Inclusive $b \to \{u, c\} \ell^- \bar{\nu}_\ell$ modes: Polarized and unpolarized Λ_b

Francesco Loparco

francesco.loparco1@ba.infn.it

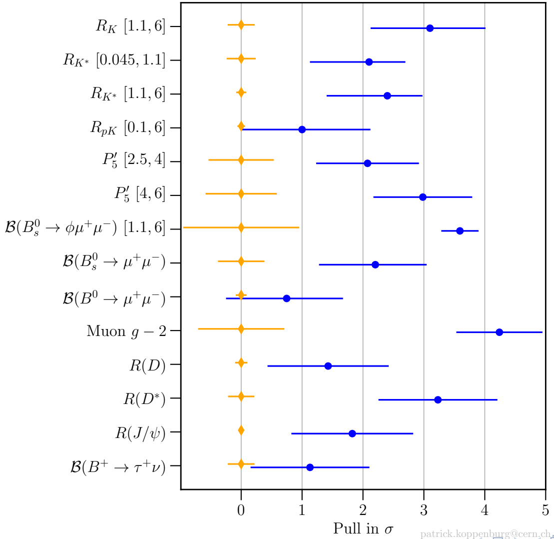
INFN Istituto Nazionale di Fisica Nucleare Sezione di Bari

Based on:

P. Colangelo, F. De Fazio, FL,
Inclusive semileptonic Λ_b decays in the Standard Model and beyond,
JHEP **11** (2020) 032, [arXiv:2006.13759]

Flavour Anomalies

Many deviations from SM predictions have been found in flavour observables.



If the tensions are due to Physics Beyond the Standard Model (BSM), instead of being related to hadronic effects, they should be observed in a coherent way in other processes induced by the same transition ($b \to c$) or by others ($b \to u$).

Heavy Quark Effective Theory (HQET)

HQET describes properties and decays of hadrons comprising a heavy quark Q ($m_Q \gg \Lambda_{\mathbf{QCD}}$) interacting with light dof exchanging soft gluons.

$$\text{HQET displays two symmetries} \Rightarrow \begin{cases} \text{Heavy Quark Spin Symmetry} \Rightarrow D \sim D^* \;, B \sim B^* \\ \text{Heavy Quark Flavour Symmetry} \Rightarrow D \sim B \end{cases}$$

The QCD field Q is divided in two components: h_v (large) and H_v (small) after removing the heavy mass components of its phase:

$$Q = e^{-i\, m_Q\, v\cdot x}\left[h_v + H_v\right] \qquad h_v = e^{i\, m_Q\, v\cdot x}\, P_+\, Q \qquad H_v = e^{i\, m_Q\, v\cdot x}\, P_-\, Q$$

$$\mathcal{L}_{\mathbf{QCD}} = \bar{Q}\,(i\,\not{D} - m_Q)\,Q = \bar{h}_v\,(i\,v\cdot D)\,h_v + \bar{h}_v\,i\,\not{D}_\perp\,H_v + \bar{H}_v\,i\,\not{D}_\perp\,h_v - \bar{H}_v\,(i\,v\cdot D + 2\,m_Q)\,H_v$$

$$\text{EoMs: } \exists \text{ relation between } h_v \text{ and } H_v \longrightarrow \begin{cases} (i\,v\cdot D)\,h_v + i\,\not{D}_\perp\,H_v = 0 \\ i\,\not{D}_\perp\,h_v - (i\,v\cdot D + 2\,m_Q)\,H_v = 0 \end{cases}$$

$$\mathcal{L}_{\mathbf{QCD}} = \underbrace{\bar{h}_v\,(i\,v\cdot D)\,h_v}_{\mathcal{L}_{\mathbf{HQET}}} + \bar{h}_v\,i\,\not{D}_\perp\,\frac{1}{i\,v\cdot D + 2\,m_Q - i\,\varepsilon}\,i\,\not{D}_\perp\,h_v$$

