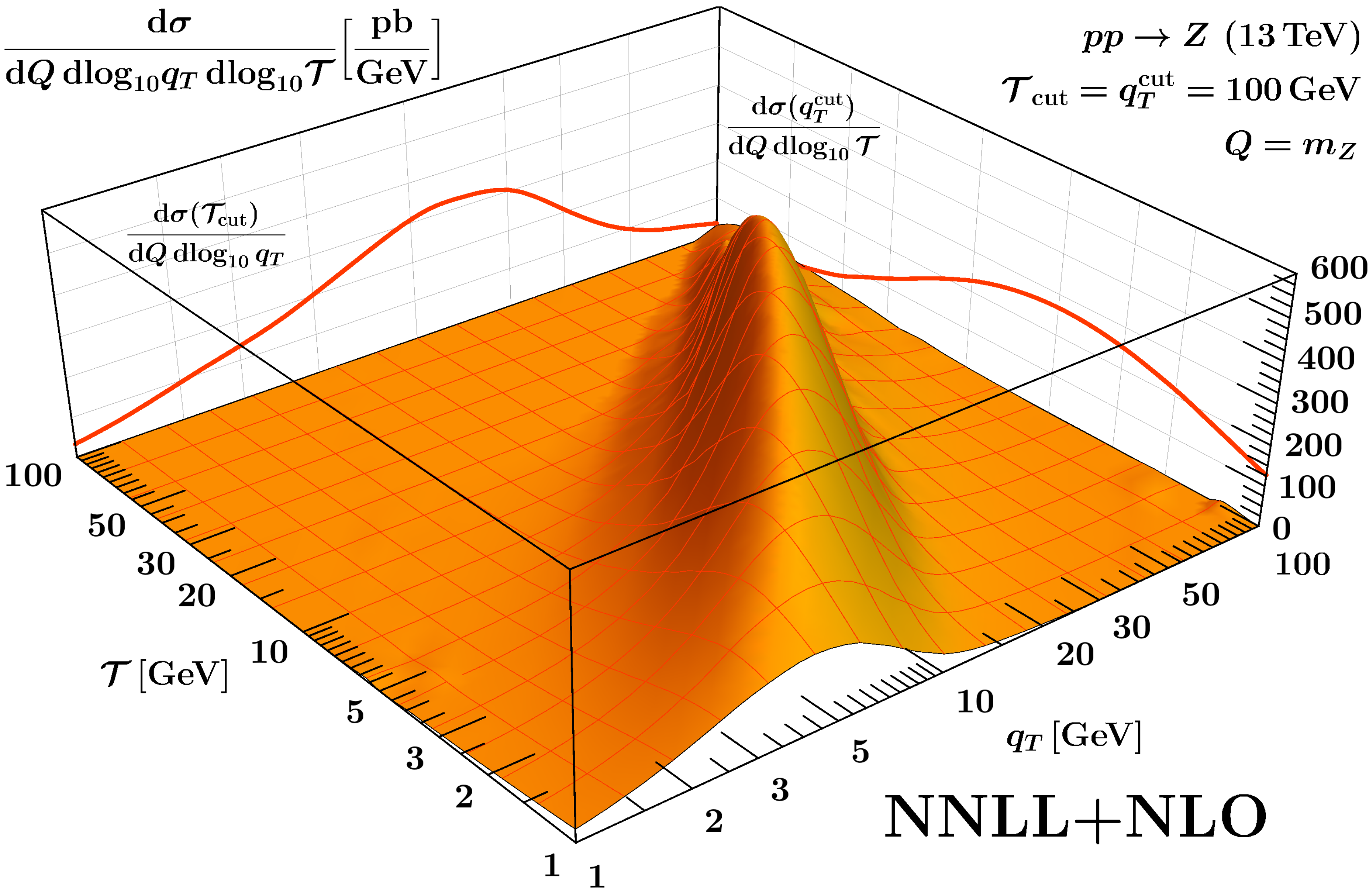
$$
\begin{aligned}
\sigma = \sigma_+(\mu) &+ [\sigma_{\rm I} - \sigma_+](\mu_{\rm I}) + [\sigma_{\rm II} - \sigma_+](\mu_{\rm II}) \\
&+ [\sigma_{\rm FO} - \sigma_{\rm I} - \sigma_{\rm II} + \sigma_+](\mu_{\rm FO})
\end{aligned}
$$

