Global reconstruction with TOE

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TOE: Tracking Of Ejectiles

➡Algorithm for concurrent track recognition and reconstruction

Stepper:

- propagation of state from plane to plane

Kalman Filter:

- combination of propagated state and measurements

Arborescence:

- division for each selected clusters if insertion requirement is met

Pattern recognition and event reconstruction in particle physics experiment, Mankel R., Rep. Prog. Phys. 67, 2004

Kalman Filter

- Kalman filter principle :

Time update: prediction

Propagated state vector:

$$\hat{x}_{k|k-1} = f(\hat{x}_{k-1|k-1})$$

Propagated covariance:

$$P_{k|k-1}$$

Measurement update: correction

m_k	measurement vector
V_k	measurement covariance
H_k	measurement matrix

Compute gain:

$$K_k = P_{k|k-1} H_k^T (H_k P_{k|k-1} H_k^T + V_k)^{-1}$$

Corrected state:

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k (m_k - H_k \hat{x}_{k|k-1})$$

Corrected covariance:

$$P_{k|k} = P_{k|k-1} - K_k H_k P_{k|k-1}$$

UKF

- Usually, linearisation of the model : EKF
- UKF:

Time update: prediction

Form 2N+1 sigma points:

$$\mathcal{X}_0 = \hat{x}_{k-1|k-1}, \quad w_0 \in [0,1)$$

$$\mathcal{X}_i = \hat{x}_{k-1|k-1} \pm \sqrt{\frac{N}{1-w_0}} A_i, \quad w_i = \frac{1-w_0}{2N}$$

Propagate all of the sigma points:

$$\mathcal{Y}_i = f(\mathcal{X}_i)$$

Form propagated state vector and covariance:

$$\hat{x}_{k|k-1} = \sum_{i=0}^{2N} w_i \mathcal{Y}_i \qquad P_{k|k-1} = \sum_{i=0}^{2N} w_i \{\mathcal{Y}_i - \hat{x}_{k|k_1}\}\{\mathcal{Y}_i - \hat{x}_{k|k-1}\}^T$$

A new extension of the Kalman Filter for non-linear systems, Julier S. et Uhlmann K., 1997

UKF

- Usually, linearisation of the model : EKF
- UKF: implementation header

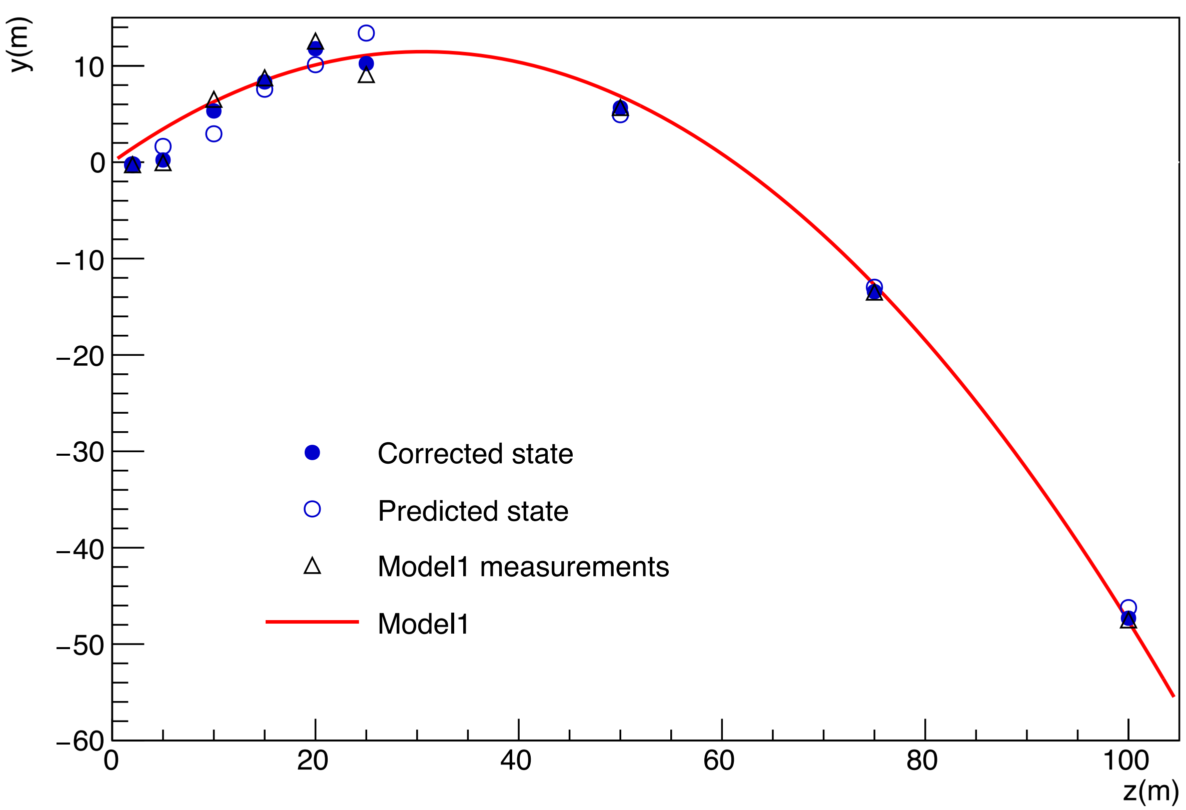
```
std::vector<sigma_state> ukf::generate_sigma_points(state ...);

template<class Predicate>
std::vector<sigma_state>
ukf::propagate_until(std::vector<sigma_state> ... , Predicate ...);

state ukf::generate_propagated_state(std::vector<sigma_state> ...);

state ukf::correct_state(state ... , measurement ...);
```