Inclusive $b \to \{ u, c \}$ semileptonic modes

Theoretical advantages for the calculation of inclusive semileptonic decays of hadrons comprising a single heavy quark:

$$H_b(p, s) \to X_{\{ u,c \}}(p_X)\,\ell^-(p_\ell)\,\bar{\nu}_\ell(p_\nu)$$

- exploits the large b-quark mass: Heavy Quark Expansion (HQE) $\to$ power series in $1/m_b$
- small theoretical uncertainties $\begin{cases} \text{requires few parameters (sometimes fixed by the experiment)} \\ \text{no form factors (that affect exclusive modes)} \end{cases}$

$$\mathcal{H}_{\mathbf{NP}} = \frac{G_F\, V_{ub}}{\sqrt{2}} \left[(1 + \epsilon_V^\ell)\, \mathcal{O}_{\mathbf{SM}} + \epsilon_S^\ell\, \mathcal{O}_{\mathbf{S}} + \epsilon_P^\ell\, \mathcal{O}_{\mathbf{P}} + \epsilon_T^\ell\, \mathcal{O}_{\mathbf{T}} + \epsilon_R^\ell\, \mathcal{O}_{\mathbf{R}} \right] + \text{h.c.}$$

Generalization of the SM Hamiltonian

$$\begin{aligned} \mathcal{O}_{\mathbf{SM,R}} &= [\bar{u}\,\gamma_\mu\,(1 \mp \gamma_5) b]\,[\bar{\ell}\,\gamma^\mu\,(1 - \gamma_5)\,\nu_\ell] \\ \mathcal{O}_{\mathbf{S}} &= [\bar{u}\, b]\,[\bar{\ell}\,(1 - \gamma_5)\nu_\ell] \\ \mathcal{O}_{\mathbf{P}} &= [\bar{u}\,\gamma_5\, b]\,[\bar{\ell}\,(1 - \gamma_5)\nu_\ell] \\ \mathcal{O}_{\mathbf{T}} &= [\bar{u}\,\sigma_{\mu\nu}\,(1 - \gamma_5)\, b]\,[\bar{\ell}\,\sigma^{\mu\nu}\,(1 - \gamma_5)\,\nu_\ell] \end{aligned}$$

Analysis at $\mathcal{O}(1/m_b^3)$ performed:

1. Dependence on the spin s_μ of the decaying baryon H_b;
2. Effects of all NP operators;
3. Non vanishing lepton masses: $m_\ell \neq 0$

General formalism adaptable to other heavy hadrons

$$d\Gamma = \underbrace{d\Sigma}_{\text{phace space}}\ \frac{G_F^2\,|V_{Ub}|^2}{4 M_H} \sum_{i,j} g_i^*\, g_j\ \underbrace{(W^{ij})_{MN}}_{\text{hadronic tensor}}\ \underbrace{(L^{ij})^{MN}}_{\text{leptonic tensor}}$$

$$d\Sigma = (2\,\pi)^4\, d^4 q\, \delta^4(q - p_\ell - p_\nu)\,[dp_\ell]\,[dp_\nu] \qquad [dp] = \frac{d^3 p}{(2\,\pi)^3\, 2\, p^0}$$

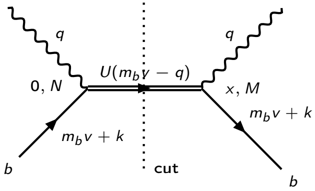
Heavy Quark Expansion (HQE)

$(W^{ij})_{MN}$ obtained from the discontinuity of the forward amplitude

$$(T^{ij})_{MN} = i \int d^4x\, e^{-i\,q\cdot x}\, \langle H_b(p,s)| \mathbb{T}\left\{ J_M^{(i)\,\dagger}(x)\, J_N^{(j)}(0) \right\} |H_b(p,s)\rangle$$

across the cut of the semileptonic process

$$(W^{ij})_{MN} = \frac{1}{\pi}\,\mathrm{Im}\left[(T^{ij})_{MN} \right] \qquad \text{(optical theorem)}$$



Hadron momentum expressed in terms of the heavy quark mass and of a residual momentum:

$$p = m_H\, v \quad \to \quad p = m_b\, v + k\,,\ \frac{|k^\mu|}{m_b} \ll 1$$

Extraction of the heavy component by the QCD field:

$$b(x) = e^{-i\,m_b\, v\cdot x}\, b_v(x) = e^{-i\,(m_b\, v + k)\cdot x}\, b_v(0)$$

$$(T^{ij})_{MN} = \langle H_b(v,s)| \bar{b}_v(0)\, \bar{\Gamma}_M^{(i)}\, \frac{1}{m_b\, \not{v} + \not{k} - \not{q} - m_U}\, \Gamma_N^{(j)}\, b_v(0) |H_b(v,s)\rangle$$