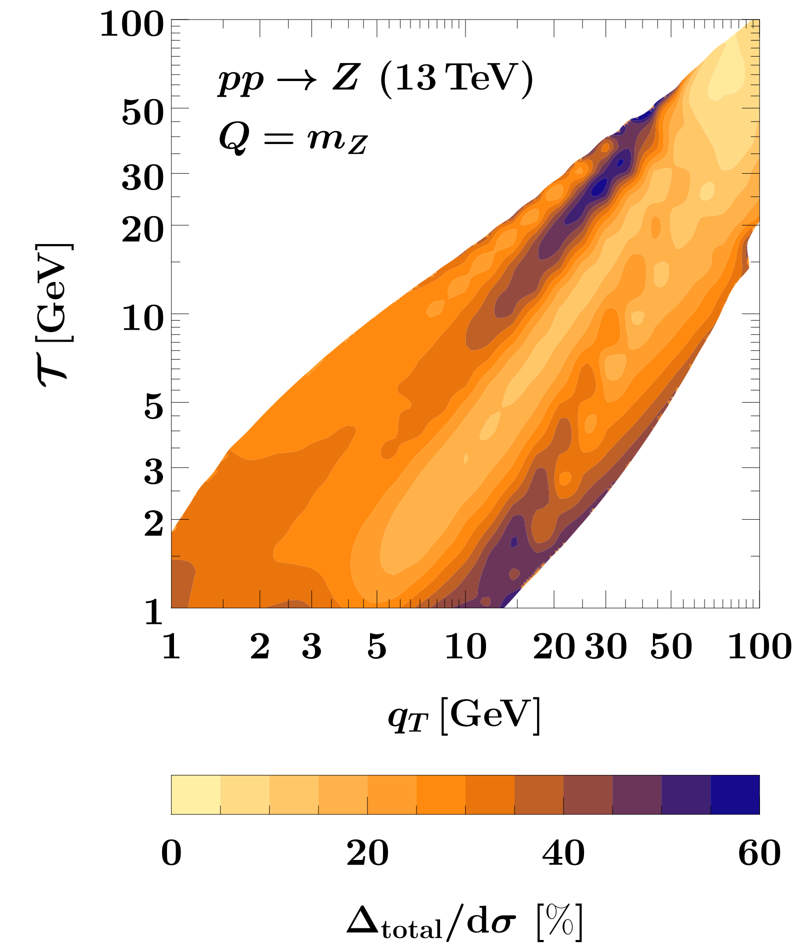
Result with only $\mathcal{T}$ or q_T resummation



- Single differential distributions (on back wall) clearly show that only one variable is resummed while the other is not
- Sharp edge in left plot due to $\mathcal{T} \leq q_T$ for one emission

Result with joint $\mathcal{T}$ and q_T resummation

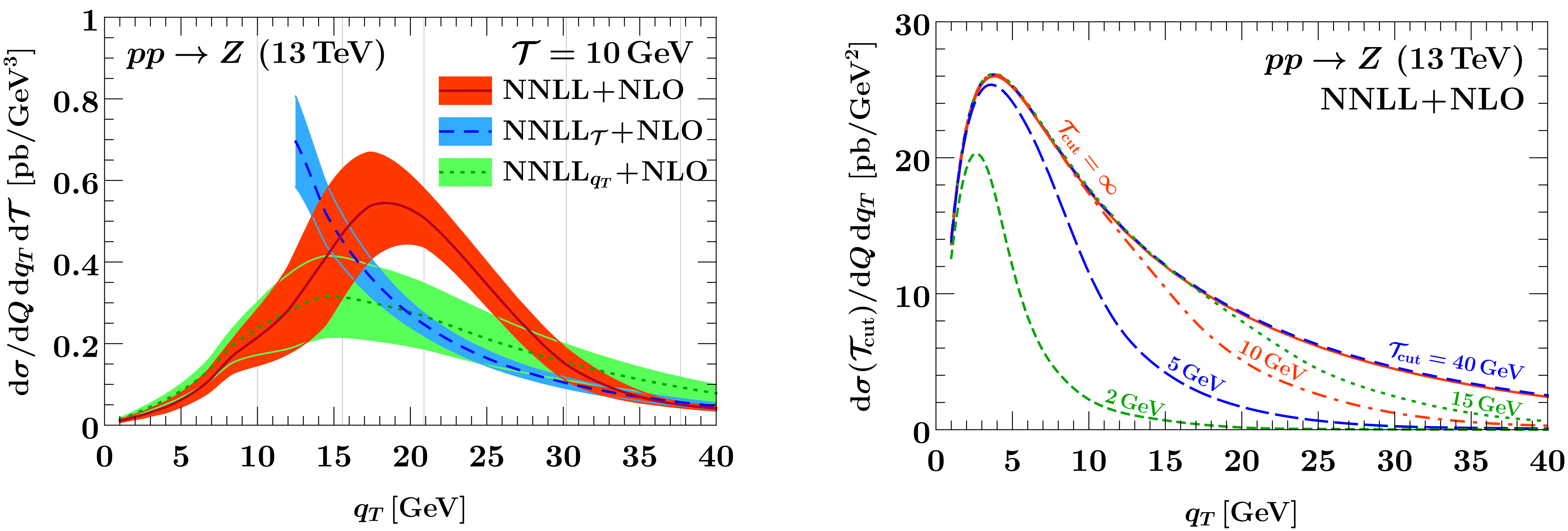




[Lustermans, Michel, Tackmann, WW]

- Resummation yields a two-dimensional Sudakov peak
- Relative uncertainties shown in heat map

Two-dimensional slices



- SCET+ interpolates between SCET$_\text{I}$ and SCET$_\text{II}$
- Cut on beam thrust $\mathcal{T}$ changes shape of q_T spectrum

Conclusions

- Collisions at the LHC involve many scales, whose ratios can give rise to large logarithms → SCET+
- SCET+ for beam thrust and q_T
  - Additional collinear-soft modes
  - Joint resummation → two-dimensional Sudakov peak
  - SCET+ interpolates between SCET_{I} and SCET_{II}
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Thank you!