Holographic Study of a $Q\bar{Q}$ Chaotic Dynamics in General Thermal Background

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Abstract

An holographic approach is applied to study chaotic behaviour of a strongly coupled $Q\bar{Q}$ pair in general thermal background. We consider two different backgrounds, one with finite temperature and baryon density, and one with finite temperature and constant magnetic field along a fixed direction. The results allow us to understand the chaotic dynamics dependence on the parameters of the background, to test the bound on chaos conjectured by Maldacena, Shenker and Standford (MSS) and a possible generalization.

Introduction

- MSS bound on chaos [1]:

$$\lambda \leqslant 2\pi T,$$

$\Rightarrow$ bound on the Lyapunov exponent characterizing the chaotic behavior of thermodynamic quantum systems.

- Systems called fast “scramblers” satisfy the conditions for the MSS bound to hold,
- black holes (BH) are the fastest scramblers in nature [2,3]

$\Rightarrow$ holographic methods in a dual geometry with BH can be used to test the bound.

- Generalization proposed for systems presenting a global symmetry [4]:

$$\lambda \leqslant \frac{2\pi T}{1-\left|\frac{\mu}{\mu_c}\right|} \qquad \mu \ll \mu_c,$$

$\mu =$ chemical potential $\mu_c =$ critical value.

Specific cases of $Q\bar{Q}$ pair in:

- finite temperature and baryon density background [5];
- constant and uniform magnetic field at finite temperature [6].

$\Rightarrow$ one allows us to test the generalized bound, and the other is relevant in different phenomenological contexts such as heavy-ion collisions.

Geometry

The dual metric for the mentioned systems is obtained solving Einstein eqs with suitable boundary conditions.

- Reissner–Nordstrom metric for the finite baryon density case:

$$ds^2 = -f(r)\, r^2 dt^2 + r^2 d\mathbf{x}^2 + \frac{1}{r^2 f(r)} dr^2,$$

$$f(r) = 1 - \frac{r_h^4}{r^4} - \frac{\mu^2 r_h^2}{r^4} + \frac{r_h^4 \mu^2}{r^6},$$

- Approximate solution of the Einstein equations [7] with magnetic field:

$$\begin{aligned} ds^2 = & -f(r)\, r^2 dt^2 + r^2 h(r)\,(dx^1)^2 \\ & + r^2 h(r)\,(dx^2)^2 + r^2 q(r)\,(dx^3)^2 \\ & + \frac{1}{r^2 f(r)} dr^2, \end{aligned}$$

$$f(r) = 1 - \frac{r_h^4}{r^4} - \frac{2B^2}{3r^4}\log\frac{r}{r_h}$$

$$q(r) = 1 - \frac{2B^2}{3r^4}\log r$$

$$h(r) = 1 + \frac{B^2}{3r^4}\log r\,.$$

$r_h =$ position of the black hole horizon, B is related to the magnetic field.

Method

- general metric

$$\begin{aligned} ds^2 = & \, g_{tt}dt^2 + g_{11}(dx^1)^2 + g_{22}(dx^2)^2 \\ & + g_{33}(dx^3)^2 + g_{rr}dr^2\,, \end{aligned}$$

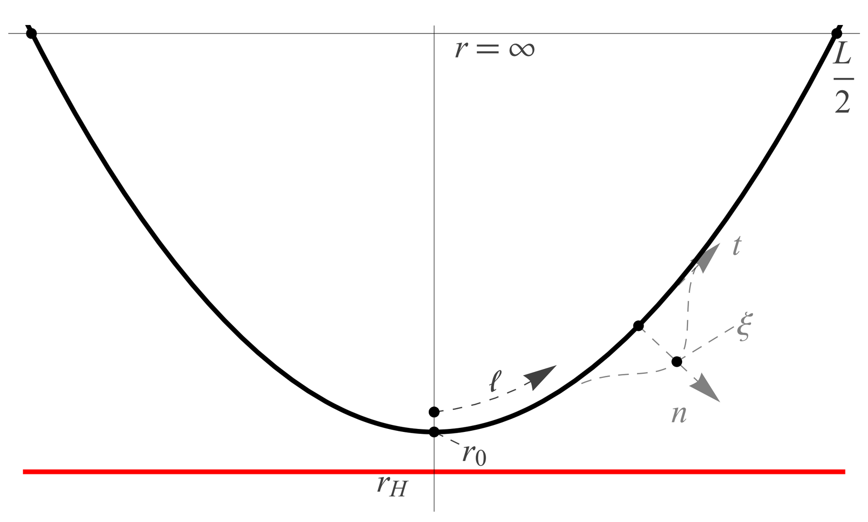
- we evaluate the static string profile from the Nambu-Goto (NG) action:

