Relations among $B_c \to J/\psi, \eta_c$ form factors

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Introduction

It has been observed that the semileptonic B_c form factors can be expressed in terms of universal functions in selected kinematical regions, on the basis of the heavy quark spin symmetry for large heavy quark masses. The relations to the universal functions have been provided at the leading order in the heavy quark mass expansion. Here we extend the analysis at the next-to-leading order in the expansion, to establish relations among the form factors and a set of universal functions based on the heavy quark spin symmetry and the power counting rules of nonrelativistic QCD (NRQCD). Using as an input the lattice QCD results for the $B_c \to J/\psi$ matrix element of the SM operator, we obtain information on these universal functions and other form factors.

Expansion of the heavy quark field and QCD Lagrangian

We want to perform an expansion of the heavy quark field in NRQCD. To construct it, the heavy quark QCD field $Q(x)$ with mass m_Q is written factorizing a fast oscillation mass term:

$$Q(x) = e^{-i\,m_Q v\cdot x}\psi(x) = e^{-i\,m_Q v\cdot x}\Big(\psi_+(x) + \psi_-(x)\Big)$$

with $\psi_\pm = P_\pm \psi(x)$ and $P_\pm = \frac{1\pm \not v}{2}$. The equation of motion allows us to relate ψ_- to ψ_+ and obtain an expansion on $1/m_Q$ for both $Q(x)$ and the QCD Lagrangian

$$Q(x) = e^{-i\,m_Q v\cdot x}\left(1 + \frac{i\not D_\perp}{2m_Q} + \frac{(-iv\cdot D)}{2m_Q}\frac{i\not D_\perp}{2m_Q} + \dots\right)\psi_+(x).$$

$$\mathcal{L}_{QCD} = \bar\psi_+(x)\left(iv\cdot D + \frac{(iD_\perp)^2}{2m_Q} + \frac{g}{4m_Q}\sigma\cdot G_\perp + \frac{i\not D_\perp}{2m_Q}\frac{(-iv\cdot D)}{2m_Q}(i\not D_\perp) + \dots\right)\psi_+(x)$$

In this expression $G_{\perp\mu\nu} = (g_{\mu\alpha} - v_\mu v_\alpha)(g_{\nu\beta} - v_\nu v_\beta)G^{\alpha\beta}$ and $D_{\perp\mu} = (g_{\mu\alpha} - v_\mu v_\alpha)D^\alpha$.

Power counting

The power counting is delicate, since there are two small parameters. On one side, the inverse of the heavy quark mass (HQET), and on another side the 3-velocity in the hadron rest frame (NRQCD). The power counting within NRQCD takes the following form:

$$D_t \sim \tilde v^2, \qquad D_\perp \sim \tilde v, \qquad \psi_+ \sim \tilde v^{3/2}, \qquad E_i = G_{0i} \sim \tilde v^3 \quad \text{and} \quad B_i = \frac{1}{2}\epsilon_{ijk}G^{jk} \sim \tilde v^4$$

where $\tilde v = |\vec v| \ll 1$ is the relative heavy quark 3-velocity in the hadron rest frame. In this work we consider a mixed approach, where we perform the expansion considering terms up to $\mathcal{O}(\tilde v^3)$ and $\mathcal{O}(1/m_Q^2)$ in our expansion. Terms of $\mathcal{O}(\tilde v^3)$ involving three derivatives ($\mathcal{O}(1/m_Q^3)$) are excluded, we assume that they provide numerically suppressed effects.

$B_c \to J/\psi, \eta_c$ form factors in the effective theory

$B_c(v) \to \eta_c(v')$ $\qquad$ $B_c(v) \to J/\psi(v', \epsilon)$

$$\langle P|\bar Q'\gamma_\mu Q|B_c\rangle = \sqrt{m_P\, m_{B_c}}\,\left[h_+(w)\,(v+v')_\mu + h_-(w)\,(v-v')_\mu\right]$$
$$\langle P|\bar Q'\sigma_{\mu\nu} Q|B_c\rangle = -i\,\sqrt{m_P\, m_{B_c}}\,h_T(w)\,(v_\mu\, v'_\nu - v_\nu\, v'_\mu)$$