Series expansion wrt $|k^\mu| \sim \Lambda_{\mathbf{QCD}} \ll 1$:

$$(T^{ij})_{MN} = \sum_{n=0}^{+\infty} \langle H_b(v,s)| \bar{b}_v(0)\, \bar{\Gamma}_{q,M}^{(i)}\, (\not{p}_U + m_U)\, (i\,\not{D}\,(\not{p}_U + m_U))^n\, \Gamma_{q,N}^{(j)}\, b_v(0) |H_b(v,s)\rangle\, \frac{(-1)^n}{\Delta_0^{n+1}}$$

Operator Product Expansion (OPE)

Trace formalism

The H_b matrix element of QCD operator dimensions increases step by step, depending on the number of the derivatives taken into account. The trace formalism allows to rewrite:

$$\langle H_b(v,s)|\bar{b}_v(0)\,\bar{\Gamma}^{(i)}_M\,(\not{p}_U+m_U)\,\underbrace{i\not{D}\,(\not{p}_U+m_U)\ldots i\not{D}\,(\not{p}_U+m_U)}_{\textbf{n times}}\,\Gamma^{(j)}_N\,b_v(0)|H_b(v,s)\rangle =$$

$$= \left[\bar{\Gamma}^{(i)}_M\,(\not{p}_U+m_U)\,\prod_{k=1}^{n}\left[\gamma_{\mu_k}\,(\not{p}_U+m_U)\right]\,\Gamma^{(j)}_N\right]_{ab}\,\underbrace{\langle H_b(v,s)|\bar{b}_v(0)\,iD^{\mu_1}\,\ldots\,iD^{\mu_n}\,b_v(0)|H_b(v,s)\rangle_{ba}}_{(\mathcal{M}^{\mu_1\cdots\mu_n})_{ba}}$$

↑ The whole calculation requires their computation ↑

↓ The higher the order of the expansion, the greater the number of the parameters ↓

$$\mathcal{O}(1/m_b^n)\ldots\begin{cases}\mathcal{O}(1/m_b^3)\begin{cases}\mathcal{O}(1/m_b^2)\begin{cases}-2\,M_H\,\hat{\mu}_\pi^2 = \langle H_b|\bar{b}_v\,iD^\mu\,iD_\mu\,b_v|H_b\rangle\\ 2\,M_H\,\hat{\mu}_G^2 = \langle H_b|\bar{b}_v\,(-i\sigma_{\mu\nu})\,iD^\mu\,iD^\nu\,b_v|H_b\rangle\end{cases}\\ 2\,M_H\,\hat{\rho}_D^3 = \langle H_b|\bar{b}_v\,iD^\mu\,(iv\cdot D)\,iD_\mu\,b_v|H_b\rangle\\ 2\,M_H\,\hat{\rho}_{LS}^3 = \langle H_b|\bar{b}_v\,(-i\sigma_{\mu\nu})\,iD^\mu\,(iv\cdot D)\,iD^\nu\,b_v|H_b\rangle\end{cases}\\ \ldots\end{cases}$$

For a heavy baryon, the dependence on the spin four-vector s_μ must be kept.

How to compute $\langle H_b(v,s)|\bar{b}_v(0)\,iD^{\mu_1}\,\ldots\,iD^{\mu_n}\,b_v(0)|H_b(v,s)\rangle$ for fixed n

$k=n$: h_v is taken into account $\longrightarrow$ EoM : $(i\,v\cdot D)\,h_v = 0$ (HQET)

$0\le k<n$: b_v is taken into account $\longrightarrow$ EoM : $(i\,v\cdot D)\,b_v = -\frac{i\not{D}\,i\not{D}}{2\,m_b}\,b_v$ (QCD)