$$\mathcal{S} = -\frac{1}{2\pi\alpha'}\int dt\, d\ell\, \sqrt{-h}\,.$$

- We perturb it with a time dependent fluctuation orthogonal in each point of the string [8]

$$\begin{aligned} r(t,\ell) &= r_{BG}(\ell) + \xi(t,\ell)\, n^r(\ell)\,, \\ x(t,\ell) &= x_{BG}(\ell) + \xi(t,\ell)\, n^x(\ell)\,, \end{aligned}$$

leaving unperturbed the endpoints.



- We expand the action until the second order in the perturbation ξ:

$$\begin{aligned} S^{(2)} = & \frac{1}{2\pi\alpha'}\int \mathrm{d}t \int_{-\infty}^{\infty} \mathrm{d}\ell \big(C_{tt}\dot{\xi}^2 + C_{\ell\ell}\xi'^2 \\ & + C_{00}\xi^2\big). \end{aligned}$$

C_{tt}, $C_{\ell\ell}$ and C_{00} depend on ℓ and on the parameters of the metric.

- we obtain an equation of motion for the perturbation and factorize it: $\xi(t,l) = \xi(l)e^{i\omega t} \Rightarrow$ e.o.m of the kind of a Sturm-Liouville equation $\partial_\ell\left(C_{\ell\ell}\,\xi'\right) - C_{00}\,\xi = \omega^2 C_{tt}\,\xi$.
- We solve it for the first two eigenvalues. The perturbation is written as a linear combination of the first two eigenfunctions with time dependent coefficients that carry out the dynamics of the system

$$\xi(t,\ell) = c_0(t)\, e_0(\ell) + c_1(t)\, e_1(\ell)\,.$$

- We evaluate the third order action using the redefined perturbation:

$$\begin{aligned} S^{(3)} = & \frac{1}{2\pi\alpha'}\int \mathrm{d}t \Big[\sum_{n=0,1}\left(\dot{c}_n^2 - \omega_n^2 c_n^2\right) \\ & + K_1 c_0^3 + K_2 c_0 c_1^2 + K_3 c_0 \dot{c}_0^2 \\ & + K_4 c_0 \dot{c}_1^2 + K_5 \dot{c}_0 c_1 \dot{c}_1\Big]. \end{aligned}$$

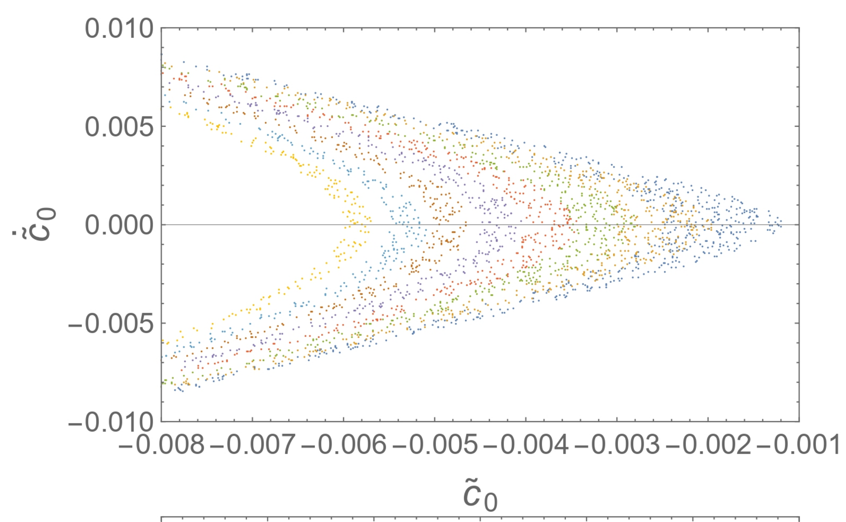
The coefficients $K_{1,\dots,5}$ depend on r_0 and on the parameters of the metric.