A new extension of the Kalman Filter for non-linear systems, Julier S. et Uhlmann K., 1997

Proof of concept: ball thrown in a gravitational field

First case:

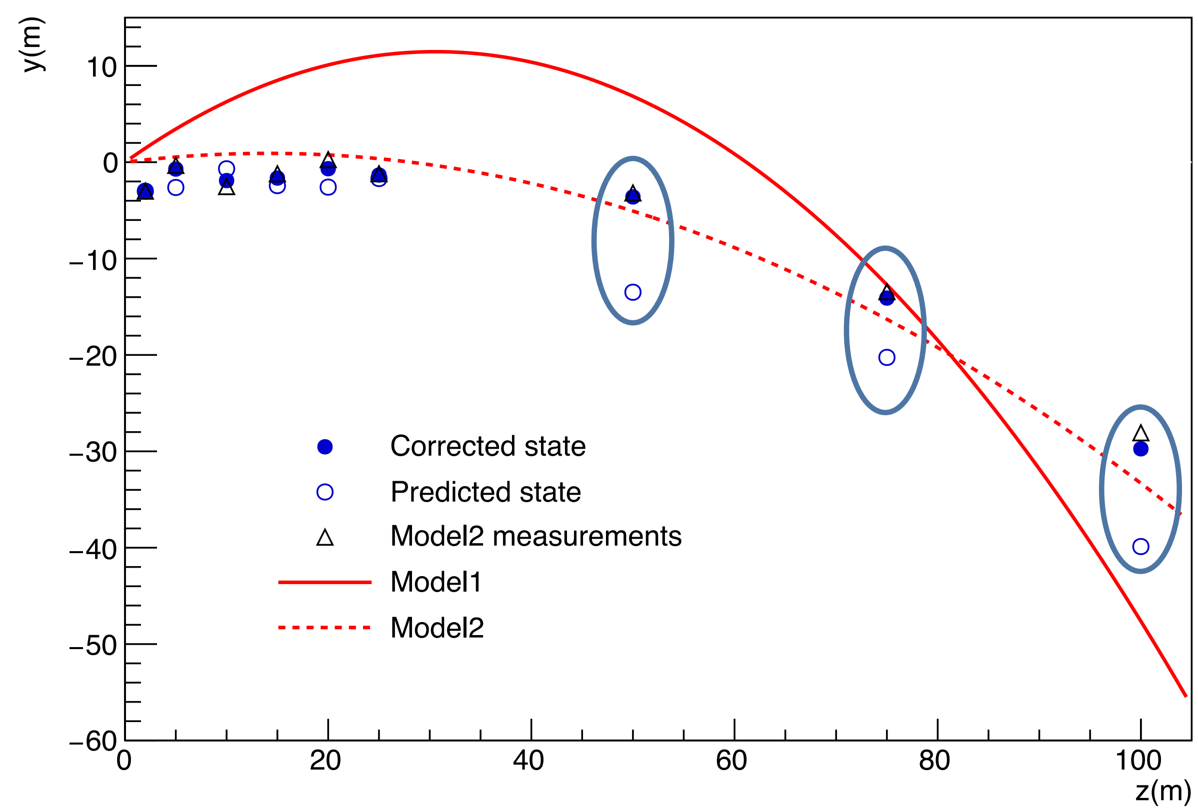
Measurements and model matches



Model1: $g = -9.81 m/s^2$; $v_{y0} = 15 m/s$; $v_{z0} = 20 m/s$

Second case:

Measurements and model do not correspond



Model2: $g = -4.91 m/s^2$; $v_{y0} = 3 m/s$; $v_{z0} = 23 m/s$

Arborescence

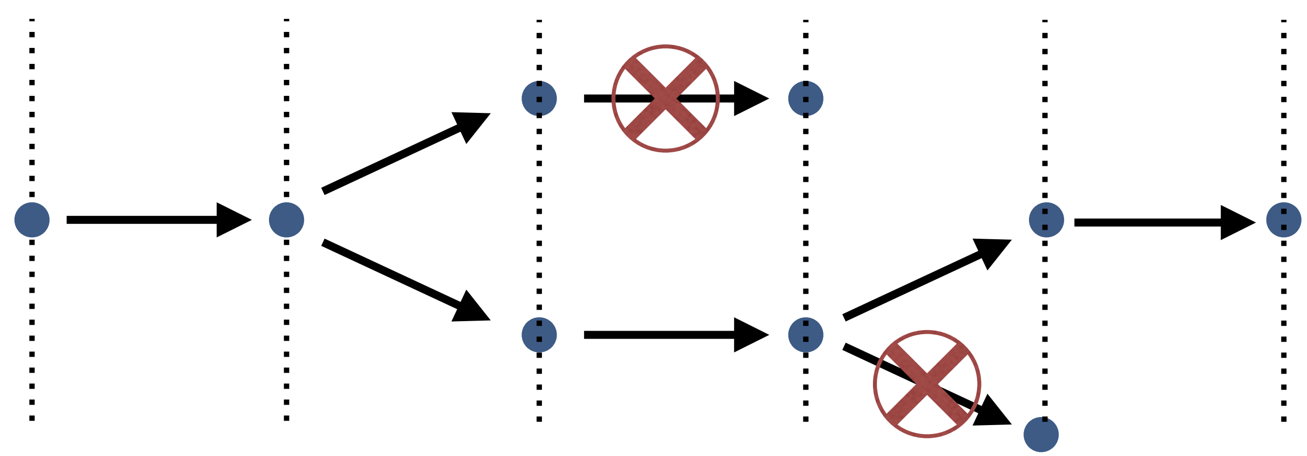
Insertion criteria:

Residuals vector and covariance:

$$r_{k|k-1} = m_k - H_k \hat{x}_{k|k-1} \qquad R_{k|k-1} = V_k + H_k P_{k|k-1} H_k^T$$

Chisquared update:

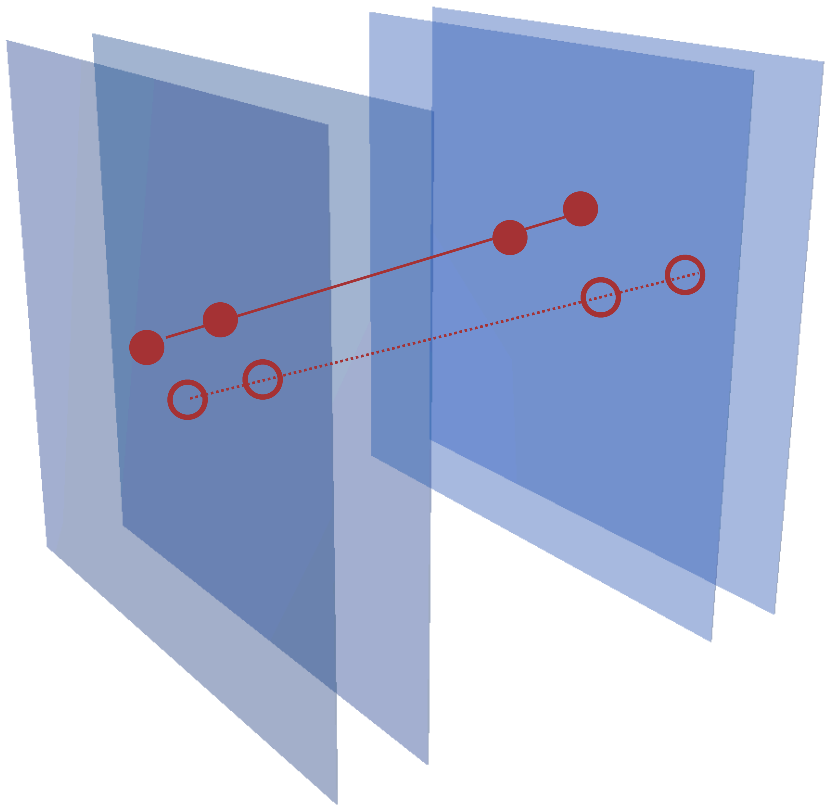
$$\chi^2_+ = r^T_{k|k-1} R_{k|k-1} r_{k|k-1} \qquad \chi^2_k = \chi^2_{k-1} + \chi^2_+$$



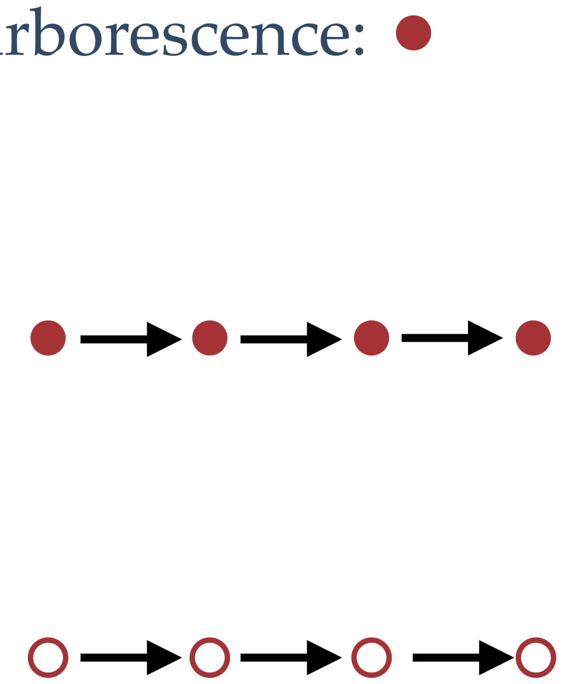
- Overview of the clusters the track could be composed of
- One node corresponds to one cluster
- One node per detection layer per branch

Application to FOOT: reconstruction of a track

Retrieve informations from TOF to form hypothesis for the track → Start propagation from first layer of vtx → Use local reconstruction to minimise possible combinations