Hadronic matrix elements

$$\mathcal{M}^{\rho\sigma\lambda} = M_H \left[\left(\frac{\hat\rho_D^3}{3} \Pi^{\rho\lambda} v^\sigma P_+ + \frac{\hat\rho_{LS}^3}{6} v^\sigma i\epsilon^{\rho\lambda\alpha\beta} v_\alpha S_\beta \right) - \left(\frac{\hat\rho_D^3}{3} \Pi^{\rho\lambda} v^\sigma s^\mu S_\mu - \frac{\hat\rho_{LS}^3}{2} v^\sigma i\epsilon^{\rho\lambda\alpha\beta} v_\alpha s_\beta P_+ \right) \right]$$

$$\begin{aligned}\mathcal{M}^{\rho\sigma} = M_H \Bigg[\Bigg(& \frac{\hat\mu_\pi^2}{3} \Pi^{\rho\sigma} P_+ + \frac{\hat\mu_G^2}{6} i\epsilon^{\rho\sigma\alpha\beta} v_\alpha S_\beta + \frac{\hat\rho_D^3 + \hat\rho_{LS}^3}{24 m_b} [4 (i\epsilon^{\rho\sigma\alpha\beta} v_\alpha S_\beta - v^\rho v^\sigma \not v) + \\ & + v^\rho (2\gamma^\sigma + \not v \gamma^\sigma - \gamma^\sigma \not v) + v^\sigma (2\gamma^\rho + \not v \gamma^\rho - \gamma^\rho \not v)] \Bigg) + \Bigg(-\frac{\hat\mu_\pi^2}{3} \Pi^{\rho\sigma} P_+ \not s \gamma_5 + \\ & + \frac{\hat\mu_G^2}{2} i\epsilon^{\rho\sigma\alpha\beta} v_\alpha s_\beta P_+ + \frac{\hat\rho_D^3}{12 m_b} [6 i\epsilon^{\rho\sigma\alpha\beta} v_\alpha s_\beta + i (v^\rho \epsilon^{\sigma\mu\alpha\beta} - v^\sigma \epsilon^{\rho\mu\alpha\beta}) v_\alpha s_\beta \gamma_\mu + \\ & + s^\rho v^\sigma \not v \gamma_5 + v^\rho s^\sigma (2\gamma_5 + \not v \gamma_5) + (2 v^\rho v^\sigma \not v - v^\rho \gamma^\sigma - v^\sigma \gamma^\rho) \not s \gamma_5] + \\ & + \frac{\hat\rho_{LS}^3}{8 m_b} [4 i\epsilon^{\rho\sigma\alpha\beta} v_\alpha s_\beta + i (v^\rho \epsilon^{\sigma\mu\alpha\beta} - v^\sigma \epsilon^{\rho\mu\alpha\beta}) v_\alpha s_\beta \gamma_\mu + \\ & + (s^\rho v^\sigma + v^\rho s^\sigma) \gamma_5 + (2 v^\rho v^\sigma \not v - v^\rho \gamma^\sigma - v^\sigma \gamma^\rho) \not s \gamma_5] \Bigg) \Bigg]\end{aligned}$$

$$\begin{aligned}\mathcal{M}^{\rho} = M_H \Bigg[\Bigg(& \frac{\hat\mu_\pi^2 - \hat\mu_G^2}{12 m_b} (v^\rho (3 + 5\not v) - 2\gamma^\rho) - \frac{\hat\rho_D^3 + \hat\rho_{LS}^3}{12 m_b^2} (4 v^\rho \not v - \gamma^\rho) \Bigg) + \\ & + \Bigg(-\frac{\hat\mu_\pi^2}{12 m_b} [(v^\rho (3 + 5\not v) - 2\gamma^\rho) \not s \gamma_5 + 4 s^\rho P_+ \gamma_5] + \frac{\hat\mu_G^2}{4 m_b} [(v^\rho (1 + 2\not v) - \gamma^\rho) \not s \gamma_5 + s^\rho \gamma_5] + \\ & + \frac{\hat\rho_D^3}{12 m_b^2} [(v^\rho (1 + 4\not v) - 2\gamma^\rho) \not s \gamma_5 + s^\rho (2 - \not v) \gamma_5] + \frac{\hat\rho_{LS}^3}{8 m_b^2} [(3 v^\rho \not v - \gamma^\rho) \not s \gamma_5 + s^\rho \gamma_5] \Bigg) \Bigg]\end{aligned}$$

$$\mathcal{M} = M_H \left[\left(P_+ - \frac{\hat\mu_\pi^2 - \hat\mu_G^2}{4 m_b^2} \right) + \left(P_+ + \frac{\hat\mu_\pi^2}{24 m_b^2} (7 + 5\not v) - \frac{\hat\mu_G^2}{8 m_b^2} (3 + \not v) - \frac{\hat\rho_D^3}{6 m_b^3} P_- \right) \not s \gamma_5 \right]$$

Red terms never computed before!