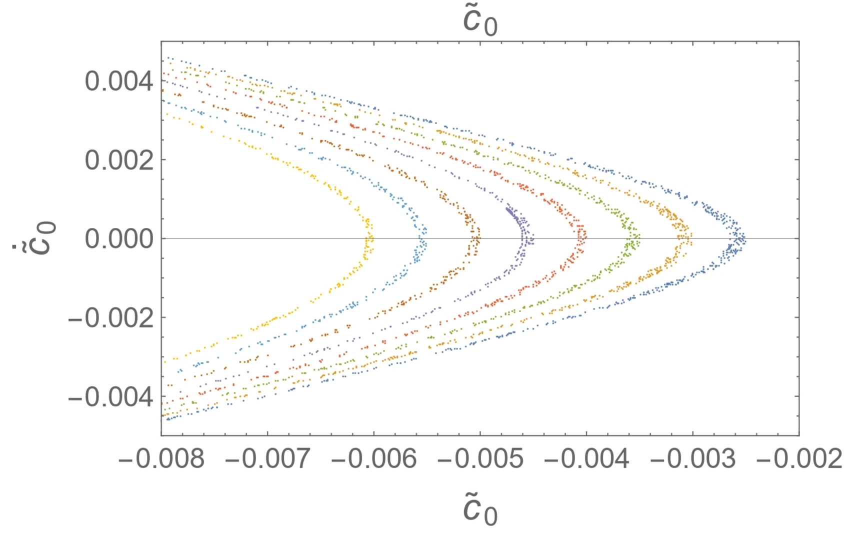
The chaotic dynamics of the system can be study by analyzing the dynamics of $c_0(t)$ and $c_1(t)$ from this action.

Results

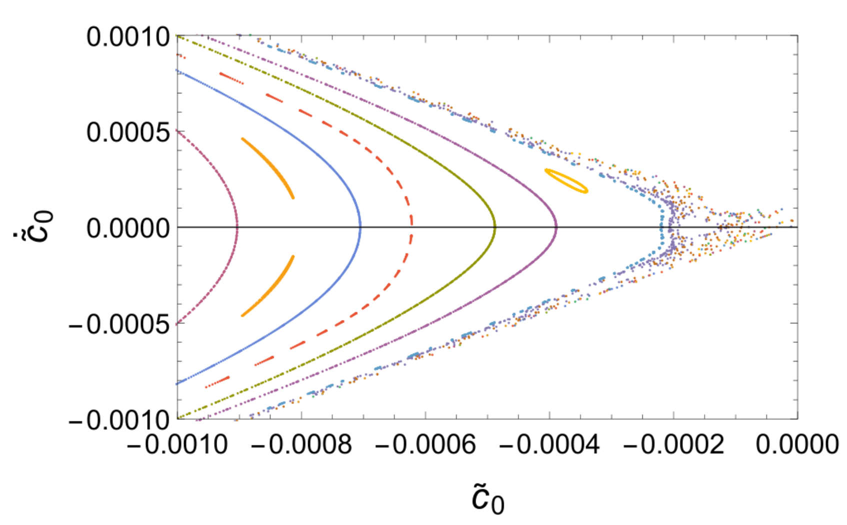
The chaotic dynamics is analyzed through the Poincaré plots and the Lyapunov exponents, evaluated at $r_0 = 1.1$ and $r_h = 1$ for different μ and B.