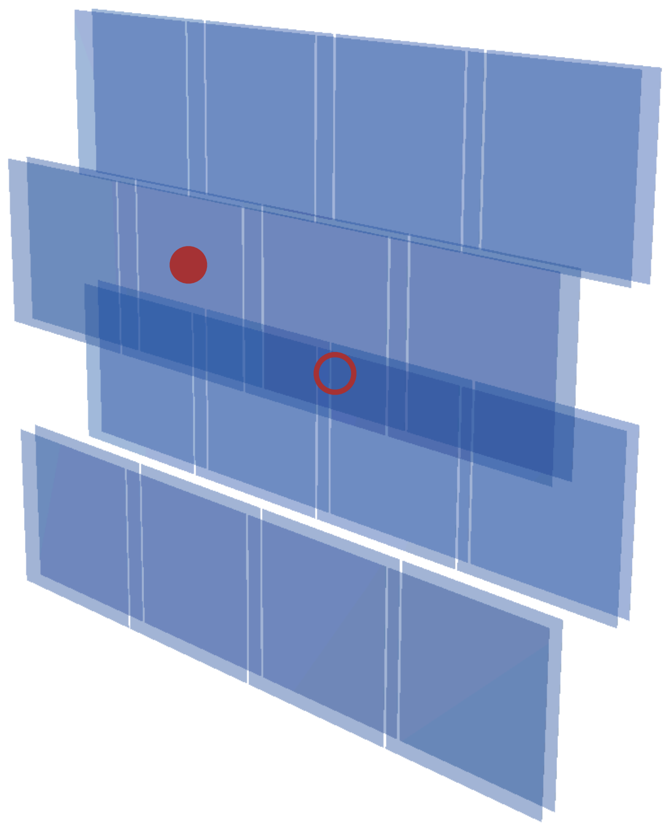


Arborescence:

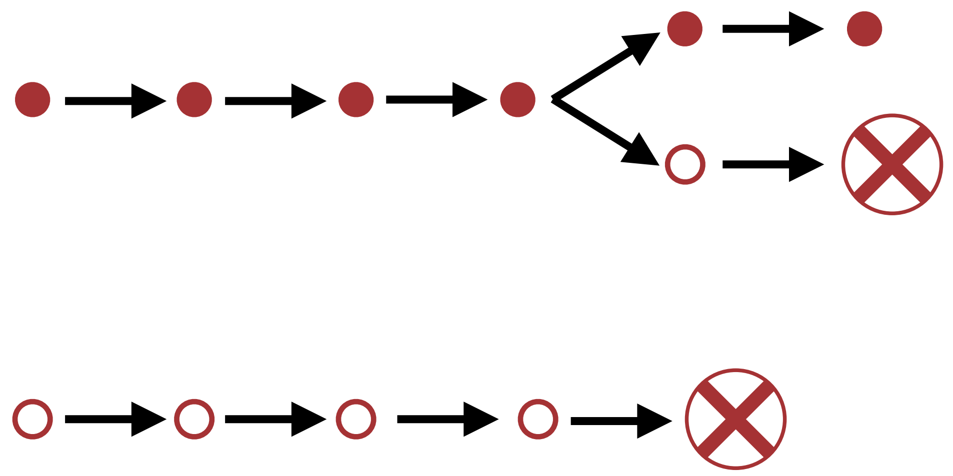


Application to FOOT: reconstruction of a track

→ Use local reconstruction to minimise possible combinations → Propagate to first layer of the inner vertex, add closest clusters → etc …