Angular analysis of $\Lambda_b \to X_{c,u}\,\ell^-\,\bar{\nu}_\ell$

Fully differential decay distribution (H_b rest frame)

$$\frac{d^4\Gamma}{dE_\ell\, dq^2\, dq_0\, d\cos\theta_P}$$

E_ℓ: lepton energy: $p_\ell = (E_\ell, \boldsymbol{p}_\ell)$

q^2: dilepton invariant mass: $q = (q_0, \boldsymbol{q})$

θ_P: angle between hadron spin $\boldsymbol{s}$ and lepton 3-momentum $\boldsymbol{p}_\ell$: $\cos\theta_P = \frac{\boldsymbol{s}\cdot\boldsymbol{p}_\ell}{|\boldsymbol{s}||\boldsymbol{p}_\ell|}$

Decay width

$$\Gamma(H_b \to X_U\,\ell^-\,\bar{\nu}_\ell) = \frac{G_F^2\,|V_{Ub}|^2\,m_b^5}{192\,\pi^3}\sum_{i,j} g_i^*\,g_j\left[\mathcal{C}_0^{(i,j)} + \frac{\hat{\mu}_\pi^2}{m_b^2}\,\mathcal{C}_{\hat{\mu}_\pi^2}^{(i,j)} + \frac{\hat{\mu}_G^2}{m_b^2}\,\mathcal{C}_{\hat{\mu}_G^2}^{(i,j)} + \frac{\hat{\rho}_D^3}{m_b^3}\,\mathcal{C}_{\hat{\rho}_D^3}^{(i,j)} + \frac{\hat{\rho}_{LS}^3}{m_b^3}\,\mathcal{C}_{\hat{\rho}_{LS}^3}^{(i,j)}\right]$$

Angular differential decay distributions and ratio

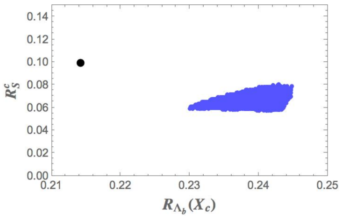
$$\frac{d\Gamma}{d\cos\theta_P} = A_\ell^U + B_\ell^U\,\cos\theta_P \qquad A_\ell^U = \frac{1}{2}\,\Gamma(H_b \to X_U\,\ell^-\,\bar{\nu}_\ell)$$

Analogously to $R(D^{(*)})$, the ratios are defined:

$$R_{\Lambda_b}(X_U) = \frac{\Gamma(\Lambda_b \to X_U\,\tau^-\,\bar{\nu}_\tau)}{\Gamma(\Lambda_b \to X_U\,\mu^-\,\bar{\nu}_\mu)} = \frac{A_\tau^U}{A_\mu^U} \qquad R_S^U = \frac{B_\tau^U}{B_\mu^U}$$

	SM	NP
$R_{\Lambda_b}(X_u)$	0.234	0.238
$R_{\Lambda_b}(X_c)$	0.214	0.240
R_S^u	0.081	0.091
R_S^c	0.100	0.074

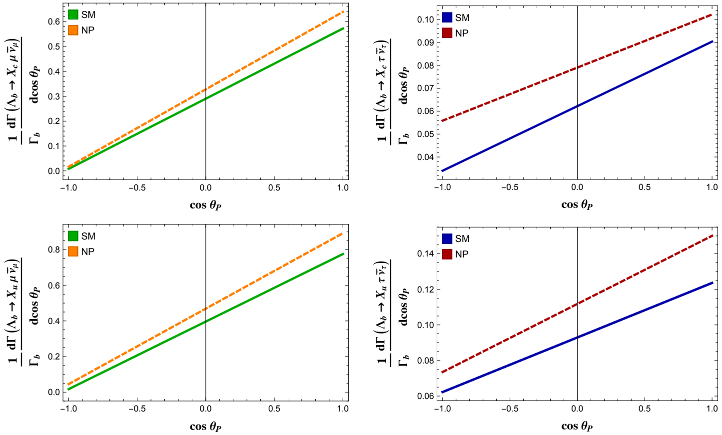
Correlation between $R_{\Lambda_b}(X_c)$ and R_S^c



Hardly measurable at LHC!
Hopefully realizable with high luminosity lepton machine