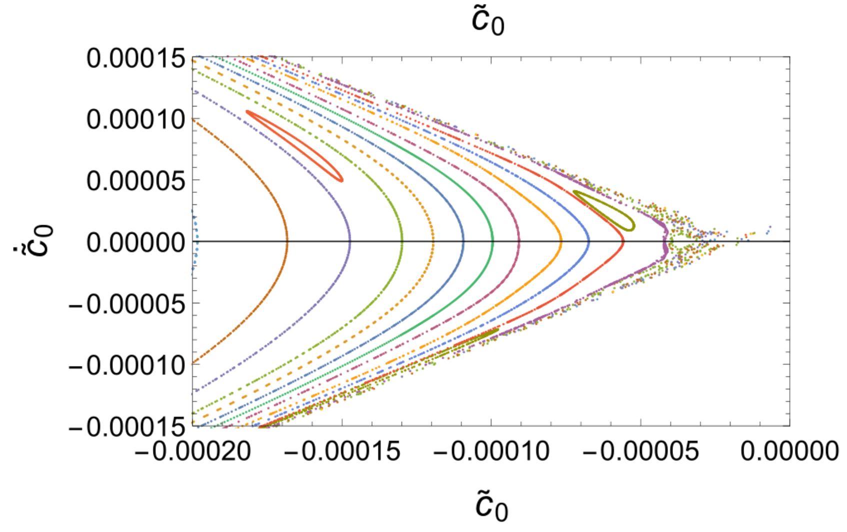
- For $\mu = 0.3$ and $\mu = 0.9$:



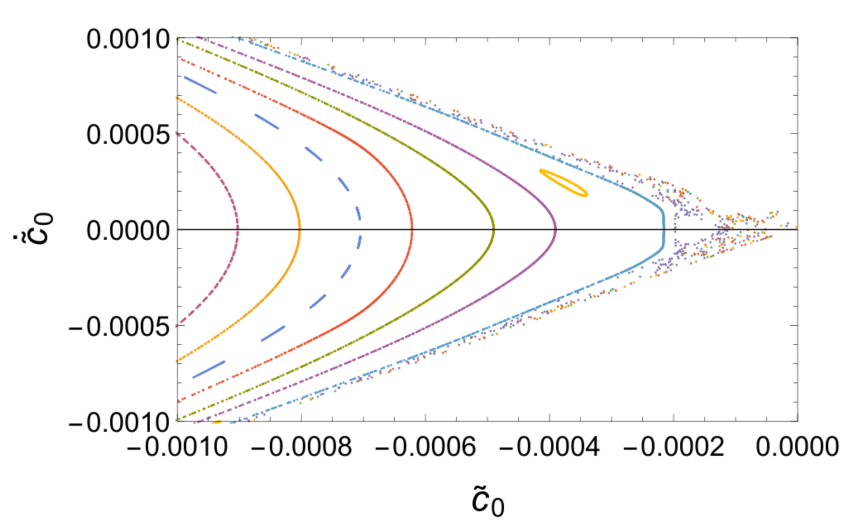


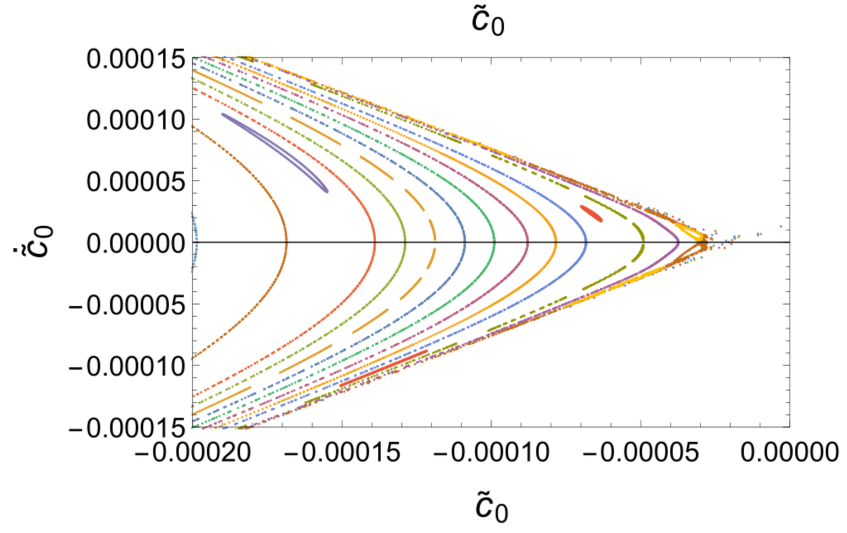
- Magnetic field along the $Q\bar{Q}$ direction, with $B = 0.3$ and $B = 1$:



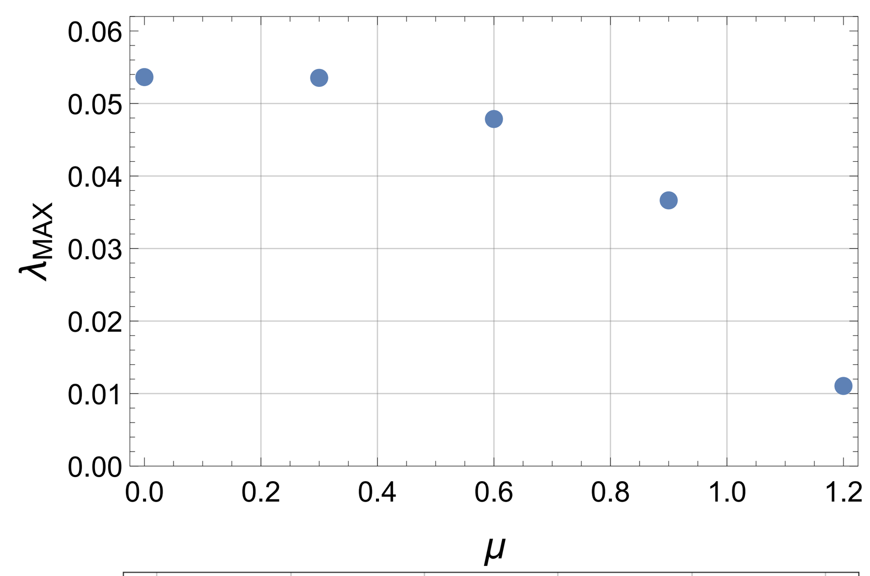


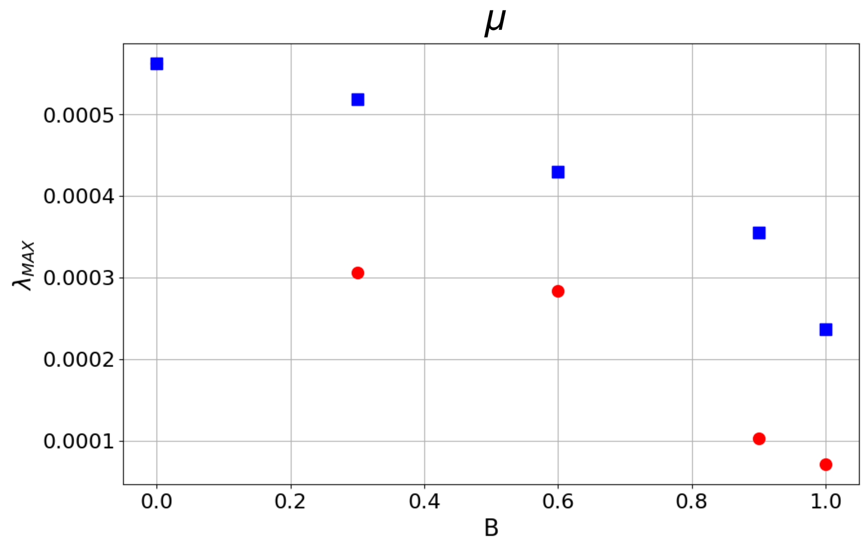
- the orthogonal direction:





Lyapunov exponents:





Bottom plot the blue squares are for the magnetic field along the string, while the red dots are for the orthogonal case.

Conclusion

- Chaos has been observed in the Poincaré plots, characterized by scattered points in the region close to the black hole horizon.
- The system becomes less chaotic increasing μ and B.
- For the magnetic field case, anisotropy effect in two different orientations of the string is found.
- The MSS bound is satisfied for the largest Lyapunov exponent and therefore, also its generalization.
- The MSS bound remains universal.

References

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