$$\langle V|\bar Q'\gamma_\mu Q|B_c\rangle = i\,\sqrt{m_V\, m_{B_c}}\,h_V(w)\,\epsilon_{\mu\nu\alpha\beta}\,\epsilon^{*\nu}\,v'^\alpha\,v^\beta$$
$$\langle V|\bar Q'\gamma_\mu\gamma_5\, Q|B_c\rangle = \sqrt{m_V\, m_{B_c}}\,\left[h_{A_1}(w)\,(1+w)\,\epsilon^*_\mu - h_{A_2}(w)\,(\epsilon^*\cdot v)\,v_\mu - h_{A_3}(w)\,(\epsilon^*\cdot v)\,v'_\mu\right]$$
$$\langle V|\bar Q'\sigma_{\mu\nu} Q|B_c\rangle = -\sqrt{m_V\, m_{B_c}}\,\epsilon^{\mu\nu\alpha\beta}\left[h_{T_1}(w)\epsilon^*_\alpha(v+v')_\beta + h_{T_2}(w)\epsilon^*_\alpha(v-v')_\beta + h_{T_3}(w)(\epsilon^*\cdot v)v_\alpha v'_\beta\right]$$

where v and v' corresponds to the velocities of the B_c and $J/\psi(\eta_c)$ mesons respectively, ϵ to the polarization vector of the J/ψ meson and $w = v\cdot v'$.
To obtain the meson form factors in the effective theory, we must expand the weak current involving two heavy quarks $\bar Q'\Gamma Q$ (local corrections), and the states $\langle M'(v')|$ and $|M(x)\rangle$ (non local corrections).

Trace formalism

The B_c and $J/\psi, \eta_c$ matrix elements of the various terms can be expressed using the trace formalism by exploiting the spin-symmetry present in the NRQCD expansion. In this formalism, the lowest-lying S-wave $\bar b c$ and $\bar c c$ bound states are described by 4×4 matrices

$$H^{c\bar b}(v) = \frac{1+\not v}{2}\left[B_c^{*\mu}\gamma_\mu - B_c\gamma_5\right]\frac{1-\not v}{2} \qquad\qquad H^{c\bar c}(v') = \frac{1+\not v'}{2}\left[\Psi^{*\mu}\gamma_\mu - \eta_c\gamma_5\right]\frac{1-\not v'}{2}$$

satisfying the relations $\not v H(v) = H(v) = -H(v)\not v$ and $H(v')\not v' = H(v') = -\not v' H(v')$. $B_c^{*\mu}$, B_c and $\Psi^{*\mu}$, η_c annihilate vector and pseudoscalar $\bar b c$ and $\bar c c$ mesons of velocity v and v', respectively.

Expanding the weak current (Local corrections)

Keeping terms up to $\mathcal{O}(\tilde v^3)$ and $\mathcal{O}(1/m_Q^2)$, the current expansion can be written as

$$\bar Q'(x)\Gamma Q(x) = J_0 + \Big(\frac{J_{1,0}}{2m_Q} + \frac{J_{0,1}}{2m_{Q'}}\Big) + \Big(-\frac{J_{2,0}}{4m_Q^2} - \frac{J_{0,2}}{4m_{Q'}^2} + \frac{J_{1,1}}{4m_Q m_{Q'}}\Big)$$

$$J_0 = \bar\psi'_+\Gamma\psi_+ \qquad J_{1,0} = \bar\psi'_+\Gamma i\overrightarrow{\not D}_\perp\psi_+ \qquad J_{0,1} = \bar\psi'_+\left(-i\overleftarrow{\not D}'_\perp\right)\Gamma\psi_+$$
$$J_{1,1} = \bar\psi'_+\left(-i\overleftarrow{\not D}'_\perp\right)\Gamma\left(i\overrightarrow{\not D}_\perp\right)\psi_+ \qquad J_{2,0} = \bar\psi'_+\Gamma\left(iv\cdot\overrightarrow{D}\right)i\overrightarrow{\not D}_\perp\psi_+ \qquad J_{0,2} = \bar\psi'_+ i\overleftarrow{\not D}'_\perp\left(iv'\cdot\overleftarrow{D}\right)\Gamma\psi_+$$

Using the trace formalism, we parametrize the matrix elements of the various terms in the expansion. For instance, the matrix element of J_0 and $J_{1,0}$ are parametrized as

$$\langle M'(v')|J_0|M(v)\rangle = -\Delta(w)\mathrm{Tr}\left[\bar H'(v')\Gamma H(v)\right]$$
$$\langle M'(v')|\bar\psi'_+\Gamma^\alpha iD_\alpha\psi_+ M(v)\rangle = -\mathrm{Tr}\left[\Delta_\alpha(v,v')\bar H'(v')\Gamma^\alpha H(v)\right]$$

and involve the form factor $\Delta(w)$ and the function $\Delta_\alpha(v,v')$ respectively

$$\Delta_\alpha(v,v') = \Delta_+(w)(v+v')_\alpha + \Delta_-(w)(v-v')_\alpha - \Delta_3(w)\gamma_\alpha$$

Expanding the states (Non-local corrections)

In addition to the corrections obtained by the expansion of the weak currents, we must consider the corrections to the states. They can be written as

$$\langle M'(v')|i\int d^4x\, T[J_0(0)\mathcal{L}_1(x)]|M(v)\rangle = -\frac{1}{2m_Q^2}\chi_1(w)\mathrm{Tr}\left[\bar H'(v')\Gamma H(v)\right] - \frac{1}{4m_Q}\mathrm{Tr}\left[\chi_{2\mu\nu}(w)\bar H'(v')\Gamma P_+\left(-\frac{i}{2}\right)\sigma^{\mu\nu}H(v)\right]$$

for the quark field and analogously for the the antiquark field.