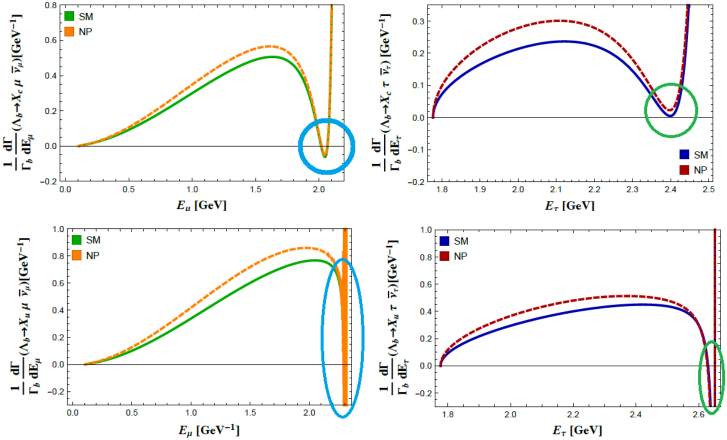
Normalized angular distributions $\frac{1}{\Gamma_b}\,\frac{d\Gamma}{d\cos\theta_P}(\Lambda_b \to X_U\,\ell^-\,\bar{\nu}_\ell)$

Linear dependence on $\cos\theta_P$:
NP modifies both the slope and the intercept



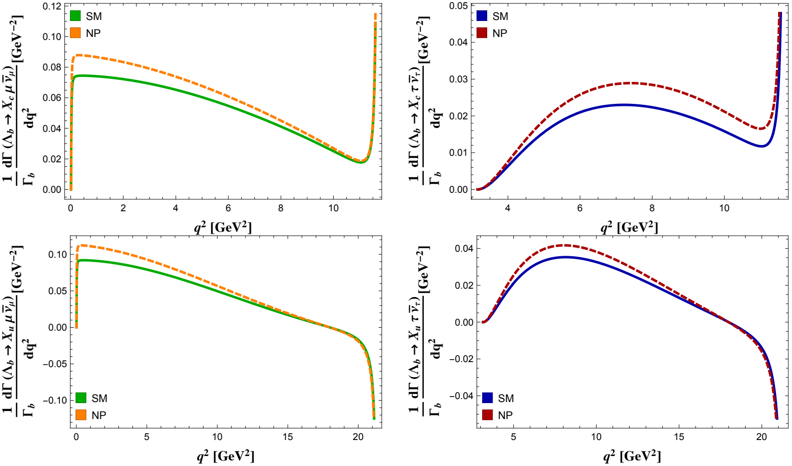
The angular distribution for Λ_b, linear in $\cos\theta_P$, is particularly sensitive to NP!

Normalized charged lepton energy spectra $\frac{1}{\Gamma_b}\frac{d\Gamma}{dE_\ell}(\Lambda_b \to X_U\,\ell^-\,\bar{\nu}_\ell)[\text{GeV}^{-1}]$



Singularities $\mapsto E_\ell^{\max} \Leftrightarrow$ U propagator on-shell!

Normalized dilepton invariant mass q^2 distributions $\frac{1}{\Gamma_b}\frac{d\Gamma}{dq^2}(\Lambda_b \to X_U\, \ell^-\, \bar{\nu}_\ell)[\mathrm{GeV}^{-2}]$



Summary

Description of the inclusive semileptonic b-baryon decays induced by the $b \to \{c, u\}$ transitions, at $\mathcal{O}(1/m_b^3)$ and including the case of polarized decaying baryon.

- Inclusive polarized baryon decay: $\mathcal{M}^{\rho\sigma\lambda}$

  Full consideration of the dependence on the spin s_μ of the decaying baryon at order $\mathcal{O}(1/m_b^3)$ (previous analyses partially done at order $\mathcal{O}(1/m_b^2)$).

- NP fully differential distribution

  Inclusion of all the NP operators at the same order $\mathcal{O}(1/m_b^3)$ (previous analyses included NP only at the leading order).

- Lepton masses

  Evaluation of the fully differential rate considering non vanishing lepton masses.

- Angular distribution $\frac{d\Gamma}{d\cos\theta_P}$

  The angular distribution for Λ_b, linear in $\cos\theta_P$, is *particularly* sensitive to NP.