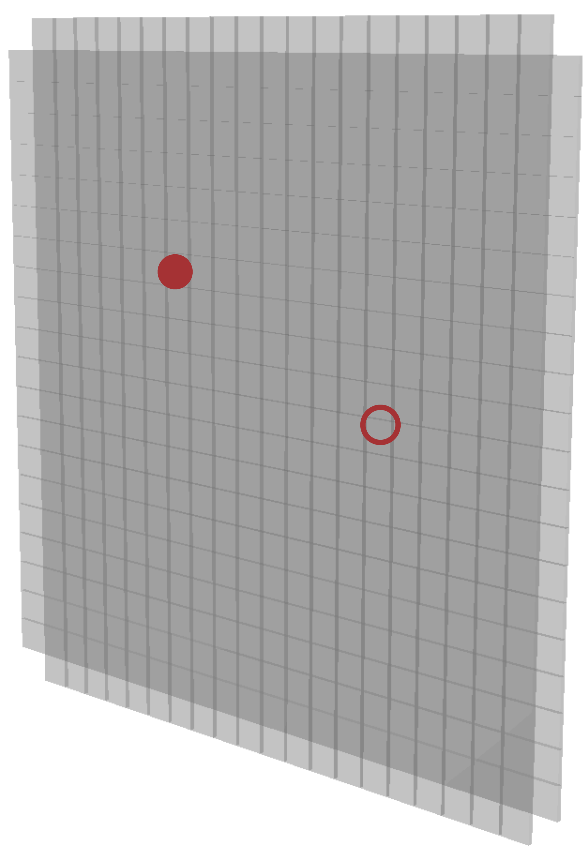


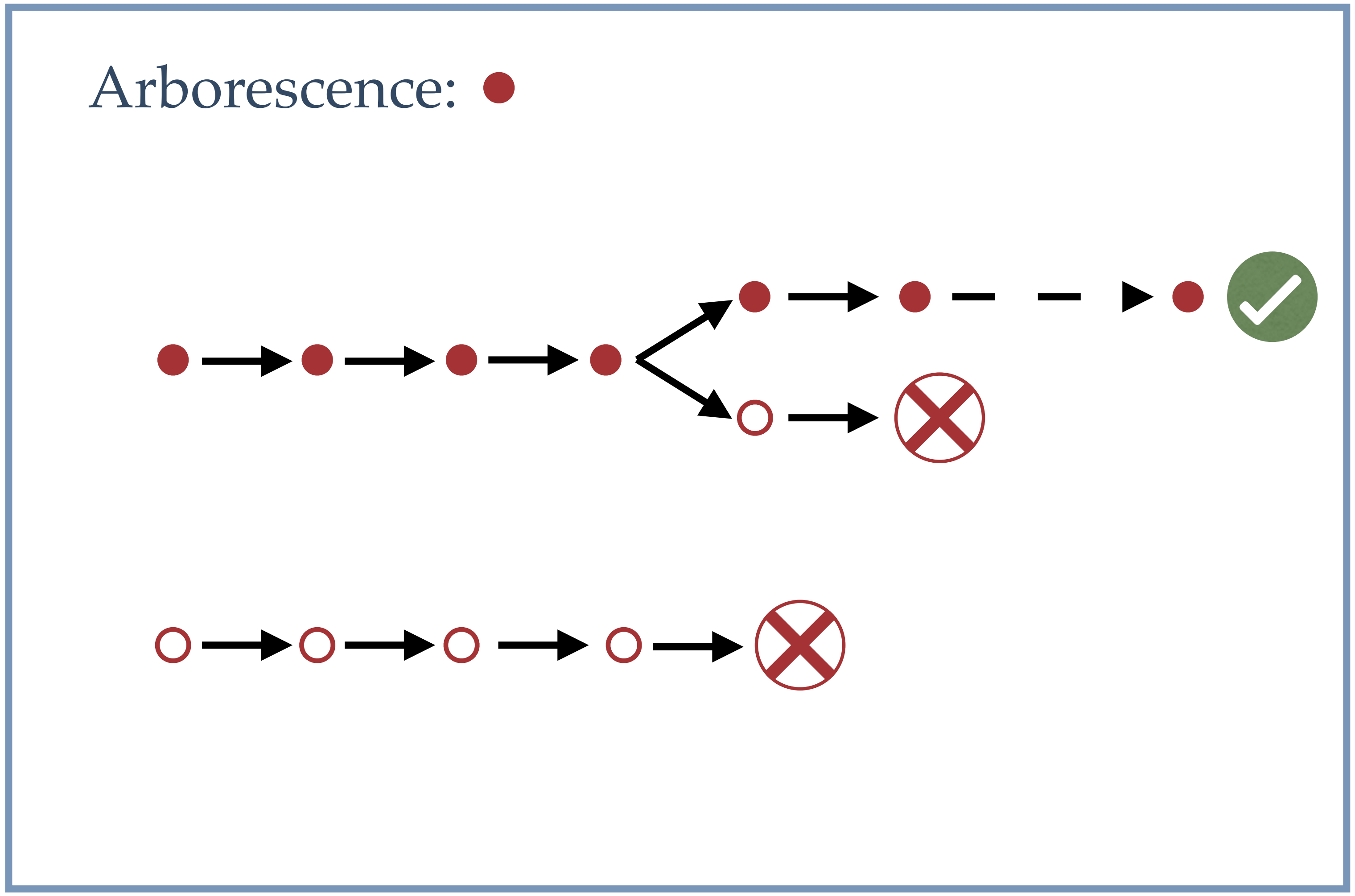
Arborescence: ●



Application to FOOT: reconstruction of a track

Propagate to first layer of the inner vertex, add closest clusters → etc … → Use TOF wall as the final cross-check : confirm that the charge matches





Thank you for your attention