Form factors in terms of universal functions

Using the expansions above and relating some of the universal functions through the EOMs, one can obtain expressions for the form factors h_i in terms of universal functions. We show here an example for the $h_{A_2}(w)$ form factor.

$$\begin{aligned}
h_{A_2} = {} & \frac{1}{m_c}\Big(\frac{1}{1+w}\left[\phi_K(w) - \Delta(w)\tilde\Lambda - \Delta_3(w)\right] + \frac{1}{2}\bar\chi_2^B(w)\Big) + \frac{1}{m_c^2}\frac{1}{2(w+1)}\left(\tilde\Lambda w - \tilde\Lambda'\right)\left[\phi_K(w) - \Delta(w)\tilde\Lambda - \Delta_3(w)\right] \\
& + \frac{1}{4m_c^2}\Big[-(1+w)\psi_2^S(w) - (w-1)\psi_3^S(w) + \psi_4^S(w) + \psi_5^S(w) - 2w\psi_6^S(w) - \psi_1^A(w) + \psi_2^A(w) + \psi_4^A(w)\Big] \\
& + \frac{1}{4m_b m_c}\Big[-\psi_1^S(w) - (1+w)\psi_2^S(w) + (w+1)\psi_3^S(w) \\
& \qquad + 3\psi_4^S(w) + 3\psi_5^S(w) - (1+w)\psi_1^A(w) + 3\psi_2^A(w) + \psi_3^A(w) + 3\psi_4^A(w)\Big]
\end{aligned}$$

Relations between form factors at $\mathcal{O}(1/m_Q)$

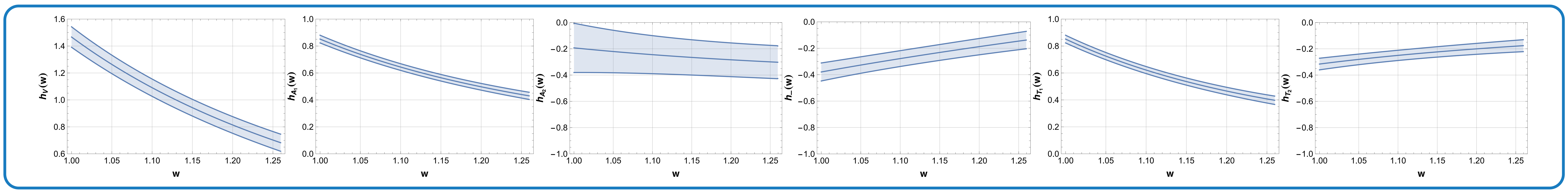
We can take advantage of this relations to relate unkown form factors, with the ones that have already been computed on the lattice. For instance, using the above expression at $\mathcal{O}(1/m_Q)$ one can derive relations between the tensor form factors for $B_c \to J/\psi$ (and similarly for $B_c \to \eta_c$) and the known vector and axial ones. These results are shown in the figures below.x

$$h_{T_1}(w) = \frac{1}{2}\Big((1+w)h_{A_1}(w) - (w-1)h_V(w)\Big)$$
$$h_{T_2}(w) = \frac{1+w}{2(m_b+3m_c)}\Big((m_b-3m_c)h_{A_1}(w) + 2m_c(h_{A_2}(w) + h_{A_3}(w)) - (m_b-m_c)h_V(w)\Big)$$
$$h_{T_3}(w) = h_{A_3}(w) - h_V(w)$$

Conclusion

Using the heavy quark expansion, the heavy quark spin symmetry and NRQCD power counting we have expressed the form factors parametrizing the matrix elements $\langle J/\psi(\eta_c)|\bar c\Gamma_i b|B_c\rangle$ in terms of universal functions near the zero-recoil point and we established relations among form factors in this kinematical range. Lattice QCD results for the matrix element of the Standard Model operator between B_c and J/ψ allow us to predict the pseudoscalar and tensor form factors and the h_- form factor for $B_c \to \eta_c$. The relations worked out here can be checked with further information from lattice QCD.

$B_c \to J/\psi, \eta_c$ form factors from Lattice QCD + NRQCD



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